

# Equivalence of RE and FSA

- FSA accepts Regular Language
- RE represents Regular Language
- $RE \equiv FSA$
- How?
- 1. Conversion of RE to FSA ( $\epsilon$ -NFA)
- 2. Conversion of FSA (DFA) to RE using Arden's theorem

# Conversion of RE to FSA ( $\epsilon$ -NFA)

## Regular Expression

1.  $\Phi$

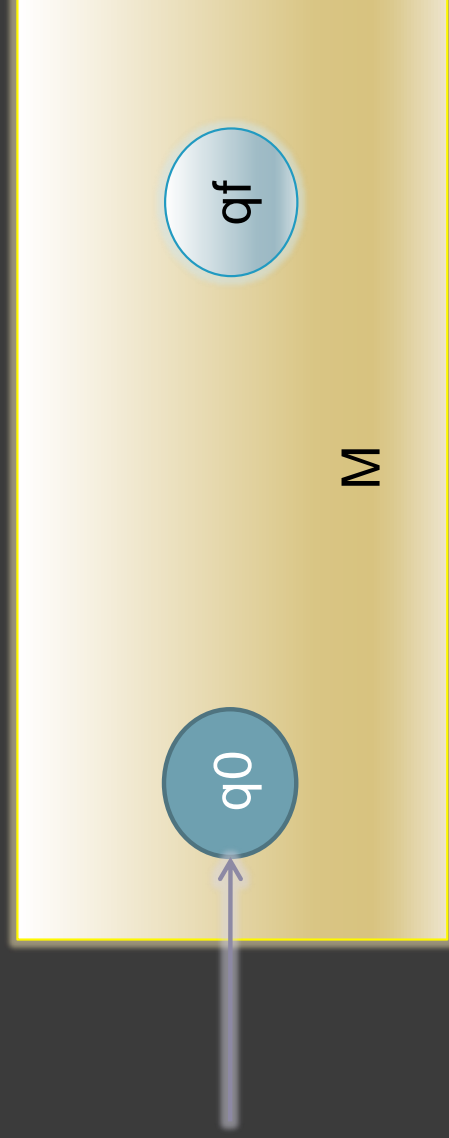
$L(\Phi) = \{ \}$

## $\epsilon$ -NFA

$M$  has no transition from initial state  $q_0$  to final state  $q_f$ .

So it can not accept any string. So language of  $M$  is  $\Phi$ .

$M$  accepts no string,  $L(M) = \{ \}$



# Conversion of RE to FSA ( $\epsilon$ -NFA)

Regular Expression

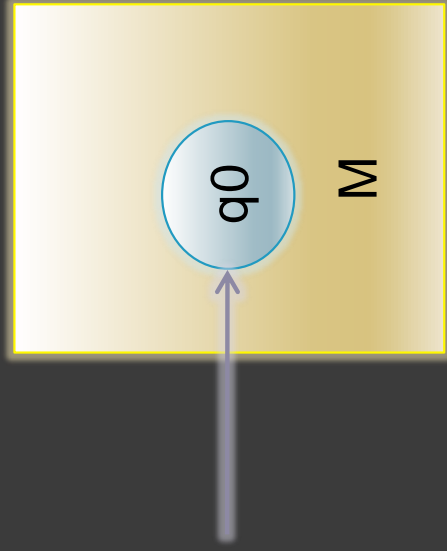
2.  $\epsilon$

$L(\epsilon) = \{\epsilon\}$

$\epsilon$ -NFA

$M$  has single state which is both initial state and final state  $q_0$ .  
So it can accept empty string. So language of  $M$  is  $\{\epsilon\}$ .

