Negation and dependencies: a dynamic approach to dependent numerals in Turkish

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1 Introduction

- I discuss the behaviour of quantificational dependencies under negation, focusing on *dependent indefinites* and *dependent numerals* (Brasoveanu and Farkas, 2011, *et seq*.).

- Dependent indefinites/numerals force a distributive reading.

(1) a. Xeqatij **ox-ox** wäy.
we-eat three-three tortilla
"We each ate three tortillas." (Kaqchikel Mayan, Henderson, 2014)

b. BOYS IX-arc-a read **one-arc-a** BOOK.
"The boys read one book each."
(American Sign Language, Kuhn, 2017)

- Dependent indefinites/numerals can occur under the scope of a quantifier.

(2) a. Chikijujunal ri tijoxela' xkiq'etej **ju-jun** tz'i'.
each the students hugged one-one dog
"Each of the students hugged a dog."
(Kaqchikel Mayan, Henderson, 2014)

b. EACH-EACH-a PROFESSOR NOMINATE **ONE-redup-a** STUDENT.
"Each professor nominated one student."
(American Sign Language, Kuhn, 2017)

- Dependent indefinites/numerals are incompatible with a singular argument.

(3) a. * Xe'inchäp **ox-ox** wäy.
I-handle three-three tortilla
"I took (groups of) three tortillas."
(Kaqchikel Mayan, Henderson, 2014)

b. * JOHN-a READ **ONE-arc-a** BOOK.
"John read one book (each time).'
(American Sign Language, Kuhn, 2017)

- Option 1: dependent indefinites have to be dependent on a quantifier upstair.

- In (1), a covert distributivity operator is inserted to satisfy this condition.

- Option 2: dependent indefinites contribute to distributivity in some way, but this distributivity condition outscopes a quantifier.

- While the previous literature focus on their behaviour in positive contexts, their behaviour under negation has been studied much less.

- Today, I examine the existing approaches to dependent indefinites/numerals in light of their interaction with negation in Turkish.

2 Dependency-based approaches

- This section is devoted to technical details. Those who are already familiar with them or not interested in them may skip this section.

- Today, I adopt *dependency*-based framework to discuss the contribution of dependent indefinites under negation.

- Among them, I choose *Dynamic Plural Logic* (DPlL) (van den Berg, 1996).

- Henderson (2014); Kuhn (2017) propose analyses based on DPlL, so this directly serves as a technical preliminary for their analyses.

- Brasoveanu and Farkas (2011) propose an analysis based on what they call *First-order logic with choice* (C-FOL), in which a formula is evaluated relative to a **set** of assignments.

- For the interest of this presentation, C-FOL can be taken as a static version of DPlL and DPlL can be taken as a dynamic version of C-FOL.

- A formula in DPlL denotes a relation between *plural information states*, i.e. sets of variable assignments (van den Berg, 1996; Nouwen, 2003; Brasoveanu, 2007, 2008, a.o.).

- I call the variables in the domain of assignment functions *discourse referent* (dref) and notate them with u with a numerical subscript.

- One can obtain a plural value by summing up the singular values of a dref under the members of a plural information state.

- i.e. $H(u) = \{x | h \in H \,\&\, h(u) = x\}$.

- Evaluation of lexical relations can be collective or distributive.

(4) a. $[\![\mathrm{R}(u_1)...(u_n)]\!] = \{\langle G, H\rangle | G = H \,\&\, \forall h \in H\,[\langle h(u_1), ..., h(u_n)\rangle \in I(\mathrm{R})]\}$ (distributive)

b. $[\![\mathrm{R}(\cup u_1)...(\cup u_n)]\!] = \{\langle G, H\rangle | G = H \,\&\, \langle H(u_1), ..., H(u_n)\rangle \in I(\mathrm{R})]\}$ (collective)

- Number restrictions are defined as collective conditions.

(5) a. $[\![\mathrm{Atom}(u)]\!] = \{\langle G, H\rangle | G = H \,\&\, \mathrm{atom}(H(u))\}$

b. $[\![\mathrm{Non\text{-}atom}(u)]\!] = \{\langle G, H\rangle | G = H \,\&\, \neg\mathrm{atom}(H(u))\}$

- Dependencies are defined as (6b).

(6) a. $G_{u_n=d} = \{g : g \in G \,\&\, g(u_n) = d\}$

b. In a state G, u_m is dependent on u_n iff
$\exists d, e \in G(u_n)\,[G_{u_n=d}(u_m) \neq G_{u_n=e}(u_m)]$ (van den Berg, 1996)

- In the following table, u_2 and u_3 are dependent on u_1, but u_4 is not.

H	u_1	u_2	u_3	u_4
h_1	a	x_2	x_3	x_4
h_2	b	y_2	x_3	x_4
h_3	c	z_2	z_3	x_4

Table 1: Dependency and co-variation

- The dynamic distributivity operator δ (van den Berg, 1996, *et seq*) introduces new dependencies to the discourse.

