

NFA - Formal Definition

- A Non Deterministic Finite State Automata M is a **5 tuple**.
- **$M = (Q, \Sigma, \delta, q_0, F)$**
- Where
- Q – is a finite set of states
- Σ – is a finite set of input alphabets
- q_0 – initial state. It belongs to set Q .
- F – set of final states/ accept states. It is subset of Q .
- δ – It is transition function or mapping from set Q and Σ to **power set of Q (2^Q)**.
- It maps a given state p and input symbol a to **zero or more next state i.e subset of Q** . It can be specified using a state transition function, table or diagram.
- **Symbolically $\delta : Q \times \Sigma \rightarrow 2^Q$**

DFA vs NFA Comparison

DFA

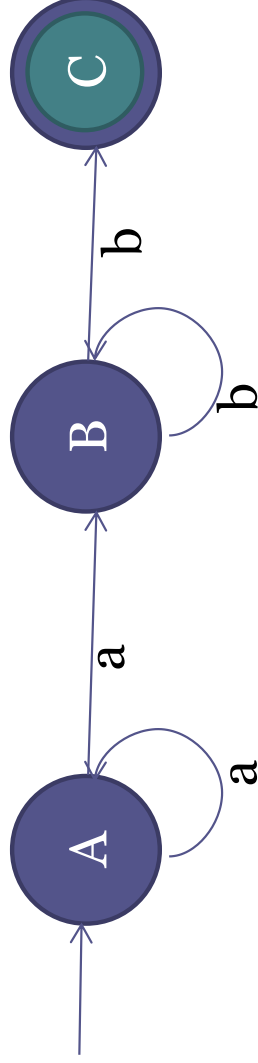
- In DFA, for **every possible state and input alphabet** (q,a) combination there is a transition to next state p.
- In DFA, there is a **unique transition** from a state and input alphabet (q,a) combination to some state p.
- Every DFA is also an NFA

NFA

- In NFA, for **every possible state and input alphabet** (q,a) combination there may not be a transition to next state. [**No Transition**]
- In NFA, there may be **multiple transitions** from a state and input alphabet (q,a) combination to states p₁,p₂,...
- Every NFA is not a DFA

