Sep. 22 (Mon)

L^2 extension with adjoint ideal sheaves

Mario Chan (Pusan National University)

I introduce in this talk the adjoint ideal sheaves of a reduced snc divisor D on a complex manifold X, which are defined via the L^2 conditions with respect to some "residue functions" incorporating information of the lc centres of the given lc pair (X, D), and can be regarded as a refinement of multiplier ideal sheaves. These ideal sheaves, together with the residue short exact sequences that they fit in, facilitate a scheme-theoretic description of the lc centres and also a function-theoretic means to relate the harmonic forms on the lc centres of different dimensions when X is compact Kähler. With these properties, I present a proof of a qualitative extension theorem with adjoint ideal sheaves which guarantees existence of extensions from lc centres of any dimension. The proof follows the strategy in the solution to Fujino's Conjecture (an injectivity theorem for lc pairs on a compact Kähler manifold) in my joint work with Young-Jun Choi and Shin-ichi Matsumura. If time allows, I will present a possible approach toward a quantitative L^2 extension theorem using the residue functions.

Continuity of singular Kähler-Einstein potential

Young-Jun Choi (Pusan National University)

In 2009, Eyssidieux-Guedj-Zeriahi proved that any compact normal Kähler variety with trivial or ample canonical line bundle admits a singular Kähler-Einstein metric, generalizing the Aubin and Yau's solution for the Calabi conejcture. The potential of such a singular Kähler-Einstein metric is smooth on the regular locus of the variety but only little is known about its behavior near the singular locus.

In this talk, we will discuss the continuity of singular Kähler-Einstein potential on the entire variety. This is joint work with Ye-Won Luke Cho.

L^2 extension of adjoint bundles

Chen Zhao (University of Science and Technology of China)

In this talk, I will present the L^2-extension construction for adjoint bundles associated with Hermitian vector bundles. By choosing appropriate Hermitian bundles, this construction specializes to several fundamental objects: the dualizing sheaf of a complex manifold, the Grauert–Riemenschneider sheaf of a reduced complex space, and higher direct images of adjoint bundles in fibered spaces. Furthermore, it generalizes

Saito' S-sheaf associated with Hodge modules, as well as its various extensions—such as the S-sheaf twisted by a multiplier ideal sheaf.

I will also discuss several key properties of the L^2-extension sheaf, including Kollar's package—torsion-freeness, vanishing theorems, injectivity theorems, and decomposition theorems—as well as the minimal extension property of its direct image sheaf. This presentation includes joint work with Junchao Shentu.

Log $\partial\bar{\partial}$-lemma and applications

Runze Zhang (Université Côte d'Azur)

The $\partial\bar{\partial}$-lemma is a powerful tool in complex and algebraic geometry. In the first part of my talk, I will establish a logarithmic $\partial\bar{\partial}$-type lemma on compact Kähler manifolds for logarithmic differential forms with values in the dual of a pseudo-effective line bundle, confirming a conjecture of X. Wan. In the second part, I will present an application to the work of L. Katzarkov–M. Kontsevich–T. Pantev on unobstructed locally trivial deformations of generalized log Calabi–Yau pairs with weights, extending their projective result to the broader Kähler setting. This talk is based on arXiv:2412.09447.

The spectrum of the Folland-Stein operator on some Heisenberg Bieberbach manifolds

Yoshiaki Suzuki (Niigata University)

In 2004, Folland studied the compact quotients of the Heisenberg group by its lattice, known as Heisenberg nilmanifolds, in the context of CR geometry. He determined the eigenvalues and eigenfunctions of the Folland-Stein operator (gives the Kohn Laplacian essentially) on Heisenberg nilmanifolds. In this talk, with the aim of extending Folland's work, we study Heisenberg Bieberbach manifolds, which are given by the quotients of Heisenberg nilmanifolds by finite groups. Using Folland's method, we calculate the eigenvalues and the dimensions of the eigenspaces of the Folland-Stein operator on some examples of Heisenberg Bieberbach manifolds.

Sep. 23 (Tue)

Some recent results on the converse L^2 theory and their applications

Jiafu Ning (Central South University)

In this talk, we will introduce some recent results on the converse L^2 theory and their applications in the study of curvature positivity of holomorphic vector bundles with (singular) hermitian metrics. Parts of this talk are based on joint works with Prof. Fusheng Deng, Zhiwei Wang and Xiangyu Zhou.

