

The two eigenvectors are linearly independent, and we can $\begin{bmatrix} -\frac{4}{3} & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -\frac{4}{3} & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 5 \end{bmatrix}$ 2) Important remarks the order in 1 as well. tragine you have a factorization 5' AS = 1, with 1 a diagonal matrix. Then 5 must be a matrix of eigenvectors.

Indeed, the equality implies AS = 51

The ith column of AS is Axi, and the ith column of SA is 1, xi, where 1; is the ith diagonal entry of 1

That means that xi is an eigenvector of A will eigenvalue to: Imagine A is an arm matrix with a distract eigenvalues

\[
\frac{1}{2} \text{ ... In the eigenvectors } \frac{2}{2} \text{ ... } \frac{2}{2} \text{ corresponding to the distract eigenvalues are linearly independent.} Here is the proof first for a 2x2 matrix and 2 eigenvectors if and is.

Let us imagine there are scalars q and cz such that cixi+czxicu

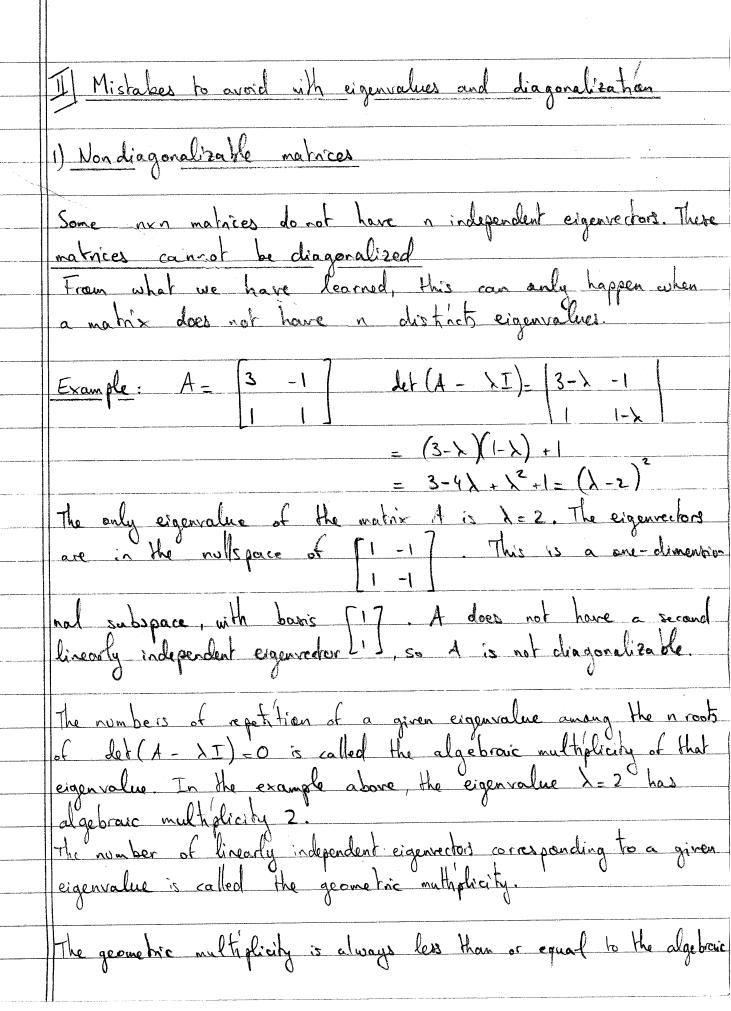
Subtracting the two equalities, we get: (1-12) c, x, =0