NFA Examples

State Transition Diagram



State Transition Function

$\delta(A,a) = \{A,B\}$

$\delta(A,b) = \Phi$

$\delta(B,a) = \Phi$

$\delta(B,b) = \{B,C\}$

$\delta(C,a) = \Phi$

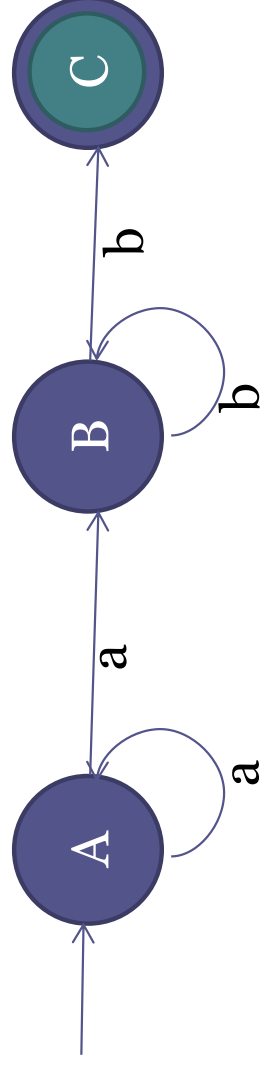
$\delta(C,b) = \Phi$

State Transition Table

δ	a	b
A	$\{A,B\}$	Φ
B	Φ	$\{B,C\}$
C	Φ	Φ

NFA Working

State Transition Diagram



State Transition Function

$\delta(A, a) = \{A, B\}$

$\delta(A, b) = \Phi$

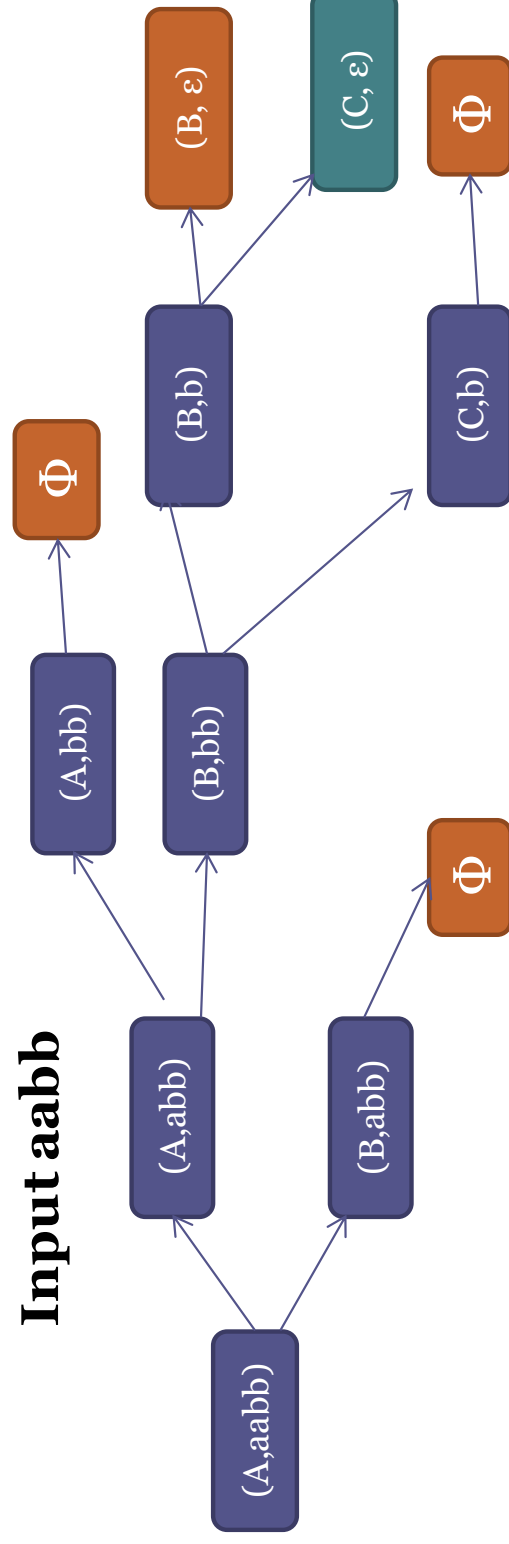
$\delta(B, a) = \Phi$

$\delta(B, b) = \{B, C\}$

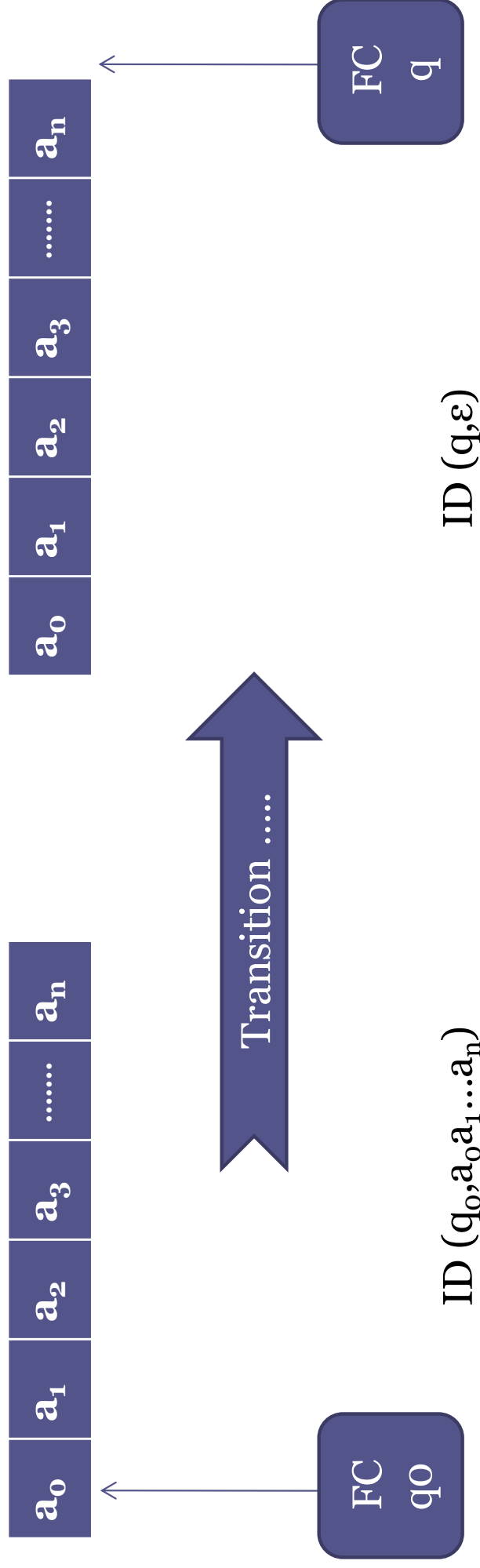
$\delta(C, a) = \Phi$

$\delta(C, b) = \Phi$

Input aabb



Acceptance of String w



