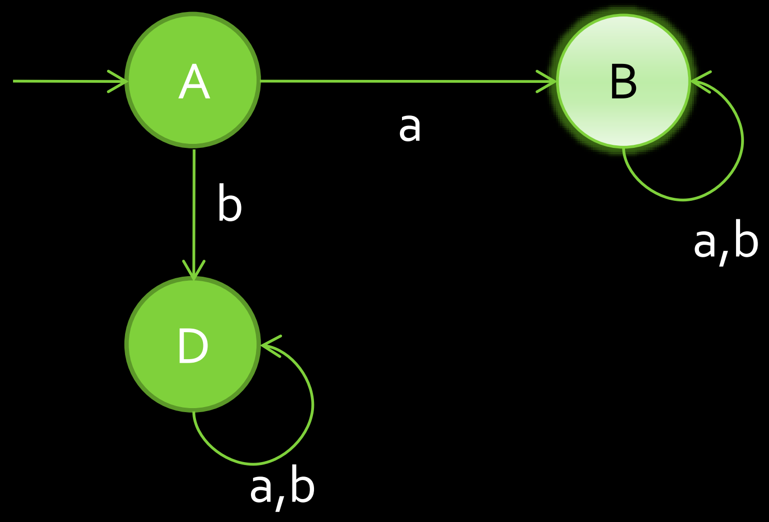
Arden’s Theorem

- If A and B are two regular expressions over ∑ and if A does not contain ε, then the following equation in X given by $X=AX+B$ has unique solution $X=A^*B$
- If A and B are two regular expressions over ∑ and if A does not contain ε, then the following equation in X given by $X=B+XA$ has unique solution $X=BA^*$
- Application – Arden’s Theorem is used to convert FSA (DFA) to RE.

Conversion of DFA to RE

- For each and every state in given DFA
- 1. Look for outgoing arcs to write equation of state. For n states we have n equations.
- 2. Repeatedly apply Arden’s theorem to solve equations of states
- Until we get Regular expression for initial state

Conversion of DFA to RE



DFA has 3 states
so we have 3 equations
A=aB+bD -----(1)
D=aD+bD -----(2)
B=aB+bB+ε ----(3)
Add ε in equation for final state

From eqn(3)
B=aB+bB+ ε
B=(a+b)B+ ε
B=(a+b)* ε=(a+b)*

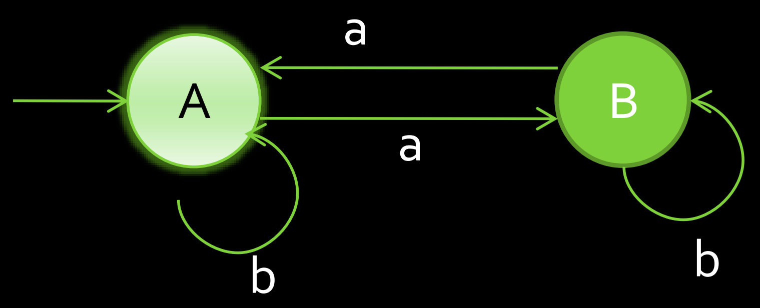
From eqn(2)
D=aD+bD
=(a+b)D+Φ
D=(a+b)* Φ = Φ

Arden's Theorem
Equation X=AX+B has unique solution X=A*B

Put B and D in equation (1)
A=aB+bD

A=a(a+b)*+bΦ=a(a+b)*

Conversion of DFA to RE



DFA has 2 states
so we have 2 equations
A=aB+bA+ ε -----(1)
B=aA+bB ----(2)
Add ε in equation for final state

Arden's Theorem
Equation X=AX+B has unique solution X=A*B

From eqn(2)
B=aA+bB
B=(b)B+ aA
B=b*aA=

From eqn(1)
A=aB+ bA + ε
=ab*aA+bA+ ε
=(ab*a+b)A+ ε
A=(ab*a+b)* ε
=(ab*a+b)*

A=(ab*a+b)*