Day 1: Dependent indefinites and quantificational dependencies

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1 Introduction

- Sentences with multiple plural arguments have several different readings.[1]

(1) Three students recited seven poems.
  a. **Collective reading**: a group of three students recited seven poems
  b. **Distributive reading**: Each of the three students recited seven (possibly) different poems
  c. **Semi-distributive reading**: Each of the three students recited the same seven poems
  d. **Cumulative reading**: each of the three students recited at least one poem and each of the seven poems are recited by at least one student.

- This series of lectures discuss expressions called *dependent indefinites*, which triggers an **obligatory** distributive reading (Choe, 1987; Gil, 1982, 1995; Farkas, 1997; Oh, 2001; Zimmermann, 2002; Balusu, 2006; Brasoveanu and Farkas, 2011; Henderson, 2014; Kuhn, 2017; Guha, 2018, a.o.).[2]

[1] I do not discuss a *paired cover reading*.

[2] Several terms with slightly different coverage compete in the market, e.g., *distributive numerals*, *share-markers*. I adopt "dependent indefinite" to focus on the expressions that trigger an obligatory distributive reading of plurals but felicitously sit under the scope of a distributive quantifier. For example, the adnominal "each" in English may be classified as a distributive numeral or a share-marker, but it is unclear if it is a dependent indefinite (However, Szabolcsi, 2010, claims that many speakers accept a binominal "each" under the scope of "every" or sometimes even under the scope of the determiner "each."). Also, Henderson (2016) distinguishes *dependent existentials*

(2) a. Xeqatij **ox-ox** wäy.
we-eat three-three tortilla
"We each ate three tortillas." (Kaqchikel Mayan, Henderson, 2014)

b. BOYS IX-arc-a read **one-arc-a** BOOK.
"The boys read one book each."
(American Sign Language, Kuhn, 2017)

- Dependent indefinites are incompatible with a singular argument.[3]

(3) a. * Xe'inchäp **ox-ox** wäy.
I-handle three-three tortilla
"I took (groups of) three tortillas."
(Kaqchikel Mayan, Henderson, 2014)

b. * JOHN-a READ **ONE-arc-a** BOOK.
"John read one book (each time).'
(American Sign Language, Kuhn, 2017)

- These observations may suggest that dependent indefinites themselves perform distributive quantification.

- Then, one would expect them to be infelicitous under the scope of another distributive quantifier, e.g., *Every student each bought two books.

- However, this is not the case.

(4) a. Chikijujunal ri tijoxela' xkiq'etej **ju-jun** tz'i'.
each the students hugged one-one dog
"Each of the students hugged a dog."
(Kaqchikel Mayan, Henderson, 2014)

b. EACH-EACH-a PROFESSOR NOMINATE **ONE-redup-a** STUDENT.
"Each professor nominated one student."
(American Sign Language, Kuhn, 2017)

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and *dependent numerals*: the former is formed with an indefinite marker or the numeral "one," and the latter is formed with numerals other than "one." While this distinction is associated with several interesting difference between them, I mostly ignore this distinction in the lectures.

[3]Dependent indefinites in some languages permit this, triggering a distributive reading with respect to events/occassions, e.g., Teligu reduplicative numerals, Korean "ssik," (Oh, 2001) Japanese "zutsu" (Champollion, 2017; Nakamura, 2021), and so on. I ignore this class until Day 3 lecture.

- Roughly speaking, there are two possible ways to capture this behaviour.

- *Dist-licensor approach*: a dependent indefinite itself does not perform distributive quantification, but it requires a distributive 'licensor' (Oh, 2001; Brasoveanu and Farkas, 2011; Guha, 2018, a.o.)

- In this view, (4) is the canonical case, and (2) involves a covert distributivity operator *Dist* which serves as a licensor.

- *Concord approach*: a dependent indefinite itself contributes to distributive quantification, but its contribution is made non-redundant with another distributive quantification (Balusu, 2006; Henderson, 2014; Kuhn, 2017).

- In this view, (2) is the canonical case in which a dependent indefinite itself perform distributive quantification, and (4) involves some mechanism to express single distribution with multiple distributive expressions.

- Both views take different stances on the treatment of distributive quantification and plural predication.

- Thus, investigation on dependent indefinites provide a window into the precise semantics of distributivity and plurality in natural language semantics.

- Day 1 lecture is devoted for an overview of these approaches.