A string w is said to be accepted by NFA M if NFA M reaches a final state after reading input string w one symbol at a time **through at least one computation path**.

Symbolically

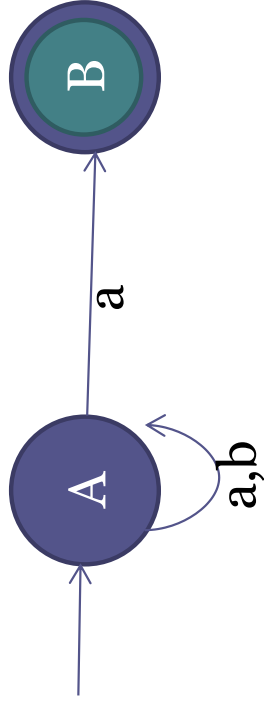
$(q_0, w) \vdash^* (q, \epsilon)$ where $q \in F$ for some computation path

Language accepted by NFA M

- The language accepted by NFA M is defined as set of all strings w belonging to Σ^* such that $(q_o, w) \vdash^*(q, \varepsilon)$ and $q \in F$ (final/accept state) **for some computation path of M**
- $L(M) = \{w \in \Sigma^* \mid (q_o, w) \vdash^*(q, \varepsilon) \wedge q \in F \text{ for some computation path of } M\}$

