Introduction to Numerical Methods and Applications Homework #2

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1. I assume diverge is that error rate greater or equal to 200% ($|\varepsilon_t| \geq 200\%$)

a. (10%) Jacobi method

Based on figure below, we can find curve is diverge after 27 iterations.

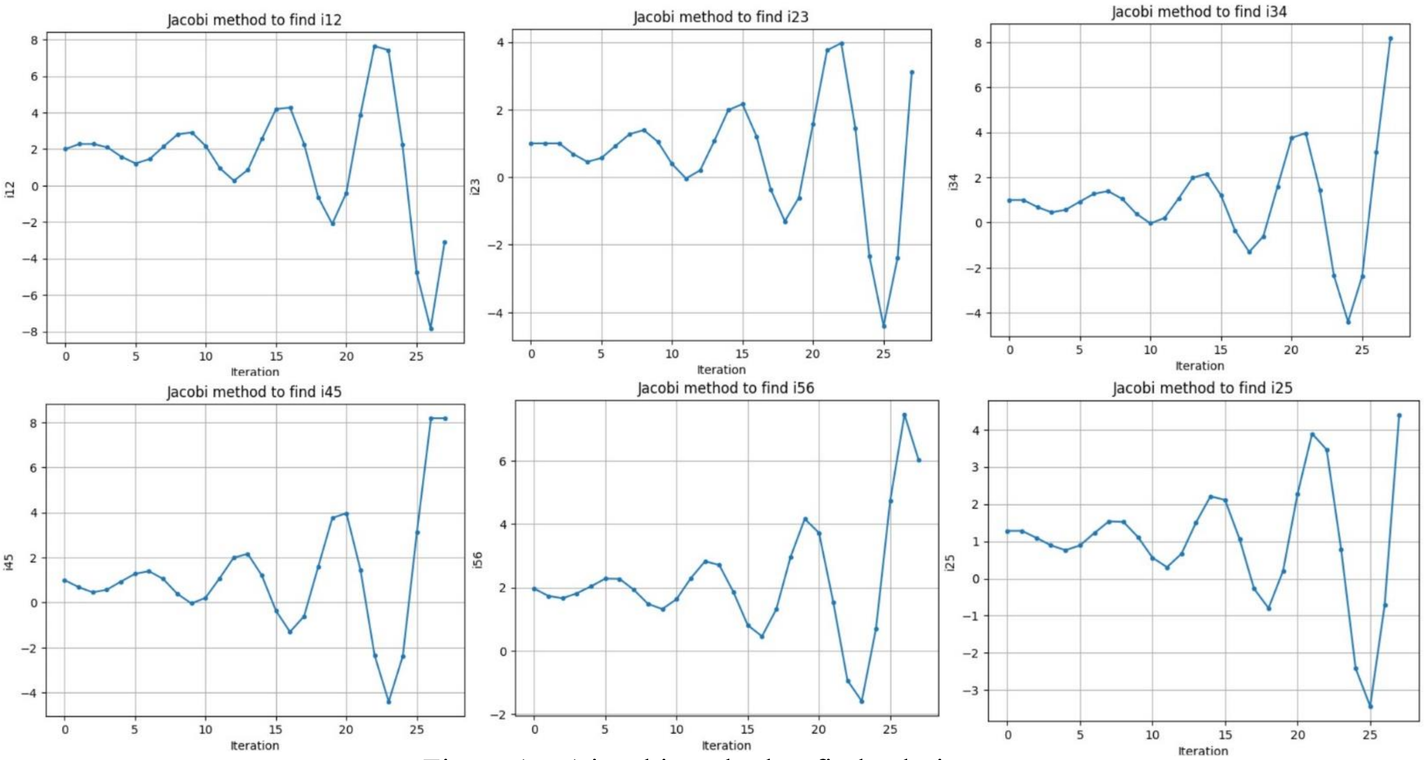

Figure 1-a-1 jacobi method to find solution

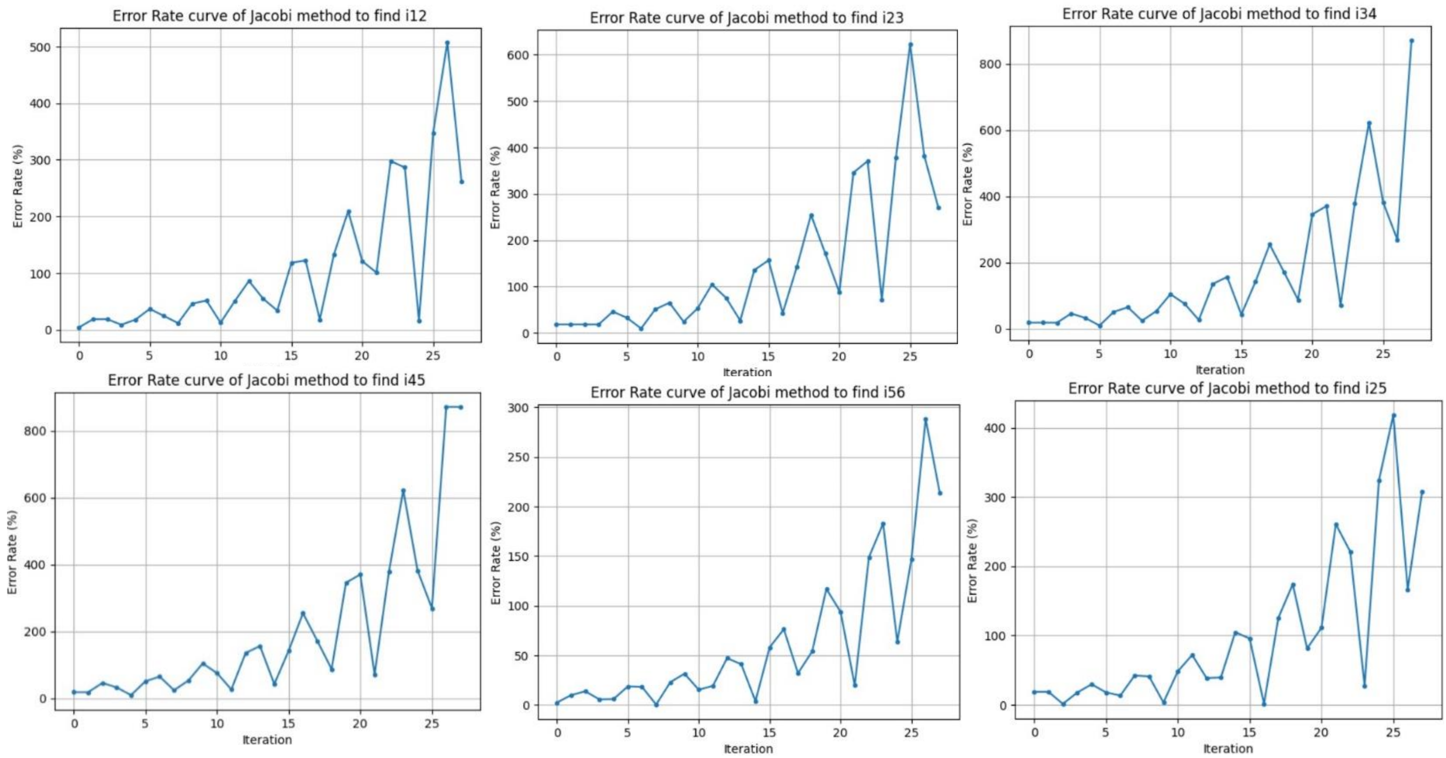

Figure 2-a-2 Error Rate of jacobi method

b. (10%)

Based on figure below, we can find curve is converge after 450 iterations.