Recent progress on the SOS conjecture

Zhiwei Wang (Beijing Normal University)

In this talk, we will introduce our recent progress on the study of the SOS conjecture (proposed by Ebenfelt), which is closely related to the Huang-Ji-Yin gap conjecture in the study of rational proper maps between the complex unit balls. This is based on joint work with Chenlong Yue and Professor Xiangyu Zhou.

L^2 extension of holomorphic functions for log canonical pairs

Dano Kim (Seoul National University)

I will discuss recent developments on L^2 extension theorems for log canonical pairs.

Ohsawa measures on singular hypersurfaces and its applications

Hoseob Seo (Seoul National University)

In L^2 extension theorems from an irreducible singular hypersurface in a complex manifold, important roles are played by certain measures such as the Ohsawa measure, which determines when a given function can be extended. In this talk, we show that the singularity of the Ohsawa measure can be identified in terms of algebraic geometry. Using this, we give an analytic proof of the inversion of adjunction in this setting. These considerations enable us to compare various positive and negative results on L^2

extension from singular hypersurfaces. In particular, we generalize a negative result of Guan and Li which places limitations on strengthening such L^2 extension by employing a less singular measure in the place of the Ohsawa measure. This is joint work with Dano Kim.

TBA

Shuho Kanda (the University of Tokyo)

TBA

Sep. 24 (Wed)

On Bott–Chern and Aeppli cohomologies of 2-dimensional toroidal groups

Jinichiro Tanaka (Osaka Metropolitan University)

A toroidal group X is a connected complex abelian Lie group without non-constant holomorphic functions, and can be regarded as a generalization of a complex torus.

The cohomology with compact support of toroidal groups has also been studied using L^2 theory.

If all the Dolbeault cohomologies $H^{p,q}(X, \mathcal{O}_X)$ are finite-dimensional, it is called a theta toroidal group.

Research by Umeno and Kazama–Takayama has revealed that theta toroidal groups behave similarly to complex tori as compact complex manifolds.

We then focus on Bott–Chern and Aeppli cohomology, which play important roles in the study of (non-Kähler) compact complex manifolds.

In this talk, we will discuss the Bott–Chern and the Aeppli cohomology of non-compact 2-dimensional theta toroidal groups.

Extension of Bergman metrics and diffusion processes for Riemann domains over $\mathbb{C}^n$

Shun Sugiyama (National Institute of Technology, Kitakyushu College)

We prove that if $n \geq 2$, a locally univalent Riemann domain (D, π) over $\mathbb{C}^n$,

bounded with respect to the canonical volume form, with $\pi(D)$ relatively compact in $\mathbb{C}^n$, whose accessible boundary satisfies the Newtonian capacity condition, and whose Bergman diffusion process is conservative, is Stein.

TBA

Sheng Rao (Wuhan University)

TBA

Multiplier submodule sheaf of singular metric on vector bundle

Hui Yang (Peking University)

We will discuss some positivity concepts of singular metric on vector bundle and its multiplier submodule sheaf and show the sheaf satisfies the strong openness property and stability for singular Nakano semipositive metrics.We will give an L^2 extension result for singular Nakano semipositive metrics.We also extend the Le Potier isomorphism theorem to isomorphism of cohomologies twisted with multiplier subdmodule sheaves for strong Nakano semipositive metrics. These works are joint with Yaxiong Liu, Zhuo Liu, Bo Xiao and Xiangyu Zhou.

L^2 estimates, L^2 extensions from non-reduced varieties and the division problem

Zhi Li (Beijing University of Posts and Telecommunications)

This talk will present recent progress in the theory of L^2 estimates. We will discuss the applications in L^2 extensions from non-reduced varieties and a converse to Skoda's L^2 division theorem. We will also discuss $\bar{\partial}$-estimates on singular complex spaces. This is joint work with Prof. Xiangyu Zhou, Zhenqian Li, Xiankui Meng, Jiafu Ning, and Wang Xu.

