A - Digit Sum of 2x

Time Limit: 2 sec / Memory Limit: 1024 MB

Score : 300 points

Problem Statement

For a positive integer x, let $f(x)$ be the sum of its digit. For example, $f(144) = 1 + 4 + 4 = 9$ and $f(1) = 1$.

You are given a positive integer N. Find the following positive integers M and x:

- The maximum positive integer M for which there exists a positive integer x such that $f(x) = N$ and $f(2x) = M$.
- The minimum positive integer x such that $f(x) = N$ and $f(2x) = M$ for the M above.

Constraints

- $1 \leq N \leq 10^5$

Input

Input is given from Standard Input in the following format:

```
N
```

Output

Print M in the first line and x in the second line.

Sample Input 1

```
3
```

Sample Output 1

```
6
3
```

We can prove that whenever $f(x) = 3$, $f(2x) = 6$. Thus, $M = 6$. The minimum positive integer x such that $f(x) = 3$ and $f(2x) = 6$ is $x = 3$. These M and x should be printed.

Sample Input 2

```
6
```

Sample Output 2

```
12
24
```

Sample Input 3

```
100
```

Sample Output 3

```
200
4444444444444444444444444
```

B - Gift Tax

Time Limit: 2 sec / Memory Limit: 1024 MB

Score : 400 points

Problem Statement

You are given positive integers a and b such that $a \leq b$, and a sequence of positive integers $A = (A_1, A_2, \dots, A_N)$.

On this sequence, you can perform the following operation any number of times (possibly zero):

- Choose distinct indices i, j ($1 \leq i, j \leq N$). Add a to A_i and subtract b from A_j.

Find the maximum possible value of $\min(A_1, A_2, \dots, A_N)$ after your operations.

Constraints

- $2 \leq N \leq 3 \times 10^5$
- $1 \leq a \leq b \leq 10^9$
- $1 \leq A_i \leq 10^9$

Input

Input is given from Standard Input in the following format:

$$
\begin{array}{l}
N \quad a \quad b \\
A_1 \quad A_2 \quad \dots \quad A_N
\end{array}
$$

Output

Print the maximum possible value of $\min(A_1, A_2, \dots, A_N)$ after your operations.

Sample Input 1

```
3 2 2
1 5 9
```

Sample Output 1

```
5
```

Here is one way to achieve $\min(A_1, A_2, A_3) = 5$.

- Perform the operation with $i = 1, j = 3$. A becomes $(3, 5, 7)$.
- Perform the operation with $i = 1, j = 3$. A becomes $(5, 5, 5)$.

Sample Input 2

```
3 2 3
11 1 2
```

Sample Output 2

```
3
```

Here is one way to achieve $\min(A_1, A_2, A_3) = 3$.

- Perform the operation with $i = 1, j = 3$. A becomes $(13, 1, -1)$.
- Perform the operation with $i = 2, j = 1$. A becomes $(10, 3, -1)$.
- Perform the operation with $i = 3, j = 1$. A becomes $(7, 3, 1)$.
- Perform the operation with $i = 3, j = 1$. A becomes $(4, 3, 3)$.

Sample Input 3

```
3 1 100
8 5 6
```

Sample Output 3

```
5
```

You can achieve $\min(A_1, A_2, A_3) = 5$ by not performing the operation at all.

Sample Input 4

```
6 123 321
10 100 1000 10000 100000 1000000
```

Sample Output 4

```
90688
```

C - K Derangement

Time Limit: 2 sec / Memory Limit: 1024 MB

Score : 600 points

Problem Statement

You are given positive integers N and K. Find the lexicographically smallest permutation $A = (A_1, A_2, \ldots, A_N)$ of the integers from 1 through N that satisfies the following condition:

- $|A_i - i| \geq K$ for all i ($1 \leq i \leq N$).

If there is no such permutation, print `-1`.

▸ What is lexicographical order on sequences?

