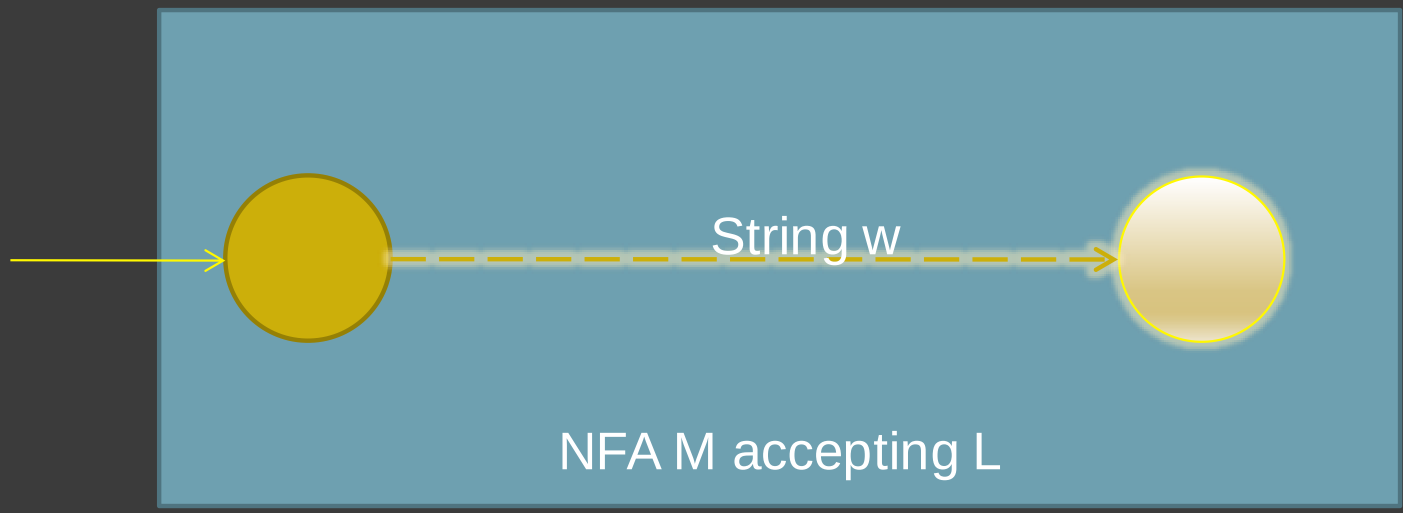
Kleene Closure operation is closed for RL

- If L is a regular language then L* is also regular.
- **Proof by construction**
- Given L is RL so there exists an NFA M such that L(M)=L
- Suppose NFA $M=(Q,\sum,\delta,q_0,q_f)$
- Now we construct a NFA $M^*=(Q,\sum,\delta',q_0,q_f)$ such that $L(M^*)=L(M)^*=L^*$