- I foreground the notion of *quantificational dependencies*, and adopt the framework that achieves an explicit treatment of their dynamics, i.e. *Dynamic Plural Logic* (DPlL) (van den Berg, 1996, *et seq*), which is an extension of *Dynamic Predicate Logic* (DPL) (Groenendijk and Stokhof, 1991).

- The other lectures address two issues in dependent indefinites which have gathered little attention in the literature to the best of my knowledge.

- Day 2 focuses on their interaction with negation.

- Day 3 focuses on their interaction with cross-sentential anaphora.

- Investigation of these issues provides further material to decide which approach works the best for the semantics of dependent indefinites.

2 Background

- I adopt a dynamic semantic approach (Kamp, 1981; Heim, 1982; Groenendijk and Stokhof, 1991, a.o.)

- I take the meaning of a sentence to be an update on the current discourse.

- *Discourse referents* (drefs) u_1, u_2,..., are addresses in which some values are stored, i.e. variables.

- *Information states* g, h,..., keep track of what entities have been mentioned in the discourse, i.e. variable assignments.

- I illustrate an information state and drefs with a table as Table 1 exemplifies.

	u_1	u_2	u_3	...
g	Alex	Bede	Chris	...

Table 1: Drefs and information states

- *Assignment extension* updates an information state by adding a new value to u_n as defined in (5): g and h minimally differ in the new value on u_n.

(5) **(Singular) Assignment Extension**:
$g[u_n]h = \forall u\,[u \neq u_n \rightarrow g(u) = h(u)]$

- An indefinite introduces a new value to an information state and a pronoun obtains its value directly from the current information state.[4]

- I adopt an ordinary logic with three types: type e for individuals, type t for truth values, and type π for *registers*, which model drefs (Muskens, 1996).[5]

[4] At this point, I assume total assignments and put aside the issues with them.

[5] Note that I take an information state as a set of register-individual pairs, while Muskens (1996); Brasoveanu (2008) takes it as primitive entities of type s entities called *states*. Muskens (1996) sets up some axioms to ensure that type s entities behave as assignment functions. Here, v is a non-logical constant that takes a state and a register and returns an individual.

(i) a. Axiom 1: $\forall g \forall v \forall x \exists h\,[g[v]h \,\&\, v(h)(v) = x]$

- Note that individuals and drefs are modelled with distinct types: a dref is the name of a **constant** of type π.[6]

- A formula denotes a relation between information states, i.e. g is updated to h with ϕ.[7]

- The denotation function $[\![\ ]\!]$ maps a formula to a relation between information states.

- $[\![\phi]\!](g)(h)$ means that ϕ takes g as its input and h as its output.

- Truth is defined relative to a pair of information states.

(6) **Dynamic Truth**: a formula ϕ is true with respect to an information state g iff there is an information state h such that $[\![\phi]\!](g)(h)$.

- The resultant system comes with the subclausal compositionality in the stype of *Compositional DRT* (Muskens, 1996), which I will come back later.

2.1 Dynamic semantics for pluralities and distributivity

- On the top of it, I add further ingredients to model pluralities and distributivity following *Dynamic Plural Logic* (DPlL) (van den Berg, 1996).

- I adopt 'pluralities of information states': a *plural information state* (PIS) G, H,..., are modelled as a set of information state.

b. Axiom 2: $u_n \neq n_m$ and u_m for any two different dref u_n and u_m

Axiom 1 says that for every state, register and individual, there is another state which is minimally different from the original one with respect to the occupant of the given register. Axiom 2 says that different names of drefs refer to different entities of type π. This is necessary to avoid a situation in which u_n and u_m happen to be the names for the same register and an update on one affects the other. Muskens (1996) postulate two more axioms concerning the distinction between *specific* and *unspecific* drefs, which is orthogonal to the main point of this lecture.

[6] This enables one to implement λ-abstraction in dynamic semantics: one may distinguish a constant of type π from a variable of type π.

[7] One may alternatively assume that (i) a context is a set of information states and (ii) a formula denotes a **function** from an input context to an output context, i.e. *Heimian update*. Both options work the same in most cases, i.e. a formula is evaluated in a point-wise fashion with respect to each input-output pair. However, see Charlow (2017) for an argument that functional updates works better than relational updates to capture the cumulative readings of non-increasing modified numerals, but making an update fully functional over-generates.

- I illustrate a PIS as a matrix as exemplified in Table 2.

H	u_1	u_2	u_3	...
h_1	Alex	Bede	Chris	...
h_2	Dan	Elin	Fausto	...
h_3	Greg	Isobel	Hideki	...
...	...	...	...	...