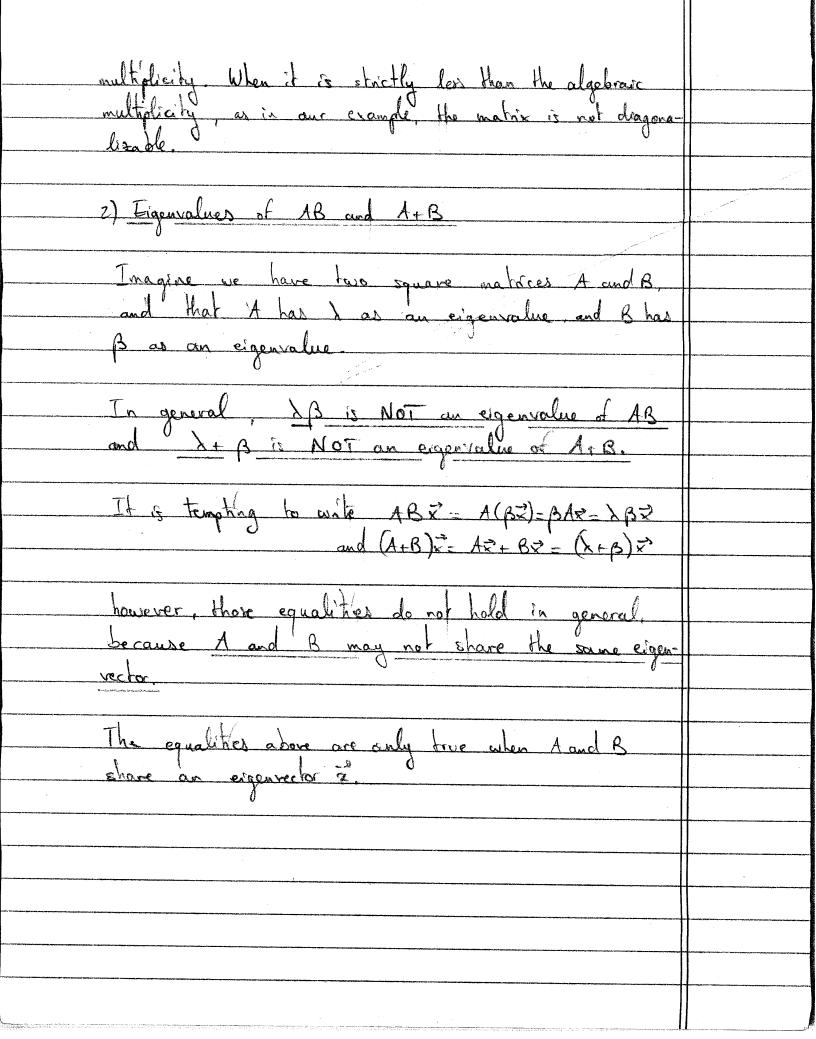
Since he and he are district c = 0, and then c=0

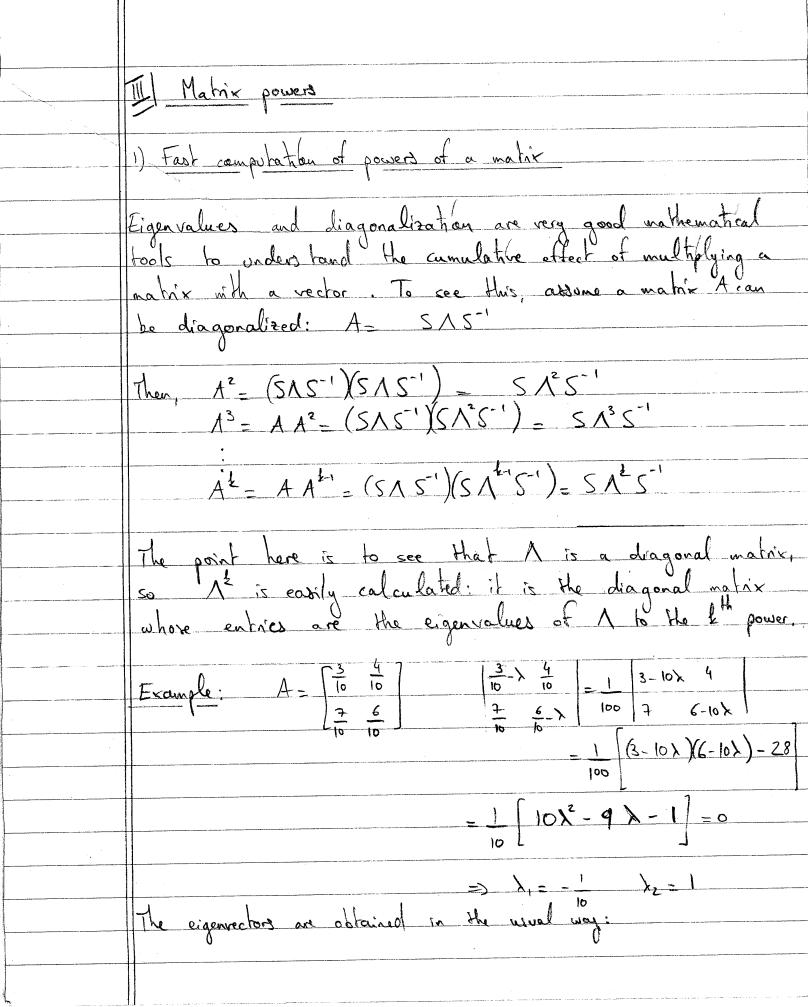
So 3, and 3, are linearly independent The proof generalizes to a eigenvector's early
Suppose CIX + GIX + -- + CIX = 0 Then applying A chilicality by has a large of the first equality by has a chilical so that a chilical so the first equality by has a chilical so that a chilical so the chilical so that a chilical so that We repeat the steps: multiply this equality Aby Annual subtract: a(1,-1, 14-1,)x,+a(1-1, 1/2-1, 1/ and we can continue the process until we find:

