HW2

Motion

① $\dot{\theta}_k = \frac{r}{L}u_\psi + w_\psi = \frac{\theta_k - \theta_{k-1}}{T} \rightarrow \theta_k = T\dot{\theta}_k + \theta_{k-1}$

② $\dot{X}_k = \begin{bmatrix}\dot{x}_k \\ \dot{y}_k\end{bmatrix} = \begin{bmatrix} r u_\omega \cos\theta_k + w_\omega \\ r u_\omega \sin\theta_k + w_\omega \end{bmatrix} \rightarrow \dot{X}_k = \frac{X_k - X_{k-1}}{T} \rightarrow X_k = \underbrace{T\dot{X}_{k-1}}_{v_k} + \underset{\uparrow A_k = I}{X_{k-1}}$ ①

$X_k = \begin{bmatrix}\hat{x}_{k-1} \\ \hat{y}_{k-1}\end{bmatrix} + T\begin{bmatrix}\dot{x}_k \\ \dot{y}_k\end{bmatrix} = f(X_{k-1}, v_k, w_k)$

③ Linearization:

- $f(X_{k-1}, v_k, w_k) \approx \check{X}_k + F_{k-1}(X_{k-1} - \hat{X}_{k-1}) + w_k'$
- $\check{X}_k = f(X_{k-1}, v_k, 0) = \begin{bmatrix}\hat{x}_{k-1} \\ \hat{y}_{k-1}\end{bmatrix} + T r u_\omega \begin{bmatrix}\cos[T\frac{r}{L}u_\psi + \theta_{k-1}] \\ \sin[\underbrace{T\frac{r}{L}u_\psi + \theta_{k-1}}_{\theta_k \text{ with no noise: } w_\psi = 0}]\end{bmatrix}$ ②
- $X_{k-1} =$ previous $\check{X}_k$

  $\hat{X}_{k-1} =$ previous $\hat{X}_k$
- $F_{k-1} = \frac{\partial f(X_{k-1}, v_k, w_k)}{\partial X_{k-1}}\Big|_{\hat{X}_{k-1}, v_k, 0} = \frac{\partial(T\dot{X}_{k-1} + X_{k-1})}{\partial X_{k-1}} = 1 = I$ ③
- $w_k = (w_\omega, w_\psi)$

  note: $w_k = \begin{bmatrix}w_\omega \\ w_\psi\end{bmatrix}$
- $w_k' = \frac{\partial f(X_{k-1}, v_k, w_k)}{\partial w_k}\Big|_{\hat{x}_{k-1}, v_k, 0} w_k = \begin{bmatrix}(\frac{\partial f}{\partial w_\omega})w_\omega & (\frac{\partial f}{\partial w_\psi})w_\psi \\ (\frac{\partial f}{\partial w_\omega})w_\omega & (\frac{\partial f}{\partial w_\psi})w_\psi\end{bmatrix}$ ④

$\frac{\partial f}{\partial w_\omega} = T\begin{bmatrix}1 \\ 1\end{bmatrix}$ ⑤ $\quad \frac{\partial f}{\partial w_\psi} = T\begin{bmatrix}-r u_\omega \sin[T(\frac{r}{L}u_\psi + \cancel{w_\psi}^{0}) + \theta_{k-1}] \times T \\ r u_\omega \cos[T(\frac{r}{L}u_\psi + \cancel{w_\psi}^{0}) + \theta_{k-1}] \times T\end{bmatrix}$

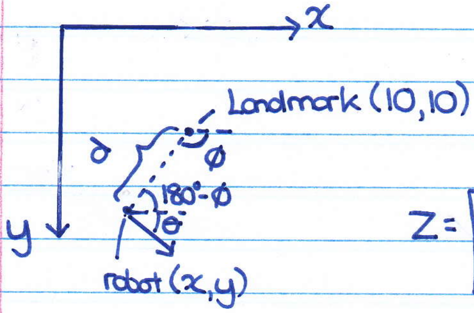
$= T^2 r u_\omega \begin{bmatrix}\sin[T(\frac{r}{L}u_\psi) + \theta_{k-1}] \\ \cos[T(\frac{r}{L}u_\psi) + \theta_{k-1}]\end{bmatrix}$ ⑥

Q_k' is defined by ④ in KF eqns

④ Measurement

- For part a), nothing changes (linear)

