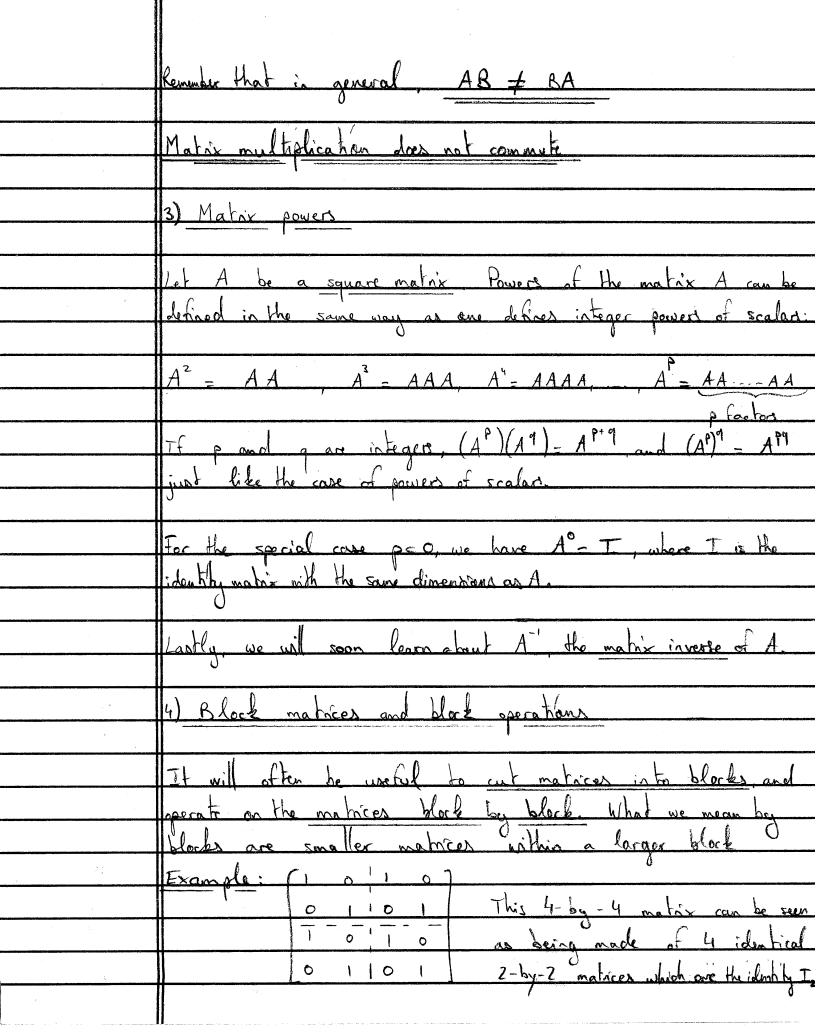
	MATH-UA 140 - Linear Algebra
	C
	Lecture 6: Inverse matrices
	I Review of matrix operations, and a useful trick
	1) <u>Revieu</u>
	In the past lectures, we have covered natrix addition: Two matrices with the same dimensions can be added, and
	Two matrices with the same dimensions can be added, and
	the sum C of the matrices A and B, C = A+B, has
	entries (; = a; +b;
	11/1 \ (1) \
	We have also seen mulhphration by a scalar c: the
	We have also seen multiplication by a scalar c: the entries of the matrix cA are caj. In other words, all the entries are multiplied by the scalar
·	all the entres are much puebl by the scalar
	lastly we also leased how to with all a whire with
	Lastly, we also learned how to multiply a matrix with another matrix. The rules were as follows:
·	If the matrix A has m cours and n columns, then
	If the matrix A has m cause and n columns, then the quantity AB only makes sense if the matrix B has
	n raws.
	If B has p columns, then the matrix C = AB has m
	If B has p columns, then the matrix C = AB has m cows and p columns. Schematically, we may write
,	