(7) $[\![\delta_{u_n}(\phi)]\!] = \{\langle G, H\rangle | G(u_n) = H(u_n) \& \forall d\, [d \in G(u_n) \rightarrow G_{u_n=d}[\![\phi]\!]H_{u_n=d})]\}$

- Note that δ allows one to adopt an **externally-dynamic** universal quantifier.

- Dref introduction in Brasoveanu (2008, 2010); Henderson (2014); Kuhn (2015) allows a new dref to be dependent to old dref.

(8) $G[u]H \Leftrightarrow$
$\forall g\, [g \in G \rightarrow \exists h\, [h \in H \,\&\, g[u]h]] \,\&\, \forall h\, [h \in H \rightarrow \exists g\, [g \in G \,\&\, g[u]h]]$

- Introduction of new plural values may introduce new dependencies: it can produce both of the following tables.

G	u_1
g_1	x_1
g_2	x_2

$\xrightarrow{G[u_2]H}$

H	u_1	u_2
h_1	x_1	y_1
h_2	x_2	y_2

Table 2: Dependent

G	u_1
g_1	x_1
g_2	x_2

$\xrightarrow{G[u_2]H}$

H	u_1	u_2
h_1	x_1	y_1
h_2	x_1	y_2
h_3	x_2	y_1
h_4	x_2	y_2

Table 3: Independent

3 Three instances of dependency-based approaches

- I briefly introduce the three proposals in the following subsections.

- In short:

- Henderson (2014) proposes that dependent indefinites encode the distributive cardinality condition and the dynamic plurality condition. The latter is evaluated after all the other conditions are evaluated, i.e. it is *post-supposed*.

- Kuhn (2017) proposes that dependent indefinites encode the distributive cardinality condition and the co-variation condition. Dependent indefinites scopes over quantifiers via Quantifier Raising (QR).

- Brasoveanu and Farkas (2011) proposes that dependent indefinites encode the co-variation condition, which requires presence of an overt quantifier or a covert distributivity operator.

- Again, those who are already familiar with them or not interested in technical details may skip the rest of this section.

3.1 Henderson (2014): post-supposed plurality condition

- Henderson (2014) proposes two conditions defined in (9).[1]

(9) a. $[\![\mathrm{Card}_n(u_n)]\!]$
$= \lambda G \lambda H\, [G = H \,\&\, \forall h \in H\, [|\{x|\, x \sqsubseteq \nu(u_n)(h) \,\&\, \mathrm{atom}(x)]\}| = n]$

b. $[\![u_n > 1]\!] = \lambda G \lambda H\, [G = H \,\&\, |\nu(u_n)(H)| > 1]$

- (9a) says that the value of y has n-cardinality in each member h of H.

- Importantly, this cardinality condition is **distributive**: the value of u_n has to have n atoms in **each member** of H.

- This part captures distributivity of dependent indefinites/numerals.

- (9b) says that the value of x is plural in H.

- It is important that (9b) counts pluralities that only arises at the level of plural information states.

[1]The notation is translated to the style of *Plural Compositional DRT* (Brasoveanu, 2008).

- (9a) counts the number of atoms, but (9b) counts **the number of sets a dref stores under a plural information states**.

- On top of that, he argues that (9b) is evaluated after all the at-issue-contents are evaluated, i.e. it is **post-supposed**.

- Intuitively, post-suppositions can be considered as the mirror image of presuppositions: they are evaluated against the output context, just like presuppositions being evaluiated against the input context.

- Formally, post-suppositions are particular types of tests which plural information states are indexed with.

- I notate a post-supposition with an overline, i.e. $\overline{\phi}$.[2]

(10) $[\![\overline{\phi}]\!](G[\xi])(H[\xi'])$ is true iff ϕ is a test, $G = H$ and $\xi' = \xi \cup \{\phi\}$

- A post-supposition does not update an input context and, instead, add a new test to the set of tests [ ], i.e. a new post-supposition ϕ is added to a set of post-suppositional test ξ and yield ξ'.

- The definition of truth incorporates the contribution of post-suppositions.

(11) **Dynamic truth with post-suppositions**: ϕ is true with respect to an input plural information state $G[\emptyset]$ iff there is an output plural information state H and a (possibly empty) set of tests $\{\psi_1, ..., \psi_m\}$ such that $[\![\phi]\!](G[\emptyset])(H[\{\psi_1, ..., \psi_m\}])$ and $[\![\psi_1 \,\&...\&\, \psi_m]\!](H[\emptyset])(H(\emptyset))$.

- The contribution of the at-issue content of ϕ remains the same.

- However, now the post-suppositions of ϕ has to be true relative to the output context for ϕ to be dynamically true.

- i.e. $[\![\psi_1 \,\&...\&\, \psi_m]\!](H[\emptyset])(H(\emptyset))$ should hold if ϕ post-supposes them.

- Since post-suppositions are evaluated relative to the output context, it 'projects' from the scope of operators.

(12) $\mathrm{Op}(A \,\&\, \overline{\phi} \,\&\, B) = \mathrm{Op}(A \,\&\, B) \,\&\, \overline{\phi}$ adopted from Kuhn (2017)

[2] In his original notation, $[\![\overline{\phi}]\!]^{\langle G[\xi], H[\xi']\rangle}$.