Generalized Levi problems

Takeo Ohsawa (Nagoya University)

In the theory of several complex variables, it is well known that a complex manifold M is holomorphically convex if there exists a locally biholomorphic map $\pi : M \to \mathbb{C}^n$ which is locally pseudoconvex in the sense that every point of $\mathbb{C}^n$ has a neighborhood whose preimage by π is holomorphically convex, or Stein equivalently in this situation. This basic fact is an immediate consequence Oka's solution of the Levi problem for Riemann domains over $\mathbb{C}^n$, which established that every connected component of the structure sheaf $\mathcal{O}_{\mathbb{C}^n}$ of $\mathbb{C}^n$ is holomorphically convex.

On the other hand, by a counterexample due to Fornaess, it is known that there exists a holomorphically nonconvex complex surface X with a locally pseudoconvex holomorphic map $p : X \to \mathbb{C}^2$ whose fibers are 0-dimensional. Roughly speaking, X is

constructed from a domain $\Omega_\varphi = \{(z,w) \in \mathbb{C}^2\,;\ |z|<1 \text{ and } e^{\varphi(z)} < |w|\}$, where φ is a subharmonic function on the disc $\mathbb{D} = \{z \in \mathbb{C}; |z|<1\}$ defined by

$$\varphi(z) = \sum_{\mu=1}^{\infty} \frac{1}{m(\mu)} \log \left| z - \frac{1}{n(\mu)} \right|$$

with $m, n : \mathbb{N} \hookrightarrow \mathbb{N}$, in such a way that $\sup \varphi(z) < 1$.

More precisely, $\mathbb{C}^2$ is blown up at the points $\left(\frac{1}{m(\mu)}, 0\right)$ by the maps

$$(u,v) \mapsto \left(uv^{m(\mu)} + \frac{1}{n(\mu)}, v\right),$$

so that one can find a neighborhood U of the intersection of the exceptional set with the proper transform of the complex lines

$$\left\{\left(\frac{1}{m(\mu)}, w\right); w \in \mathbb{C}, \mu = 1,2,\dots\right\}$$

such that U can be patched with $\Omega_\varphi \backslash V$ for some neighborhood V of $\left\{\left(\frac{1}{m(\mu)}, 0\right); \mu = 1,2,\dots\right\}$ to define a complex surface X with a locally pseudoconvex holomorphic map $p : X \to \mathbb{C}^2$ in such a way that $p^{-1}(z)$ are finite for all $z \in \mathbb{C}$ and X contains complex curves which are mapped biholomorphically onto

$$L_\mu := \left\{\left(\frac{1}{m(\mu)}, w\right); w \in \mathbb{C}\right\} \quad (\mu = 1,2,\dots)$$

by p.

Holomorphic nonconvexity of X is an immediate consequence of the maximum modulus principle applied to the restrictions of holomorphic functions on X to $p^{-1}(L_\mu)$.

This example suggests, as well as counterexamples to the Serre problem on the Steinness of analytic fiber bundles with Stein fibers and bases, that there remains something to be explored on those non-Stein manifolds.

From such an interest, it might be still worthwhile to see whether or not the above mentioned patching procedure does not destroy the separatedness of the manifolds by holomorphic functions. This point is closely related to the following question which was raised by P. A. Griffiths in 1977.

Question. Let S be a locally closed complex submanifold of $\mathbb{C}^n$. Is S holomorphically convex if the inclusion map $S \hookrightarrow \mathbb{C}^n$ is locally pseudoconvex ?

The main purpose of the talk is to show that Fornaess's example can be modified to yield a negative answer to Griffiths's question.

More explicitly, we shall prove the following.