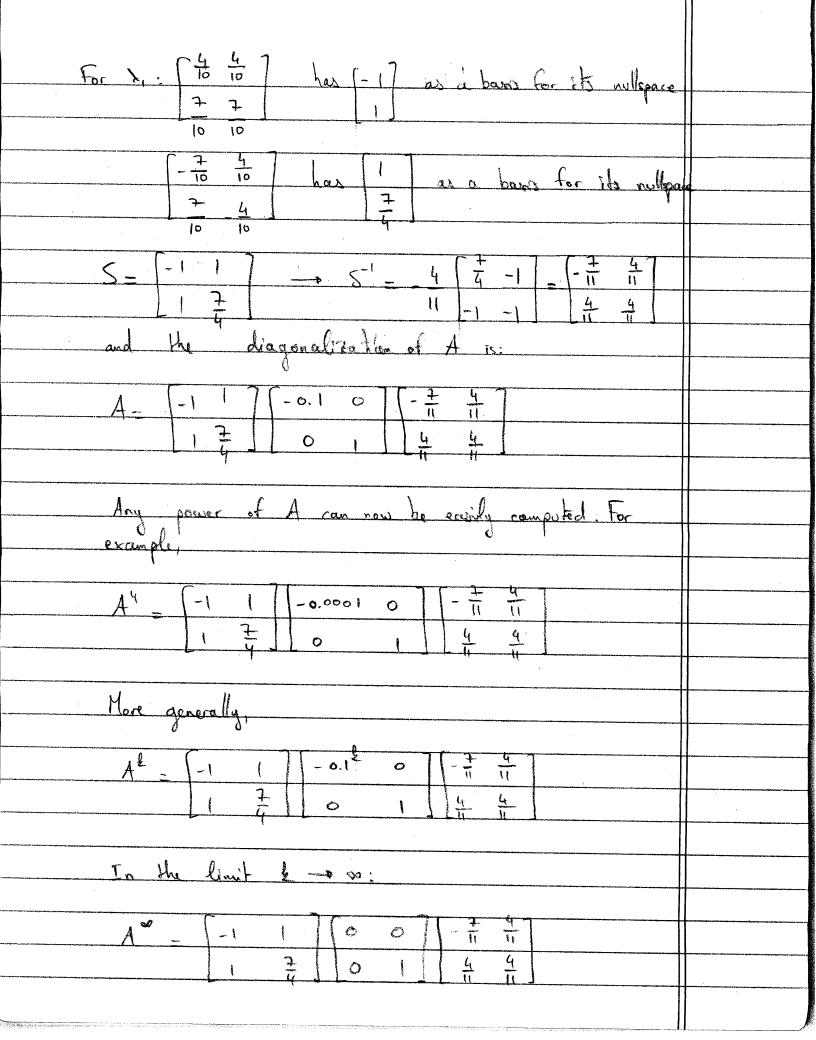
(1-2) (1-2) (1-2) (1-2) (1-2) (1-2)

and all the c: must then be zero going back one
equality at a time. Bottom line. Any matrix with a district eigenvalues









In this way, we also see that if A is diagonalizable,

At -+ O if all its eigenvalues & are such that 12/<1 2) Fast computation of sequences of vectors If A is diagonalizable, then it = SAS' is. This is already quite nice. However, there is an even more in withhe way to visualize and compute this expression. First, wite no in the eigenvector books: 10 = 4 x + 4 x x + - + Cn x n

This is is = 5 = 5 till Multiply each ci by his

This corresponds to $\Lambda \vec{c} = \Lambda S' \vec{u}_0$ Construct the linear countries then of the \vec{x}_i with λ_i^i ci as the multiplying Scalars
This is $S \wedge \vec{c} = S \wedge S / \vec{u}_0 = \vec{u}_0$ We conclude that $\vec{u_e} = A \vec{u_{e_1}} = A^{\frac{1}{2}} \vec{u_e} = c_1 + \frac{1}{2} + c_2 + \frac{1}{2} + \cdots + c_n + \frac{1}{2} = c_n + \frac{1}{2} + \cdots + c_n + \frac{1}{2} + \cdots + c_n + \frac{1}{2} = c_n + \frac{1}{2} + \cdots + c_n + \frac{1}{2} = c_n + \frac{1}{2} + \cdots + c_n + \frac{1}{2} = c_n + \frac{1}{2} + \cdots + c_n + \frac{1}{2} + \cdots + c_n + \frac{1}{2} = c_n + \frac{1}{2} + \cdots + c_n + \frac$ Example Let $\vec{n}_0 = (1,0)$. Compute \vec{A} \vec{n}_0 for $\vec{A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

which has $k=-1$ and $k=3$ as eigenvalues, and $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ as eigenvectors.	
We have $\vec{u} = \begin{bmatrix} 1 & -1 & -1 & 1 & 1 \\ 0 & -2 & 1 & 2 & 1 \end{bmatrix}$	
$\vec{u}_{2} = \frac{1}{2} (-1)^{2} \begin{bmatrix} -1 \\ -1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 3^{2} \\ 1 \end{bmatrix}$	