- Now, dependent indefinites are defined as (13).[3]
  - I adopt the composition in the style of *(Plural) Compositional DRT* Muskens (1996); Brasoveanu (2008), where E stands for a dref and T stands for a relation between plural information states.[4]

(13) $[\![\text{two-two}]\!] = \lambda \mathrm{P}_{\langle ET \rangle}\, \lambda \mathrm{Q}_{\langle ET \rangle}\, [u_m|];\mathrm{P}(u_m);\mathrm{Q}(u_m);[|\overline{u_m > 1}];[|\mathrm{Card}_2(u_m)]$

- The plurality condition $u_m > 1$ is post-supposed and thus it is evaluated against H, i.e. after all the at-issue contents are evaluated.
  - Accordingly, (9b) 'projects' from the scope of quantifiers.
  - This does not affect anything in non-quantificational environments, but this plays a crucial role in quantificational environments.

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[3] Henderson (2014) does not clarify how a dependent indefinite is combined with a lexical relation. One may define it in different ways, e.g., a modifier type.

(i) $[\![\text{two-two}]\!] = \lambda \mathrm{P}_{\langle ET \rangle}\, \lambda v_E\, \overline{v > 1};\mathrm{Card}_2(v);\mathrm{P}(v)$

[4] More specifically, I adopt a logic with four types: t for truth values, e for individuals, π for *registers* and s for *states*. Registers and states are used to model drefs and variable assignments, respectively. Note that drefs are defined as **constants** of type π and variable assignments are modeled as a primitive entity. Muskens (1996) postulates several axioms which ensure that states behave as variable assignments and define the non-logical operator ν that tells the value of a dref under a state. Accordingly, $g(u)$ can be taken as an abbreviation of $\nu(u)(g)$. On the top of that, Muskens (1996) provide appreviation conventions.

(i) Abbreviation 1 (Conditions):
  a. $R\{u_1, u_2, ..., u_n\} \Leftrightarrow \lambda g\, [R(\nu(u_1)(g))(\nu(u_2)(g))...(\nu(u_n)(g)))]$
  b. $u_1 = u_2 \Leftrightarrow \lambda g\, [\nu(u_1)(g) = \nu(u_2)(g)]$

(ii) Abbreviation 2 (DRS): $[u_1, ..., u_n, | C_1, C_2, ..., C_n]$
$= \lambda g \lambda h\, [g[u_1, ..., u_n]h \,\&\, C_1(h) \,\&\, C_2(h) \,\&\, ... \,\&\, C_n(h)]$

(iii) Abbreviation 3 (Sequencing): $K;K' = \lambda g \lambda h \exists k\, [K(g)(k) \,\&\, K'(k)(h)]$

These ensure that one may express DRT-representations as abbreviation of formulae in this logic.

(iv) a. $[u_1|\mathrm{dog}\{u_1\}, \mathrm{smiled}\{u_1\}]$
  b. $= \lambda g \lambda h\, [g[u_1]h \,\&\, \mathrm{dog}(\nu(u_1)(h)) \,\&\, \mathrm{smiled}(\nu(u_1)(h))]$

Now, type e and t in static system corresponds to type π type $\langle s, \langle st \rangle \rangle$, respectively. Denotations in the standard compositional semantics can be rewritten with these capital types.

- Consider the following toy examples originally from Kuhn (2017).

(14) a. Three students saw two-two zebras.
b. Each student saw two-two zebras.

- A PCDRT-style denotation of (14a) is given in (15).

(15) $[u_1|]; [|\text{students}(u_1)]; [|\text{three}(u_1)]; [u_2|]; [|\overline{u_2 > 1}]; [|\text{Card}_2(u_2)]; [|\text{zebras}(u_2)]; [|\text{saw}(u_1)(u_2)]$

- Here, the post-suppositional status of $u_m > 1$ does not matter.

- (15) requires that (i) u_2 stores multiple sets under H and (ii) the value of u_2 has 2 atoms in each member h of H.

- An example output of (15) is given in Table 4.

H	u_1	u_2
h_1	student$_1$	zebra$_1$+zebra$_2$
h_2	student$_2$	zebra$_3$+zebra$_4$
h_3	student$_3$	zebra$_5$+zebra$_6$

Table 4: Output for dependent indefinites

- In this plural information state, u_1 stores multiple sets under H, i.e. {zebra$_1$,zebra$_2$}, {zebra$_3$,zebra$_4$} and {zebra$_5$,zebra$_6$}.

- Also, the value of u_2 has two atoms in each of h_1, h_2 and h_3,

- Importantly, a plural information state with just a single information state does not satisfy $u_2 > 1$.

- In Table 5, u_1 only stores one set under H, i.e. {zebra$_1$+zebra$_2$}.