Table 2: A plural information state

- I model singular individuals with singleton sets, e.g., $a = \{a\}$, and plural individuals with non-singleton sets, e.g., $a + b = \{a, b\}$.

\- One may obtain plural individuals by summing up the values spread across a PIS, e.g., $H(u_1) = \{\{\text{Alex}\},\{\text{Dan}\},\{\text{Greg}\}\}$ and $\cup H(u_1) = \{\text{Alex,Dan,Greg}\}$.

(7) **Value projection**:
$G(u_n) = \{x : g \in G \,\&\, g(u_n) = x\}$

\- One may obtain subsets of a plural assignments along with particular values of a dref u_n as given in (8).

(8) **Subset assignments**:
$G_{u_n=d} = \{g : g \in G \,\&\, g(u_n) = d\}$

- Now, one may assume that (7) is the only way to obtain plurals, or assume that an information state g itself may assign plural values to a dref.

(9) **Choice point I**:

a. **Single source view**: an information state g may only assign an atomic value to a dref u.

b. **Double source view**: an information state g may assign an atomic or a non-atomic value to a dref u.

- With (8), *dependencies* are defined in terms of co-variation as given in (10): u_m is dependent on u_n iff their values co-vary (van den Berg, 1996).

(10) **(Quantificational) Dependency**:
In a plural information state G, u_m is dependent on u_n iff
$\exists d, e \in G(u_n)\,[G_{u_n=d}(u_m) \neq G_{u_n=e}(u_m)]$

\- For example, u_2 is dependent on u_1 in I and J, but not in K in Table 3.

$$\begin{array}{|c|c|}\hline G & u_1 \\ \hline g_1 & x_1 \\ \hline g_2 & x_2 \\ \hline \end{array} \xrightarrow[{[u_2]}]{} \begin{array}{|c|c|c|}\hline I & u_1 & u_2 \\ \hline i_1 & x_1 & y_1 \\ \hline i_2 & x_2 & y_2 \\ \hline \end{array} \Big/ \begin{array}{|c|c|c|}\hline J & u_1 & u_2 \\ \hline j_1 & x_1 & y_1 \\ \hline j_2 & x_1 & y_2 \\ \hline j_3 & x_2 & y_2 \\ \hline \end{array} \Big/ \begin{array}{|c|c|c|}\hline K & u_1 & u_2 \\ \hline k_1 & x_1 & y_1 \\ \hline k_2 & x_1 & y_2 \\ \hline k_3 & x_2 & y_1 \\ \hline k_4 & x_2 & y_2 \\ \hline \end{array}$$

Table 3: Dependency

- The dynamic distributivity operator δ (van den Berg, 1996, *et seq*) introduces new dependencies to the discourse.

(11) $[\![\delta_{u_n}(\phi)]\!] = \lambda G \lambda H [G(u_n) = H(u_n) \& \forall d [d \in G(u_n) \rightarrow [\![\phi]\!](G_{u_n=d})(H_{u_n=d})]]$

- While it is not controversial that distributive quantification introduces new dependencies, it is controversial if plural predication may do so, too.

- Here lies another theoretical choice point.

(12) **Choice point II**:

a. **Dependencies via plural predication**: new dependencies can be introduced with either distributive quantification or plural predication.

b. **Dependencies via distributivity**: new dependencies can only be introduced with distributive quantification.

- One may choose a definition of plural assignment extension that may introduce new dependencies (Brasoveanu, 2008, *et seq*), or a definition of it that may not (van den Berg, 1996; Nouwen, 2007, *et seq*).

- (13) is defined as the point-wise generalisation of assignment extension $g[u]h$, and it may introduce new dependencies between u and old drefs.

(13) **Plural assignment extension (dependent)**:
$G[u]H \Leftrightarrow \forall g \in G \exists h \in H [g[u]h]] \& \forall h \in H \exists g \in G [g[u]h]$

- In contrast, the definition in (14) disallows u to be dependent on old drefs.

(14) **Plural assignment extension (dependency-free)**:
$G[u]H \Leftrightarrow \exists D [H = \{h | \exists g \exists d [g[u]h \& h(u) = d \& g \in G \& d \in D]\}]$

- In Table 3, (13) may produce all the PISs, but (14) may only produce K.

