Area Under the Bell Curve

In addition to exotic but familiar functions like $\ln x$, we can also use definite integrals and Riemann sums to get truly *new* functions.

Example: The solution to $y' = e^{-x^2}$; $y(0) = 0$ is:

$$F(x) = \int_0^x e^{-t^2}\, dt$$

The graph of e^{-x^2} is known as the bell curve, and $F(x)$ describes the area under the curve. This function is extremely useful for computing probabilities.

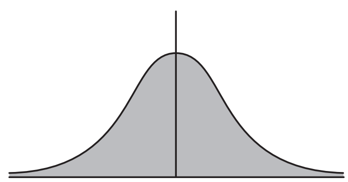

Figure 1: Graph of e^{-x^2}.

The exciting thing about $F(x)$ is that although we have a geometric definition and can compute it using Riemann sums, we can't describe it in terms of any function we've seen previously, including logarithmic and trigonometric functions. It's a completely new function. The problem of describing F is analogous to the problem of calculating the value of π — the area of a circle with radius 1. The number π is transcendental; it is not the root (zero) of an algebraic equation with rational coefficients.

Using definite integrals we can define a huge class of new functions, many of which are important tools in science and engineering.