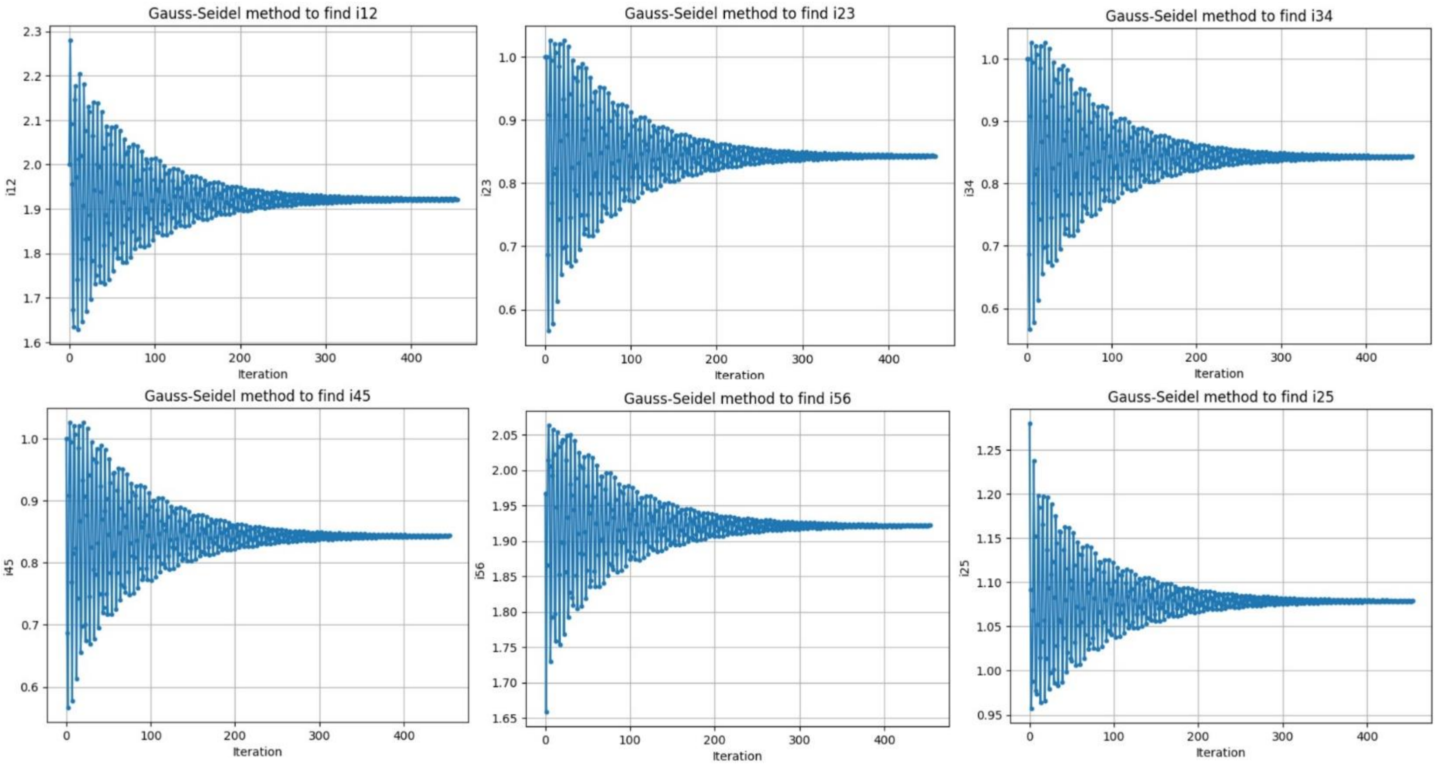

Figure 3-b-1 Gauss-Seidel method to find solution

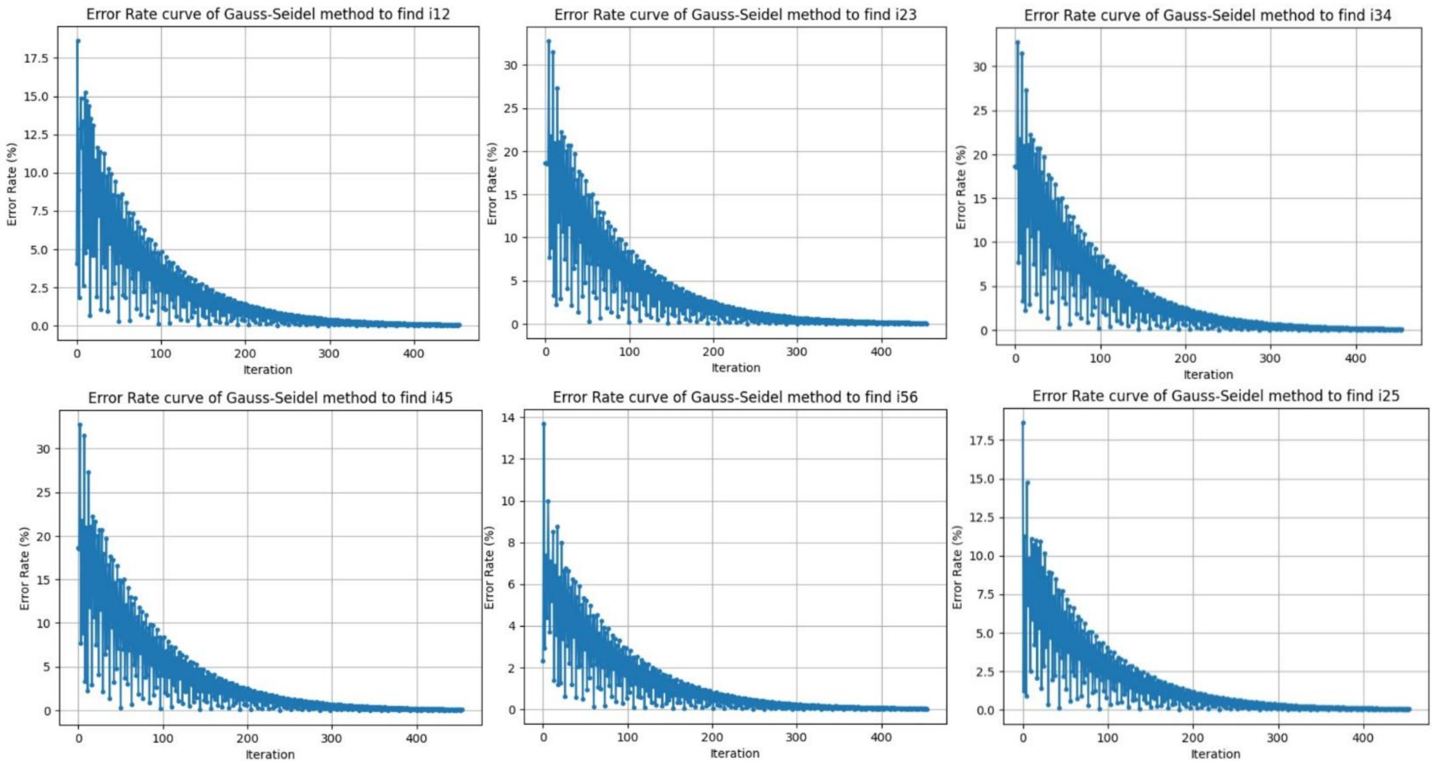

Figure 4-b-2 Error Rate of Gauss-Seidel method

c. (20%)

Because part 1-b Guass-Seidel method converge, I keep using it in part c.

(1). $\lambda = 0.3$

By using $\lambda = 0.3$, the curve converge after 68 iterations, faster than without relaxing.