NFA Examples I

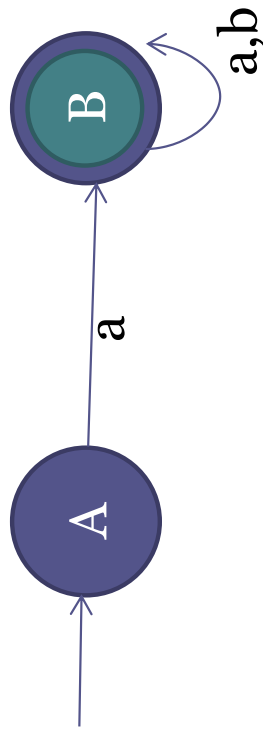
State Transition Diagram



$L(M) = \{w \in \{a,b\}^* \mid w \text{ ends with 'a'}\}$

NFA Examples II

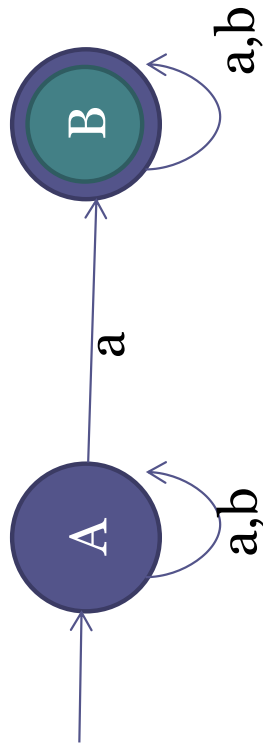
State Transition Diagram



$$L(M) = \{w \in \{a,b\}^* \mid w \text{ starts with 'a'}\}$$

NFA Examples III

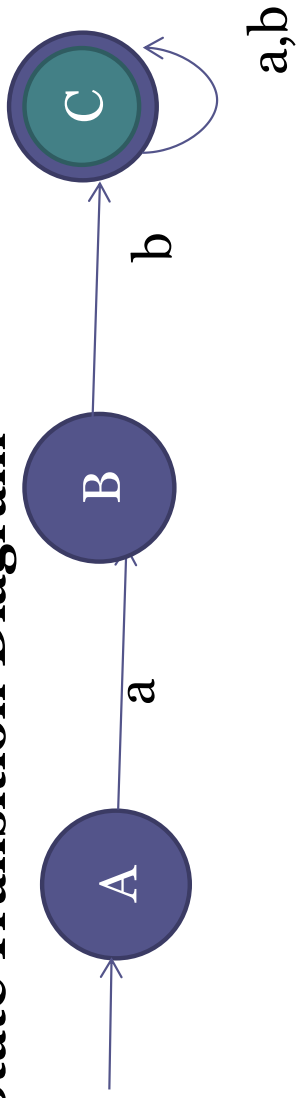
State Transition Diagram



$$L(M) = \{w \in \{a,b\}^* \mid w \text{ contains 'a'}\}$$

NFA Examples IV

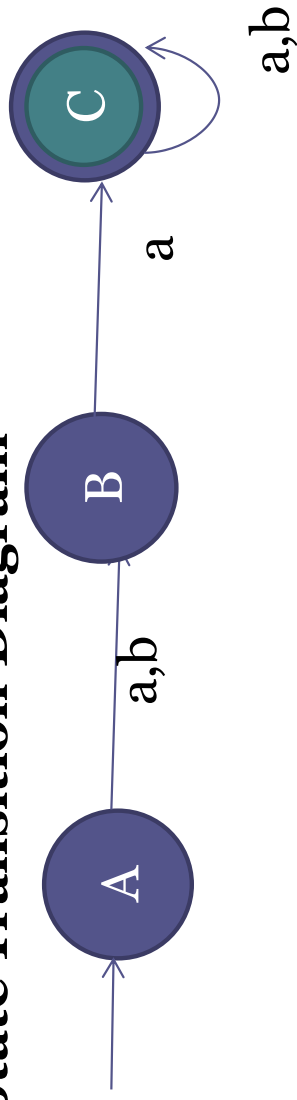
State Transition Diagram



$$L(M) = \{w \in \{a.b\}^* \mid w \text{ begins with 'ab'}\}$$

NFA Examples V

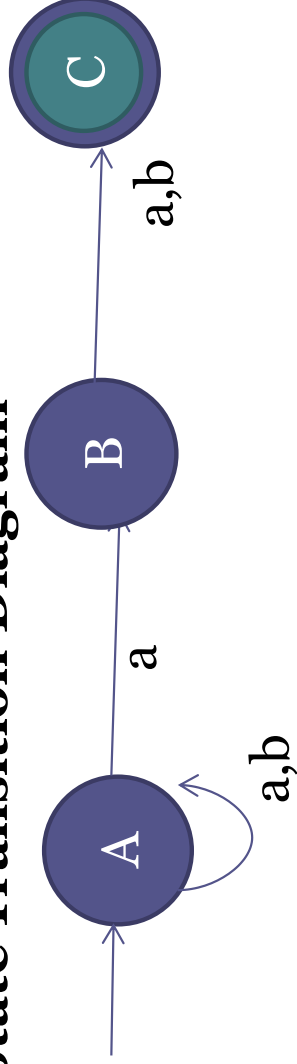
State Transition Diagram



$L(M) = \{w \in \{a.b\}^* \mid \text{second symbol of } w \text{ from LHS is 'a'}\}$

NFA Examples VI

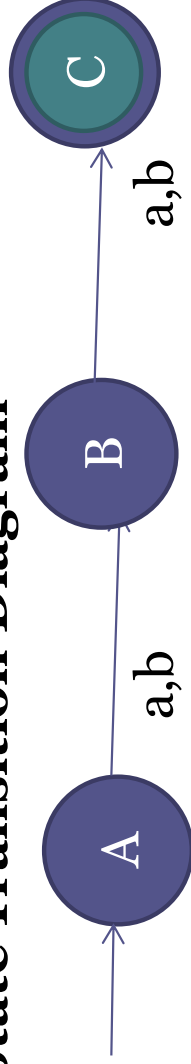
State Transition Diagram



$L(M) = \{w \in \{a.b\}^* \mid \text{second symbol of } w \text{ from RHS is 'a'}\}$