$M$  accepts empty string,  $L(M) = \{\epsilon\}$



# Conversion of RE to FSA ( $\epsilon$ -NFA)

Regular Expression

3. a

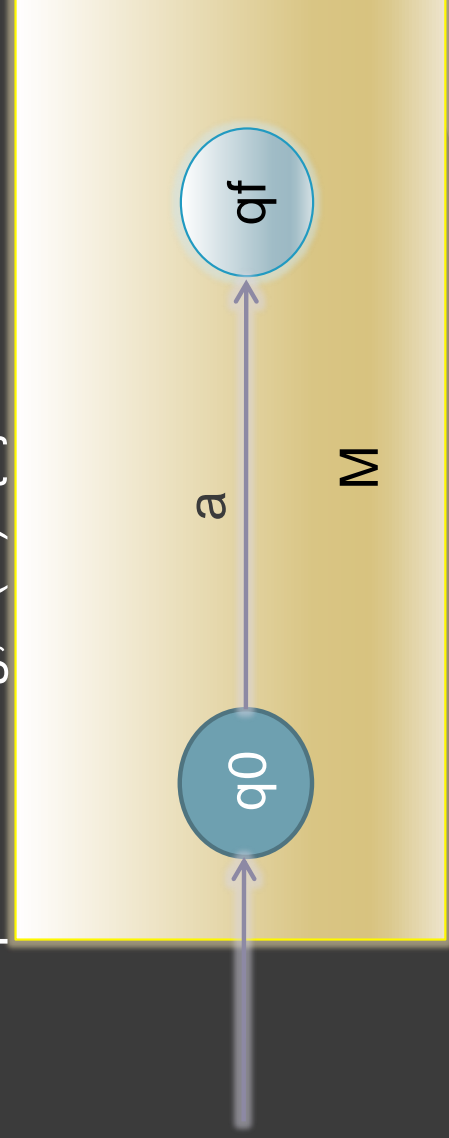
$L(a) = \{a\}$

$\epsilon$ -NFA

M has a single transition from initial state  $q_0$  to final state  $q_f$  on input symbol a.

So it can not accept only one string a. So language of M is  $\{a\}$ .

M accepts no string,  $L(M) = \{a\}$



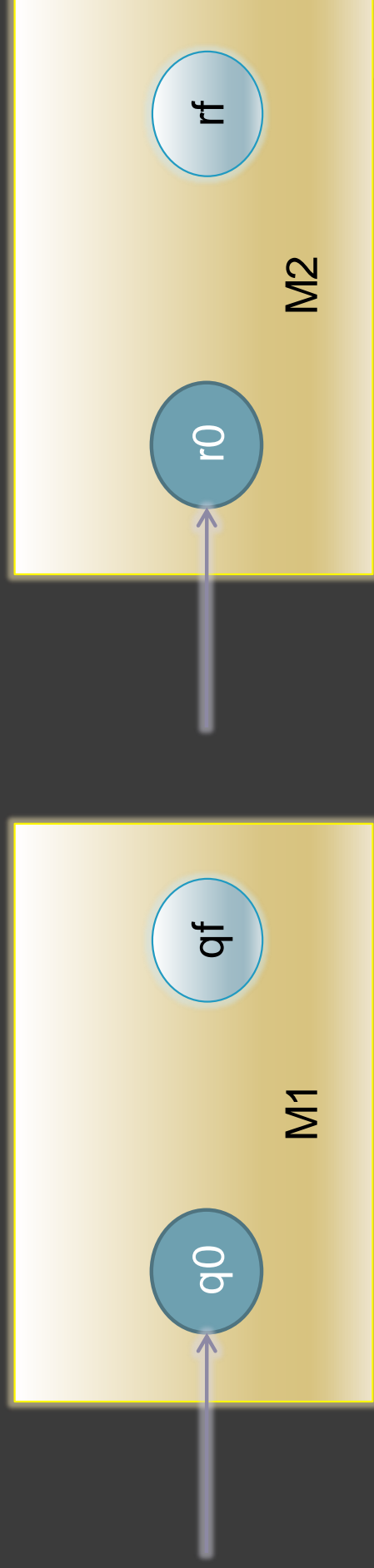
# Conversion of RE to FSA ( $\epsilon$ -NFA)

Regular Expression

3.  $r_1 + r_2$

$L(r_1 + r_2) = L(r_1) \cup L(r_2)$

Suppose RE  $r_1$  and  $r_2$  is represented by NFA  $M_1$  and  $M_2$  respectively as follows