Part b: Derive the measurement model



$180 - \phi = 90 - \theta$

$\phi = 90 + \theta$ ⑦

$d = \sqrt{(x-10)^2 + (y-10)^2}$ ⑧

$Z = \begin{bmatrix}10 + d\cos\phi \\ 10 + d\sin\phi\end{bmatrix}$ ⑨

↓

- The is noise in both the measurement d and bearing $\emptyset$

$$Z_k = \underbrace{\begin{bmatrix} 10+(d+n_d)\cos(\emptyset+n_\emptyset) \\ 10+(d+n_d)\sin(\emptyset+n_\emptyset) \end{bmatrix}}_{g(x_k,n_k)} \quad (9) \qquad \begin{matrix} n_d \sim N(0,0.1) \\ n_\emptyset \sim N(0,0.01) \end{matrix} \quad (10)$$

$\rightarrow n_k = (n_d, n_\emptyset)$

- $g(x_k,n_k) \approx \check{z}_k + G_k(x_k-\check{x}_k)+n_k'$

defined by (8)

- $$G_k = \left.\frac{\partial g(x_k,n_k)}{\partial x_k}\right|_{\check{x}_k,0} = \left.\frac{\partial \begin{bmatrix} 10+d\cos\emptyset \\ 10+d\sin\emptyset \end{bmatrix}}{\partial x_k}\right|_{\check{x}_k} \qquad d=((x-10)^2+(y-10)^2)^{\frac{1}{2}}$$

$$= \left.\begin{bmatrix} \frac{1}{2}[(x-10)^2+(y-10)^2]^{-\frac{1}{2}}(2x-20)\cos\emptyset & \frac{1}{2}[(x-10)^2+(y-10)^2]^{-\frac{1}{2}}(2y-20)\cos\emptyset \\ \frac{1}{2}[(x-10)^2+(y-10)^2]^{-\frac{1}{2}}(2y-20)\sin\emptyset & \frac{1}{2}[(x-10)^2+(y-10)^2]^{-\frac{1}{2}}(2y-20)\cos\emptyset \end{bmatrix}\right|_{\check{x}_k}$$

$$= \frac{1}{\sqrt{(\check{x}-10)^2+(\check{y}-10)^2}} \begin{bmatrix} (\check{x}-10)\cos\emptyset & (\check{y}-10)\cos\emptyset \\ (\check{x}-10)\sin\emptyset & (\check{y}-10)\sin\emptyset \end{bmatrix} \quad (11)$$

$\rightarrow$ form is: $g=\begin{bmatrix} g_x \\ g_y \end{bmatrix}$ $X=\begin{bmatrix} x \\ y \end{bmatrix}$

$$\frac{\partial g}{\partial x} = \begin{bmatrix} \frac{\partial g_x}{\partial x} & \frac{\partial g_x}{\partial y} \\ \frac{\partial g_y}{\partial x} & \frac{\partial g_y}{\partial y} \end{bmatrix}$$

- $$n_k' = \left.\frac{\partial g(x_k,n_k)}{\partial n_k}\right|_{\check{x}_k,0} n_k = \left.\begin{bmatrix} \frac{\partial g_x}{\partial n_d} & \frac{\partial g_x}{\partial n_\emptyset} \\ \frac{\partial g_y}{\partial n_d} & \frac{\partial g_y}{\partial n_\emptyset} \end{bmatrix}\right|_{\check{x}_k,0} n_k$$

$$= \left.\begin{bmatrix} \cos(\emptyset+n_\emptyset) & -(d+n_d)\sin(\emptyset+n_\emptyset) \\ \sin(\emptyset+n_\emptyset) & (d+n_d)\cos(\emptyset+n_\emptyset) \end{bmatrix}\right|_{n_d,n_\emptyset=0} n_k$$

$$= \begin{bmatrix} \cos\emptyset & -d\sin\emptyset \\ \sin\emptyset & d\cos\emptyset \end{bmatrix} \begin{bmatrix} n_d \\ n_\emptyset \end{bmatrix} = \begin{bmatrix} n_d\cos\emptyset & -dn_\emptyset\sin\emptyset \\ n_d\sin\emptyset & dn_\emptyset\cos\emptyset \end{bmatrix} \quad (12)$$

- Use (12) as R_k in EKF