MATH-UA 140- Linear Algebra Lecture 25: Similar Matrices In Lecture 22, we saw the factorization A = 5'AS for matrices A which are diagonalizable. In this lective, we generalize this type of factorization, even if the matrix A does not have a set of a linearly independent eigenvectors. In that case, the equivalent of the matrix 1 is of course not diagonal. But it can come fairly classe, as we will tee. Il Similar matrices 1) Definition Let M be any invertible matrix. The matrix B such that B = M' AM is said to be similar to A. Note that if B is similar to A, then A is similar to B. Indeed, B- M'AM => MBM' = A

(=) (M') B M' = A

and M' is as acceptable a matrix as M.

The zero natrix is similar only to shall: M'OM = O

For diagonalizable matrices, A = 5'AS, so A and A

are similar. The word "similar" was chosen an purpose: we will see that similar matrices have much in rommon. We short with eigenvalues.

2) Eigenvaluer of similar matrices	
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Similar matrices A and M'AM have the same eigenva-	CONSIDERATION OF THE SECTION OF THE
lue de la companya della companya de	and the second of the second o
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Their eigenvectors, however, are different: if is an eigenvector of A, then M'z is an eigenvector of M'AM.	a talah merendak talah salah serbangka kendalah di sebendah di selah pendamban pendamban pendamban sebagai men
	See Debug the 2000 Commission of Company or resident and proposed the secretary and an extra commission of Commiss
Here is the proof: Assume & is an eigenvector of A.	Colombian Security Colombian (1985), S. S. S. March Security (1987), Secur
there is the proof: Assume it is an eigenvector of A. with corresponding eigenvalue &: Ai = \frac{1}{2} Counsider B such that B = M'AM = A = MBM'	
Counder B such that B = M'AM & A = MBM'	and the second s
MBM-17 = XX (-> B (M-17) - X (M-17)	ZZSOWACE (1853: ZZW CES), N. Estadolomona (1855; empario), Ar John Anna Antographon a sa
	anu i menintana kangan kemendali intangan diang pangangan ang pangangan dianggan dianggan dianggan dianggan di
is an eigenvalue of B. The corresponding eigenvector	
	and activities and activities and activities are applying the constitution of the cons
Example: Let A- [2] Its eigenvalues and	and the second contract of the second se
Land 3, so it is diagonalizable: t-SNS-', and	ADDRING I KITATION OF CONTROL OF
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	and an executive of the state o
Now consider M-10 M-10	SSEALUS (MISSELLE) (SMISSELLE) (SMISSELLE) (SMISSELLE) (SMISSELLE) (SMISSELLE) (SMISSELLE) (SMISSELLE) (SMISSELLE)
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B=M-1 AM- 10 12 1 1 10 1- 14 1	с тин ання на применя в применя на п
-2 1 1 2 2 1 -3 0	
B is similar to A and has eigenvalues Land 3 like A.	

Other quantités that are commen to similar matrices
Le have just seen that timilar matrices have the same eigenva-
Similar natrices thus have the same determinant, which is the product the eigenvalues. They also have the same trace, which is the sum of the eigenvalues.
ince the eigenvectors of a similar matrix are M'z', with M invertible (so fell rand), similar matrices have the same number of linearly independent eigenvectors.
Similar matrices also have the same rant. There are many ways to see this. One way is as follows, for B= M"/AM. rank (B) = rank (M"AM) < rank (A) by Problem 2, HWS rank (A) = rank (MBM") < rank (B) by Problem 2, HWS
so rank (A)= rank B. I Jordan form of a matrix
When an nxn matrix has a linearly independent eigenvectors, it can be diagonalized: $A = SAS^{-1}$, with A diagonal. A is similar to A , with $M = S$.
If an nxn matrix does not have a linearly independent eigenvertors,

it cannot be diagonalized. However, a theorem	
called Jordan's Theorem (named after French mathematical)	- Constitution
tician Camille Jordan) tells is that this matrix is	2572.16
similar to a matrix I which is as close to	
diagonal as possible there is the exact statement	2000
at the theorem, tollowed by an example	-
Jordan's theorem: If an axa matrix A has d	TO SERVICE
independent eigenvectors then it is similar to a	
matrix I that has a black In I am it	
diagonal, where each black J: has the eigenva-	
lue is an the diagonal, and is just above	D. C.
the diagonali	200000
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Each Ji has one eigenvector	
The Jordan form of A is	ana pica
	a deposition of
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A matrix A is similar to a matrix B if and only	
if they share the some Jordan form	
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The Jordan form I has an off-diagonal I for each misting
The Jordan form J has an off-diagonal I for each missing eigenvector. So there are n-d off-diagonal I's.
In this class, we will not learn to compute Jordan forms, but it is important to understand that every matrix is vinitar to a Jordan matrix. That means that we can always unite
A= MJM' Then, A2= MJM' MJM' = MJ2 M-1, and more generally, A2= MJ2 M-1
Computing JE is not as simple as computing 1th, but still more convenient than computing 1th
Example: $A = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$
We saw in Lecture 22 that A had only $k=2$ as its eigenvector.
A is similar to $J=\begin{bmatrix}2\\0\\2\end{bmatrix}$