- Evaluation of lexical relations can be collective or distributive.[8]

(15) a. $[\![\mathrm{R}(u_1)...(u_n)]\!] = \lambda G \lambda H\, [G = H \,\&\, \forall h \in H\, [\langle h(u_1),...,h(u_n)\rangle \in I(\mathrm{R})]]$ (distributive)

b. $[\![\mathrm{R}(\cup u_1)...(\cup u_n)]\!] = \lambda G \lambda H\, [G = H \,\&\, \langle H(u_1),...,H(u_n)\rangle \in I(\mathrm{R})]]$ (collective)

- Number restrictions are (often) defined as collective conditions.[9]

(16) a. **Static atomicity**:
$\mathrm{atom}(x) \Leftrightarrow \forall y\, [y \subseteq x \rightarrow y = x]$

b. $[\![\mathrm{Atom}(u)]\!] = \lambda G \lambda H\, [G = H \,\&\, \mathrm{atom}(H(u))]$

c. $[\![\mathrm{Non\text{-}atom}(u)]\!] = \lambda G \lambda H\, [G = H \,\&\, \neg\mathrm{atom}(H(u))]$

- One may define the cardinality condition analogously (van den Berg, 1996; Brasoveanu, 2008), but Henderson (2014); Kuhn (2017); Guha (2018) define it distributively to model the semantics of dependent indefinites.

- As for plain indefinites, they make sure that their cardinality conditions are collectively evaluated. See Section 3 for details.

2.2 Subclausal compositionality

- The version of DPlL defined above comes with the classical Heim-and-Krazter style subclausal compositionality (Brasoveanu, 2007, 2008).

[8] It is also a theoretical choice point: van den Berg (1996) only adopt the collective evaluation of relations. The same holds for Law (2020); Roelofsen and Dotlačil (2023) but they additionally assume lexical cumulativity.

[9] Not everyone adopts this approach, though. Minor (2022); Dotlačil and Roelofsen (2021); Roelofsen and Dotlačil (2023) define the atomicity condition distributively, i.e. $\mathrm{Atom}(u)$ requires the value of u in each g of G to be atomic. This allows cases in which u is atomic in each g but plural in G. However, they adopt an additional requirement that the values of u across g in G to be constant. The conjunction of the distributive atomicity condition and this constanct condition emulates the collective atomicity condition. See Minor (2022); Dotlačil and Roelofsen (2021); Roelofsen and Dotlačil (2023) for motivations to this approach.

- The type logical language defined above plays the role of *intermediate language*, which expressions of an object language is mapped to and whose expressions function as **abbreviations** of dynamic meaning representations.

- Take the example (17a) and its DRT-style representation (17b) often called a *discourse representation structure* (DRS).

(17) a. A dog smiled.
b. $[x|\text{dog}(x), \text{smiled}(x)]$

- (17b) can be expressed as abbreviations of the type-logical object language with the following abbreviation conventions.

(18) **DRS abbreviation**:
$[u_1, ..., u_n, |C_1, C_2, ..., C_n]$
$= \lambda G \lambda H [G[u_1, ..., u_n]H \,\&\, C_1(H) \,\&\, C_2(H) \,\&\, ... \,\&\, C_n(H)]$

- A DRS is an abbreviation of a relation between PISs so that (i) these minimally differ in newly introduced drefs and (ii) conditions check if an input PIS satisfies these.[10]

- Dynamic conjunction is defined in (19) with the sequencing operator ';'.

(19) $D;D' = \lambda G \lambda H \exists K [D(G)(K) \,\&\, D'(K)(H)]$

- Now, (17a) is written as (20a), which is an abbreviation of (20b).

(20) a. $[u_1|\text{dog}(u_1),\text{Atom}(u_1),\text{smiled}(u_1)]$
b. $= \lambda G \lambda H [G[u_1]H \,\&\, \text{dog}(H(u_1)) \,\&\, \text{atom}(H(u_1)) \,\&\, \text{smiled}(H(u_1))]$

- One can define "meta-type" (Brasoveanu, 2008), which makes types in this logic and types in H&K semantics homomorphic.

- I use E for π, and T for $\langle st, \langle st, t \rangle \rangle$, relation between PISs.

[10] There is another abbreviation concerning negation, disjunction and material implication. They are essentially the same as their DPL counterparts.