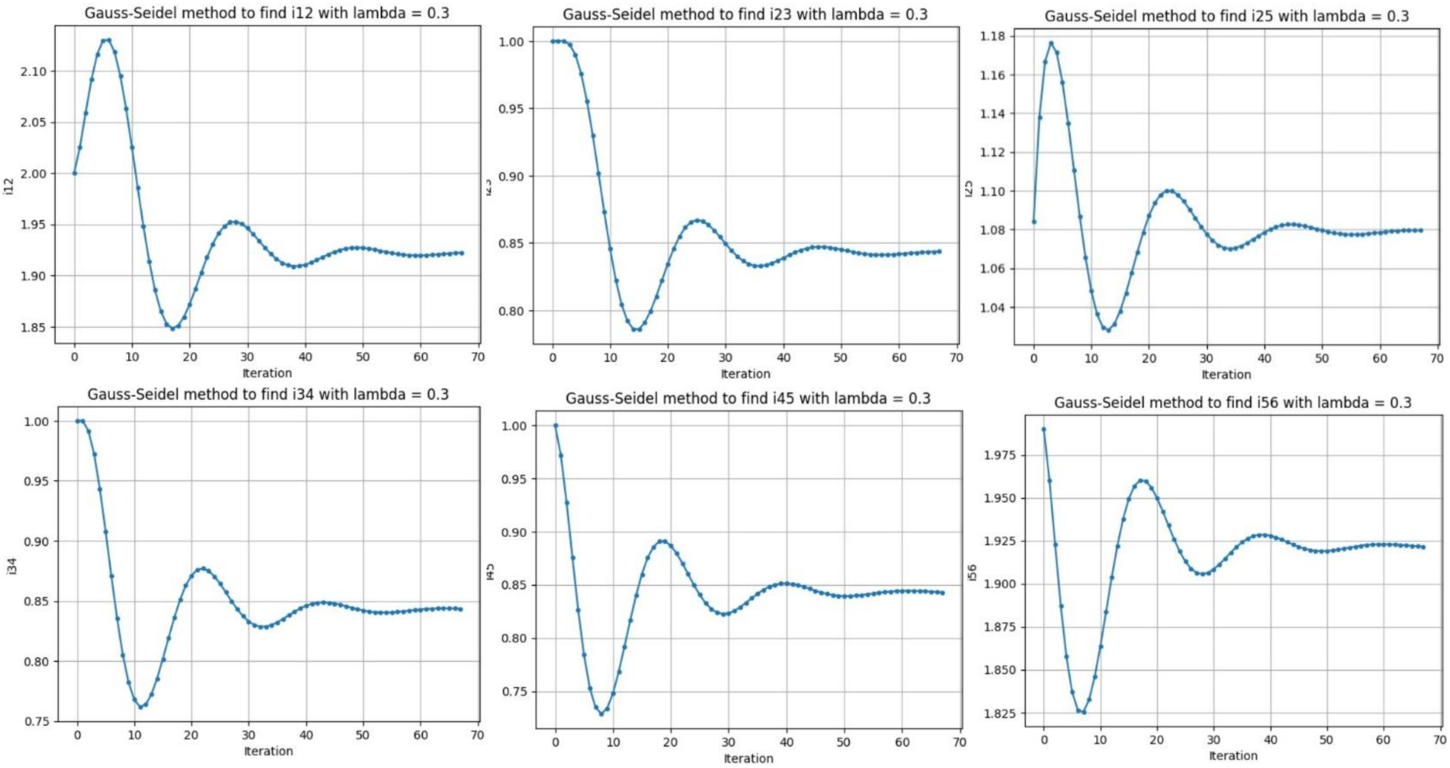

Figure 5-c-1 Gauss-Seidel method with relax $\lambda = 0.3$

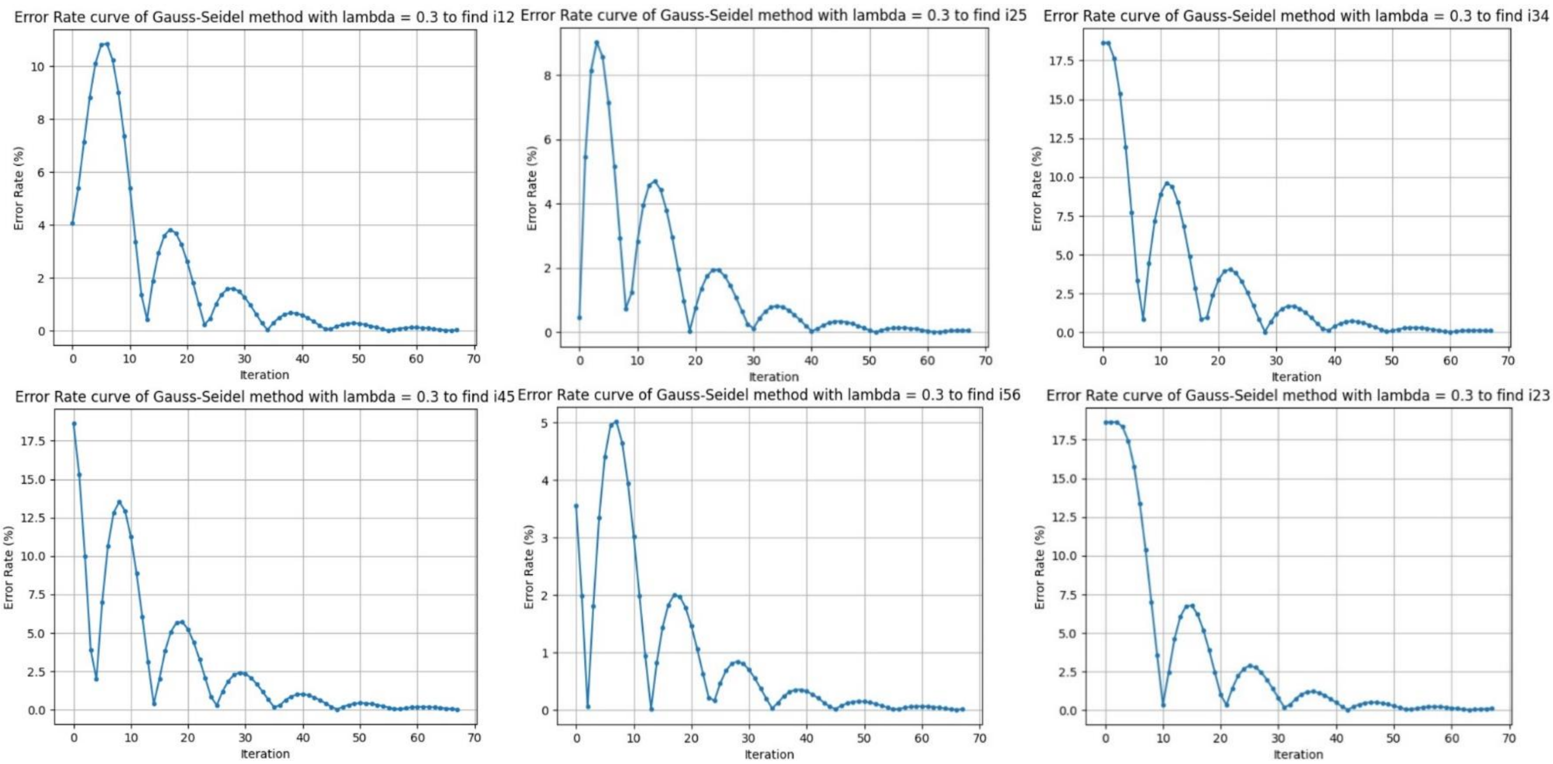

Figure 6-c-2 Error Rate of Gauss-Seidel method with relax $\lambda = 0.3$

(2). $\lambda = 0.7$

By using $\lambda = 0.7$, the curve converge after 48 iterations, even faster than $\lambda = 0.3$.