NFA Examples VII

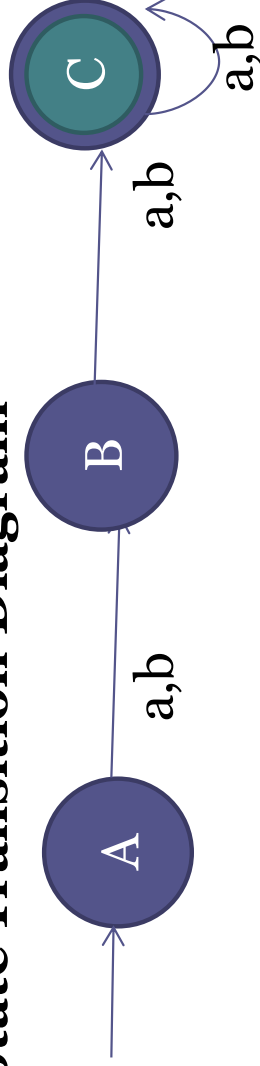
State Transition Diagram



$$L(M) = \{w \in \{a,b\}^* \mid |w| = 2\}$$

NFA Examples VIII

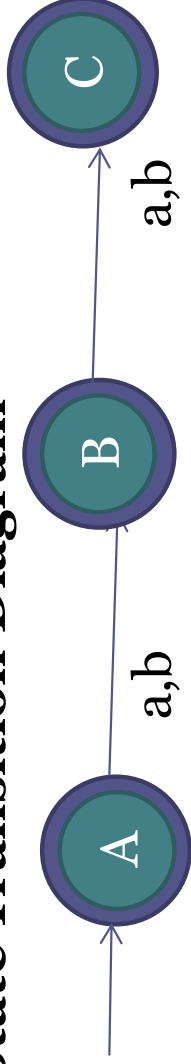
State Transition Diagram



$$L(M) = \{w \in \{a,b\}^* \mid |w| \geq 2\}$$

NFA Examples IX

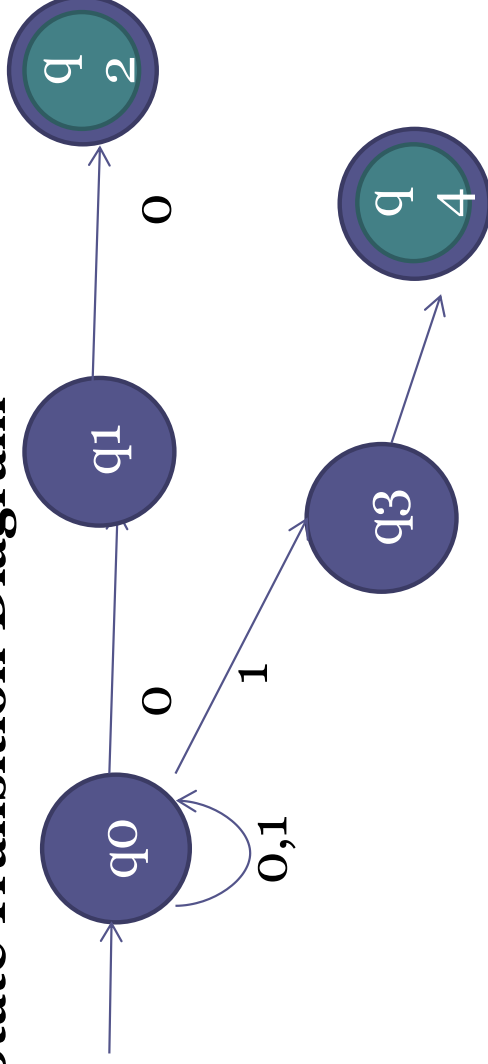
State Transition Diagram



$$L(M) = \{w \in \{a,b\}^* \mid |w| \leq 2\}$$

NFA Examples X

State Transition Diagram



$L(M) = \{w \in \{0,1\}^* \mid w \text{ ends with '00' or '11'}\}$