(i) a. not $D = \lambda G \neg \exists K [D(G)(K)]$ c. $D \Rightarrow D' = \lambda G \forall I [D(G)(I) \rightarrow \exists J [D'(I)(J)]]$
b. D or $D' = \lambda G \exists K [D(G)(K) \vee D'(G)(K)]$

- e.g., $\langle et\rangle$ corresponds to $\langle ET\rangle$ and $\langle et,\langle et,t\rangle\rangle$ corresponds to $\langle ET,\langle ET,T\rangle\rangle$.

- Now, (20b) is compositionally obtained in the H&K way as shown in (21).

(21)

- $[u_1|];[|\,\mathrm{dog}(u_1)];[|\,\mathrm{Atom}(u_1)];[|\,\mathrm{smiled}(u_1)]$
  - $\lambda \mathrm{Q}\,[u_1|];[|\,\mathrm{Atom}(u_1)];[|\,\mathrm{dog}(u_1)];\mathrm{Q}(u_1)$
    - $\lambda \mathrm{P}\lambda \mathrm{Q}\,[u_1|];[|\,\mathrm{Atom}(u_1)];\mathrm{P}(u_1);\mathrm{Q}(u_1)$
      - a
    - $\lambda v\,[|\,\mathrm{dog}(v)]$
      - dog
  - $\lambda v\,[|\,\mathrm{smiled}(v)]$
    - smiled

- A sequence of DRSs can be simplified with *Merging Lemma*.

(22) **Merging Lemma** (Muskens, 1996): If $u'_1, ..., u'_n$ do not occur in any of $C_1, ..., C_n$, then the following two DRSs are equivalent.

a. $[u_1, ..., u_n|C_1, ..., C_n];[u'_1, ..., u'_n|C'_1, ..., C'_n]$

b. $[u_1, ..., u_n, u'_1, ..., u'_n|C_1, ..., C_n, C'_1, ..., C'_n]$

- Then, (21) is equivalent to (20a), which is an abbreviation of (20b).

3 Approaches to dependent indefinites

- Let's come back to the puzzle of dependent indefinites.

- The puzzle is that they seem to perform distributive quantification with a plural term but their contribution seems inert with a distributive quantifier.

- Here, I review three previous approaches to dependent indefinites.

- Henderson (2014): dependent indefinites encode the distributive cardinality condition and the dynamic plurality condition. The latter is evaluated after all the other conditions are evaluated, i.e. it is *post-supposed*.

- Kuhn (2017): dependent indefinites encode the distributive cardinality condition and the co-variation condition. Dependent indefinites scopes over quantifiers via *Quantifier Raising* (QR).

- Guha (2018): dependent indefinites encode the co-variation condition, which requires presence of an overt quantifier or a covert distributivity operator.

- I take Henderson (2014); Kuhn (2017) as instances of the Concord approach, and Guha (2018) as an instance of the Dist-licensor approach.[11]

3.1 Henderson (2014): post-supposed plurality condition

- He adopts the **double source view**, and **dependent plural extension**.

- As for the cardinality condition for plain indefinites, he combines the distributive cardinality condition with collective atomicity/plurality condition, which he calls *evaluation plurality*.

(23) a. **Distributive cardinality condition**:
$[\![\mathrm{Card}_n(u_n)]\!] = \lambda G \lambda H [G = H \,\&\, \forall h \in H [|\{x | x \subseteq h(u_n) \,\&\, \mathrm{atom}(x)]\}| = n]$

b. **Evaluation atomicity**:
$[\![u_n = 1]\!] = \lambda G \lambda H [G = H \,\&\, |H(u_n)| = 1]$

c. **Evaluation plurality**:
$[\![u_n > 1]\!] = \lambda G \lambda H [G = H \,\&\, |H(u_n)| > 1]$

- Notice that (23a) counts the number of atoms, but (23c) counts **the number of sets a dref stores under a PIS**, i.e. a set of sets.

- A plain indefinite is defined with the distributive singularity and the evaluation atomicity.[12]

- $[| u_n = 1]$ ensures that the value of u_1 is singular under the output PIS.

(24) $[\![\text{one}]\!] = \lambda P_{\langle ET \rangle} \lambda Q_{\langle ET \rangle} [u_n|]; [| u_n = 1]; [| \mathrm{Card}_1(u_1)]; [| P(u_1)]; [| Q(u_1)]$

- In contrast, a dependent indefinite is defined with the distributive cardinality condition plus the evaluation **plurality** condition.[13]

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[11] Brasoveanu and Farkas (2011) is another instance of the Dist-licensor approach. Their theory is based on what they call *First-order logic with choice* (C-FOL), in which a formula is evaluated relative to a **set** of assignments. Thus, C-FOL can be taken as a static version of DPlL and DPlL can be taken as a dynamic version of C-FOL for our purpose.