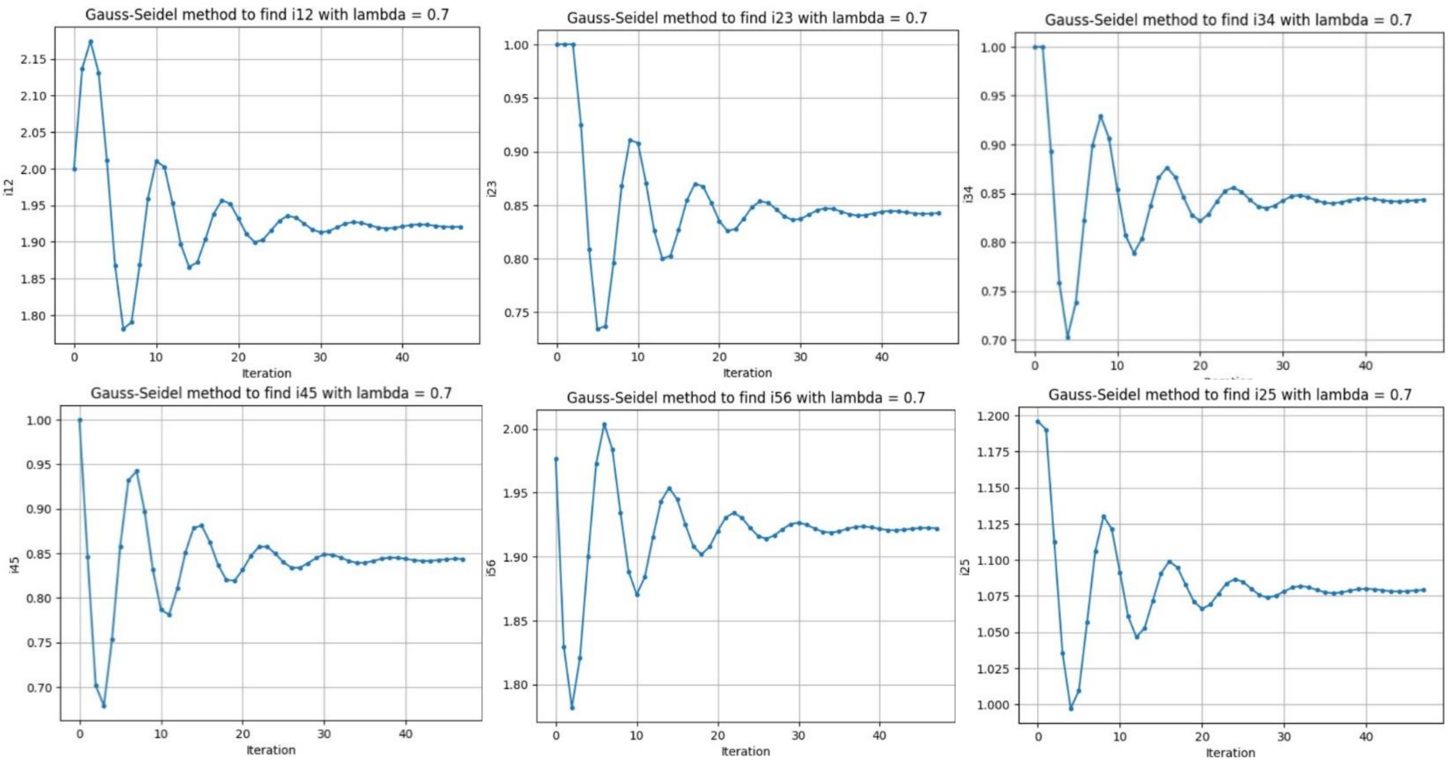

Figure 7-c-3 Gauss-Seidel method with relax $\lambda = 0.7$

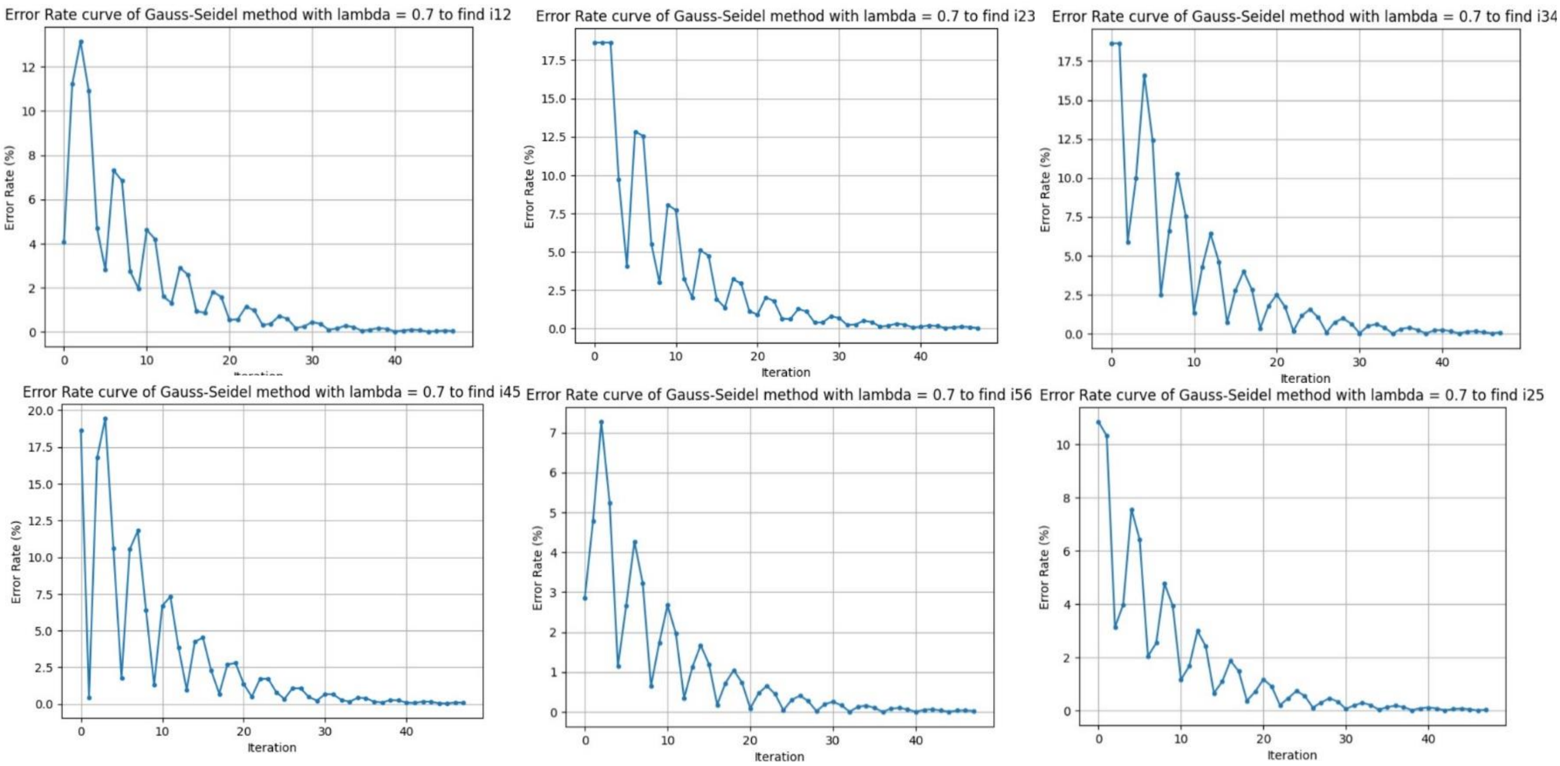

Figure 8-c-4 Error Rate of Gauss-Seidel method with relax $\lambda = 0.7$

(3). $\lambda = 1.2$

However, by using $\lambda = 1.2$, the curve diverge quickly, worse than without relaxtion.

