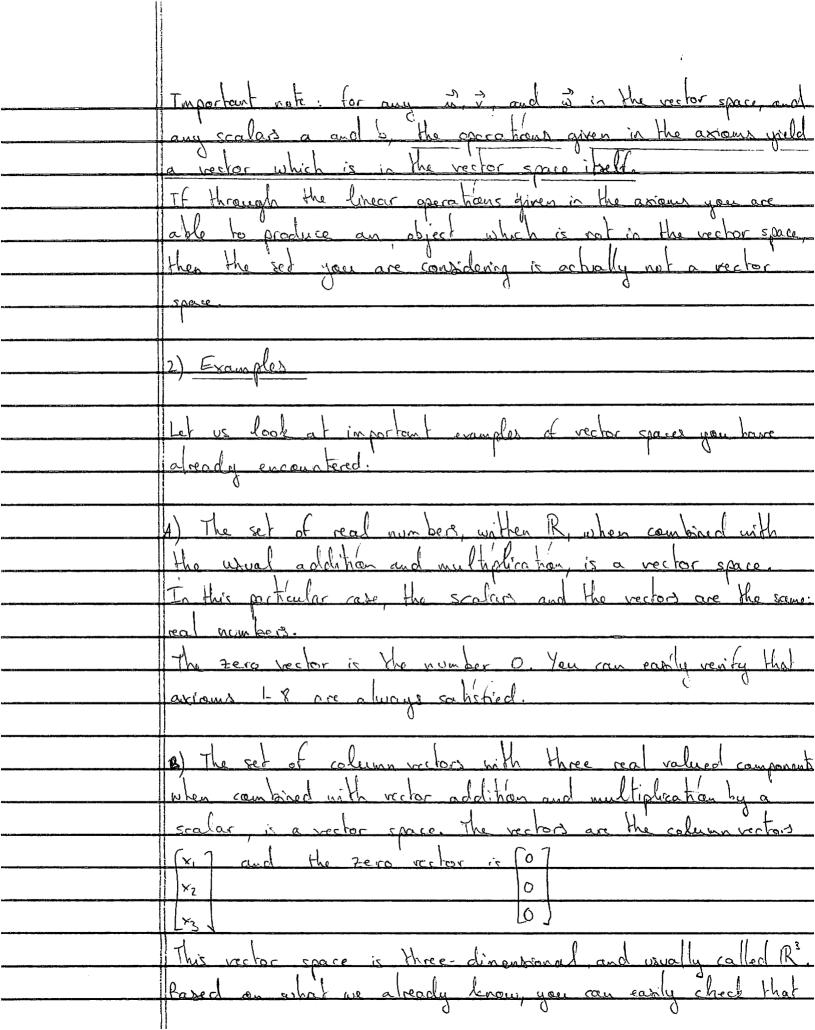
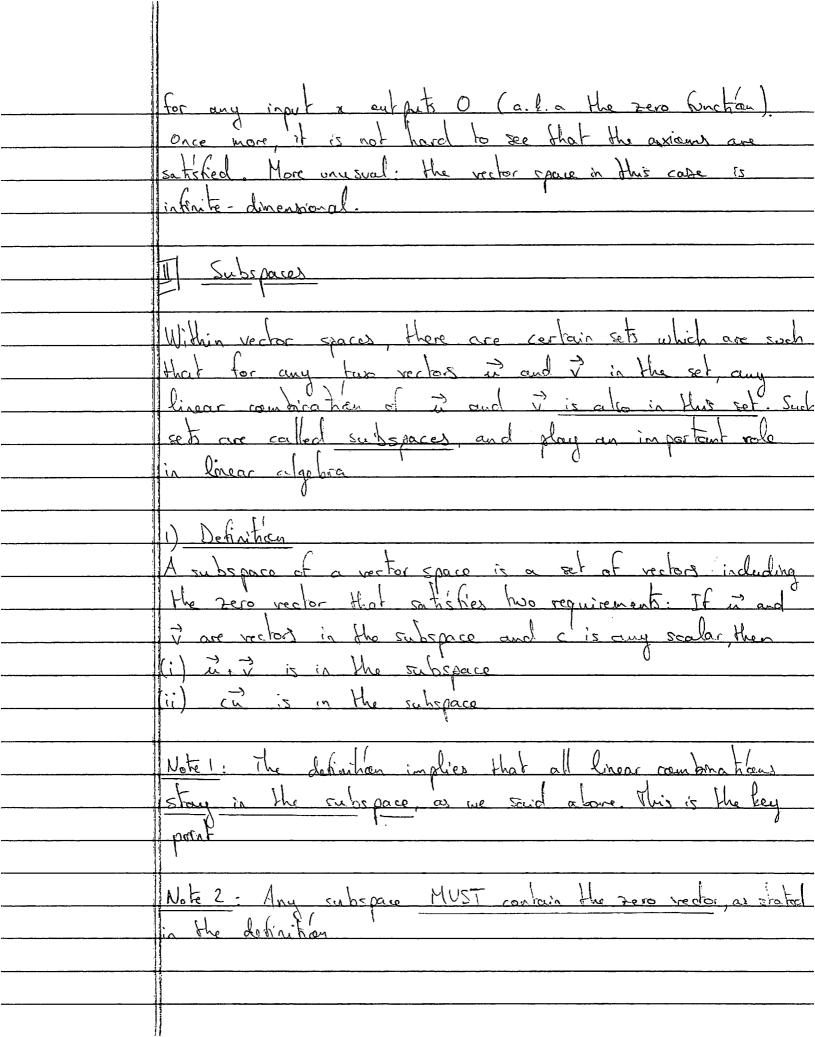
MATH-UA 140 - Linear Algebra Lecture 9: Vector Spaces Thus for, we have loaned about vectors and matrices with a "bottom up" approach : we have defined these objects in isolation, and then looked at operations that could be applied to them For the next few lectures, we will thift gears learn about the general spaces vertors and matrices live is called vector spaces, and present the axioms defining rules and operations in these spaces. This may seem a little abstract at dist but will be very helpful very soon to better understand when systems have solutions and whey they I Vector spaces