[12] Henderson (2014) does not discuss a compositional implementation of his analysis, and (24) is my rendering of his analysis of indefinite in the style of PCDRT.

[13] If one prefers, one may also define it as a modifier.

(i) $[\![\text{two-two}]\!] = \lambda \mathrm{P}_{\langle ET \rangle} \lambda v_E [| \overline{v > 1}]; [| \mathrm{Card}_2(v)]; [| P(v)]$

(25) $[\![\text{one-one}]\!] = \lambda P_{\langle ET\rangle}\lambda Q_{\langle ET\rangle}[u_n|];[|u_n > 1];[|\text{Card}_1(u_1)];[|P(u_1)];[|Q(u_1)]$

- $[|u_n > 1]$ ensures that u_1 stores **multiple sets** under the output PIS.

• The idea is that (23c) makes sure that a dependent indefinite requires pluralities or quantification to be licensed.

- However, if it is evaluated under the scope of δ, it cannot be satisfied, i.e. each subset of a PIS contains at most one set.

- To circumvent it, he evaluates (23c) all the at-issue-contents are evaluated, i.e. it is **post-supposed**.

• Intuitively, post-suppositions can be considered as the mirror image of presuppositions: they are evaluated against the output context, just like presuppositions being evaluiated against the input context.

- Formally, post-suppositions are particular types of tests which plural information states are indexed with, which I mark with an overline, i.e. $\overline{\phi}$.[14]

(26) $[\![\overline{\phi}]\!](G[\xi])(H[\xi'])$ is true iff ϕ is a test, $G = H$ and $\xi' = \xi \cup \{\phi\}$

- A post-supposition does not update an input context and, instead, add a new test to the set of tests [ ], i.e. a new post-supposition ϕ is added to a set of post-suppositional test ξ and yield ξ'.

• The definition of truth incorporates the contribution of post-suppositions.

(27) **Dynamic truth with post-suppositions**: ϕ is true with respect to an input plural information state $G[\emptyset]$ iff there is an output plural information state H and a (possibly empty) set of tests $\{\psi_1,...,\psi_m\}$ such that $[\![\phi]\!](G[\emptyset])(H[\{\psi_1,...,\psi_m\}])$ and $[\![\psi_1 \& ... \& \psi_m]\!](H[\emptyset])(H(\emptyset))$.

• The contribution of the at-issue content of ϕ remains the same.

- However, now the post-suppositions of ϕ has to be true relative to the output context for ϕ to be dynamically true.

- i.e. $[\![\psi_1 \& ... \& \psi_m]\!](H[\emptyset])(H(\emptyset))$ should hold if ϕ post-supposes them.

[14] In his original notation, $[\![\overline{\phi}]\!]^{\langle G[\xi],H[\xi']\rangle}$.

- Since post-suppositions are evaluated relative to the output context, it 'projects' from the scope of operators.

(28) $\text{Op}(A \,\&\, \overline{\phi} \,\&\, B) = \text{Op}(A \,\&\, B) \,\&\, \overline{\phi}$ adopted from Kuhn (2017)

- The entry of dependent indefinites is refined as (29).

(29) $[\![\text{two-two}]\!] = \lambda \text{P}_{\langle ET \rangle} \lambda \text{Q}_{\langle ET \rangle} [u_n|]; [|\overline{u_n > 1}]; [|\text{Card}_2(u_n)]; [|P(u_n)]; [|Q(u_n)]$

- The 'post-supposed' evaluation plurality $u_n > 1$ is evaluated against the output context H, i.e. after all the at-issue contents are evaluated.

- This does not affect anything in non-quantificational environments, but this plays a crucial role in quantificational environments.

- Consider the following toy examples originally from Kuhn (2017).

(30) a. Three students saw two-two zebras.
b. Each student saw two-two zebras.

- (31) shows the DRS of (30a). Here, the post-suppositional status of $u_2 > 1$ does not matter.

(31) $[u_1|]; [|\text{students}(u_1)]; [|\text{three}(u_1)]; [u_2|]; [|\overline{u_2 > 1}]; [|\text{Card}_2(u_2)]; [|\text{zebras}(u_2)]; [|\text{saw}(u_1)(u_2)]$

- (31) requires that (i) u_2 stores multiple sets under H and (ii) the value of u_2 has 2 atoms in each member h of H.