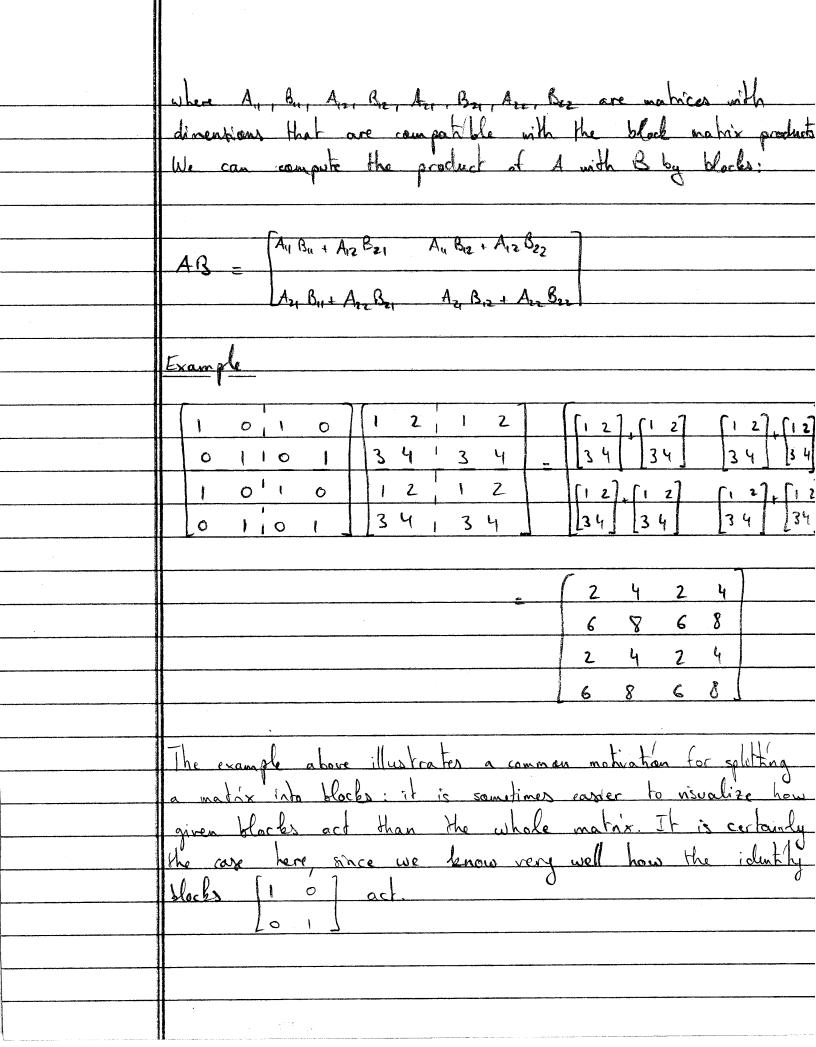
(m-by-n)(n-by-p) => (m-by-p) To multiply two matrices A and B, we take the det product of each row of A with each column of R: The entry c; of C-AR is (row i of A). (column j of B) 2) Properties of matrix operations In the following, we assume that the matrices

A. B. and Chave dimensions such that the

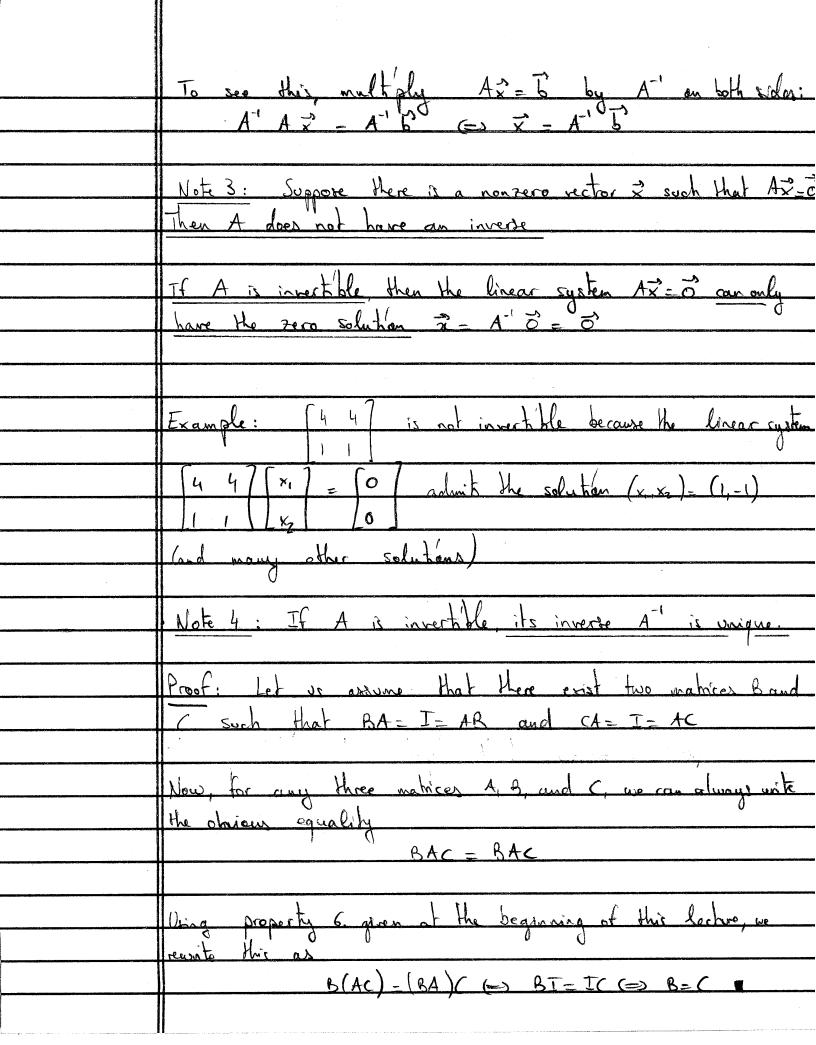
operations shown belown are allowed. c is a scalar. The following properties hold: 1. A+B - B+A (matrix addition commutes) 2. c(A+B) = cA+cB = (A+B)e (distributive law) 3. A+(B+C) - (A+B)+ ( (associative law) 4. C(A+B) = CA + CB (distributive law from the left) 5. (A+B)C = AC+BC (distributive law from the right) 6. A(RC) - (AB)C 7. AT - TA = A where T is the identity matrix.