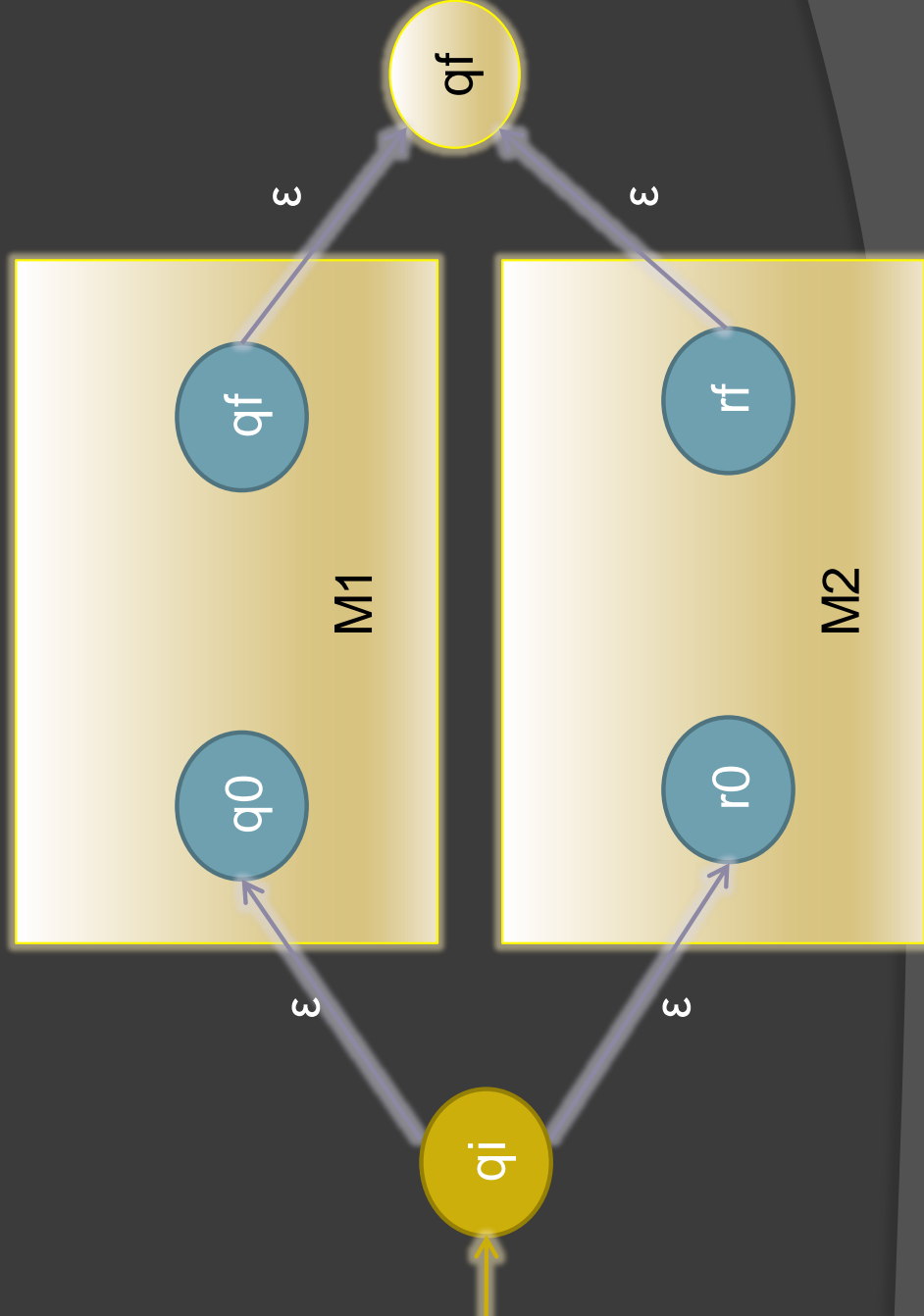


# Conversion of RE to FSA ( $\epsilon$ -NFA)

## Construction of NFA for $r_1 + r_2$

### $\epsilon$ -NFA M Idea

1. Make initial states  $q_0, r_0$  of  $M_1$  and  $M_2$  non initial in  $M$
2. Make final states  $q_f, r_f$  of  $M_1$  and  $M_2$  non final in  $M$
3. Introduce a new initial state  $q_i$
4. Introduce a new final state  $q_f$
5. Add  $\epsilon$ -transitions from new initial state  $q_i$  to old initial states  $q_0, r_0$  of  $M_1$  and  $M_2$
6. Add  $\epsilon$ -transitions from old final states  $q_f, r_f$  of  $M_1$  and  $M_2$  to new final state  $q_f$ .