Constraints

- $2 \leq N \leq 3 \times 10^5$
- $1 \leq K \leq N - 1$

Input

Input is given from Standard Input in the following format:

$$N \quad K$$

Output

Print the lexicographically smallest permutation $A = (A_1, A_2, \ldots, A_N)$ of the integers from 1 through N that satisfies the condition, in the following format:

$$A_1 \quad A_2 \quad \ldots \quad A_N$$

If there is no such permutation, print `-1`.

Sample Input 1

```
3 1
```

Sample Output 1

```
2 3 1
```

Two permutations satisfy the condition: $(2, 3, 1)$ and $(3, 1, 2)$. For instance, the following holds for $(2, 3, 1)$:

- $|A_1 - 1| = 1 \geq K$
- $|A_2 - 2| = 1 \geq K$
- $|A_3 - 3| = 2 \geq K$

Sample Input 2

```
8 3
```

Sample Output 2

```
4 5 6 7 8 1 2 3
```

Sample Input 3

```
8 6
```

Sample Output 3

```
-1
```

D - AND OR Equation

Time Limit: 2 sec / Memory Limit: 1024 MB

Score : 700 points

Problem Statement

You are given positive integers N and K. Find the number, modulo 998244353, of integer sequences $\bigl(f(0), f(1), \dots, f(2^N - 1)\bigr)$ that satisfy all of the following conditions:

- $0 \leq f(x) \leq K$ for all non-negative integers x ($0 \leq x \leq 2^N - 1$).
- $f(x) + f(y) = f(x \,\mathrm{AND}\, y) + f(x \,\mathrm{OR}\, y)$ for all non-negative integers x and y ($0 \leq x, y \leq 2^N - 1$)

Here, AND and OR denote the bitwise AND and OR, respectively.

Constraints

- $1 \leq N \leq 3 \times 10^5$
- $1 \leq K \leq 10^{18}$

Input

Input is given from Standard Input in the following format:

```
N K
```

Output

Print the number, modulo 998244353, of integer sequences that satisfy the conditions.

Sample Input 1

```
2 1
```

Sample Output 1

```
6
```

The following six integer sequences satisfy the conditions:

- $(0, 0, 0, 0)$
- $(0, 1, 0, 1)$
- $(0, 0, 1, 1)$
- $(1, 0, 1, 0)$
- $(1, 1, 0, 0)$
- $(1, 1, 1, 1)$

Sample Input 2

```
2 2
```

Sample Output 2

```
19
```

Sample Input 3

```
100 123456789123456789
```

Sample Output 3

```
34663745
```

E - GCD of Path Weights

Time Limit: 2 sec / Memory Limit: 1024 MB

Score : 800 points

Problem Statement

You are given a directed graph G with N vertices and M edges. The vertices are numbered $1, 2, \ldots, N$. The i-th edge is directed from Vertex a_i to Vertex b_i, where $a_i < b_i$.

The **beautifulness** of a sequence of positive integers $W = (W_1, W_2, \ldots, W_N)$ is defined as the maximum positive integer x that satisfies the following:

- For every path $(v_1, \ldots, v_k)$ ($v_1 = 1, v_k = N$) from Vertex 1 to Vertex N in G, $\sum_{i=1}^{k} W_{v_i}$ is a multiple of x.

You are given an integer sequence $A = (A_1, A_2, \ldots, A_N)$. Find the maximum beautifulness of a sequence of positive integers $W = (W_1, \ldots, W_N)$ such that $A_i \neq -1 \implies W_i = A_i$. If the maximum beautifulness does not exist, print `-1`.

Constraints

- $2 \leq N \leq 3 \times 10^5$
- $1 \leq M \leq 3 \times 10^5$
- $1 \leq a_i < b_i \leq N$
- $(a_i, b_i) \neq (a_j, b_j)$ if $i \neq j$
- In the given graph G, there is a path from Vertex 1 to Vertex N.
- $A_i = -1$ or $1 \leq A_i \leq 10^{12}$

Input

Input is given from Standard Input in the following format:

$$
\begin{array}{l}
N \quad M \\
a_1 \quad b_1 \\
\vdots \\
a_M \quad b_M \\
A_1 \quad A_2 \quad \ldots \quad A_N
\end{array}
$$

Output

Print the maximum beautifulness of a sequence of positive integers W. If the maximum beautifulness does not exist, print `-1`.

