Experiment: Millikan-Fletcher Oil Drop

Goals

- Collect extensive and detailed data to accurately determine the elementary charge of an electron.
- Calculate the elementary charge of the electron using multiple approaches.
- Evaluate the efficacy of the experiment by considering the impact of unmeasured factors on the results.

Personal Protective Equipment & Safety

In addition to the standard safety rules for Second-year Physics Lab Courses, this lab involves handling electronic equipment, a glass plate, and aerosolized watch oil. The Physics Undergraduate Labs (UGL) will supply personal protective equipment upon request. The following safety rules will be in effect for this lab:

- No food or drink will be allowed in the labs.
- Wear long pants, a dress, or the equivalent.
- Wear close-toed shoes.
- You may request safety goggles from your TA or bring your own. We require that you use safety goggles when handling the glass plate.
- If you would like a mask, request them from your TA. We recommend you consider using a mask when spraying the watch oil.
- If you would like Nitrile gloves, request them from your TA. We recommend you consider using Nitrile gloves when handling the glass plate.

Required, Suggested, & Optional Equipment for Students

- Lab Book #1 (A01-A & A03) or #2 (A01-B)
- Pens (Required)
- USB key (Required)
- Ruler (Suggested)
- Safety Goggles (Required); Either UGL-Supplied or Student-Owned
- Mask (Suggested); Either UGL-Supplied or Student-Owned
- Nitrile Gloves (Suggested); Either UGL-Supplied or Student-Owned
- Lab Coat (Optional); Either UGL-Supplied or Student-Owned
- Laptop (Optional)

1. Background

The elementary charge of the electron is one of the most important and fundamental physical constants ever found. J. J. Thomson discovered the electron by observing that particles with a large charge to mass ratio boiled out of hot metal wires, and that the q/m ratio was identical within his experimental uncertainty for any wire material used. An approximate electron charge and mass had been determined by droplet methods and were reported in Thomson's 1906 Nobel Lecture. Later, Robert Millikan and Harvey Fletcher made the first accurate determination of the elementary charge by refining the droplet technique to allow long-term (hours-long) observations of individual, sub-micrometre diameter watch-oil mist droplets produced with a DeVilbiss perfume atomizer (both procured by Fletcher at a neighbourhood store in Chicago). The precision evidence Millikan presented convinced the scientific community that electric charge was a quantized property.

Theory

The fundamental principle behind the Millikan-Fletcher oil drop experiment is to charge small droplets with limited amounts of charge. A fine oil can be separated into such droplets using an atomizer. In our current setup, the drops obtain their charge from friction in the nozzle that typically deposits more than one electron on the drop. A light shone on the droplets allows them to be viewed through a small telescope as they fall through a capacitor. The capacitor grants control over the charged droplets, granting them the ability to either undergo free fall or rise against gravity.

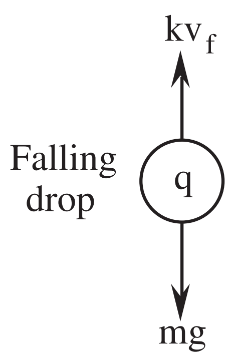

Figure 1: The forces on a charged oil drop with the electric field E turned off and the drop is falling at terminal velocity.

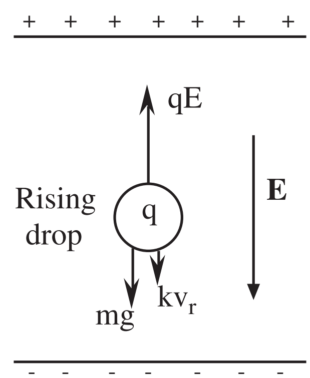

Figure 2: The forces on a charged oil drop with the electric field E turned on and the drop is rising at terminal velocity.

As long as the oil droplets are small, the charge on an individual droplet can be determined using net force calculations. A small droplet moving through a fluid typically moves at terminal velocity due to the exertion of a viscous force, $F_v = kv$, where k is a calculable constant and v is the speed of the particle, that resists the motion of the droplet. This viscous force is large enough to balance out other forces causing the droplet to move through the air, meaning the droplet falls at a constant terminal velocity.