Kleene Closure operation is closed for RL

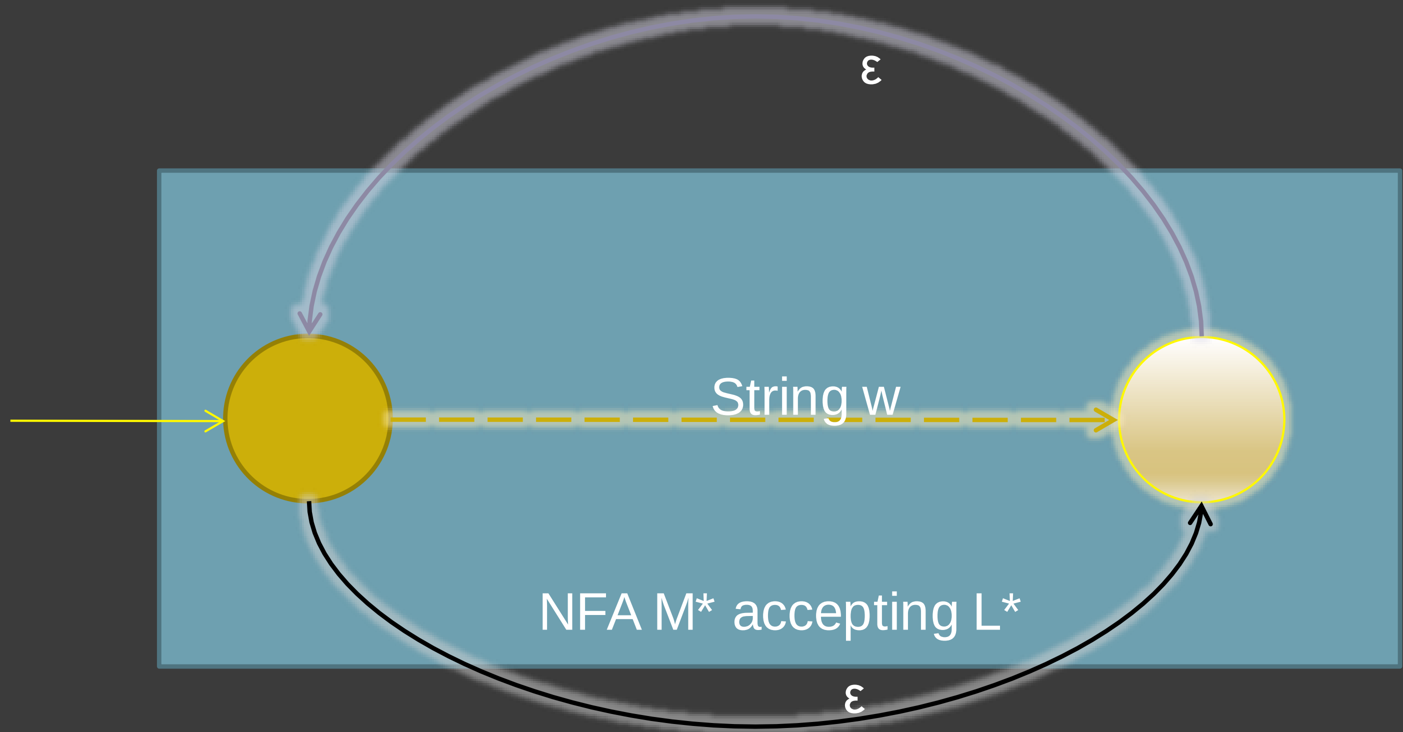
- We assume M has a single initial state q_0 and single final state q_f
- Note – we can convert any NFA with multiple final states to an NFA with single final states by
- Creating a new final state q_f and adding ε-transition from old final states to new final state q_f. Make old final states non final.
- Idea for M*
- 1. Add ε-transition from q_f to q_0. $\delta'(q_f,\varepsilon)=q_0$
- 2. Add ε-transition from q_0 to q_f. $\delta'(q_0,\varepsilon)=q_f$
- So δ' has all the transition of M and above two ε-transitions.

Kleene Closure operation is closed for RL



If there exist a string w ε L, we will have a sequence of transitions from initial to final state in M

Kleene Closure operation is closed for RL



If there exist a string w1 ε L, we will have a sequence of transitions from initial to final state in M*

Now after reaching final state we can make ε transition to initial state and Read another string w2 in L. So M* can accept L.L (L^2)
And this process can be repeated infinite times. Hence M* can accept L^1, L^2, L^3…..
M* can accept L^0 or ε. So clearly $L(M^*)=L^0 \cup L^1 \cup L^2 \cup L^3 \cup \ldots\ldots = L^*$

Reversal operation is closed for RL

- If L is a regular language then L^R is also regular.
- Definition of Reversal operation
- 1. Suppose we have a
- string $w=a_1a_2a_3\ldots\ldots a_{n-1}a_n$ then reversal of string w is defined as $w^R=a_na_{n-1}\ldots\ldots a_3a_2a_1$
- 2. Suppose we have a language L then reversal of language L is defined as
- $L^R=\{w^R \mid w \varepsilon L\}$
- In other words by reversing each and every string in the language L we get reversal of language L^R

