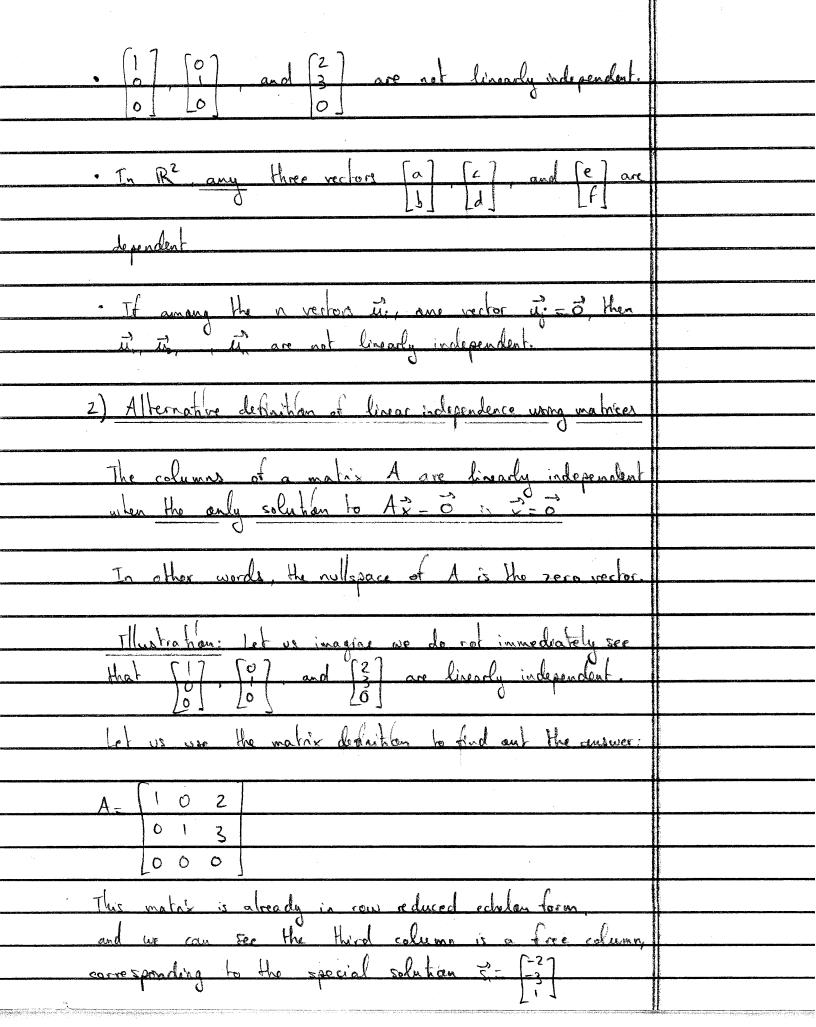
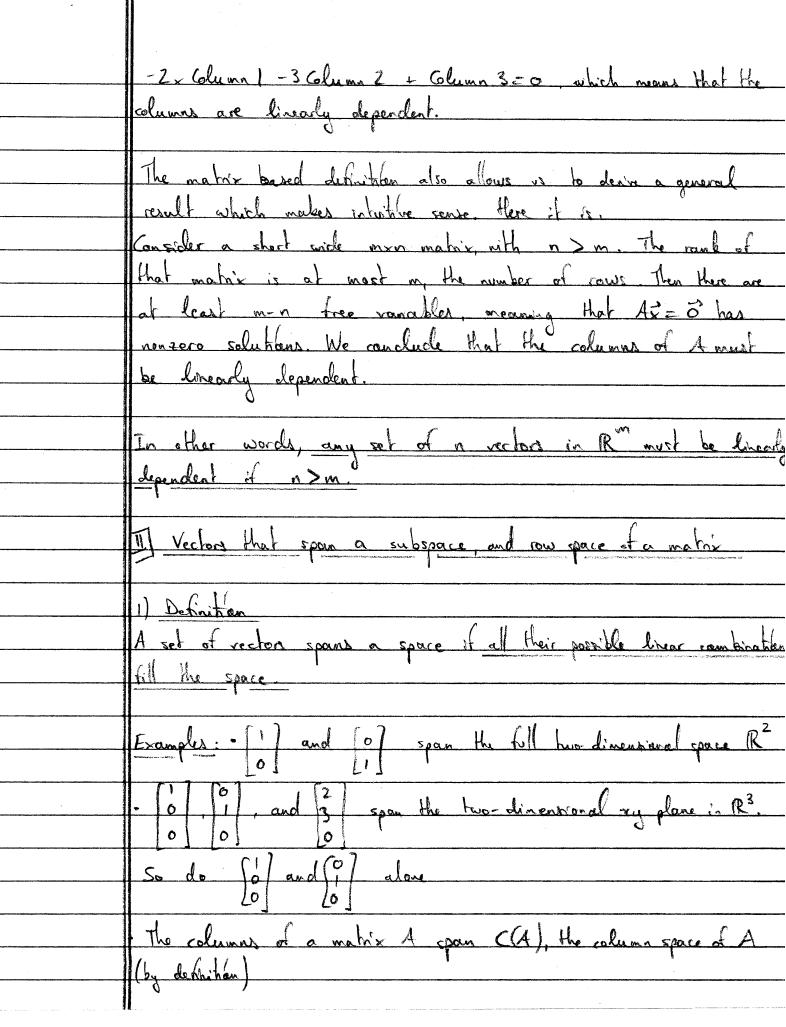
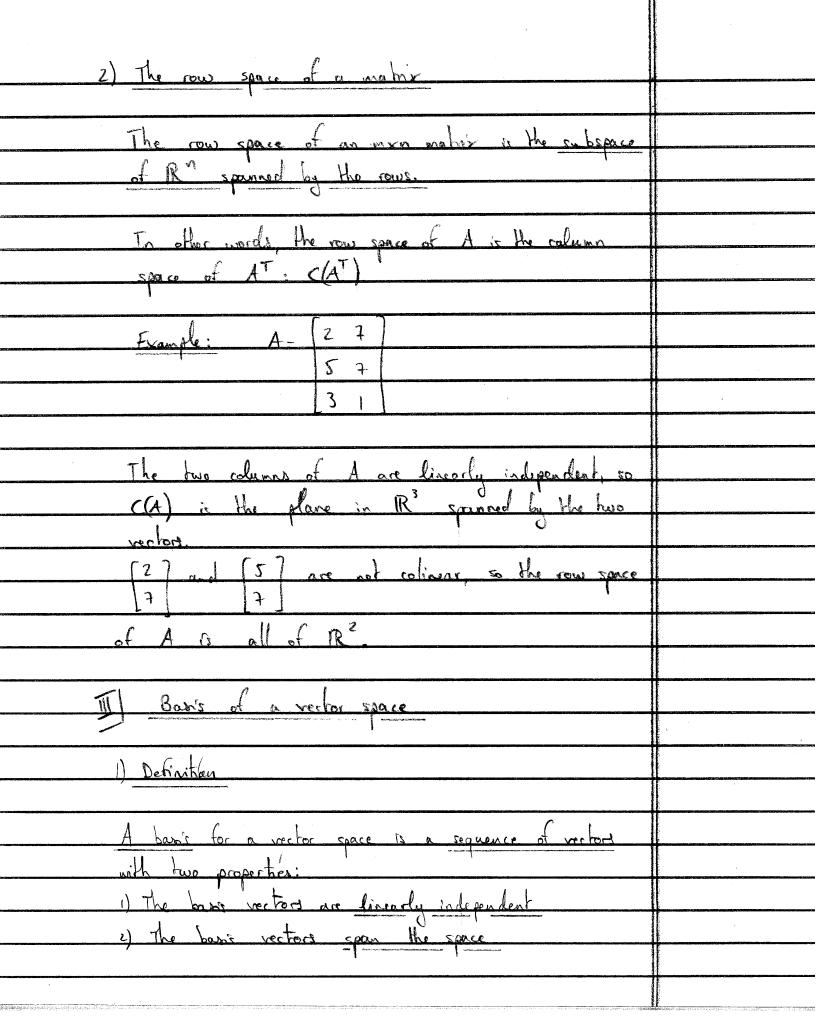
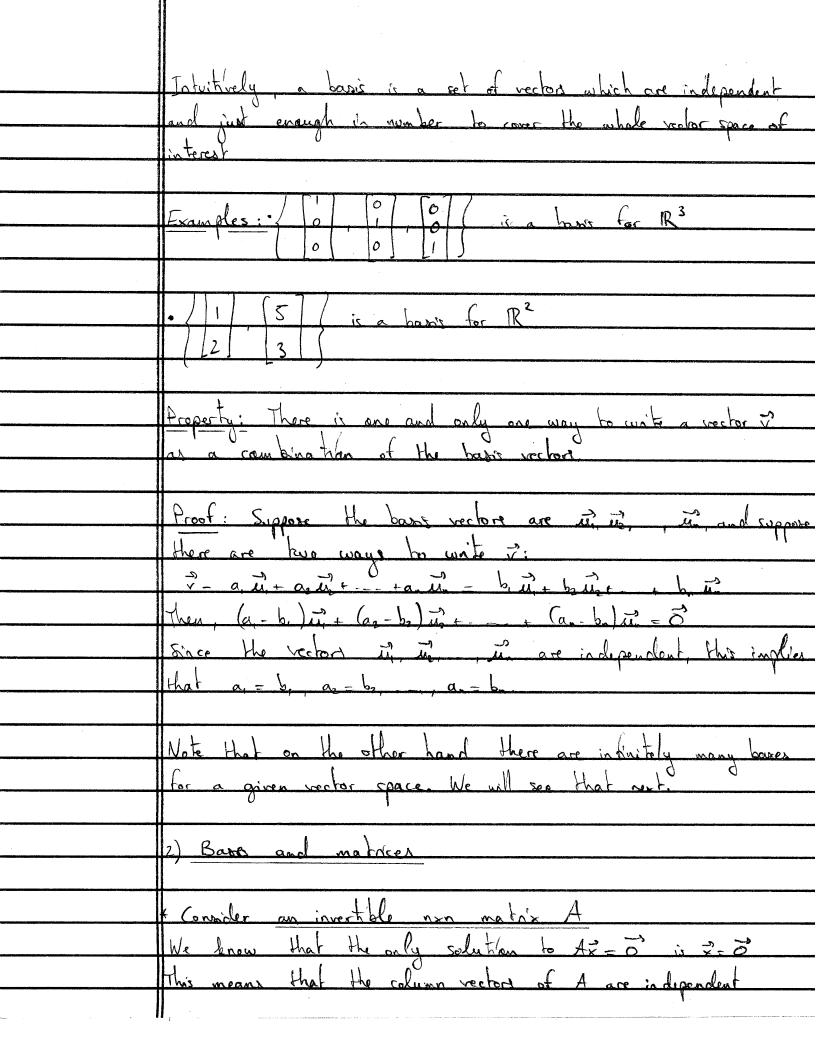
MATH-UA 140 - Linear Algebra Lecture 12: Independence, Baris and Almention In the last tew lectures, we have encountered a few important subspaces, including the column space of a matrix and the nullepace of a matrix. We have also loarned how to characterize and represent them. The purpose of this lecture is to learn how one determines the dimension of any subspace, and what is the minimum amount of Information required to represent the subspace. We will see that the key is to find a basis of way bowards defining what a baris is. I Linear independence A sequence of vectors ii, ii, is linearly independent if In other words, the only linear combination which gives the zero rector is the trivial linear combination with all scalars being o Examples: - [1] and [1] are linearly independent · [1], [0], and [0] are linearly independent









Furthermore, for any b in R" Ax-I has the solution x- A' b'. So the column vectors of A span R". We just proved that: The rectors is in are a basis of R if and only if they are the columns of an nxn