For a drop that is falling through the plates with terminal velocity v_{fall}, our net force equation becomes:

$$mg = kv_{\text{fall}}, \tag{1}$$

where g is the gravitational field applied to the particle.

In contrast, for a drop that is rising with a terminal velocity v_{rise} due to an applied electric field E, as shown in Figure 2, the net force equation becomes:

$$mg + kv_{\text{rise}} = qE, \tag{2}$$

where q is the charge contained on the drop.

The viscous force applied to a particle is detailed by Stokes' law, which goes on to state that the value of k for a small sphere of radius r moving in a fluid of viscosity η is given by $k = 6\pi r\eta$. However, a correction factor is required to allow for the size of the oil drop being comparable to the mean free path of the air molecules, yielding:

$$k = \frac{6\pi r\eta}{(1 + \frac{b}{Pr})}, \tag{3}$$

where P is the atmospheric pressure (in Hg), and b is a constant equalling 6.17×10^{-5} m · mm Hg or 8.23×10^{-3} m · Pa.

Given that gravitational force is involved, knowing the mass of the droplet is highly important. Assuming the oil drop has a consistent radius and a density ρ, the mass m of the spherical droplet is found to be:

$$m = \frac{4}{3}\pi\rho r^3. \tag{4}$$

Thus, the radius r of the droplet must be found to complete the calculation of the oil droplet's charge. An equation for r can be obtained by substituting Equation 3 and Equation 4 into the free-fall equation, Equation 1, yielding:

$$r = -\frac{b}{2P} + \sqrt{\frac{b^2}{4P^2} + \frac{9\eta v_{\text{fall}}}{2\rho g}}. \tag{5}$$

The final goal is to calculate the charge of the oil droplet. An equation for q can be derived by substituting Equation 1 twice and Equation 4 into Equation 2:

$$q = \left(\frac{v_{\text{fall}} + v_{\text{rise}}}{v_{\text{fall}}}\right)\frac{4\pi\rho g}{3E}r^3, \tag{6}$$

where v_{fall}, v_{rise}, r, and E are calculated, while ρ and g are measurable.

Methods

Apparatus

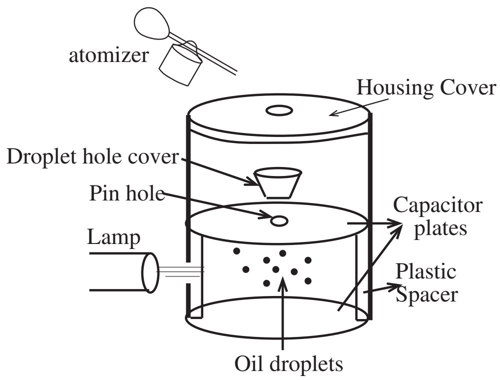

Figure 3: Inside the capacitor housing.

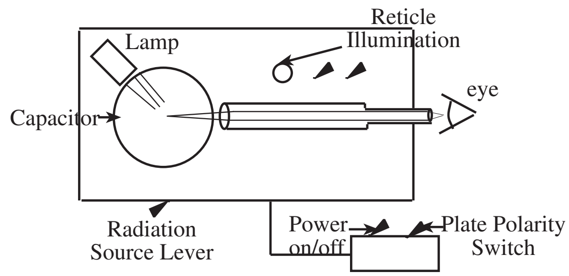

Figure 4: The Millikan Oil Drop apparatus schematic.