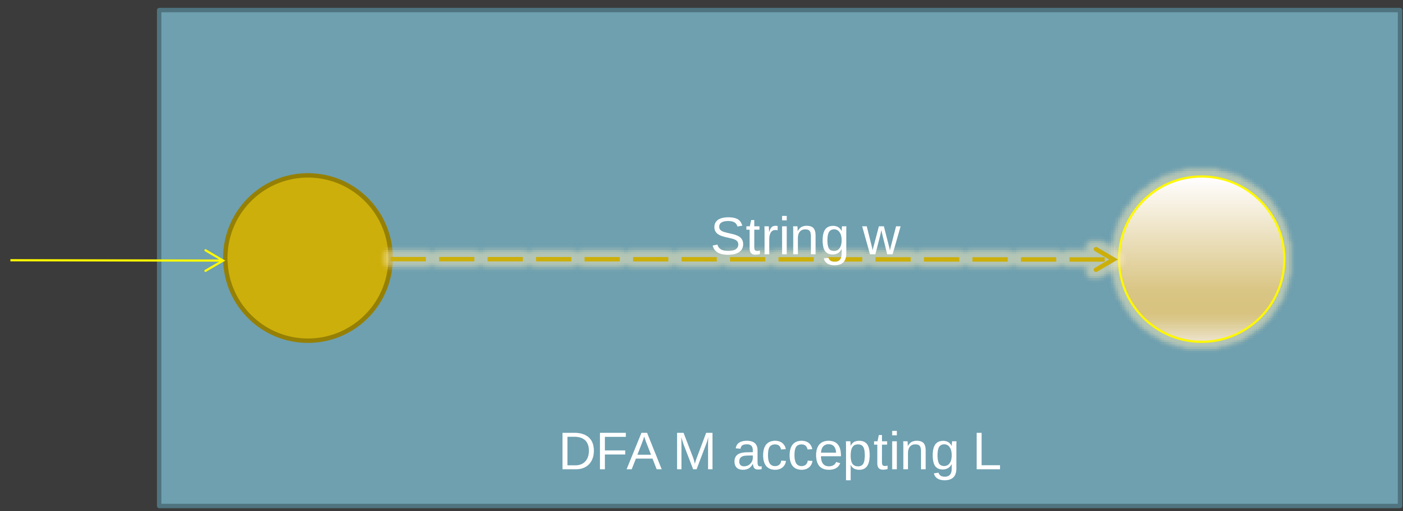
Reversal operation is closed for RL

- If L is a regular language then L^R is also regular.
- **Proof by construction**
- Given L is RL so there exists a NFA M such that $L(M)=L$
- Suppose NFA $M=(Q,\sum,\delta,q_0,q_f)$
- Now we construct an NFA $M^R=(Q,\sum,\delta',q_f,q_0)$ such that $L(M^R)=L(M)^R=L^R$

Reversal operation is closed for RL

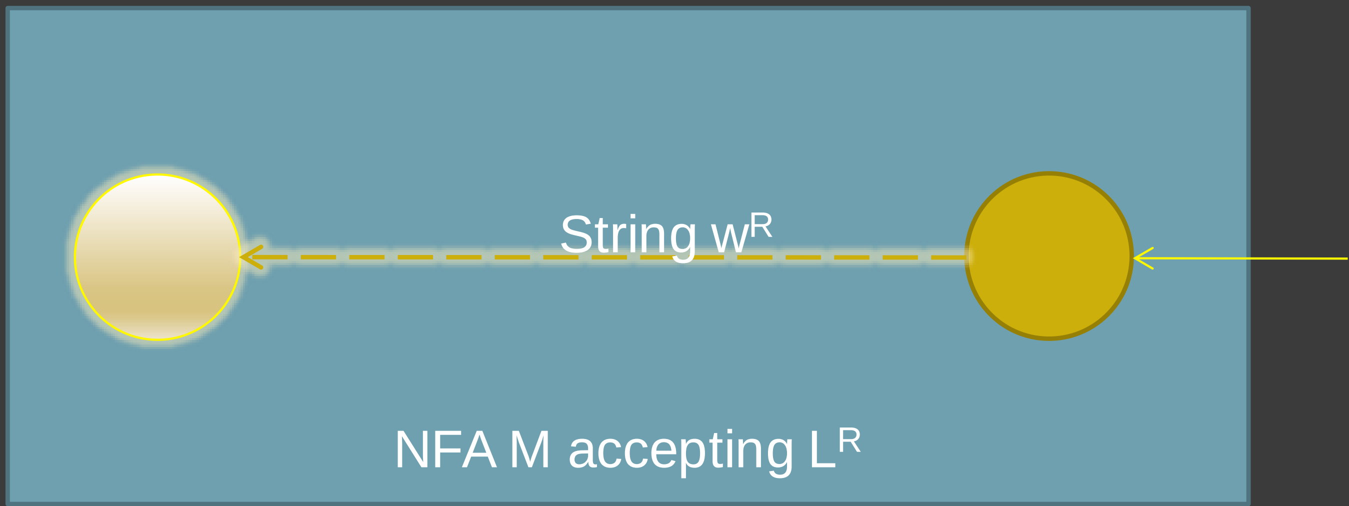
- We assume M has a single initial state q_0 and single final state q_f
- Note – we can convert any NFA with multiple final states to an NFA with single final states by
- Creating a new final state q_f and adding ε-transition from old final states to new final state q_f. Make old final states non final.
- Idea for M^R
- 1. Interchange final state q_f and initial state q_0.
- 2. Reverse the direction of each and every transition.
- $\delta'(q,a)=p$ if $\delta(p,a)=q$

Reversal operation is closed for RL



If there exist a string w ε L, we will have a sequence of transitions from initial to final state in M

Reversal operation is closed for RL



If there exist a string w ε L, we will have a sequence of transitions from initial state q_0 to final state q_f in M

Now for a string w^R , we will have a sequence of transitions from initial state q_f to final state q_0 in M^R
So $L(M^R)=L^R$