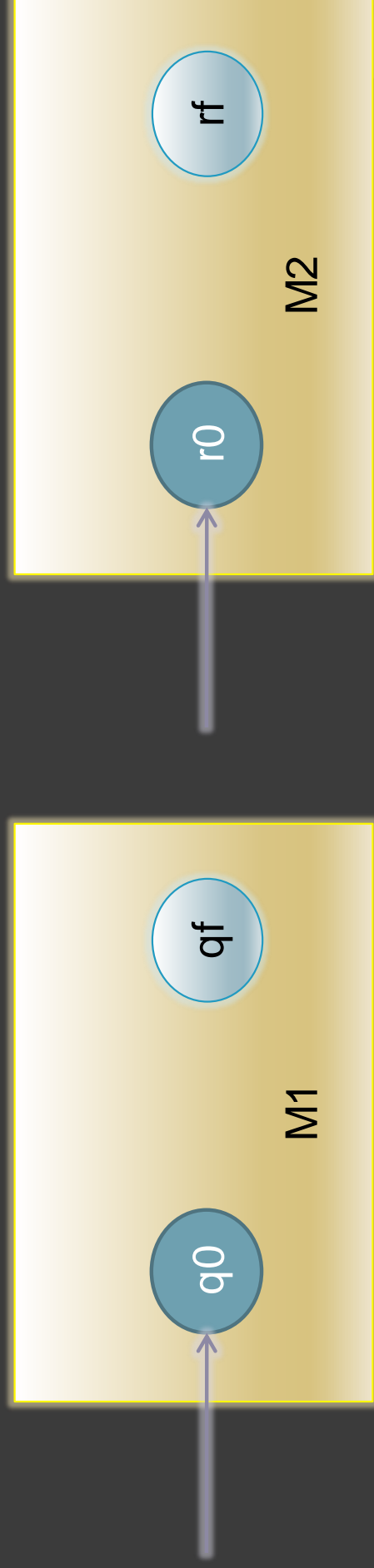
# Conversion of RE to FSA ( $\epsilon$ -NFA)

Regular Expression

5.  $r_1r_2$

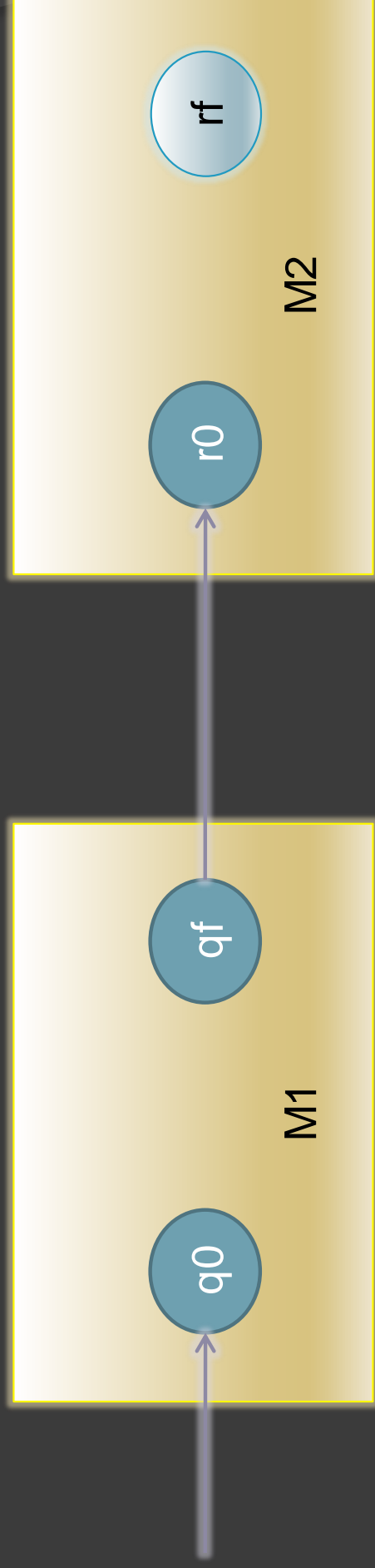
$L(r_1r_2) = L(r_1)L(r_2)$

Suppose RE  $r_1$  and  $r_2$  is represented by NFA  $M_1$  and  $M_2$  respectively as follows



