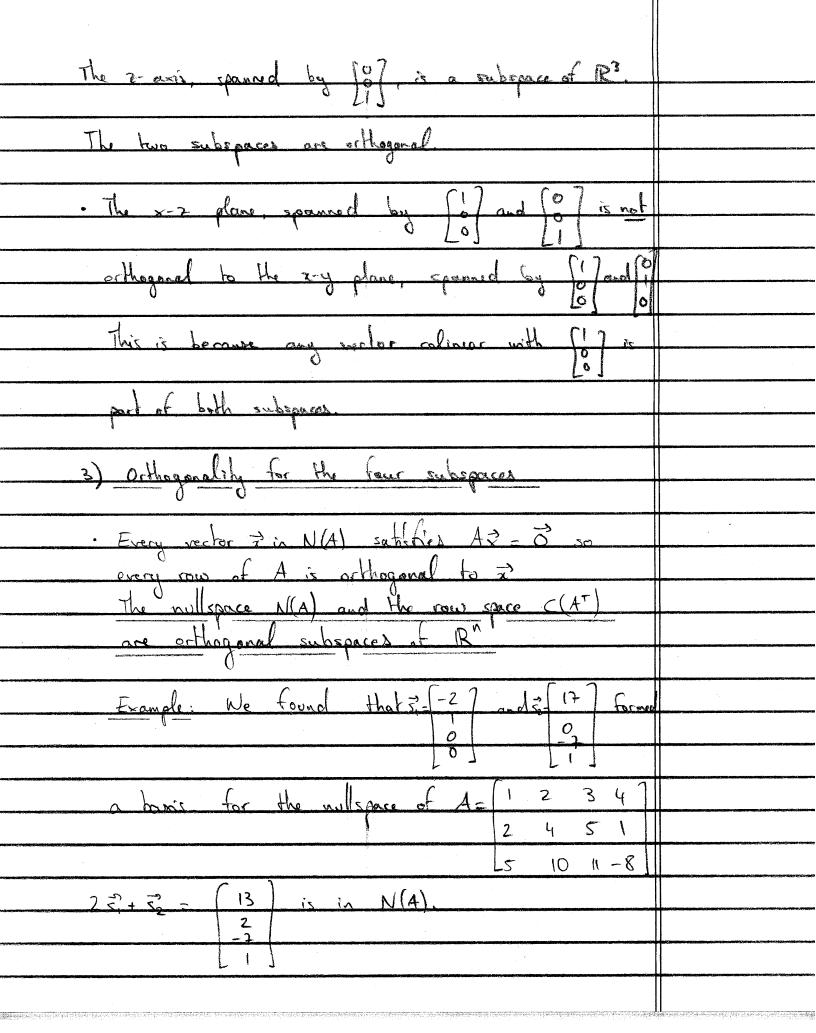
	MATH- VA 140 - Linear Algebra
	Lecture 14: Orthogonality of the Four Emboraces
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	You may have noticed in the previous earlier that the fordamental
	theorem of linear algebra was not complete: we wrote (Part 1).
	The reason for this is that there is more one can say about the
	four subspaces and this has to do with orthogonality properties.
	This is the purpose of this lecture.
	I Orthogonal subspaces
	1) Brief minaul
	Two rectors is and it are orthogonal if is it - m
	Two rectors is and is are esthagonal if is. $\vec{v} = 0$ In linear algebra language, we would write $\vec{x}^T \vec{v} = 0$
	2) Definition
	Two subspaces U and V of a rector space are orthogonal if
	every victor is in V is orthogonal to every vector in V
	Note: If a vector w is in two orthogonal subgrasses, it is orthogo-
	Note: If a vector $\vec{w}$ is in two orthogonal subspaces, it is orthogonal to itself clearly, one then has $\vec{w} = \vec{0}$
	Example of orthogonal subspaces
	The ray plane, spanned by [6] and [6] is a subspace of R.



 $\begin{bmatrix}
 5 & 10 & 11 - 8 \end{bmatrix}
 \begin{bmatrix}
 13 \\
 2 \\
 -7 \\
 -1
 \end{bmatrix}
 = 0$ Applying the same reasoning to AT, we can say that the left nullspace N(AT) and the column space C(A) are orthogonal in RM. The orthogonality of the four subspaces and the first part of the fundamental theorem of linear algebra (previous lecture are nicely summarized in the fext-book with the figures 4.2 and 4.3 I The fundamental theorem of linear algebra, Part Z 1) Octhogonal complements Definition: The orthogonal complement of a subspace V contains every rector that is perpendicular to V. This subspace is often withen V. (1 is the "perp" symbol). Consider ((AT), the row space of A. Any rector in (AT)

satisfies  $A\vec{x} = \vec{o}$ . It is therefore in N(A) (convertely, any

vector  $\vec{x}$  in N(A) is orthogonal to each row of A. It

is therefore orthogonal to any vector in  $C(A^T)$ , so  $\vec{x}$  is in  $C(A^T)$ . We conclude that N(A)= C(AT) This is the second point of

