Introduction to Numerical Analysis and Applications

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Homework #3 (**DUE**: May 9th)

This homework includes the materials from the past weeks, including *Fourier analysis and Interpolation*.

1. (20%) The two-point Gauss-Legendre formula has a truncation error $\sim f^{(4)}(\xi)$. Validate this statement by integrating the following polynomials over [0 0.8]: $f_1(x) = 0.2 + 25x - 200x^2 + 675x^3$ and $f_2(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4$. What are the true percent relative error ($\varepsilon_t = \frac{true\ value - approx.}{true\ value}$) in each case?

2. The next exercise is about the well-known *Einstein-Monroe illusion*. If you look closely into Figure 1, you can find a friendly Albert Einstein; however if you move further away from the figure, Marilyn Monroe shows up. In this exercise, you are requested to generate such illusion with any two faces: You could either find the images (*make it 384×384*) you would like to try, or use the images provided on the Cyber University.

Figure 1 The Einstein-Monroe Illusion.

   a. (10%) This illusion consists of two parts: the fine detail (Albert) and the coarse contour (Marilyn). The fine detail of an image comes from the combination of high spatial frequency components, and on the other hand, the low spatial frequency components determine the coarse contour (or the main image contrast). In a Figure, show an image consisted of its high spatial frequency components using image $I_{1,high}$, and an image of its low spatial frequency components using image $I_{2,low}$. As a starter, separate the whole spectrum (frequency distribution) into two segments to define the low- and high-frequency components. Meanwhile to make your figures less confusing, you could show the magnitude of your images, for the pixel values could be complex. Please also include a color bar for your images.

   b. (20%) The above images are less than optimal, and let's improve them with filters. A bell-shaped band-pass filter is commonly adopted for this purpose. Design a 2D Gaussian filter (Full-width-half-maximum = 64) to preserve the low-frequency components. Show your filter and the improved $I_{2,low}$ in a Figure.

c. (15%) On the other hand, using the knowledge you gain from (b), design a high-pass filter (HPF) that preserves the high-frequency components. Show such HPF and the improved $I_{1,high}$ in a Figure.

d. (30%) Basically if you add your results in part a, you can generate your own X-Y illusion. In your code you should have a knob to adjust the cut-off threshold for the low- and high-frequency components (for example, the FWHM in part b). Show your best results in a figure and the threshold value of your choice. How did you determine this threshold? You are encouraged to explore more options regarding the spectral filters. If so, please carefully include all details regarding your filters.

e. (5%) What's your observation in this exercise? How would you improve if it falls below your expectation?