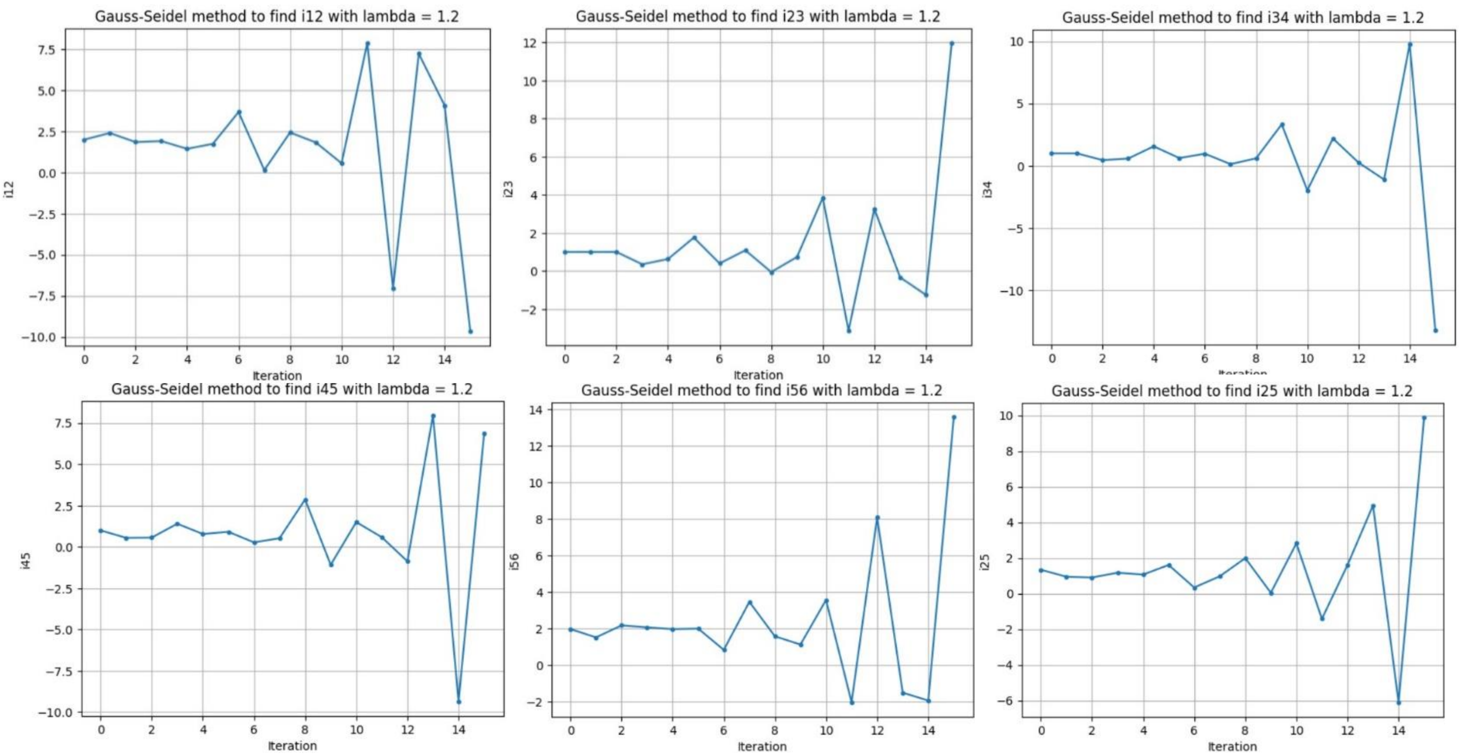

Figure 9-c-5 Gauss-Seidel method with relax $\lambda = 1.2$

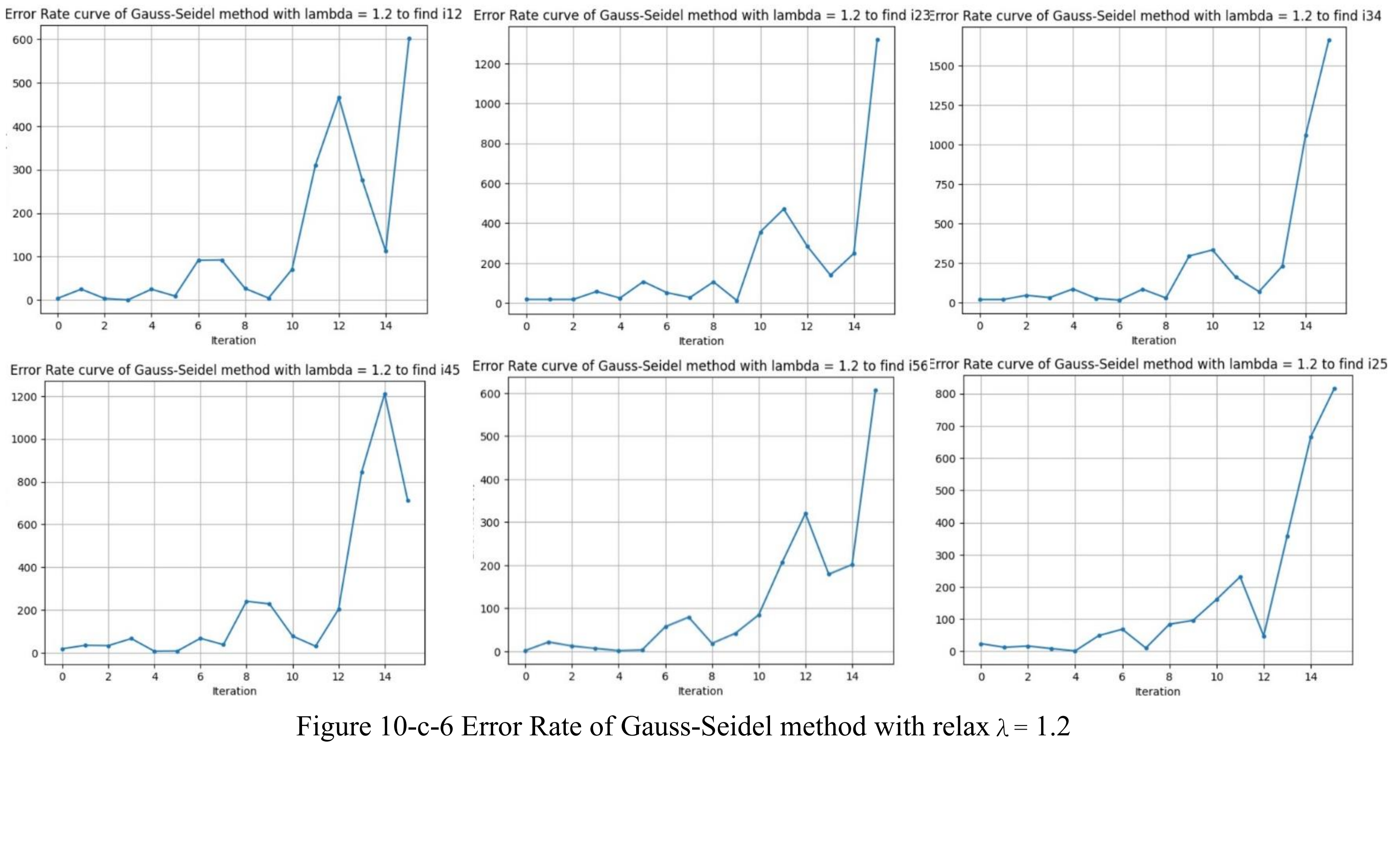

Figure 10-c-6 Error Rate of Gauss-Seidel method with relax $\lambda = 1.2$

Discussion: Comparing the result above of $\lambda = 0.3$, $\lambda = 0.7$, $\lambda = 1.2$ and without relaxation, only $\lambda = 1.2$ diverge, the others result converge successfully. And for converge rate, $\lambda = 0.7 >$ $\lambda = 0.3 >$ without relaxation. Thus, I conclude that Guass-Seidel method converge toward correct way, but it oscillates too largely. By relaxation between 0 and 1 (including previous result), we can avoid the result oscillate, and the larger λ, the faster converging the result (except for λ>1).

