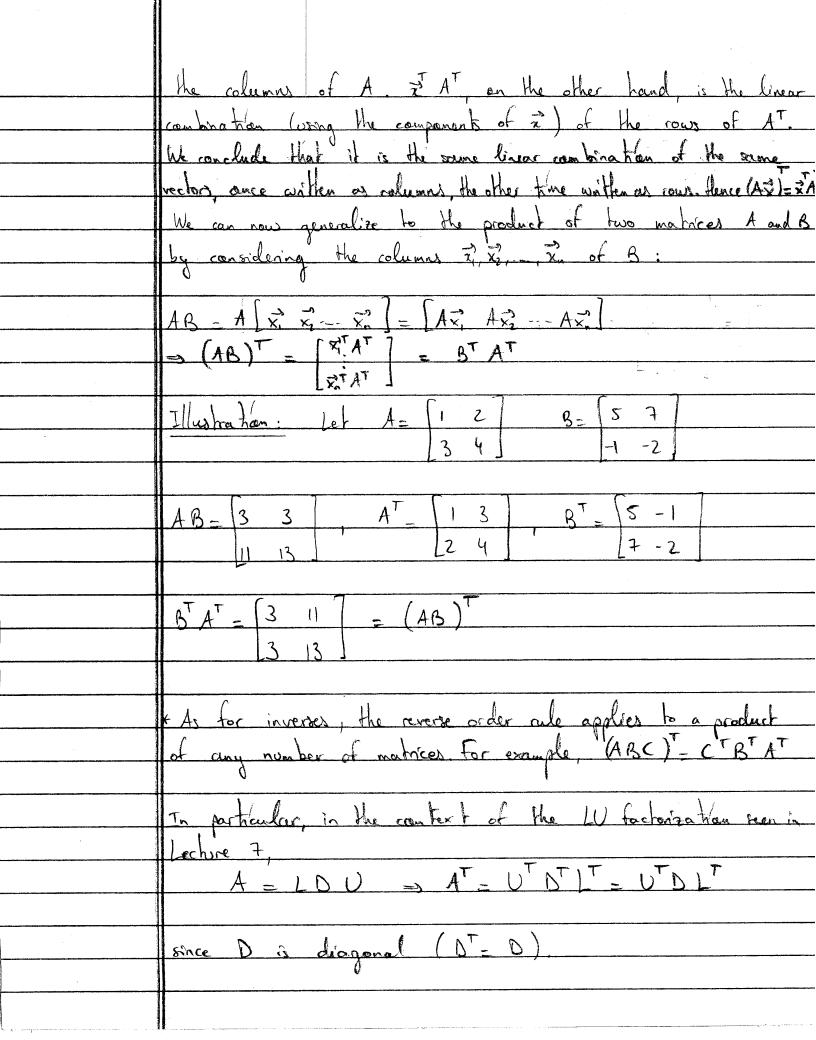
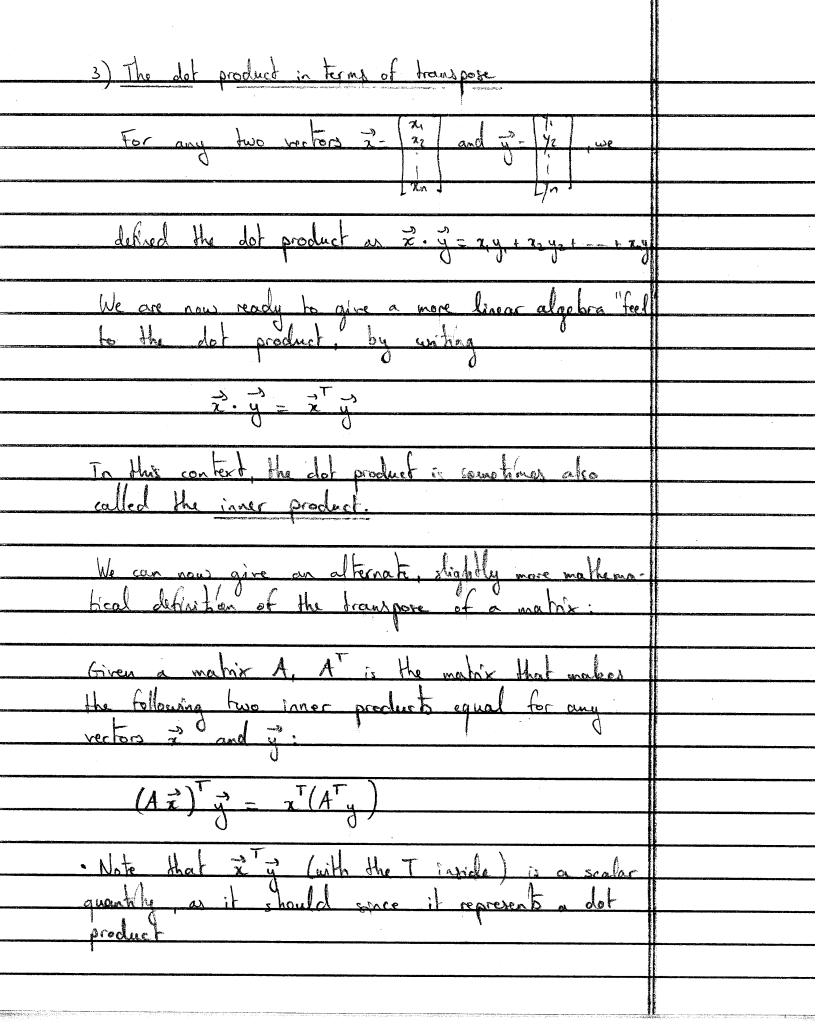
	MATH-UA 140 - Linear Algebra
	Lecture 8: Transposes and Permutations
	assumentation of the control of the
	I Matrix transpose
·	
	1) De finition
	Let A be an m-by-n matrix, with entries a: The
	branspose of A withen A is the n-by-m matrix with
	brand page of A withen A, is the n-by-m matrix with entries a: The
	<u>'3</u>
	In other words, the entry in row i column jost AT comes
	from row j, column i of the original A.
	3'
	Example: A = [3 1 4] AT = [3-1]
·	-1 8 10 1 8
-1	4 10
	$\star \text{ Note 1: } (A^{\top})^{\top} = A$
	* Note 2: The transpose of a lower triangular matrix
	* Note 2: The transpose of a lower triangular matrix