invertible \* What can we say about matrices that are not invertible? To that case the columns are not linearly independent However we have seen in the previous lecture that the pivot columns are linearly independent and spon the column space. The pirot column of a matrix A are a bases for its column space Note that even though the pivot columns of A are at the same place as the pivot columns of R-rref(A). The corresponding column rectors are not the same, so that in general, the column space of A is not the same as the column space of R. Example: A= 4 12 -0 R- 13

The second column of A & colored with U. Gat 10
The second column of A is colinear with the first column of A so the column space of A is a line in R2 with [47] as a bount. [12] is also a possible bount, as it any scalar [1]
as a basis. [12] is also a possible basis as is any scalar.
multiple of [4]
The column space of his also a line, but with direction vector
The column space of his also a line, but with direction vector [1]. This is a different line in R2
* Thinking in terms of At it is clear that the private rows of a matrix A are a basis for its row space.
rows of a mains of are a band for its row sporce.
What is more the givet rows of A- crof(A) are also
What is more, the pivot rows of R- rrof(A) are also a basis for the row space of A
Going back to the example above, we indeed see that [4]
12
and I are two possible bases for the same line in R2
3
QUESTION: Find bases for the column and row space of
0-120-177
10017
0000

Summary of the section	
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Given a set of vertors 2, 7 in R. how does	
are find a basis for the subspace the motor space?	) Springer transportation on the state of th
There are two equivalent methods:	
Method I Construct a matrix A whose rows are the	
vector x', x' Compute R = cref(A); the nonzero rows of	,
R are a basis for the desired subspace	
	,
Method 2 Construct a matrix A whose columns are	
the vectors \$2 , \$2. Use climina tran to find the pirot	
columns of A. The vectors in these columns are a	
basis for the desired subspace.	
IV Dimension of a Vector Space	
1) Theorem - Definition	
All bases for a given vector space contain the same	
number of vector.	
The dimension of a vector space is the number of	
vector is every base	
Note: An instructive proof of the theorem is given in	
the best book	

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