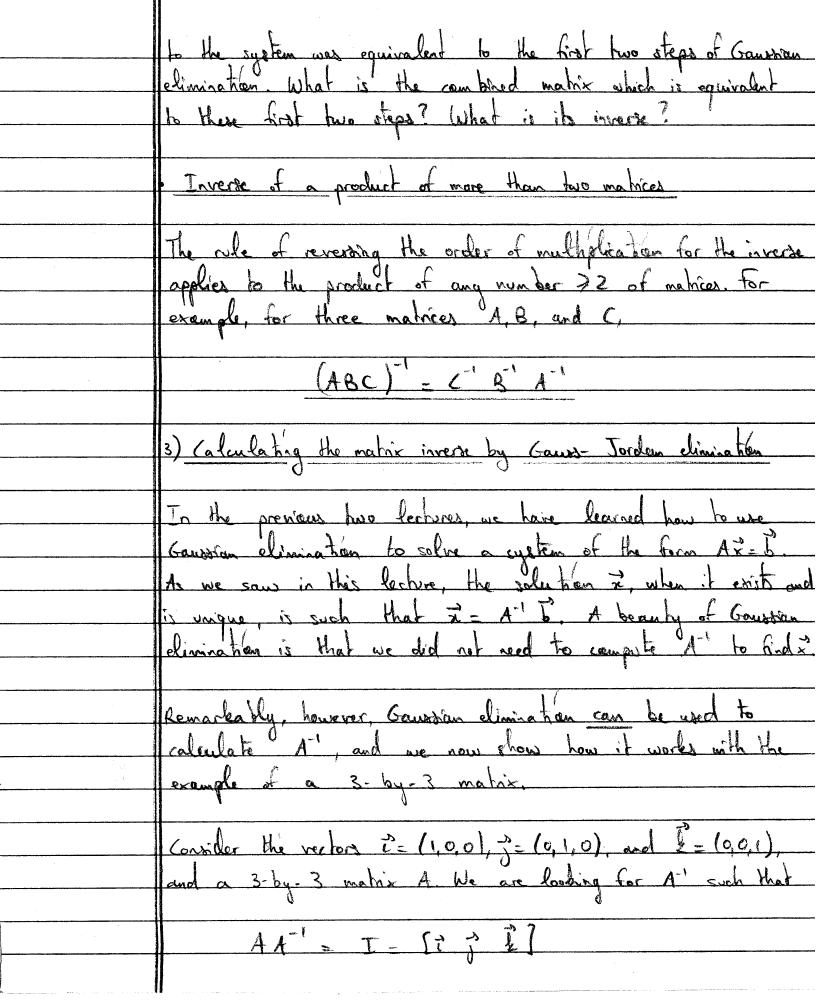
* Blocks and matrix addition	
The state of the s	
Since matrix addition is defined by the addition	
of the entries with the same row and column numbers	
it is clear that matrix addition can be calcu-	
lated a block at a time.	
Frangle: 21,34 10,01	
-1-122,01,11	
TT V211e 2-2,37	
0 1   -3 -4   -5 41 2 1 ]	
_ (3 1 3 5	
-1 0 , 3 3	
2+1T V2-21 4 e+7	
-5 5 1 -1 -3	
* Blocks and makix multiplication	
	and ration the structure research and the control of the structure of the control
If we cut two matrices A and B and if the	
cut between columns of A match the cuts between	
rows of B then AB can be evaluated by	
block multiplication.	MATERIAL CONTROL CONTR
Consider the black motorces	
$A = A_{11}$ Az and $B = B_{12}$	
Azy Azz By Bzz	
li di	