- An example output of (31) is given in Table 4.

H	u_1	u_2
h_1	student_1	$\text{zebra}_1 + \text{zebra}_2$
h_2	student_2	$\text{zebra}_3 + \text{zebra}_4$
h_3	student_3	$\text{zebra}_5 + \text{zebra}_6$

Table 4: Output for dependent indefinites

- Here, u_2 stores sets of two atomic zebras in each member of H, i.e. {zebra$_1$, zebra$_2$}, {zebra$_3$, zebra$_4$} and {zebra$_5$, zebra$_6$}.[15]

- Also, $H(u_2)$ = {{zebra$_1$, zebra$_2$},{zebra$_3$, zebra$_4$},{zebra$_5$, zebra$_6$}}.[16]

- Thus, the evaluation plurality condition is met: $|H(u_2)| = 3 > 1$.

- On this point, notice that one may consider an output PIS such as Table 5 since Henderson (2014) adopts the double source view.

H	u_1	u_2
h_1	student$_1$+student$_2$+student$_3$	zebra$_1$+zebra$_2$

Table 5: an illicit output for dependent indefinites

- However, such 'singleton' PISs never satisfy $u_2 > 1$: u_1 only stores one set under H, i.e. {zebra$_1$+zebra$_2$} in Table 5.

- Such PISs are discarded due to violation of post-supposed $u_2 > 1$ test.

- Now, I turn to cases with an overt distributive quantification.

- The DRS of (30b) is given in (32).[17]

(32) $[u_1|]; \delta_{u_1}([|\text{student}(u_1)]; [|\text{Card}_1(u_1)]; [u_2|]; [|\overline{u_2 > 1}];$
$[|\text{Card}_2(u_2)]; [|\text{zebras}(u_2)]; [|\text{saw}(u_1)(u_2)])$

- In this case, the post-suppositional status of $u_2 > 1$ becomes important.

- Assuming that there are three students, take the same PIS repeated below.

[15]Note that Card$_2(u_2)$ requires each information state to store a plurality of two zebras, and an alternative PIS in Table 7 violates Card$_2(u_2)$because each information state only stores an atomic zebra. Thus, (23a) is incompatible with the single source view.

[16]If each member of PIS assigns the same value to u_2 under H, $|H(u_2)| = 1$ and thus $u_n > 1$ forces 'unselective' co-variation condition.

[17]More precisely, quantificational determiners involves maximisation of the restrictor set and the scope set van den Berg (1996); Nouwen (2003); Brasoveanu (2008), but I omit it as it is orthogonal to the main point.

H	u_1	u_2
h_1	student$_1$	zebra$_1$+zebra$_2$
h_2	student$_2$	zebra$_3$+zebra$_4$
h_3	student$_3$	zebra$_5$+zebra$_6$

Table 6: Output for dependent indefinites

- If $u_2 > 1$ were at-issue, it is evaluated with respect to H_{u_1}, i.e. h_1, h_2 and h_3.
- These information state each store only one set of values, i.e. {zebra$_1$,zebra$_2$} in u_1, {zebra$_3$,zebra$_4$} in u_2 and {zebra$_5$,zebra$_6$} in u_3.
- Thus, $u_2 > 1$ is not met.

• On the other hand, if $u_2 > 1$ is post-supposed, it is evaluated against the output context H, i.e. it projects through the scope of δ, in effect.

(33) (32)=$[u_1|]; \delta_{u_1}([|\text{student}(u_1)]; [|\text{Card}_1(u_1)]; [u_2|];$
$[|\text{Card}_2(u_2)]; [|\text{zebras}(u_2)]; [|\text{saw}(u_1)(u_2)]); [|\overline{u_2 > 1}]$

- As a result, $u_n > 1$ is collectively evaluated against H in which u_2 stores three sets of values.
- Thus, it correctly predicts that dependent indefinites are felicitous under the scope of quantifiers while being **in**felicitous with a singular argument.

• Note that Henderson (2014) does not need a covert distributivity operator to license a dependent indefinite, but it is compatible with it.

- (34) is a variant of (31) with a covert δ.

