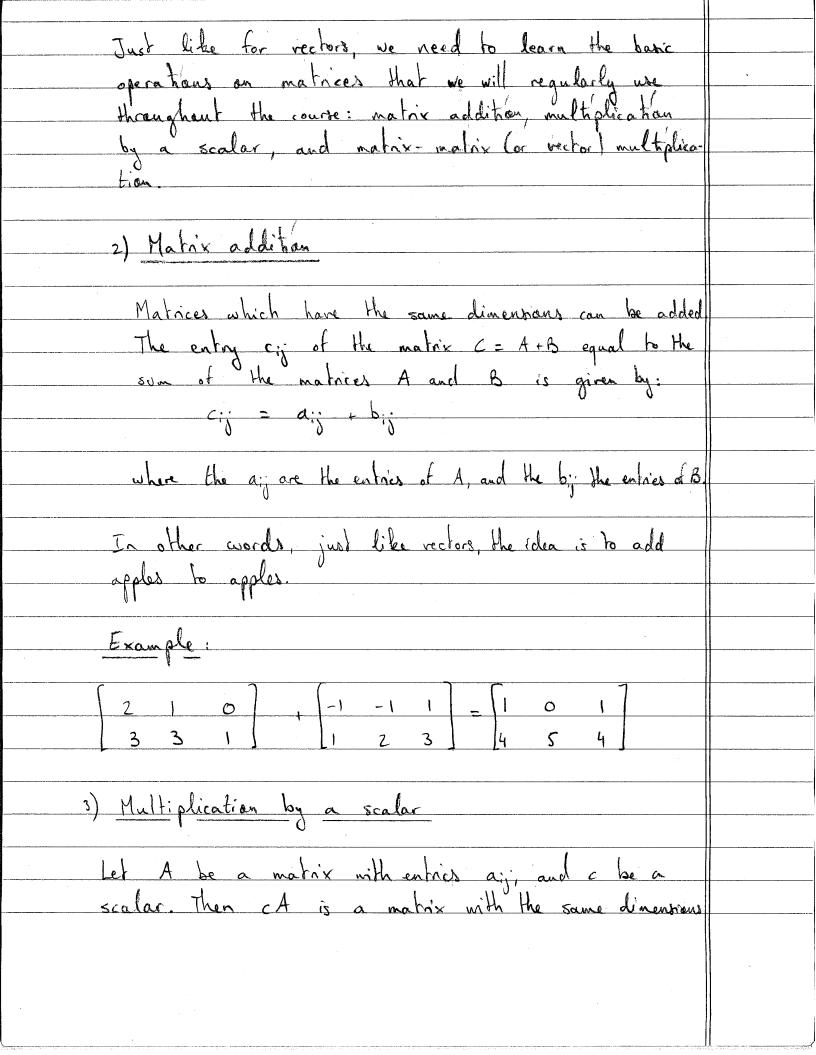
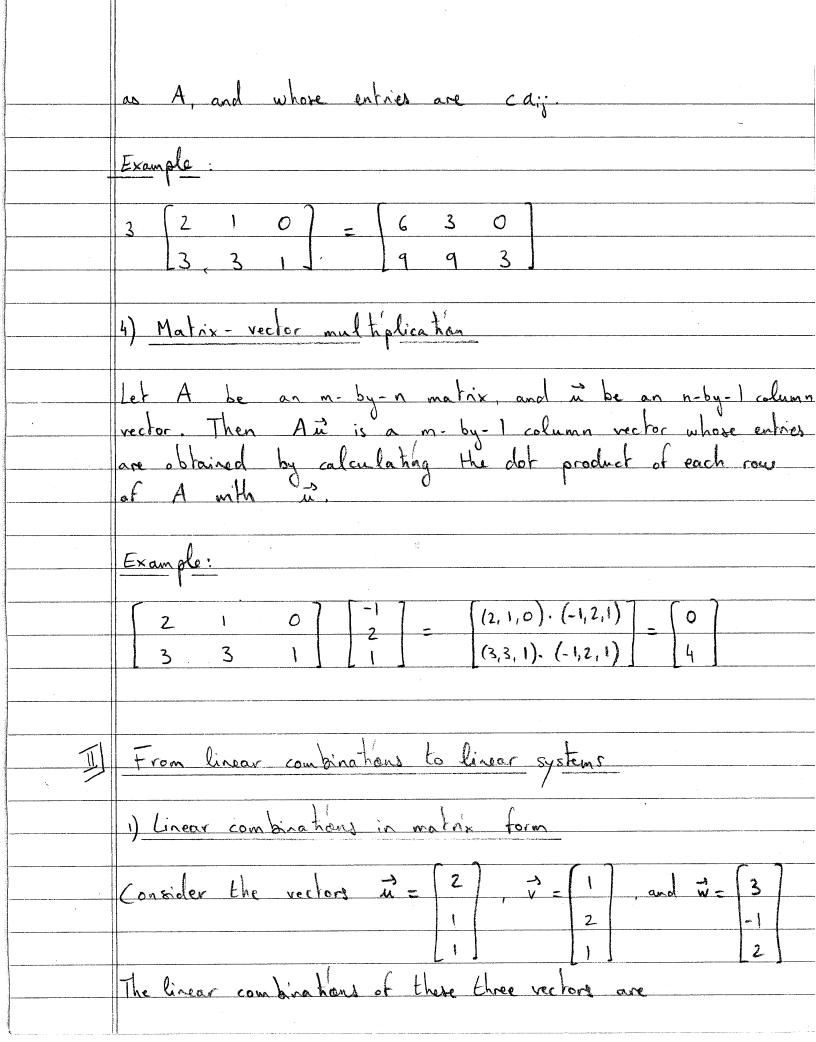
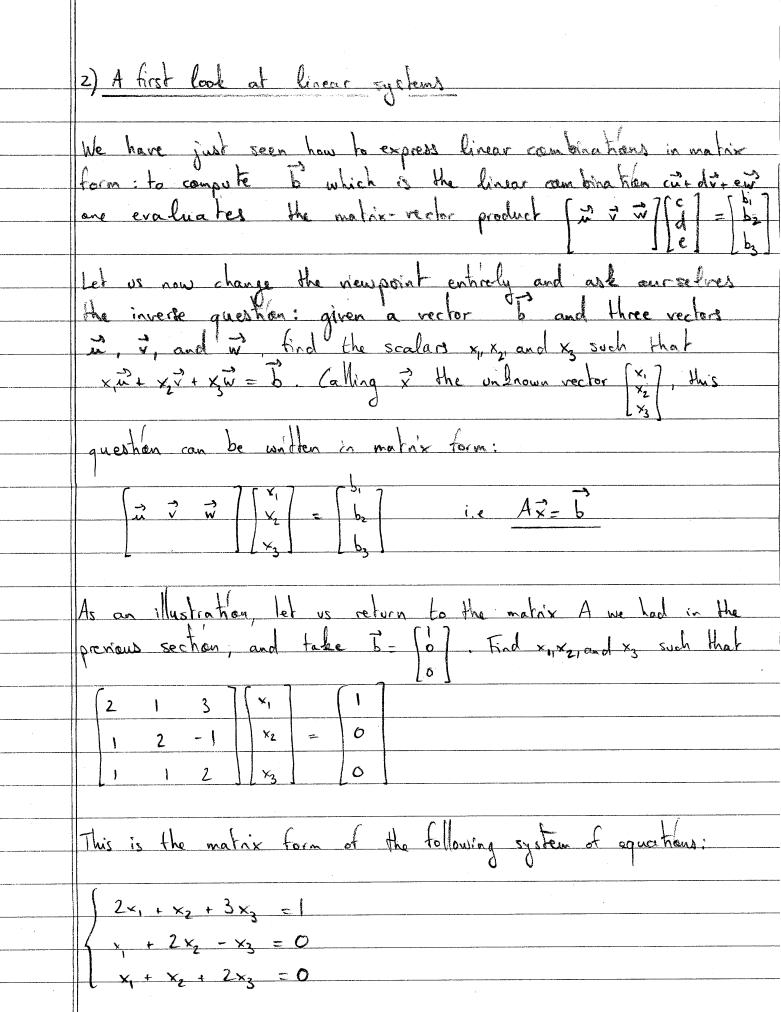
MATH-UA 140 - Linear Algebra
Lecture 3: A brief introduction to matrices
A central application of linear algebra is to efficiently solve linear systems of equations. In the next lecture, we will learn
how to write such systems in matrix form, and later we will learn methods to determine whether a given system has solutions,
As a preparation for this key part of the course, we will introduce here all the ideas we need to be comfortable with for where lecture
I Matrices
1) Definition
A matrix is a rectangular array of numbers arranged in m rows and n columns, where m and n are positive integers.
The matrix entries are usually written in the following format: a: where i is a positive integer representing the row number of the entry, and j representing the column number.
Example: The generic 3-by-3 matrix A = \[a_{11} \ a_{22} \ a_{23} \]
Laz, azz azz
Note that in this class, we will only consider entries a; which are real numbers