# Conversion of RE to FSA ( $\epsilon$ -NFA)

Construction of  $\epsilon$ -NFA for  $r_1r_2$



## $\epsilon$ -NFA M Idea

1. Make final state  $q_f$  of  $M_1$  non final in  $M$  and initial state  $r_0$  of  $M_2$  non initial in  $M$
2. Add  $\epsilon$ -transitions from final state  $q_f$  of  $M_1$  to initial state  $r_0$  of  $M_2$



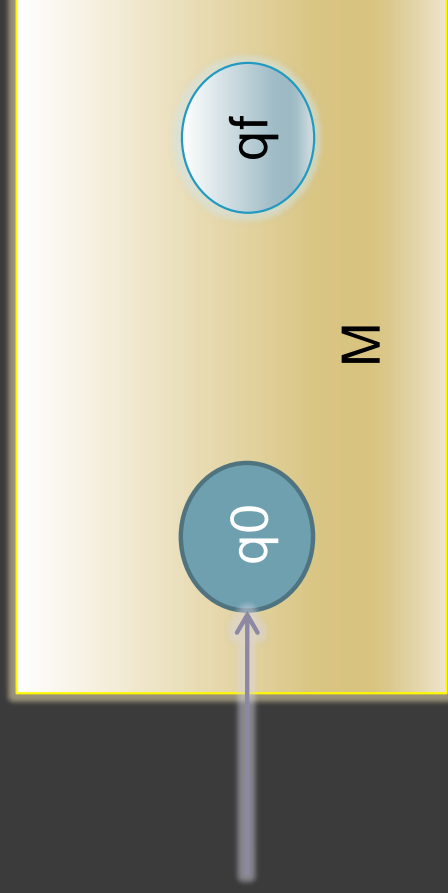
# Conversion of RE to FSA ( $\epsilon$ -NFA)

Regular Expression

6.  $r^*$

$$L(r^*) = L(r)^* = L^0 U L^1 U L^2 U L^3 \dots$$

Suppose RE  $r$  is represented by NFA  $M$  as follows



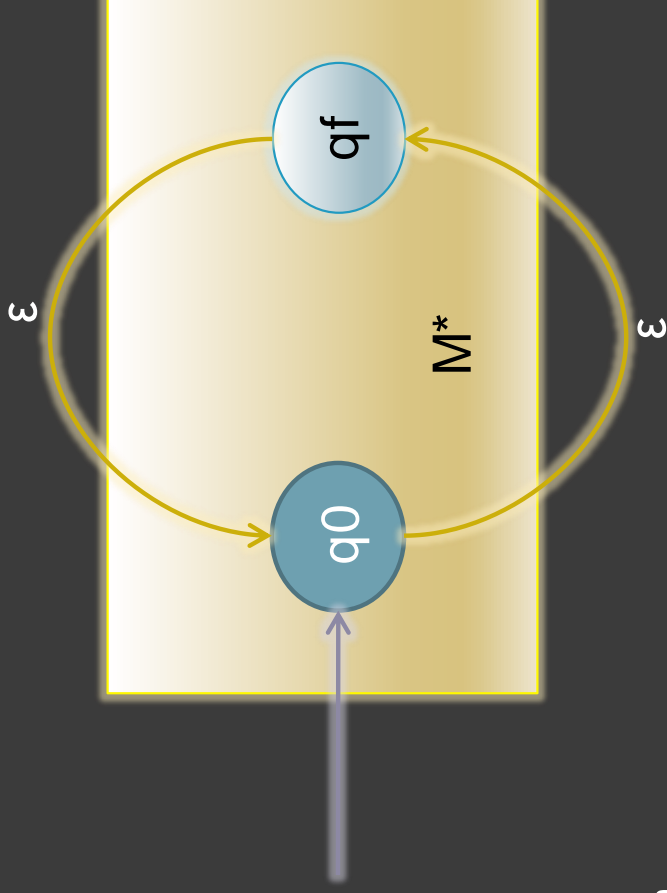
# Conversion of RE to FSA ( $\epsilon$ -NFA)