2. (15%)

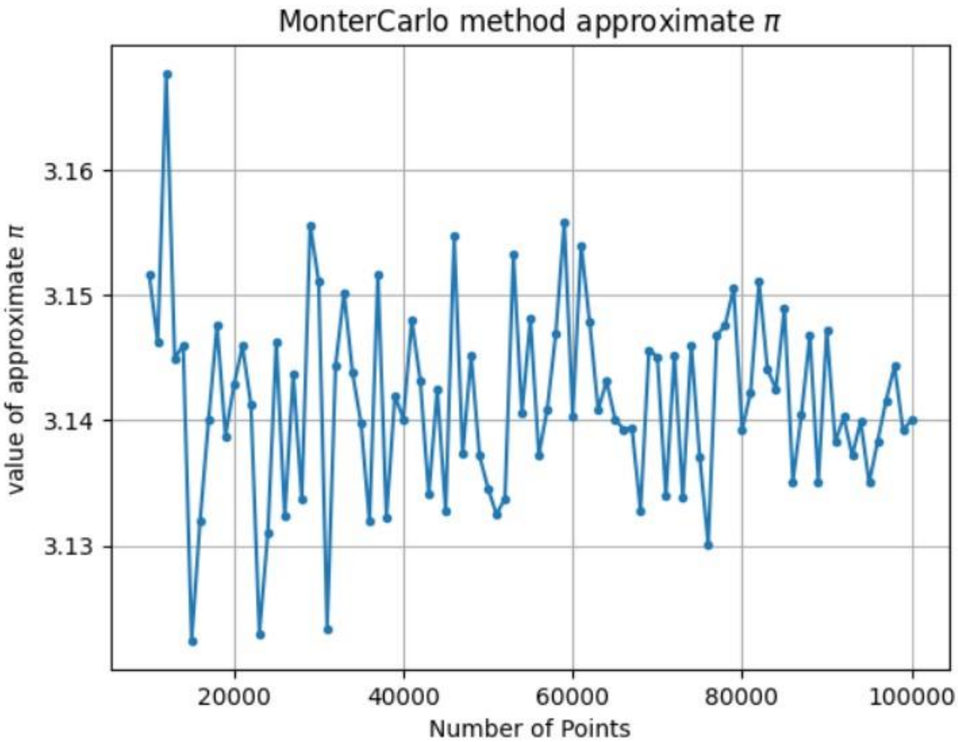

Figure 2 Monte Carlo method to approximate π

Investigate the curve above, I found that it seems that just oscillate around π, but actually amplitude is becoming smaller with more points. Comparing range (0, 40000), (40000, 70000) and (70000, 100000), the amplitude of oscillation going to coverage.

3.

a. (5%)

I use the function “random.normalvariate” in python package to generate a group of random numbers random. “normalvariate” requires two arguments, the first one is mean and the second one is standard deviation, so my RNG is calling normalvariate(0, 1).

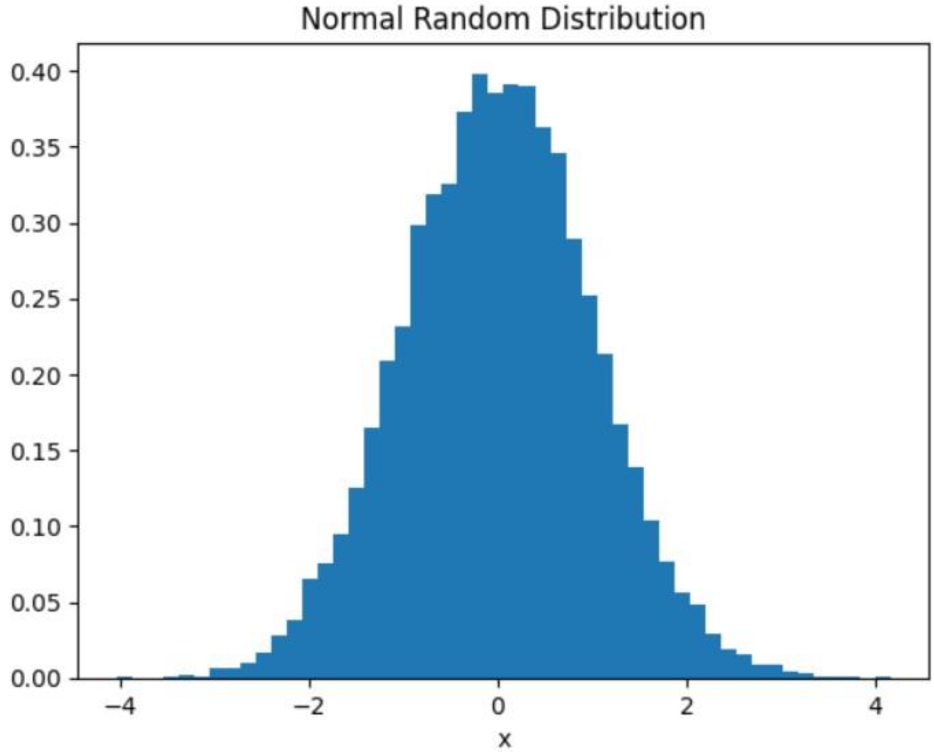

Figure 3-a histogram of RNG result with 20000 points

b. (5%)

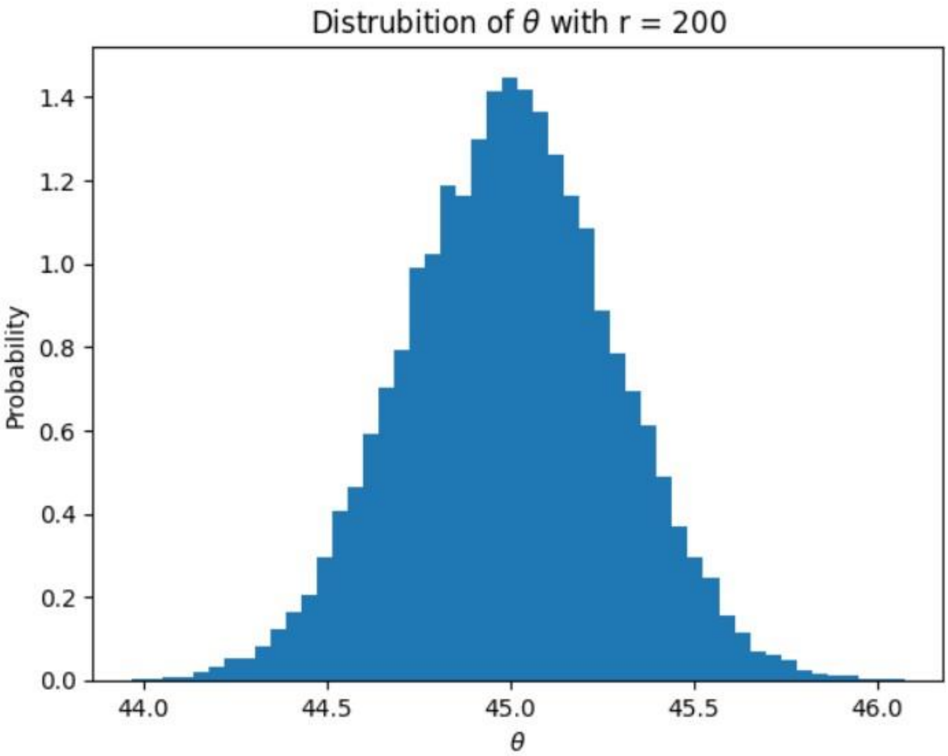

Figure 3-b histogram of θ with r = 200

c. (10%)