THEOREM 0.1. *Let $\varphi(z) = \sum_{\mu=1}^{\infty} 2^{-\mu} \log|z - 2^{-\mu}|$ and let $\Omega'_\varphi = \{(z,w) \in \mathbb{D} \times \mathbb{C}; e^{\varphi(z)} < |w| < e\}$. Then, Ω'_φ is biholomorphically equivalent to a dense open subset of a holomorphically nonconvex locally closed submanifold S of $\mathbb{C}^3$ such that the inclusion map $\Omega'_\varphi \hookrightarrow \mathbb{C}^2$ is continuously extended to S by this correspondence as a locally pseudoconvex map $q : S \to \mathbb{C}^2$ satisfying $q^{-1}((2^{-\mu}, 0)) \cong \mathbb{D}$ for all μ.*

Corollary 0.2. *There exists a locally pseudoconvex but holomorphically nonconvex Riemann domain over* $\mathbb{C}^2$ *which is embeddable into* $\mathbb{C}^3$ *as a locally closed complex submanifold.*

Sep. 25 (Thu)

Mean curvature of direct image bundles

Kuang-Ru Wu (National Tsing Hua University)

Let $E \to X$ be a vector bundle of rank r over a compact complex manifold X of dimension n. It is known that if the line bundle $O_{P(E^*)}(1)$ over the projectivized bundle $P(E^*)$ is positive, then $E \otimes \det E$ is Nakano positive by the work of Berndtsson. In this talk, we give a subharmonic analogue. Let $p : P(E^*) \to X$ be the projection and α be a Kähler form on X. If the line bundle $O_{P(E^*)}(1)$ admits a metric h with curvature Θ positive on every fiber and $\Theta^r \wedge p^*\alpha^{n-1} > 0$, then $E \otimes \det E$ carries a Hermitian metric whose mean curvature is positive.

As an application, we show that the following subharmonic analogue of the Griffiths conjecture is true: if the line bundle $O_{P(E^*)}(1)$ admits a metric h with curvature Θ positive on every fiber and $\Theta^r \wedge p^*\alpha^{n-1} > 0$, then E carries a Hermitian metric with positive mean curvature.

Generalized Suita conjecture and minimal L^2 integrals

Zheng Yuan (Academy of Mathematics and Systems Science, Chinese Academy of Sciences, CAS)

In 1972, Suita posted a conjecture on the relation between the Bergman kernels and the logarithmic capacity on open Riemann surfaces. Following the establishment of the optimal L^2 extension theorems, the inequality part of the conjecture was proved by Blocki (for bounded planar domains) and Guan-Zhou (for open Riemann surfaces). By adjusting the gain functions in the optimal L^2 extension theorem, Guan-Zhou proved the equality part of the conjecture, thus completely solving the Suita Conjecture. In this talk, we recall some results on minimal L^2 integrals related to multiplier ideal sheaves, which were used as the main tools in the study of generalized Suita conjectures, and then present some solutions to generalized Suita conjectures. This talk is based on joint works with Qi'an Guan, Zhitong Mi and Xun Sun.

Microscopic stability thresholds and constant scalar curvature Kähler metrics

Takahiro Aoi (Wakayama National College of Technology)

Fujita-Odaka proved that a uniformly Gibbs stable Fano manifold is uniformly K-stable (so, there exists a Kähler-Einstein metric). After their work, Berman gave a direct and analytic proof that uniformly Gibbs stable Fano manifold has a Kähler-Einstein metric. I will talk about a generalization of Berman's result: if the microscopic stability threshold for a polarized manifold satisfies some inequality, then there exists a constant scalar curvature Kähler (cone) metric. This is an analogue of Kewei Zhang's application of the delta-invariant to constant scalar curvature Kähler metrics. Our proof uses Berman's idea which applies the positivity of the direct image sheaf by Berndtsson.

Sep. 26 (Fri)

TBA

Shota Kikuchi (National Institute of Technology, Suzuka College)

TBA

On the sharp L^2-estimates of Skoda division theorem

Masakazu Takakura (Tokyo Metropolitan University)

The classical division problem asks: given holomorphic functions $f, g_1, \ldots, g_r$ on a complex manifold X, when does f belong to the ideal generated by $g_1, \ldots, g_r$? In this talk, we consider a generalized division problem: given plurisubharmonic functions $\varphi_1, \ldots, \varphi_r$, under what conditions does f belong to the sum of the multiplier ideal sheaves $\sum_{i=1}^{r} \mathcal{I}(\varphi_i)$? We establish an existence theorem for solutions to this generalized division problem with sharp L^2 estimates. This result refines and extends the classical Skoda-type division theorem. Furthermore, I will present applications to the L^2 extension theorem and to the openness conjecture for multiplier ideal sheaves. The talk is based on `arXiv:2505.05938`.