## Regular Expression

6.  $r^*$

$$L(r^*) = L(r)^* = L^0 U L^1 U L^2 U L^3 \dots$$

Suppose RE  $r$  is represented by NFA  $M$  as follows

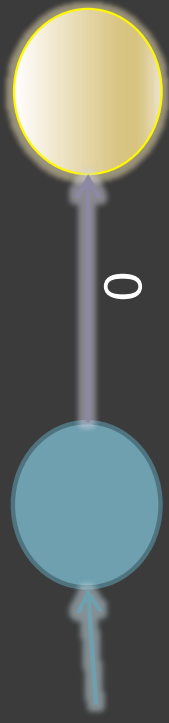


## $\epsilon$ -NFA $M^*$ Idea

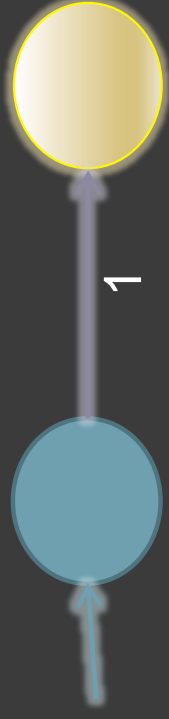
1. Add  $\epsilon$ -transition from final state  $q_f$  of  $M$  to initial state  $q_0$  of  $M$
2. Add  $\epsilon$ -transition from initial state  $q_0$  of  $M$  to final state  $q_f$  of  $M$

# Construct $\epsilon$ -NFA for RE $(0+01)^*$

NFA for RE 0



NFA for RE 1

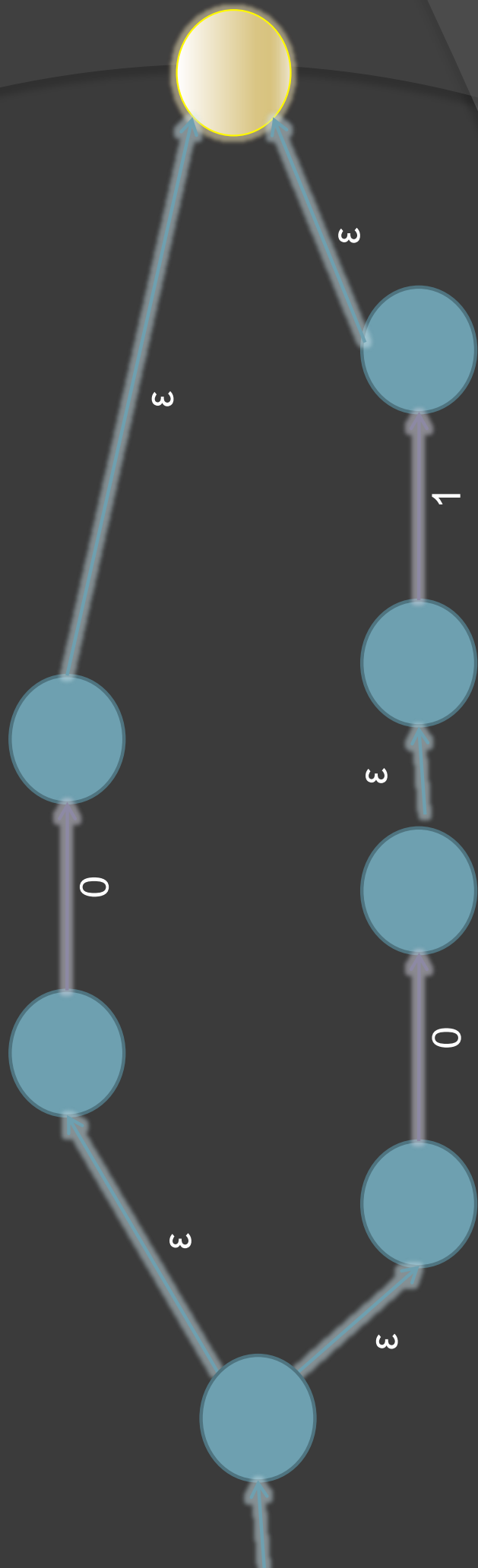


NFA for RE 01



# Construct $\epsilon$ -NFA for RE $(0+01)^*$

NFA for RE  $0+01$



# Construct $\epsilon$ -NFA for RE $(0+01)^*$

NFA for RE  $(0+01)^*$