Sample Input 1

```
4 4
1 2
1 3
2 4
3 4
-1 3 7 -1
```

Sample Output 1

```
4
```

There are two paths from Vertex 1 to Vertex N: $(1,2,4)$ and $(1,3,4)$. For instance, $W=(5,3,7,8)$ has a beautifulness of 4. Indeed, both $W_1+W_2+W_4=16$ and $W_1+W_3+W_4=20$ are multiples of 4.

Sample Input 2

```
4 5
1 2
1 3
2 4
3 4
1 4
-1 3 7 -1
```

Sample Output 2

```
1
```

There are three paths from Vertex 1 to Vertex N: $(1,2,4)$, $(1,3,4)$, and $(1,4)$. For instance, $W=(5,3,7,8)$ has a beautifulness of 1.

Sample Input 3

```
4 4
1 2
1 3
2 4
3 4
3 -1 -1 7
```

Sample Output 3

```
-1
```

For instance, $W = (3, 10^{100}, 10^{100}, 7)$ has a beautifulness of $10^{100} + 10$. Since you can increase the beautifulness of W as much as you want, there is no maximum beautifulness.

Sample Input 4

```
5 5
1 3
3 5
2 3
3 4
1 4
2 -1 3 -1 4
```

Sample Output 4

```
9
```

F - Arithmetic Sequence Nim

Time Limit: 2 sec / Memory Limit: 1024 MB

Score : 900 points

Problem Statement

You are given a positive integer m, a non-negative integer a ($0 \leq a < m$), and a sequence of positive integers $A = (A_1, \dots, A_N)$.

A set X of positive integers is defined as $X = \{x > 0 \mid x \equiv a \pmod{m}\}$.

Alice and Bob will play a game against each other. They will alternate turns performing the following operation, with Alice going first:

- Choose a pair (i, x) of an index i ($1 \leq i \leq N$) and a positite integer $x \in X$ such that $x \leq A_i$. Change A_i to $A_i - x$. If there is no such (i, x), the current player loses and the game ends.

Find the number, modulo 998244353, of pairs (i, x) that Alice can choose in her first turn so that she wins if both players play optimally in subsequent turns.

Constraints

- $1 \leq N \leq 3 \times 10^5$
- $0 \leq a < m \leq 10^9$
- $\max(1, a) \leq A_i \leq 10^{18}$

Input

Input is given from Standard Input in the following format:

$$
\begin{array}{llll}
N & m & a & \\
A_1 & A_2 & \dots & A_N
\end{array}
$$

Output

Print the number, modulo 998244353, of pairs (i, x) that Alice can choose in her first turn so that she wins if both players play optimally in subsequent turns.

Sample Input 1

```
3 1 0
5 6 7
```

Sample Output 1

```
3
```

We have $X = \{1, 2, 3, 4, 5, \ldots\}$. Three pairs (i, x) satisfy the condition: $(1, 4)$, $(2, 4)$, $(3, 4)$.

Sample Input 2

```
5 10 3
5 9 18 23 27
```

Sample Output 2

```
3
```

We have $X = \{3, 13, 23, 33, 43, \ldots\}$. Three pairs (i, x) satisfy the condition: $(4, 23)$, $(5, 3)$, $(5, 13)$.

Sample Input 3

```
4 10 8
100 101 102 103
```

Sample Output 3

```
0
```

Alice cannot win even if she plays optimally. Thus, zero pairs (i, x) satisfy the condition.

Sample Input 4

```
5 2 1
111111111111111 222222222222222 333333333333333 444444444444444 555555555555555
```

Sample Output 4

```
943937640
```

833333333333334 pairs (i, x) satisfy the condition. Print the count modulo 998244353.