H	u_1	u_2
h_1	student$_1$+student$_2$+student$_3$	zebra$_1$+zebra$_2$

Table 5: an illicit output for dependent indefinites

- On the other hand, a PCDRT-style denotation of (14b) is given in (16).[5]

(16) $[u_1|]; \delta_{u_1}([|\text{student}(u_1)]; [|\text{atom}(u_1)]; [u_2|]; [|\overline{u_2 > 1}]; [|\text{Card}_2(u_2)]; [|\text{zebras}(u_2)];$
$[|\text{saw}(u_1)(u_2)])$

- In this case, the post-suppositional status of $u_2 > 1$ becomes important.
- Consider Table 4, assuming that there are three students.
- If $u_2 > 1$ were at-issue, it is evaluated with respect to H_{u_1}, i.e. h_1, h_2 and h_3.
- These information state each store only one set of values, i.e. {zebra$_1$,zebra$_2$} in u_1, {zebra$_3$,zebra$_4$} in u_2 and {zebra$_5$,zebra$_6$} in u_3.
- Thus, $u_2 > 1$ is not met.
- However, since $u_2 > 1$ is a post-supposition, it is evaluated against H, i.e. it projects through the scope of δ.
- As a result, it is evaluated against H in which u_2 stores three sets of values.
- Thus, it is predicted that dependent indefinites/numerals are felicitous under the scope of quantifiers while predicting them to be infelicitous with a singular argument.

- Before closing this section, let me point out that the δ operator in Henderson (2014) is defined so that it passes through post-suppositions under its scope.
- However, Brasoveanu (2013) defines the δ operator so that post-suppositions are **discharged** under its scope, i.e. δ is a 'post-supposition filter'.

[5]More precisely, quantificational determiners involves maximisation of the restrictor set and the scope set van den Berg (1996); Nouwen (2003); Brasoveanu (2008), but I omit it as it is orthogonal to the main point.

- I do not discuss their motivations here, but let me note that if post-suppositions project or not depends on the definition of the δ operator.[6]

3.2 Kuhn (2017): covert syntactic movement

- Kuhn (2017) proposes that a dependent indefinite introduces a new dref whose value can be divided into subpluralities each of which satisfies the cardinality condition.

  - He defines the 'outside' plurality condition as shown in (17).[7]

(17) $[\![\textbf{outside}(u_m/u_n)]\!] = \lambda G\,\lambda H\,[G = H\,\&\,|\{S : \exists d \in \nu(u_n)(H)\,[H_{u_m=d} = S\,]\}| > 1]$

  - Recall the definition of dependency: (17) checks if u_m is dependent on u_n.[8]

- Analogously, he defines the 'inside' cardinality condition as shown in (18), which checks the cardinality of each of the relevant subpluralities.

(18) $[\![\textbf{inside}(u_m/u_n) = n)]\!]$
$= \lambda G\,\lambda H\,[G = H\,\&\,\forall T \in \{S : \exists d \in \nu(u_n)(H)\,[H_{u_m=d} = S\,]\}\,[|T| = n]]$

  - Note that (9a) and (18) subtly differ:

  - (9a) distributes over each **member** of a plural information state while (18) distributes over **particular subsets** of a plural information states.

- Now, dependent indefinites are defined as (19).

[6] Brasoveanu (2013) proposes that the cardinality of (non-increasing) modified numerals are post-positional. Then, the following data suggests that their post-supposed cardinality condition has to be evaluated under the scope of δ: the cardinality of "exactly five" has to be evaluated last under the scope of δ, but not after "a cake."

(i) Every student δ(ate exactly five cupcakes) and I drank a coke.

However, see Charlow (2017) for an argument that one does not need post-supposition nor any scoping mechanism for the behaviour of (non-increasing) modified numerals.

[7] One key aspect of Kuhn (2017) is that dependent indefinites come with an anaphoric index, i.e. u_n in (17) and (18). This is motivated by a visible anaphoric signature of dependent indefinites in ASL. Although this has a great empirical importance, I will not go into details today.

[8] Note that Henderson (2014) does not require co-variation. In this sense, (17) is stronger than (9b).

- Note that it carries an anaphoric index u_n.

(19) $[\![\text{two-two}_{u_n}]\!]$
$= \lambda P \lambda Q\, [u_m|];P(u_m);Q(u_m);[|\,\text{outside}(u_m/u_n) > 1];[|\,\text{inside}(u_m/u_n) = 2]$

- On top of that, Kuhn (2017) proposes that a dependent indefinite undergoes (obligatory) quantifier raising (QR).[9]

- Here, Kuhn (2017) differs from Henderson (2014): Henderson (2014) adopt an in-situ syntax with semantic pseudo scoping mechanism while Kuhn (2017) a covert movement without semantic pseudo scoping mechanism.

- Consider the following toy examples again.

(14a) Three students saw two-two zebras.

(14b) Each student saw two-two zebras.

- I assume the following (simplified) LFs.[10]

(20) a. two-two zebras$_{u_1}$ λv [Three students^{u_1} saw t_v].
b. two-two zebras$_{u_1}$ λv [Each student^{u_1} saw t_v].

- On this point, two notes are in order.

- First, λ-abstraction is definable in (P)CDRT.[11]

- Second, the 'antecedent' of "two-two zebras" occurs below it, i.e. it is backward or left-to-right accosiation.

- Kuhn (2017) takes it as a welcome consequence of dynamic binding.