1) Definition A vector space is a set of objects called vectors, which may be added together and multiplied by a scalar. The operation of vector addition and scalar multiplication must ratify the following arious, wither for three vectors is, i, and is in The vector space, and two scalars a and b: 1. it (v+ i) - (i+v)+w Associating of addition 3. There exists an element of in the vector space, called the zero frechor, such that it of = in for any vertor in the vector space

4. For every element in in the vector space, there exists an element - in the vector space, called the addition inverse of in, such that in + (-ii)=0 Distribution of scalar multipli-7. a( ii + x)= au + av 8. (a+b) i - n i + bi Note 1: You probably recognize many of those around in the form of properties for vectors and unabsces we have seen premounty. This is in agreement with what I said at the beginning of this lockere: vectors and matrices live is well-known vector spaces. Note 2: In principle, there are many possibilities for the space the multiplicative scalars live in: rational nombers, real numbers, complex numbers etc. In this course, the scalars will always be real numbers Note 3: The word "rector" in the definition has to be understood in it generic sense, associated with vector spaces. As we will soon see, the rection we are used to are vectors of particular vector spaces, but so are matrices, burchens of one variable etc.



axioms 1 to 8 are so tistied. c) More generally for any portive integer of the set of column vectors with a real valued components is a vector space when equipped with vector addition and scalar multiplication. The zero lled R, and it has a dimensions 1) The set of real 3 by 3 matrices is a victor space when equipped with matrix addition and muthtoces far as as I and the zero vector is a31 a32 a33 [0 0 0] Once more, you can easily consince yourself that the axioms are satisfied. The dimension of the gace is 32=9 E) The set of all real furchase f(x) of one variable is a vector space when equipped with the usual addition and multiplication between



2) Examples · Inside the vector space of all 3 by 3 matrices the set of upper triangular matrices (a b a subspace: (0 0 0 ) the zero rector is upper matrices is upper tranquelar, as is the multiplication of an upper tranquelar matrix he a scalar. Topicle the rector space R of the real numbers the positive real numbers do NOT form a rector space. The zero rector O belongs to the set, but for any vector in the set, say IT+e, the multiplication of this rector by the scalar c=-1, -TT-e, is not in the set. Condition (ii) fails · Any plane through the origin (0,0,0) is a subspace of 123: it is clear that the zero vector (0,0,0) is in the set, and any two vectors ii-(u, u, u) and v-(v, u, v) in the plane, the sum u+v is in the plane, as is air for any scalar o

III The Column Space of A We are now cooly to make the link between solutions to a system and subspaces. We start with another important, somewhat abstract concept namely the span of a set of vectors, and make the connection with systems subsequently, when we consider the columns of a maker. 1) Subspace spanned by a set S of vectors Definition: Let S be a set of vectors in a spector space which is not necessarily a subspace. The subspace SS of all the linear combinations of the vectors in S is called the span of S. QUESTION: Can you prove that SS is indeed a subspace? Property: SS is the smallest subspace containing S. 2) Column space of a matrix Definition: The column space of a matrix A is the subspace spanned by the columns of A. It is often denoted by C(A) In other words, C(A) is the subspace of all livear rambrate of the columns of A