2) Properties Let A and B be two natices with the appropria 1. $(A+B)^{T} - A^{T} + B^{T}$ 2. $(AB)^{T} - B^{T} A^{T}$ 3. $(A^{-1})^{T} = (A^{T})^{-1}$ * Property 3 follows from property 2. Indeed, let A be an invertible matrix. Then Taking the tremspose on both sides, we have

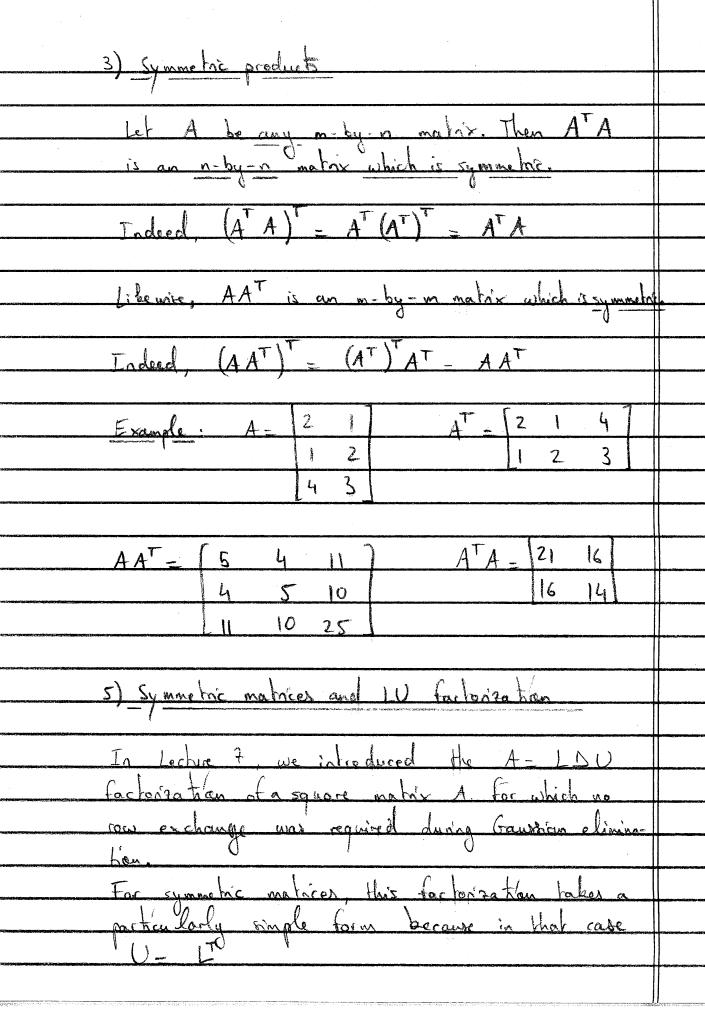
(A A-1) T - IT - I

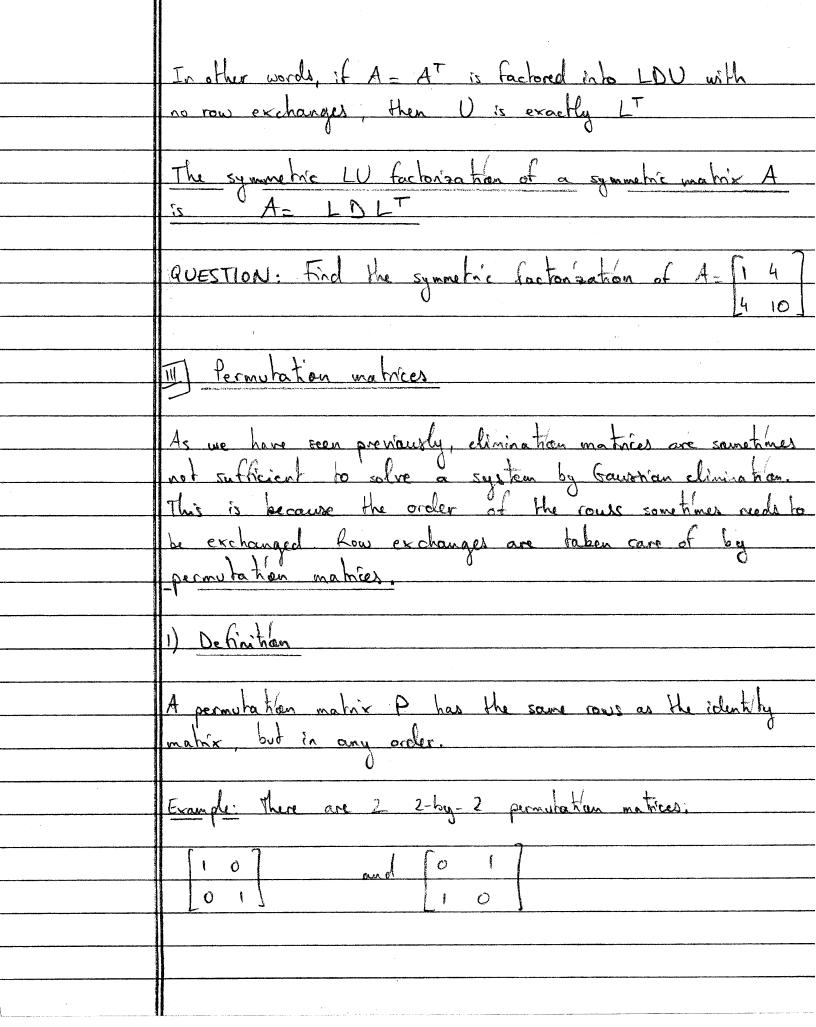
Voing property 2, we can rewrite the left-hand
side which proves that the inverse of AT is (A')T Note that property 3 also implies that AT invertible whenever A is * To understand Property 2 first consider the athenthen
of a matrix vector multiplication. A?
We have seen that A? can be seen as the
linear combination (uping the component of ?) of





	There exists another product, with the Toutside, called the rank one product or outer product:
	The state of the s
	It is the product of an n-by-1 matrix with a 1-by-n
	It is the product of an n-by-1 matrix with a 1-by-n matrix; it thus results is a square n-by-n matrix.
	II Symmetrie matrices
	1) Définition
	A symmetric matrix A is such that $A = A$. In other words, its entries of are such that $a_{ij} = a_{ji}$.
	To other words, its entires dig are such that dig = aj:
	Example: [3-1] [10] 0 10 are symmetric matrix
	Example: 3-1 10 00 are symmetric montre
	2) Tamediate property
	The inverse of a symmetric matrix is also symmetric.
	Property 3 tren confier
	Proof: (A') = A'
9	+11 \)([3 \]-1 [4 \]-1 [1 \]-1
	Illustration: 3 - 1
	1-1 4 1 1 1 1 1 3 1 0 3 1 0 3





There are 6 3-by-3 permutation natices: In general, there are no permutation matrices of dimension n-by-n. Here! means "factorial".

n: is the number $n \times (n-1) \times (n-2) \times \cdots \times 3 \times 2 \times 1$ 2! = 2x1 = 2 3! = 3x2x1 = 6Permutation matries are invertible and their invertes P' are also permutation matries. In fact, we can say even more: If P is a permutation matrix, P' = PT In other words, PPT = I