[9]For this reason, it is difficult to refine (19) with a non-quantificational type under Kuhn's (2017) system.

[10]Kuhn (2017) also assumes that the subject also undergoes QR. This QR is harmless with a non-quantificationla subject, but it reduces the scope possibility with a quantificational subject. Specifically, the subject has to scope below a dependent indefinite: if the subject takes the wide scope, it violates the outside condition of the dependent indefinite. For reasons of time, I will not discuss them.

[11]This is because drefs are modelled as **constants** of type π and traces can be defined as **variables** of type π.

- Furthermore, he assimilate it to the anaphoric component of "same," which does not obey weak crossover.[12]

(21) The same_{u_n} waiter served everyone^{u_n}.

- Now, the denotation of (14a) is composed as shown in (22)

(22) a. $[\![\text{three students saw } t_v]\!] =$
$[u_1|]; [|\text{students}\{u_1\}]; [|\text{three}\{u_1\}]; [|\text{saw}\{u_1\}\{v\}]$
b. $[\![\lambda v\,[\text{three students saw } t_v]]\!] =$
$\lambda v\,[u_1|]; [|\text{students}\{u_1\}]; [|\text{three}\{u_1\}]; [|\text{saw}\{u_1\}\{v\}]$
c. $[\![\text{two-two zebras}(\lambda v\,[\text{three students saw } t_v])]\!] =$
$[u_2|]; [|\text{zebras}\{u_2\}]; [u_1|]; [|\text{students}\{u_1\}]; [|\text{three}\{u_1\}]; [|\text{saw}\{u_1\}\{u_2\}];$
$[|\text{outside}(u_2/u_1) > 1]; [|\text{inside}(u_2/u_1) = 2]$

- Consider a possible output context in Table 6.

H	u_1	u_2
h_1	student_1	zebra_1
h_2	student_1	zebra_2
h_3	student_2	zebra_3
h_4	student_2	zebra_4
h_5	student_3	zebra_5
h_6	student_3	zebra_6

Table 6: Output for dependent indefinites

- The outside condition is met here: u_2 is dependent on u_1.

- The inside condition is also met: the value of u_2 in each co-varying pair has 2 cardinality.

- Now, consider (14b): its denotation is composed as shown in (23).[13]

[12] See Brasoveanu (2011) for a similar discussion on "different."
[13] Again, I ignore maximisation and the restrictor set for an expository reason.

(23) a. ⟦each student saw t_v⟧ =
$[u_1|]; \delta_{u_1}([|\text{student}\{u_1\}]; [|\text{atom}\{u_1\}]; [|\text{saw}\{u_1\}\{v\}])$

b. ⟦λv [each student saw t_v]⟧ =
$\lambda v\, [u_1|]; \delta_{u_1}([|\text{student}\{u_1\}]; [|\text{atom}\{u_1\}]; [|\text{saw}\{u_1\}\{v\}])$

c. ⟦two-two zebras(λv [each student saw t_v])⟧ =
$[u_2|]; [|\text{zebras}\{u_2\}]; [u_1|]; \delta_{u_1}([|\text{student}\{u_1\}]; [|\text{atom}\{u_1\}]; [|\text{saw}\{u_1\}\{u_2\}]);$
$[|\text{outside}(u_2/u_1) > 1]; [|\text{inside}(u_2/u_1) = 2]$

- Here, QR of the dependent indefinite successfully let the dependent indefinite escape from the scope of δ.

- Thus, Kuhn (2017) can be thought of as a syntactic implementation of Henderson (2014) without post-suppositions.

- Again, consider the same output Table 6, assuming that there are three students.

- The outside condition is met because u_2 is dependent on u_1.

- The inside condition is also met because the value of u_2 in each co-varying pair has 2 cardinality.

3.3 Brasoveanu and Farkas (2011): co-variation condition without built-in distributivity

- Brasoveanu and Farkas (2011) proposes that a dependent indefinite is an ordinal indefinite that comes with a co-variation condition.

- Their account does not take dependent indefinites to contribute to distributivity by themselves.

- (24) is a simplified version of their condition on dependent indefinites.

(24) ⟦dep(u_m/u_n)⟧
$= \{\langle G, H\rangle | G = H \,\&\, \exists h, h' \in H\, [h(u_n) \neq h'(u_n) \,\&\, h(u_m) \neq h'(u_m)]$

- With (24), the denotation of dependent indefinites is given as (25)

(25) ⟦two-two$_{u_n}$⟧ = $\lambda \text{P} \lambda \text{Q}\, [u_m|]; \text{P}(u_m); \text{Q}(u_m); [|\text{dep}(u_m/u_n)]$

- Essentially, (25) can be taken as a light version of Kuhn (2017),
  - i.e. it requires co-variation, but it does not contribute to distributivity.
  - Instead, it requires insertion of the covert δ operator when there is no overt quantifier above it.

4 Dependent indefinites in Turkish

- This is the main section, in which I discuss Turkish data.
  - Turkish has a suffix "(ş)Ar," which attaches to cardinals.
  - The unit NUM-şAr shows several signature of dependent indefinites.
- First, they force a distributive reading when they occur below a plural argument as exemplified in (26).