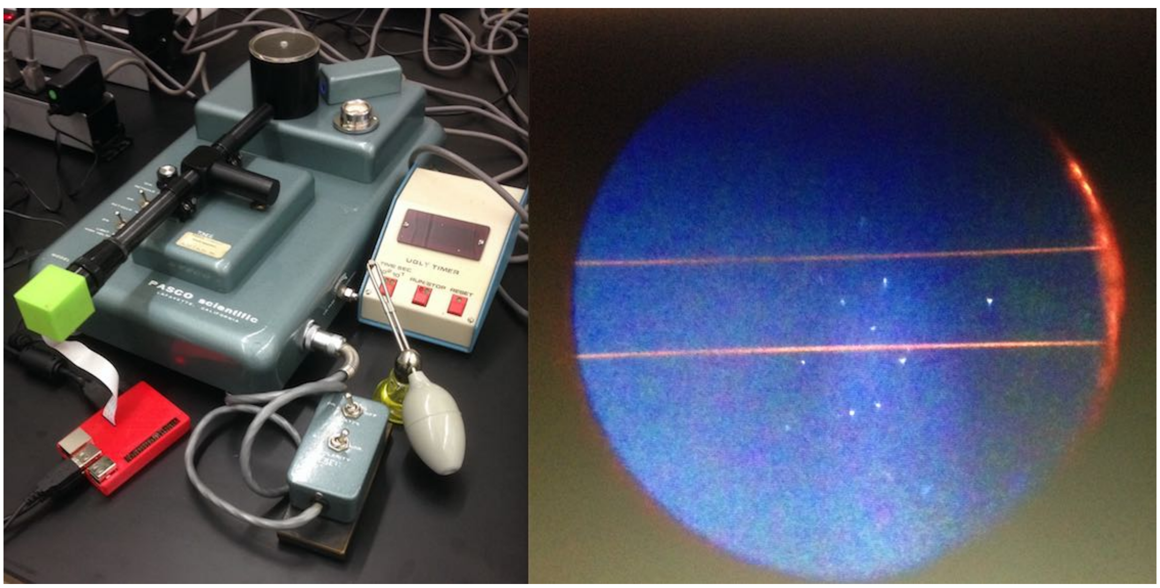

Figure 5: A photograph of the setup and a sample image of droplets as seen using the Raspberry Pi camera.

Cleaning Equipment

An air syringe (*turkey baster*) will be provided for cleaning the apparatus. Prior to running the experiment, the air syringe is used to clear all openings in the plates.

Capacitor

The capacitor is on the top-left of the Millikan-Fletcher apparatus within the plastic chamber. Within the chamber, there is a small green cap atop three plates. The two metal plates form the capacitor, while the glass plate acts as a spacer: the spacer's thickness must be measured to find the electric field between the metal plates applied by the voltage source, as per the equation $E = V/d$. For this experiment a blind analysis is recommended, in which the voltage, V, is left as an unknown and only entered at the very end, *after* the lowest common multiple in the data has been identified. If you prefer to perform the analysis 'unblind' (warning: it will be really hard not to be influenced by what you already know is the value of the electron charge!), ask your TA or professor for the capacitor voltage and voltage uncertainty. For the blind analysis you may assume that an exact voltage will be given at the end, and not worry about the voltage uncertainty.

To find the thickness of the spacer it must be sandwiched it between two square glass pieces. Use a micrometer to find the width of the three pieces in several places around the centre of the spacer. After, doing so remove the spacer from between the glass piece and then repeat the measurements for the two glass pieces. Once these data are collected, the width of the spacer can be calculated.

Millikan Unit

The Millikan-Fletcher apparatus, as shown in Figures 3 and 4, is a three-legged platform containing a beta decay source and the voltage source for the capacitor. To resolve the oil droplets the unit must be properly levelled. A bullseye spirit level has been attached beside the capacitor for this purpose. Use thumbscrews located on the legs until the bubble sits inside the prescribed circle.

Once the unit is properly levelled and the plates clean, replace the green cap with one of the wire pegs. With the telescope properly aligned and focussed an edge of the wire should be clearly visible from reflecting some of the light from the LED into the telescope. If not, try slightly rotating the plexiglas disc to reduce background scattered light. If there is still a problem, make sure that the spacer's interior is still painted black. A black cloth can also be placed over the telescope to reduce ambient light.

A box containing the controls for the capacitor must be attached to the side of the unit. The PLATES switch is used to turn the electric field between the plates on by connecting the voltage supply to the capacitor. The POLARITY switch controls the direction of the electric field between the plates: the 'normal' setting makes the top plate positive and the bottom plate negative with the 'reverse' position switching the charge on each plate.