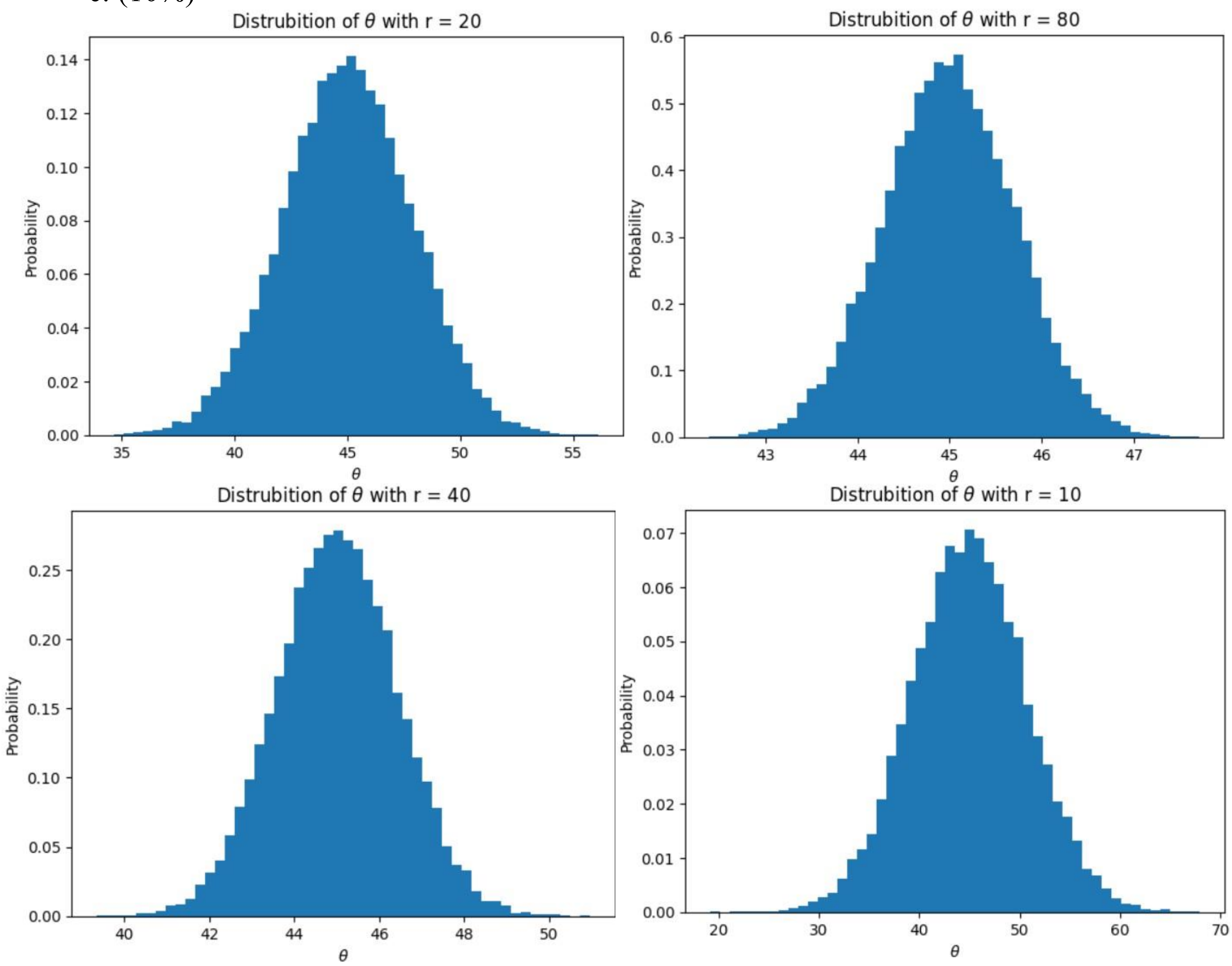

Figure 3-c histogram of θ with different r

d. (5%)

Consider two points A B in Figure 3-d, which vector length are r_1 and r_2, and the radius of the red cycle is equal to 1. Because standard deviation of random noise is one, the points with noise would distribute in the red cycle mostly. The blue line is vertical with $\overline{OA}$ and $\overline{OB}$. Assume σ_1 and σ_2 is $\angle R_1OA$ and $\angle R_2OB$, because red cycle is where first standard deviation, I consider $\angle R_1OA$ and $\angle R_2OB$ is standard deviation of θ distribution in r_1 and r_2.

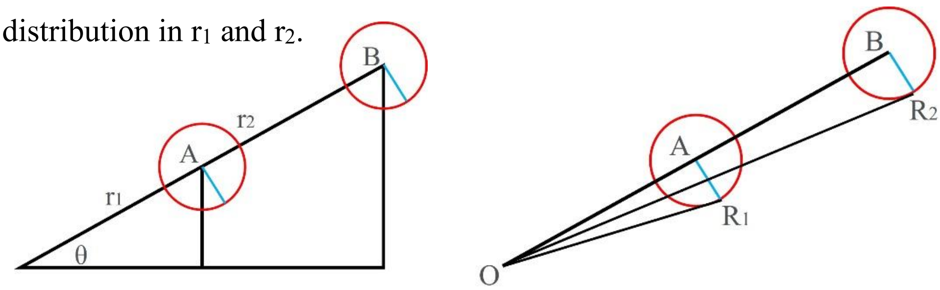

Figure 3-d relation of r and θ

Thus, $\sigma 1 : \sigma 2 = \angle R1OA : \angle R2OB = arctan(1/r1) : arctan(1/r2) \approx 1/r1 : 1/r2$ where $arctan(kx) \approx k * arctan(x)$ with kx is small.