(26) Yedi çocuk üç-**er** oyuncak seç-ti.
seven child three-şer toy choose-past
"Seven children picked three toys each."

- At the same time, if NUM-şAr occurs in a sentence without any plural argument or if no plural argument occurs above it, it is unacceptable as exemplified in (27a) and (27b).[14]

(27) a. ?? Bir çocuk üç-**er** oyuncak seç-ti.
One child three-şer toy choose-past
"{A / one} child picked three toys each."

b. ?? Yedi-**şer** çocuk üç oyuncak seç-ti.
Seven-şer children three toy choose-past
(Intended) "Each group of seven children picked three toys."

- Lastly, NUM-şAr can occur under the scope of distributive universal quantifier without being redundant as exemplified in (28).

(28) Her çocuk üç-**er** oyuncak seç-ti.
Every child three-şer toy choose-past
"Every child picked three toys each."

[14] However, one speaker accepted (27a) and (27b).

- These observations suggest that NUM-şAr in Turkish patterns with dependent indefinites in languages such as Kaqchikel Mayan and ASL.

- Now, I explore its behaviour under negation.

- Negative examples are given in (29a) and (29b).[15]

(29) a. Bu yedi öğrenci üç-**er** müze-ye git-me-di.
this seven student three-şer museum-dat visit-neg-past
"These seven students did not visit three museums each."

b. Her öğrenci üç-**er** müze-ye git-me-di.
Every student three-şer museum-dat visit-neg-past
"Every student did not visit three museums each."

- Typical cases to use (29a) and (29b) are exemplified below.

(30) Context: seven students took a seminar of modern art. The lecturer told them that they should visit at least three museums in the city to see the general art style here. Now, it is the end of the semester and the lecture is asking about the students' visiting to museums.

a. Scenario 1: four of the seven students visited three museums during the semester, but the other three just visited one or two museums.
→ (29a) and (29b) are **true**

b. Scenario 2: three of the seven students visited three museums, but the other four didn't visit any museums. → (29a) and (29b) are **true**

- Atypical cases are exemplified below.

- Here, some students did not visit any museum, which makes (29a) and (29b) sound less natural than in Scenario 1 and 2.

(31) a. Scenario 3: none of the seven students visited any museums.
→ (29a) and (29b) are **true**

b. Scenario 4: all the seven students visited at least one museum, but none of them visited more than two museums.
→ (29a) and (29b) are **true**

- In the rest of this section, I discuss three aspects of dependent indefinites under negation, namely scope, co-variation and distributivity.

[15] I added "bu" (this) to the subject in (29a). This is to force a definite reading of the subject and eliminate possible readings in which the subject indefinite scopes under negation.

4.1 Intermediate scope of dependent indefinites

- In this section, I discuss the intermediate scope construal of dependent indefinites under negation.

- All the three accounts assume that dependent indefinites introduce a new value to a dref.

- This dref introduction takes place in-situ in Brasoveanu and Farkas (2011); Henderson (2014), but it takes place at the QR landing site in Kuhn (2017).

- Thus, the former is compatible with $\forall > \exists > \neg$ scope order, but the latter is not: **dependent indefinites have to outscope a quantifier** in Kuhn (2017).

- This intermediate scope reading is available in Turkish.

(32) Scenario 5: seven students took a seminar of modern art. The lecturer told them that they should visit several museums in the city to see the general art style here. This city has seven museums and the students may visit any of them. Now, it is the end of the semester and the lecture is asking about the students' visiting to museums. Interestingly, all the seven students visited exactly four museums. $\rightarrow$ (29a) and (29b) are **true**

- This intermediate scope reading in (29b) poses a problem for Kuhn (2017).

- In his account, a dependent indefinite has to be raised above a quantifier so that the dependency condition is satisfied.

- This means that the dependent indefinite introduce a new value above the scope of the quantifier, precluding the possibility of $\forall > \exists 3 > \neg$ scope order.

- Indeed, discourse anaphora is possible only under the $\forall > \exists 3 > \neg$ reading.

- First of all, anaphoric reference to a dref introduced under negation is not possible.

(33) Mary does not have a car. #It is red.

- (34) shows that a pronoun may pick up the value of dref that is introduced with "üç-er müze" (three museums each).

(34) O yedi öğrenci^{u_1} üç-er müze-ye^{u_2} git-me-di. Onlar-ı$_{u_2}$ beğen-m-iyor-lar.
That seven student three-şer museum-dat go-neg-past they-acc like-neg-pres-3pl
"Those seven students visited three museums each. They don't like them."

- In this continuation, it is most coherent if the students did not visit some museums **because** they do not like the museums.

- In contrast, (35) shows the opposite of (34).

(35) # O yedi öğrenci^{u_1} üç-er müze-ye^{u_2} git-me-di. (Ama) onlar-ı$_{u_2}$
That seven student three-şer museum-dat go-neg-past (but) they-acc
çok beğen-di-ler.
much like-past-3pl
"Those seven students didn't visit three museums each. (But) They liked them very much."

- In this continuation, it is most coherent if the students visited some museums and, **as a result**, they like the museums.