Oil Aspirator

There will be an aspirator containing fine watch-oil with the apparatus (Fletcher hit a home run in his first at-bat; the density of the oil, $\rho = 890 \pm 3\,\text{kg/m}^3$ at room temperature). The aspirator is used to pass the oil through an atomizer, releasing a fine mist that will be aimed into the chamber. The height of the chamber will allow the viscous force of air to reach terminal velocity before entering the capacitor. To measure this force, the viscosity η of air must be known. Use the following chart to determine the viscosity of air given the temperature of the laboratory.

t [°C]	17	18	19	20	21	22	23	24
$\eta[10^{-6}\text{N.s.m}^{-2}]$	18.23	18.27	18.31	18.35	18.39	18.44	18.48	18.52

Barometer & Thermometer

The temperature and pressure of the laboratory can be measured one of two ways with this lab.

- ◇ The temperature and pressure of the laboratory can be recorded and measured by using the thermometer and barometer installed in CCIS L2-010 on the wall. Ensure that the barometer is zeroed before recording a measurement. Take several measurements over the course of the experiment. Make sure to estimate the uncertainty of your measurements.

  Note: To zero the barometer, adjust the height of the mercury in the bottom well until it just touches the pointer tip visible inside. The height of the mercury can be changed by turning the thumb screw behind the well.

- ◇ The temperature and pressure of the laboratory can be recorded and measured by using an electronic device that will be present in the room where the experiment is being performed. This device incorporates a Bosch Sensortech BMP 280 Barometric Pressure Sensor, which reports pressure in units of kiloPascals, temperature in Celsius, and the relative humidity. As incorporated into our instrument, the pressure is measured with a resolution of 0.01 kPa and absolute accuracy of 0.1 kPa. As incorporated into our instrument, the temperature is measured with a resolution of 0.01°C (beware the device may display an extra significant digit that the sensor does not really measure well) and an absolute accuracy of 0.5°C.

Timer

A timer capable of measuring up to a thousandth of a second is also provided. Practice timing until you feel capable of accurately recording a droplet's fall time.

Other equipment available

- ◇ Raspberry Pi 3 Computer
- ◇ Raspberry Pi Camera
- ◇ 2 x Glass Plates (2 Setups on the center table)
- ◇ Micrometer (2 Setups on the center table)

Procedure

1. Squeeze the atomizer in the top of the chamber until oil droplets are visible in the viewing area. This might take a few attempts; try a firm squeeze at first, not too soft, but not with full force. Limit the number of squeezes to avoid producing a thick film of oil that will block the channel into the capacitor. When droplets are seen they can be *captured* by turning the PLATES on, modifying the POLARITY until the drop rises.

2. A preferred droplet will fall *slowly* while the PLATES are off and rise *slowly* (8 seconds or longer) while the electric field is on.

3. Time the rise and fall of the selected oil droplet using the illuminated reticle lines in the telescope, the separation of which is labelled on the apparatus. Measure the rise and fall times of each droplet at least 10 times. To ensure that the droplet reaches terminal velocity the droplet should move past the reticles lines in each trial. This also provides a moment to reset the timer for the next rise or fall.

4. To record video for additional post-analysis (optional), type the following command in a terminal on a Raspberry Pi:

   raspivid -o <filename> -t 100000 -fps 25

   where you replace filename with a meaningful name. An example of a meaningful name includes the initials of your lab partners, the date, a counter that you increment for each capture you make, and the proper 'avi' file extension. For example 'WMGS_20191017_03.avi' would be the third drop captured by the Raspberry Pi from October 17, 2019 taken by William Morrish and Gregory Sivakoff. Do not rely on this solely! Use this only to try to improve precision of measurements, follow additional droplets, etc. Data transfer can be slow or worse for big files. **Consider bringing a USB stick to transfer data off the Raspberry Pi.**

5. Repeat Steps 1–4 for as many drops as possible. The more droplets you measure, the better your analysis will be. If other droplets remain in view at the end of the trial they may be used in place of introducing more oil via the aspirator.

