Introduction to Numerical Analysis and Applications

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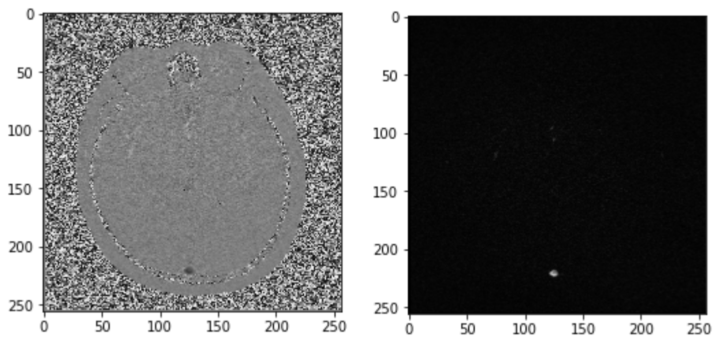
Homework Bonus (**DUE**: June 6^{th})

This homework includes the materials from the past weeks, including *Curve fitting, Interpolation, Numerical Integration,* and *Numerical Differentiation*.

1. (15%) The two-point Gauss-Legendre formula has a truncation error $\sim f^{(4)}(\xi)$. Validate this statement by integrating the following polynomials over [0 0.8]: $f_1(x) = 0.2 + 25x - 200x^2 + 675x^3$ and $f_2(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4$. What are the true percent relative error ($\varepsilon_t = \frac{true\ value - approx.}{true\ value}$) in each case?

2. Many of the integration techniques are already implemented, so let's give them a shot. You are very much encouraged to write your own codes; nonetheless the scipy.integrate sub-package could be found in the following link and used in this exercise.
   https://docs.scipy.org/doc/scipy/reference/tutorial/integrate.html
   As for Matlab users, *INTEGRAL* would be your friend; and Octave users, please look up *QUAD*, *TRAPZ*, *CUMTRAPZ, … etc.*

   A series of MRI phase-contrast images is packed into "pcMRI.zip", with dimension (x, y, t) of 256×256×51. The field-of-view of the images is 220mm×220mm. The pixel value represents the through-plane velocity in cm/s (value within ±40), while the frame rate of dataset is 50 frame/s. Another set of data is helpful to identify the region of the superior sagittal sinus (SSS, the vein in interest). The "magMRI.zip" gives the magnitude information of the same data, and the pixel value represents how likely this pixel contains flowing blood. The following two figures show the velocity map and the magnitude image, respectively. You should find the same images after you successfully load the data.



a. (15%) Perform spatial integration over the SSS region, and plot the average blood flow velocity with respect of time.

b. (15%) Perform the integration over the whole time interval (i.e. 1 s), and find out the average blood flow rate in ml/s. Use composite trapezoidal rule and composite Simpson's rules and how different are your answers?

c. (20%) With the result of part (b), when does the flow velocity reach to the maximum for each pixel? Show a spatial profile indicating the moment when maximal velocity occurs, and then find the mean value over such profile. On the other hand with the result of part (a), when does the maximal velocity occur? Which of the above two methods do you think is more reasonable? *Hint: a 3rd-order polynomial that fits the first half of the time series could indicate the time when it reaches to the maximal flow velocity.*

3. (15%) Derive the following centered finite-difference formula for 1st derivative with high-accuracy $O(h^4)$:

$$f'(x_i) = \frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h}$$

4. Use the following data to find the velocity and acceleration at $t = 10$ seconds.

t (s)	0	2	4	6	8	10	12	14	16
x (m)	0	0.7	1.8	3.4	5.1	6.3	7.3	8.0	8.4

Apply the second-order correct for the following methods.

a. (10%) Forward finite-difference method.

b. (10%) Centered finite-difference method.