- The intended reading is that the students like the museums which they visit.

- This reading can be expressed with an overt noun phrase + a relative clause.

(36) O yedi öğrenci üç-er müze-ye git-me-di. (Ama) git-tik-leri
That seven student three-şer museum-dat go-neg-past (but) go-nomin-poss3pl
müze-ler-i çok beğen-di-ler.
museum-pl-acc much like-past-3pl
"Those seven students didn't visit three museums each. (But) they liked the museums they visited very much."

- Thus, the infelicity of (35) is most likely to be because negation scopes over the indefinite, i.e. (33) and (35) are bad for the same reason.

- However, Kuhn (2017) cannot predict this because "NUM-şer" always introduce a new value above negation.

4.2 Co-variation

- In this section, I discuss co-variation with dependent indefinites.

- Brasoveanu and Farkas (2011); Kuhn (2017) requires co-variation, but Henderson (2014) does not.

- If a co-variation condition is evaluated above negation, e.g., it scopes over negation or it projects from the scope of negation, it runs into a problem:

- when a dependent indefinite takes scope under negation, the co-variation condition outside negation cannot access this value.

- This predicts that dependent indefinites are always infelicitous under negation, which is falsified with (30).

- Another option is to assume that a co-variation condition is evaluated under negation as an at-issue content.

- Then, absence of co-variation should suffice to verify (29a) and (29b).

- However, it does not seems to be the case.

(37) Scenario 6 (accidental absence of co-variation): all the seven students independently visited the same three museums.

$\rightarrow$ (29a) and (29b) are **false**

- Thus, absence of co-variation does not verify (29a) and (29b).

- On this point, one may pursue for the possibility that dependent indefinites require **possible** co-variation, instead of **actual** co-variation.

- To check this, I test cases in which even possible co-variation is not possible.

- In (38), co-variation is contextually ruled out, i.e. there are only three LOR films. Here, use of NUM-şAr is judged infelicitous.[16]

(38) Scenario 7 (co-variation impossible): three students watched all the Lord of the Rings films.

[16] I deeply thank to Deniz Özyıldız for providing me this context. Note that one speaker judged the positive counterpart of (38b) true.

a. Her öğğrenci üç-er yüzüklerin efendisi filmi izle-me-di.
every student three-şAr lord of the rings film watch-neg-past
"Every student did not watch three LOR films each." → **infelicitous**

b. Bu üc öğğrenci üç-er yüzüklerin efendisi filmi izle-me-di.
this three student three-şAr lord of the rings film watch-neg-past
"These three students did not watch three LOR films each."
→ **infelicitous**

- This suggests that dependent indefinites in Turkish require possible co-variation, but it tolerates accidental absence of co-variation.

- Henderson (2014) make a too weak prediction: it should tolerate (38).

- Brasoveanu and Farkas (2011); Kuhn (2017) make a too strong prediction: they require **actual** co-variation.

• Interestingly, this co-variation condition seems to behave differently in different contexts.

- Here, consider two examples that minimally differ in polarity.

(39) a. Bu üç öğrenci üç-**er** müze-ye git-ti.
this three student three-şer museum-dat visit-past
"These three students visited three museums each."

b. Bu üç öğrenci üç-**er** müze-ye git-**me**-di.
this three student three-şer museum-dat visit-neg-past
"These three students did not visit three museums each."

- Then, consider two contexts that differ in whether co-variation is relevant.

- The question is, is the editor or the PhD student **truthfully speaking** to the professor in each of the following scenarios?

(40) Context 1 (co-variation relevant): three students are writers of a school biweekly newspaper in a university of fine art. A professor recommended their editor that the three students should visit 3-er museums so that they can write about as many museums as possible in this semester. A few weeks later, the students had a meeting with the editor. The editor told the professor that these three students {visited / did not visit} 3-er museums.

a. Scenario 1 (total co-variation): they each visited a different set of three museums, i.e. nine museums are visited in total.
b. Scenario 2 (partial co-variation): they each visited three museums, but they overlap in some museums. Between three and nine museums are visited in total.
c. Scenario 3 (no variation): they each visited three museums, but they happened to visit the same three museums.

(41) Context 2 (co-variation irrelevant): three BA students are supervised by a PhD student in the department of art. The supervisor of the PhD student asked her to ask the three students to visit three museums and write an essay on them. A few weeks later, the BA students had a meeting with the PhD student. The PhD student told the professor that these three students {visited / did not visit} 3-er museums.
a. Scenario 1 (total co-variation): they each visited a different set of three museums, i.e. nine museums are visited in total.
b. Scenario 2 (partial co-variation): they each visited three museums, but they overlap in some museums. Between three and nine museums are visited in total.
c. Scenario 3 (no variation): they each visited three museums, but they happened to visit the same three museums.

- Although the judgement does not seem super crisp, judgments from two informants differ based on the relevance of co-variation.

- The judgements I have gathered at this point is summarised in Table 7.

- I boldface the part in which informants disagree.