σ_θ and r is relation of inverse ratio.

e. (10%)

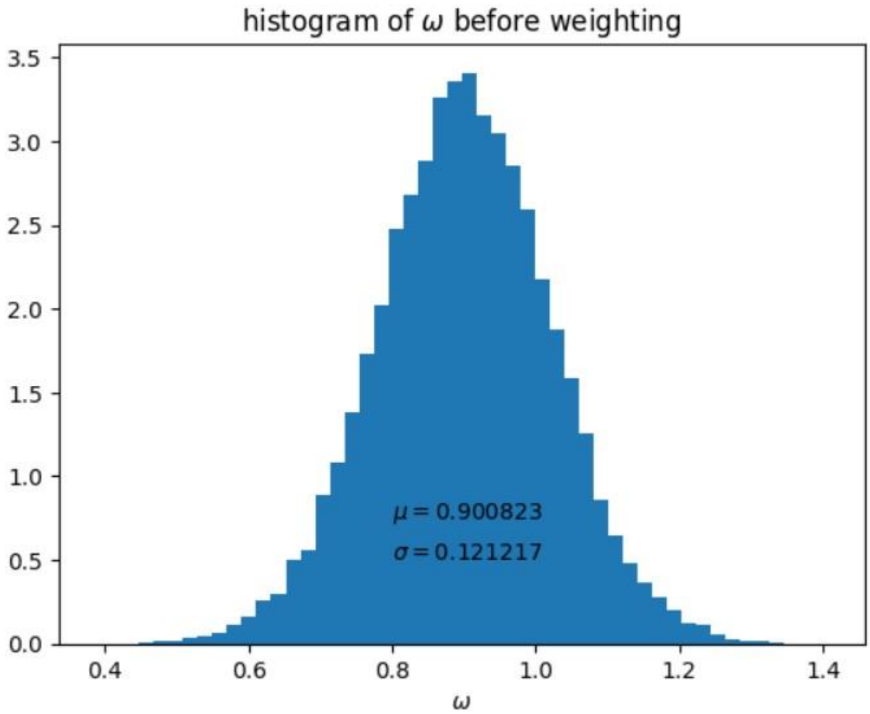

Figure 3-e histogram of ω (Slope of regression result)

f. (10%)

Observing the histograms of signal phase in that at each different t distribution.

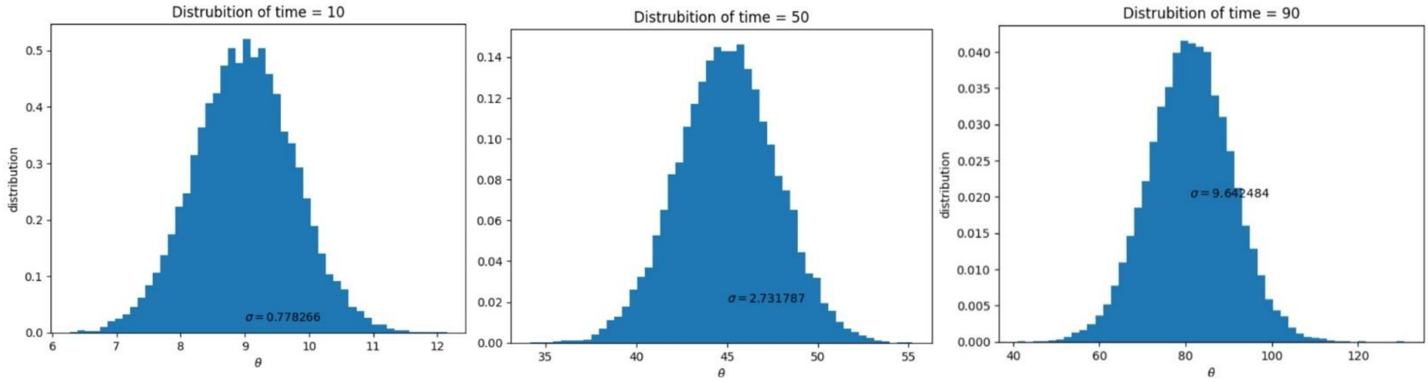

Figure 3-f-1 histogram of θ with different t before weighting

Although the distribution look like normal distribution, the value of peak is not same. With t grows, the peak value go down and σ go up, so I guess the relation of this two is inverse ratio. Thus, I try weighting t with σ calculated above, the result shows follow.

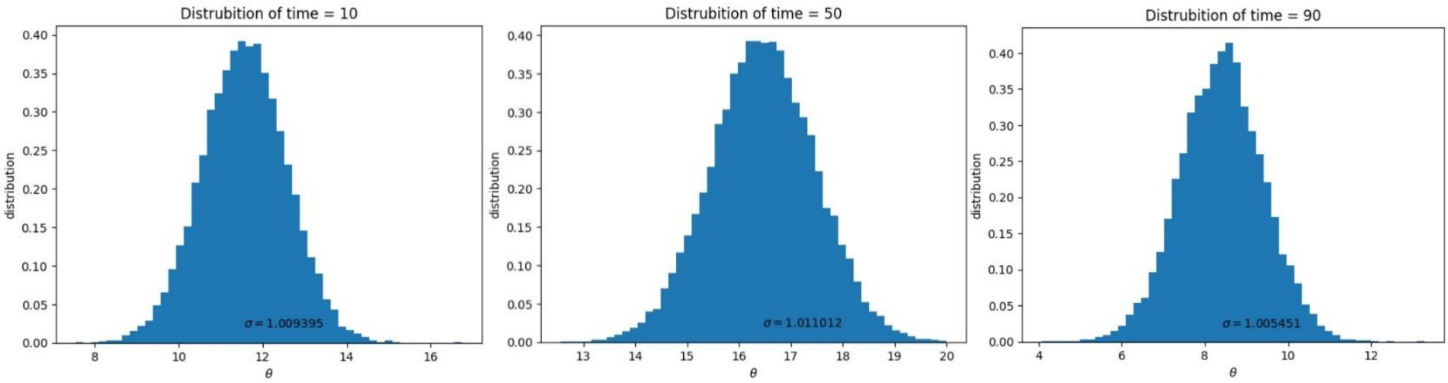

Figure 3-f-2 histogram of θ with different t after weighting

After weighting input with σ, the distribution of different t show the same result.

Finally redo regression of part e.

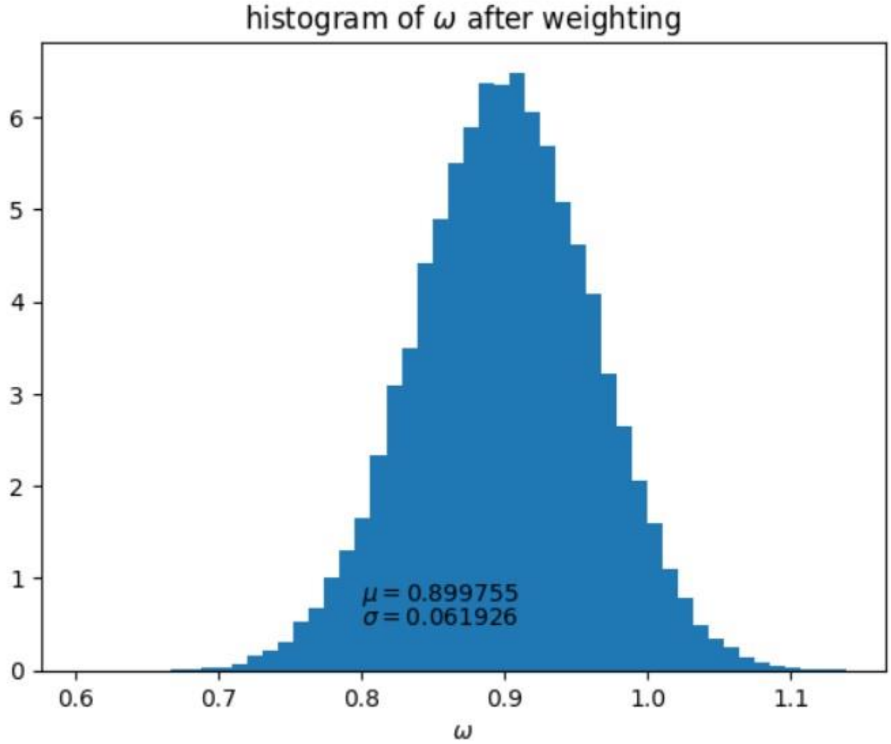

Figure 3-f histogram of ω (Slope of regression result) with weighting