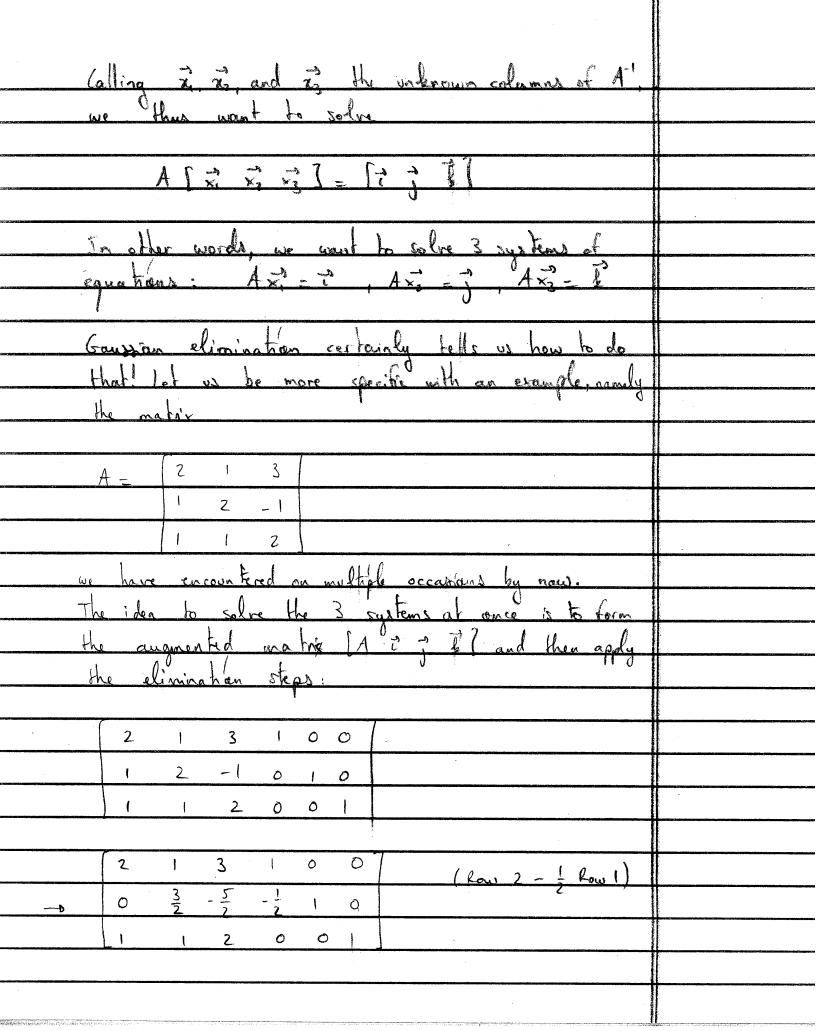


## I Inverse matrices For any scalar c = 0, c' is called its inverte It is such that cc' = c'c = 1. The purpose of this section is to learn about the idea of invertes for matrices. As we will see the silvation is slightly more complex for matrices than it is for scalar, and therefore more interesting! 1) Definition and immediate proporties A square matrix A is invertible if there exists a matrix A', called "A inverse", such that $A^{-1}A = T$ and $AA^{-1} = T$ · Note 1: Not all natrices have invested In this course, we will leave several methods to find out if a matrix is investible or not. Many methods will not even require calculating 1-1 Note 2: An n-by-n native has an inverse if and only if elimination produces a pivot (i.e. nonzero prot) (Row exchanges are allowed as necessary) If A is invertible the one and only solution is to the linear system $A\vec{x} - \vec{b}$ is $\vec{z} = A^{-1}\vec{b}$