cût dit eû, with c, d, and e scalar: $\begin{bmatrix}
 2 \\
 1
 \end{bmatrix}
 + d \begin{bmatrix}
 2 \\
 1
 \end{bmatrix}
 + e \begin{bmatrix}
 3 \\
 -1
 \end{bmatrix}
 = \begin{bmatrix}
 2e + d + 3e \\
 c + 2d - e
 \end{bmatrix}$ $c + 2d - e
 \end{bmatrix}$ We may rewrite this in matrix form as follows: of in consise form in \vec{v} \vec{w} $\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 2c + d + 3e \\ c + 2d - e \\ e \end{bmatrix}$ If we call A the matrix [2 1 3] = [] = [] we see that the multiplication of A with the vector [c] is the same as the linear resulting han cut direct of the three columns of A. This is another perspective for understanding matrix vector product: in section I/4), we viewed it in terms of the rous of A; here, we view it in terms of the rolumns of A.



This system of equations is said to be linear, because the unknown x, x2, and x3 appear linearly, i.e. there is no x2, or Jx, or x2, or any such term that is non linear. Any linear system of equations can be withen in the matrix form $A\vec{x} = \vec{b}$ In this particular example, fairly straight forward algebra leads to the solution: x = 5, x = 3, x = 1 In general, systems cannot be solved in such a straight forward manner. We will soon learn a robust method to solve linear systems that makes we of the matrix representation. 3) Linear dependence and independence Given a matrix A = | i v v v , are may wonder if for any vector $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ the linear system A = 5 has a solution = [x2]. We will answer this question in detail in betwee lectures Still one can obtain very good inhitton regarding the answer by rephrasing the question in the following terms: given 3 vectors in the

win 3-D space, can one always find, for any rector b= |62], a linear combination such that $x_1\vec{u} + x_2\vec{v} + x_3\vec{w} = \vec{b}$ Remembering Lecture 1, if w is in the plane of it and if the answer is NO, since all linear combinations x, in + 2 2 + 2, in only fill that plane in that case, and cannot be equal to vectors to that are not in that plane. On the other hand, if is not in the plane of in and it, then
the answer is YES, since linear countrinations xinc xxv + xx in
fill the whole space. When is not in the plane of is and is, we say that is, is, and is are linearly independent. No linear combination except où + où + où = o gives b = o When we is in the plane of it and i, we say that it, i, and is are linearly dependent. There are non-trivial linear combinations such that cut disenses