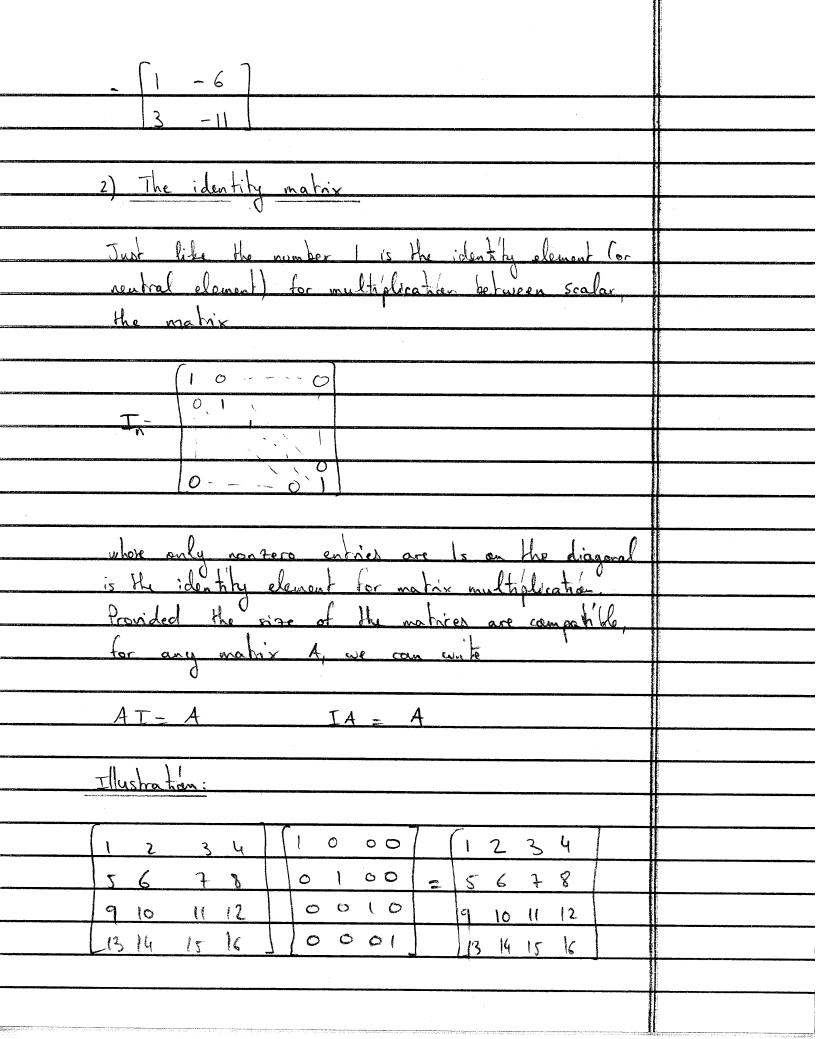
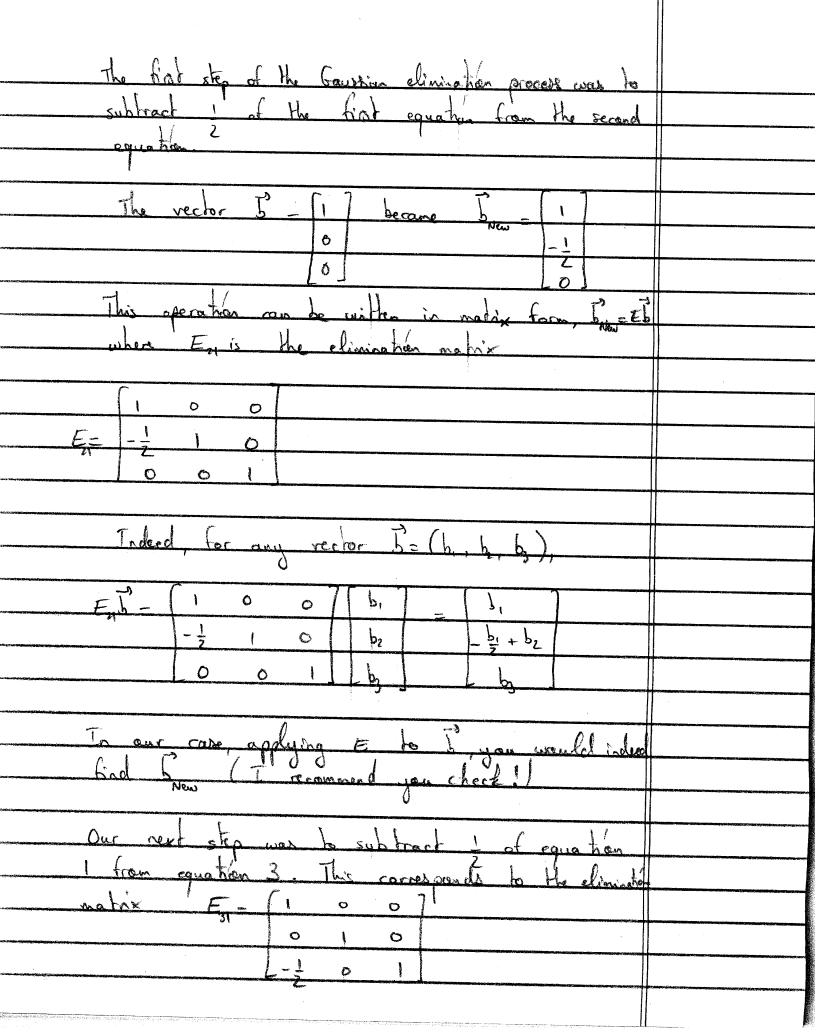
	MATH-UA 140 - Linear Algebra
	Lecture 5: Flimination in terms of matrix operations
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	The same of the sa
	The purpose of this lecture is to response the two ideas covered
	in the previous lecture - Gaussian elimination and permutation -
	in terms of simple operations on the matrix A for the system.
	To do so, we start by learning a new bey operation:
· · · · · · · · · · · · · · · · · · ·	multiplication of a matrix by another matrix
	Il Matrix multiplication
ì	1) Definition
	Let A be an m-by-n matrix, and B be an n-by-p matrix,
	with m, n, and p positive integers. Then C = AB is
	an m-by-p matrix given by:
	$C - AB = A \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \cdots & \vec{b}_p \end{bmatrix} = \begin{bmatrix} A\vec{b}_1 & A\vec{b}_2 & \cdots & A\vec{b}_p \end{bmatrix}$
	where the vectors by bo Bo are the columns of the matrix
	B and the vector matrix products Ab, Ab, have been define
	is 1450 pg 3.
	Eva da
ann gall and a garage and a second or had to a be a be a be a beautiful and a second of the second o	LAUMAN
	$\begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} -2 \end{bmatrix}$
	3 3 1 -1 -2 3 3 1 -2
	123 3 1 1 2 2 1 2 3 7 3 1 1 1 1



