Linear Regression Example

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Linear Regression

No Restriction on Standard Error

First we conduct regression without specifying the types of the standard error.

Set Up Data

```
data(CASchools)
# student teacher ratio
CASchools$STR <- CASchools$students / CASchools$teachers
# average test score
CASchools$score <- (CASchools$read + CASchools$math)/2
This is same as: "reg score STR" in Stata. In R language we always need to specify which data will be used.
fit1 <- lm(score ~ STR, data = CASchools)</pre>
summary(fit1)
##
## Call:
## lm(formula = score ~ STR, data = CASchools)
##
## Residuals:
##
     \mathtt{Min}
              1Q Median
                             3Q
                                   Max
## -47.73 -14.25
                  0.48 12.82 48.54
##
## Coefficients:
               Estimate Std. Error t value
##
                                                         Pr(>|t|)
                               9.47
                                      73.82 < 0.000000000000000 ***
## (Intercept)
                 698.93
## STR
                  -2.28
                               0.48
                                      -4.75
                                                       0.0000028 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 18.6 on 418 degrees of freedom
## Multiple R-squared: 0.0512, Adjusted R-squared: 0.049
## F-statistic: 22.6 on 1 and 418 DF, p-value: 0.00000278
Present the equations.
equatiomatic::extract_eq(fit1)
```

$$score = \alpha + \beta_1(STR) + \epsilon$$

```
equatiomatic::extract_eq(fit1, use_coefs = TRUE)
```

```
score = 698.93 - 2.28(STR) + \epsilon
```

We can include further controls in the equation.

```
fit2 <- lm(score ~ STR + english + income, data = CASchools)
summary(fit2)
##
## Call:
## lm(formula = score ~ STR + english + income, data = CASchools)
## Residuals:
     Min
##
             1Q Median
                          3Q
                                Max
## -42.80 -6.86
                 0.27
                         6.59
                              31.20
##
## Coefficients:
##
              Estimate Std. Error t value
                                                   Pr(>|t|)
## (Intercept) 640.3155
                          5.7749 110.88 < 0.0000000000000000 ***
               -0.0688
                          0.2769
                                   -0.25
                                                        0.8
## STR
               -0.4883
                          ## english
                                   ## income
                1.4945
                          0.0748
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 10.3 on 416 degrees of freedom
## Multiple R-squared: 0.707, Adjusted R-squared: 0.705
## F-statistic: 335 on 3 and 416 DF, p-value: <0.00000000000000000
equatiomatic::extract_eq(fit2)
                     score = \alpha + \beta_1(STR) + \beta_2(english) + \beta_3(income) + \epsilon
equatiomatic::extract_eq(fit2, use_coefs = TRUE)
```

Cluster Robust Standard Error

In empirical work we always deal with correlation within a group by clustering SE. Above lm code conduct regression under the assumption of homoskedasticity just like reg and without robust option in Stata. We now use the estimatr package.

 $score = 640.32 - 0.07(STR) - 0.49(english) + 1.49(income) + \epsilon$

```
## Coefficients:
##
            Estimate Std. Error t value
## (Intercept) 640.3155
                       6.2866 101.855
## STR
             -0.0688
                       0.2898 -0.237
## english
             -0.4883
                       0.0309 -15.779
## income
              1.4945
                       0.1023 14.604
##
                                                     Pr(>|t|)
## STR
            0.81352732429668439539938162852195091545581817626953125000\\
            0.000000000000000000930987090849287295359433197705020740\\
## english
## income
            0.000000000000000175194413389987415730081432545853544527\\
##
            CI Lower CI Upper DF
             627.646
                    652.985 44
## (Intercept)
## STR
              -0.653
                      0.515 44
              -0.551
                     -0.426 44
## english
## income
               1.288
                      1.701 44
##
## Multiple R-squared: 0.707 , Adjusted R-squared: 0.705
```

We estimated the same model as fit2, but clustered SE at the county level (we do not know whether this is the best unit). Clustered SE for STR increased from 0.277 to 0.299.