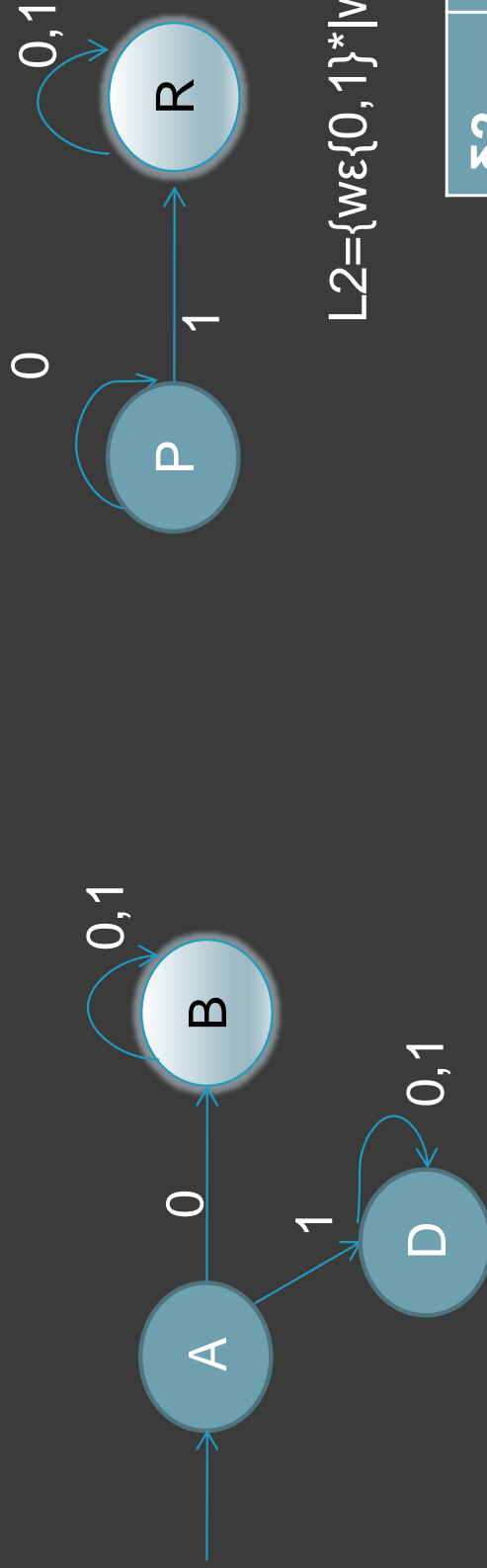


# Union operation is closed for RL

- If  $L_1$  and  $L_2$  are regular sets then  $L_1 \cup L_2$  is also regular.
- **Proof by construction**
- Given  $L_1$  and  $L_2$  are RL so there exist a DFA  $M_1$  and  $M_2$  respectively such that  $L(M_1) = L_1$  and  $L(M_2) = L_2$
- Suppose DFA  $M_1 = (Q_1, \Sigma, \delta_1, q_{01}, F_1)$  and  $M_2 = (Q_2, \Sigma, \delta_2, q_{02}, F_2)$
- Now we construct FSA  $M = (Q, \Sigma, \delta, q_0, F)$  such that  $L(M) = L(M_1) \cup L(M_2) = L_1 \cup L_2$
- $Q = Q_1 \times Q_2$
- $\delta: \delta([q, p], a) = [\delta_1(q, a), \delta_2(p, a)]$  where  $q \in Q_1$  and  $p \in Q_2$
- $q_0 = [q_{01}, q_{02}]$
- $F$  contains pair  $[q, p]$  such that either  $q \in F_1$  or  $p \in F_2$

# Union operation is closed for RL



$L_1 = \{w \in \{0,1\}^* \mid w \text{ starts with '0'}\}$

$\delta: \delta([q,p],a) = [\delta_1(q,a), \delta_2(p,a)]$

$\delta_2$	0	1
P	P	R
R	R	R

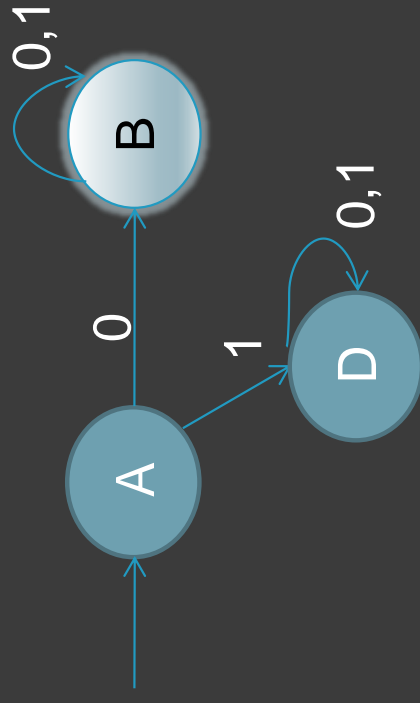
$\delta$	0	1
AP	BP	DR
AR	BR	DR
BP	BP	BR
BR	BR	BR
DP	DP	DR
DR	DR	DR

$\delta_1$	0	1
A	B	D
B	B	B
D	D	D

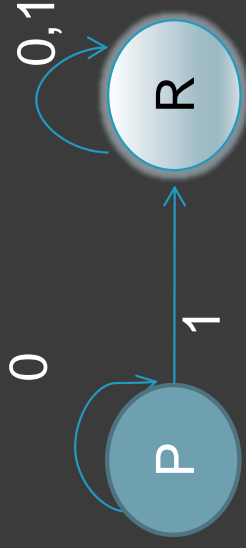
# Intersection operation is closed for RL

- If  $L_1$  and  $L_2$  are regular sets then  $L_1 \cap L_2$  is also regular.
- **Proof by construction**
- Given  $L_1$  and  $L_2$  are RL so there exist a DFA  $M_1$  and  $M_2$  respectively such that  $L(M_1) = L_1$  and  $L(M_2) = L_2$
- Suppose DFA  $M_1 = (Q_1, \Sigma, \delta_1, q_{01}, F_1)$  and  $M_2 = (Q_2, \Sigma, \delta_2, q_{02}, F_2)$
- Now we construct FSA  $M = (Q, \Sigma, \delta, q_0, F)$  such that  $L(M) = L(M_1) \cap L(M_2) = L_1 \cap L_2$
- $Q = Q_1 \times Q_2$
- $\delta: \delta([q, p], a) = [\delta_1(q, a), \delta_2(p, a)]$  where  $q \in Q_1$  and  $p \in Q_2$
- $q_0 = [q_{01}, q_{02}]$
- $F$  contains pair  $[q, p]$  such that both  $q \in F_1$  and  $p \in F_2$

# Intersection operation is closed for RL



$L1 = \{w \in \{0,1\}^* \mid w \text{ starts with '0'}\}$



$L2 = \{w \in \{0,1\}^* \mid w \text{ contains with '1'}\}$

$\delta: \delta([q,p],a) = [\delta1(q,a), \delta2(p,a)]$

$\delta1$	0	1
A	B	D
B	B	B
D	D	D

$\delta$	0	1
AP	BP	DR
AR	BR	DR
BP	BP	BR
BR	BR	BR
DP	DP	DR
DR	DR	DR

$\delta2$	0	1
P	P	R
R	R	R