   Notes: If the atomizer fails to introduce more droplets into the capacitor the plates need to be cleaned again. A change in the oil drop charge can result from collisions with air molecules. These collisions usually result in a charge transfer of $\pm e$.

Analysis

Lab Book and Lab Report Evaluation

For this experiment you will be completing standard analysis in your lab book, where this will be evaluated under the standard “Lab Book” rubric. You will also be completing a “Full Lab Report”, where this will be evaluated under the standard “Lab Report” rubric.

Important Notes: Read carefully the Lab Book Guidelines to include In-Lab and Post-Lab Notes. Make sure you add your observations about the experiment and discussion/conclusion requested for each tasks. In addition, don’t forget to include setup detail including diagram (with appropriate and clear labels/captions), uncertainties including apparatus uncertainty and justifications. The Post-Lab notes after the first afternoon must include preliminary analysis of the data your obtained during the first session, and a plan of work for the second afternoon session.

Required Analyses

1. Determine the mean rise time, fall time, and the associated uncertainty for each droplet. This can be refined if you take data with the Raspberry Pi camera.
2. Calculate the charge q on each drop (or, for blind analysis the product qV, of the charge q on each drop and the voltage V across the capacitor, where the voltage may be assumed to have negligible uncertainty). To calculate for each drop the uncertainty δq (or equivalently $\delta(qV) = V\delta q$ in blind analysis) use the simplified error provided below. Equation 7 is derived by assuming the only significant errors are in timing, and that the ratio of $\dfrac{b}{2P}$ is small enough to be neglected in the calculation.
$$\delta q = q\sqrt{\left(\frac{\delta v_r}{v_f + v_r}\right)^2 + \left(\frac{1}{2v_f} + \frac{1}{(v_f + v_r)}\right)^2 \delta v_f^2} \tag{7}$$
3. Look at the qV values of all rises and falls for each drop. Check if the standard deviations appear to be arising from random scatter and not a systematic error like the charge changing during a series or measurements (e.g., from collisions of the drop with air molecules). If a drop appears to be affected by a larger systematic error or appears to be a clear outlier from other drops, you may remove that drop from further consideration in the analysis (but be specific about what you did in the lab report).
4. Make an estimation of the elementary charge of the electron from your data. There are two methods that can work here.
   - ◇ With measurements of enough droplets in hand (and especially droplets chosen for their likelihood to harbour small net charge), create a histogram plot of the charges q (or, for blind analysis the product qV). You will want (largely or completely) empty bins between islands of data. (Hint, start with a scatter plot of charge versus trial number and then ‘project’ the results horizontally onto a histogram by binning the y-axis into appropriate sub-ranges.) Look for the minimum separation between statistically-significant bins of data. This will be a good candidate to correspond to the charge of one electron of the charge product due to one electron .
   - ◇ You can explore the data manually, looking for a lowest common multiple of charge (or qV) for the oil drops by using the smallest charge as a divisor for the larger charges. If this does not yield integer multiples, divide the smallest charge by 2, 3, 4, ... and use the new result as a divisor for the larger charges. The intent of this method is to look for a charge (or qV) that is an integer factor of all the other charges (or measurements of qV).

5. If you were doing a blind analysis (which was recommended), now is the time to remove the voltage from the product qV.

6. Once the most likely assignment of integer multiples of the electronic charge has been created for each measured charge q_i, divide by the integer multiple for charge q_i to get a single value of the experimental value of a single electron e_i. Make sure to think about δe_i.

7. Use the method of weighted averages to determine the best experimental value of charge e and its uncertainty δe.

Additional tasks you should address in the lab report

1. Rather than taking measurements for both rising and falling velocities, one could simply let the oil drop remain stationary by balancing its weight with the electric field. Discuss why this proposal is not a valid way to determine e.

2. The buoyant force, $F_B = \frac{4}{3}\pi r^3 \rho_A g$, of the air acting on the oil drop has been neglected in Equation 1 and Equation 2, where ρ_A is the density of the air. Discuss if this a reasonable assumption to make. Provide proof using the values of r and q.