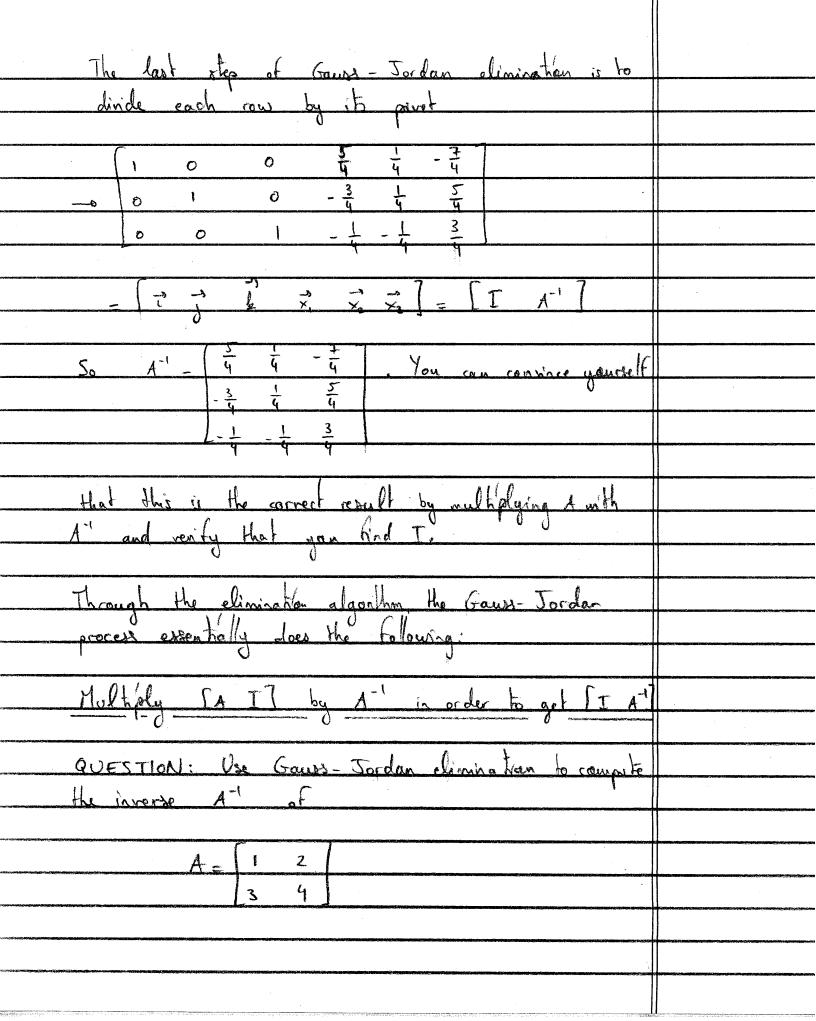


QUESTION: Consider the plining from matrix By interpreting E in terms of operations on rows of a system of equations, find E' Verify that you obtained the correct answer by direct computation of E' E. 2) Inverse of the product of matrices Theorem: If A and B are two square matrices with the same dimensions which are investible then AR is also invertible. Furthermore, the inverse of AB is (AB)" - B" A"  $\frac{\text{Proof:}}{\text{Proof:}} \left( AR \right) R^{-1} A^{-1} = A \left( BR^{-1} \right) A^{-1} = AA^{-1} = T$ Hence B' A' is indeed the inverse of AB 





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Ζ 2 2
(2 1 3 1 0 0
$-0$ 0 $\frac{3}{2}$ $-\frac{5}{2}$ $-\frac{1}{2}$ 10 (Row 3 - 1 Row 2)
0 0 4 1 1 1
3 3 3
At that point, we are done: back substitution wrong the
4th column would give us x, back substitution using the
5th column would give x2 and back substitution when the
6th column would give zo This is Gauss' idea to get A-1
However, Jordan came up with the idea of continuing with
elimination so as to make A' appear in the augmented matrix.
The idea is to use elimination to turn the nonteros entires
above the diagonal in the list three columns into zero entries
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$-\frac{1}{8} = \frac{3}{8} = \frac{3}{8} = \frac{15}{8} = $
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Lo 0 \frac{3}{3} - \frac{1}{3}



The Gauss-Jordan process always works if A has a pivots it A has a pivots it is invertible. One can ask eneself the reverse question: is it true that if A does not have a pivots, then A is not invertible? The answer is yes, and the Gauss-Jordon process gives a hint for why this is true, because when a pivot is zero, we cannot proceed with elimination, and counset divide the rows by the fivots. We will soon provide a crisper, more rigorous proof. For the moment, it is sufficient to remember the important statement: A square nation A is invertible if and only if it has a pieces