	Scenario 1	Scenario 2	Scenario3
Context 1 Aff	true	**??/true?**	**??/true?**
Context 1 Neg	false	true	**??/true**
Context 2 Aff	true	true	true
Context 2 Neg	false	**?/false**	**?/false**

Table 7: Distribution of judgements on “-şer”

- I need to gather more data to find out a clear tendency here, but there are two things that should be emphasised.

- First, (39a) uttered without (total) co-variation is regarded less truthful in Context 1, but it is more or less constantly truthful in Context 2.

- Second, (39b) uttered without (total) co-variation is judged **true** or infelicitous in Context 1, but is judged **false** or infelicitous in Context 2.

- On the other hand, "farklı" (different) require co-variation more actively.

- Note that it can even co-occur with "-şer."[17]

(42) a. Bu üç öğrenci üç(-**er**) **farklı** müze-ye git-ti.
this three student three(-şer) different museum-dat visit-past
"These three students visited three different museums (each.)"

b. Bu üç öğrenci üç(-**er**) **farklı** müze-ye git-**me**-di.
this three student three(-şer) different museum-dat visit-neg-past
"These three students did not visit three different museums (each.)"

- The judgement I could obtain at this point is summarised in Table 8.

- Crucially, the contribution of "farklı" (different) does not vary depending on the relevance of co-variation.

- Also, note that "farklı" (different) seems to require total co-variation.

	Scenario 1	Scenario 2	Scenario3
Context 1 Aff	true	false	false
Context 1 Neg	false	true	true
Context 2 Aff	true	false	false
Context 2 Neg	false	true	true

Table 8: Distribution of judgements on "(-şer) farklı"

[17] An informant reported that they prefer to put "-şer" because its presence eliminates another possible reading of "different" with respect to an entity mentioned in the prior discourse, an *external reading*.

- The important observations are:

- “(-şer) farklı” does not lead to infelicity due to redundancy,

- when co-variation is relevant, lack of co-variation leads to infelicity of “-şer,” and it does not matter, otherwise.

- Although much more work is necessary to say anything strong, this behaviour of ⟦-şer⟧ might be analogous to **non-maximal readings** of definite plurals that is sensitive to **Q(uestion) U(nder) D(iscussion)** (Malamud, 2012; Krifka, 1996; Križ, 2015, 2016; Križ and Spector, 2021, a.o.)

(43) A: The windows are open.

a. Scenario 1: A and B went on a trip. A storm is coming up. B asks whether the house will be safe. Only half the windows are closed.

→ (43) is **adequate**

b. Scenario 2: A and B hired painters to paint their house. They cannot start working until all the windows are open. A asks B whether the house is ready. Half of the windows are still closed.

→ (43) is **inadequate**

(Malamud, 2012)

- Križ (2015, 2016) propose that definite plurals involve **truth-value gap**, i.e. their truth conditions and falsity conditions are non-complementary.

- Those conditions in the grey zone are **pragmatically** grouped up with truth or falsity conditions, depending on how the context is partitioned.

- Furthermore, addition of “all” removes the truth-value gap (Brisson, 1998; Križ, 2016, a.o.).

- The contribution of “farklı” to “-şer” might be taken as something analogous to this contribution of “all” to definite plurals.

- Stay tune to see if this line of consideration is on the right track!

4.3 Distributivity

- In this section, I discuss distributivity condition of dependent indefinites.
- All the three accounts force distributivity in different ways.

- For Brasoveanu and Farkas (2011), distributivity comes from an overt quantifier or a covert δ operator.
- For Henderson (2014), it comes from the combination of the cardinality condition and the dynamic plurality condition.
- Similarly for Kuhn (2017), it comes from the combination of the outside condition and the inside condition.

- As their predictions are complicated, I start with data.

(44) Scenario 8 (collective action): three children collaborated to carry desks, e.g., they each hold a part of a desk to lift, carry and put down, and they carried five desks in this way.

a. O üç çocuk beş-er masa taşı-dı.
that three child five-şAr table carry-past
"Those three children carried five tables (each)."

→ **false** or **infelicitous**

b. O üç çocuk beş-er masa taşı-ma-dı.
that three child five-şAr table carry-neg-past
"Those three children did not carry five tables (each)."

→ **true** or **infelicitous**

- Judgement varies between true and infelicitous.

- This suggests that distributivity is hard-wired either as an at-issue condition or a not-at-issue condition.

- This result is most straightforward for Henderson (2014): post-supposed condition is evaluated under the scope of negation.

- The variation is just a matter of whether post-supposition violation leads to falsity or infelicity.

- It is tricky for Brasoveanu and Farkas (2011) because they do not have a way to just negate distributivity.

- In their account, dependent indefinites have to be under the scope of a (c)overt quantifier because of its dependency condition and distributivity itself comes from the (c)overt quantifier.

- Alternatively, one may claim that the oddness of (44b) is due to absence of co-variation.

- Its validity depends on whether the anomaly in (38) differs from the anomaly in (44b).

- Kuhn (2017) makes right predictions if QR lands below negation.

- Then, if (44b) is judged false or infelicitous depends on whether the co-variation condition is at-issue or not-at-issue.

- However, this requirement of QR landing site contradicts with (32).

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