	3) Properties of matrix multiplication
	Let A, B, and C be three matrices whose dimensions are compatible with the operations below, c be a scalar, and
	compatible with the operations below, a be a scalar and
·	I the identity matrix (also with rempatible dimensions). We
	have:
	1. A(BC) - (AB)C 2. c(AB)= (cA)B= A(cB)
	3. TA-A, AI-A
	Note that in general, AB + BA
	Indignation of the state of the
	Frangle: A-[1] B=[0-1]
	AB-[-1 0 while BA=[-1 -1
	-100
	III Gaussian climina han in terms of matrix operations
	1) Elimination matrices
	Let us retiren to the example
	(2 1 3 X
	$\begin{vmatrix} 1 & 2 & -1 & y & = & 0 \end{vmatrix}$
	[1 1 2] [2]
	laen in Lephure 4



	You short to see the general pattern: the identity making
	I such that IB=B for any vector B has I's on
	the diagonal and zeros otherwise; the climinatia matrix
	En that subtracts a multiple l'efron j' from rowi
	has the extra nonzero entry - l'in the inj position, ain
	addition to the diagonal of lie
	·
	We did not only apply the elimination algorithm to be of course; we also applied it to the left hand side, i.e the matrix A. Let us see how that looks in matrix form:
	course: we also applied it to the left hand side, in the
¢ '	matrix A Let us see how that looks in matrix form:
	[100]213 213
	$\begin{vmatrix} -\frac{1}{2} & 1 & 0 & 1 & 2 & -1 & = & 0 & \frac{3}{2} & -\frac{5}{2} \end{vmatrix}$
	$ \begin{bmatrix} 1 & 0 & 0 & 2 & 1 & 3 & 2 & 1 & 3 \\ -\frac{1}{2} & 1 & 0 & 1 & 2 & -1 & = & 0 & \frac{3}{2} & -\frac{5}{2} \\ 0 & 0 & 1 & 1 & 1 & 2 & 1 & 1 & 2 \end{bmatrix} $
	The matrix on the right hand side indeed corresponds to the intermediate system we had after the first elimination operation.
	the intermediate system we had after the first elimination
	operation.
	2) Permutation marries
	We sow that exchanging rows at a system was sometimes receive
	for the elimination algorithm to truction property. Let us see
	Juhat row exchanges look like in matier form Specifically.
	what matrix Prexchanges row I and row 2? The answer is
	abbained by exchanging row and 2 of the identity weak
	[0 1 0]
	P12 = 1100
	1001

Indeed, for any vector $\vec{b} = (b_1, b_2, b_3)$,	
P ₁₂	
0 0 b ₂ b ₁ b ₃ b ₃	
Proposition of the column	
Per exchanges the rows land 2 of the column rector I. Consequently it also exchanges rows land 2 of any matrix:	
0 1 0 7 0 6 5 7 2 4 1 7 1 0 0 2 4 1 = 0 6 5	
0 0 2 4 1 = 0 6 5	
In the example above, Por does exactly what it was made for: it ask on (0 6 5) to him	
il into an apper triangular matrix.	
General cule for Pij: The permulation matrix P: which exchanges rows i and j of a matrix A when multiplied to A is 11.	
cous i and j in the identity matrix	