(34) $[u_1|]; [|\text{students}(u_1)]; [|\text{three}(u_1)]; \delta_{u_1}([u_2|]; [|\overline{u_2 > 1}]; [|\text{Card}_2(u_2)];$
$[|\text{zebras}(u_2)]; [|\text{saw}(u_1)(u_2)])$
$= [u_1|]; [|\text{students}(u_1)]; [|\text{three}(u_1)]; \delta_{u_1}([u_2|]; [|\text{Card}_2(u_2)];$
$[|\text{zebras}(u_2)]; [|\text{saw}(u_1)(u_2)]); [|\overline{u_2 > 1}]$ (post-supposition)

3.2 Kuhn (2017): covert syntactic movement

- He adopts the **single source view**, and **dependent plural extension**.
  - As for the cardinality of plain indefinites, he defines a collective cardinality condition, $\text{inside}(u) = n$.

(35) $[\![\text{inside}(u) = n]\!] = \lambda G\, \lambda H\, [G = H \,\&\, |H(u)| = n]$

- Kuhn (2017) adopts a condition similar to $u > 1$ in Henderson (2014).
  - Instead of adopting two types of pluralities, he divides the value of a dref into subpluralities and require each of them to contain pluralities.
  - This is defined as the 'outside' plurality condition as shown in (36).

(36) $[\![\text{outside}(u_m/u_n)]\!]$
$= \lambda G \lambda H\, [G = H \,\&\, |\{S : \exists d \in H(u_n)\, [H_{u_n=d}(u_m) = S\,]\}| > 1]$

  - Recall the definition of dependency: (36) checks if u_m is dependent on u_n.

- Analogously, he defines the dependent version of $\text{inside}(u) = n$ as shown in (37), which checks the cardinality of each of the relevant subpluralities.

(37) $[\![\text{inside}(u_m/u_n) = n]\!]$
$= \lambda G \lambda H\, [G = H \,\&\, \forall T \in \{S : \exists d \in H(u_n)\, [H_{u_n=d}(u_m) = S\,]\}\, [|T| = n]]$

- While Henderson's (2014) distributive cardinality condition distributes over each **member** of a PIS, (37) distributes over **particular subsets** of a PIS.
  - This difference comes from their difference in Choice point I.

- Now, dependent indefinites are defined as (38).

(38) $[\![\text{two-two}_{u_n}]\!] =$
$\lambda \mathrm{P} \lambda \mathrm{Q}\, [u_m|]; [|\, P(u_m)]; [|\, Q(u_m)]; [|\, \text{outside}(u_m/u_n) > 1]; [|\, \text{inside}(u_m/u_n) = 2]$

  - Note that it carries an anaphoric index u_n that occurs in (36) and (37).
  - It is one key aspect of Kuhn (2017).[18]

[18] This is motivated by a visible anaphoric signature of dependent indefinites in ASL. Although this has a great empirical importance, I will not go into details today.

- Then, Kuhn (2017) also has a mechanism to deal with a dependent indefinite under the scope of an overt distributive quantifier.

- He proposes that a dependent indefinite undergoes (obligatory) quantifier raising (QR).[19,20]

- i.e. Henderson (2014) takes an in-situ syntax with pseudo scope mechanism, but Kuhn (2017) takes a covert movement without such a mechanism.

- Consider the following toy examples again.

(30a) Three students saw two-two zebras.

(30b) Each student saw two-two zebras.

- I assume the following (simplified) LFs.[21]

(39) a. two-two zebras$_{u_1}$ λv [Three students^{u_1} saw t_v].
b. two-two zebras$_{u_1}$ λv [Each student^{u_1} saw t_v].

- On this point, notice that the antecedent of "two-two zebras" occurs below it, i.e. it is backward or left-to-right accociation.

- Kuhn (2017) takes it as a welcome consequence of dynamic binding.

- Furthermore, he assimilate it to the anaphoric component of "same," which is not sensitive to weak crossover effect.[22]

(40) The same$_{u_n}$ waiter served everyone^{u_n}.

- I am not sure if this is a welcome result, though.

- Guha (2018) reports that a dependent indefinite in Bangla cannot be licensed at the subject position even if the object is a licit licensor.

[19] If one wishes to define dependent indefinite with a modifier type, one has to assume that indefinites under go QR just like 'canonical' quantifiers.

[20] Recall that λ-abstraction is definable in (P)CDRT because drefs are modelled as **constants** of type π and traces can be defined as **variables** of type π.

[21] Kuhn (2017) also assumes that the subject also undergoes QR. This QR is harmless with a non-quantificationla subject, but it reduces the scope possibility with a quantificational subject. Specifically, the subject has to scope below a dependent indefinite: if the subject takes the wide scope, it violates the outside condition of the dependent indefinite.

[22] See Brasoveanu (2011) for a similar discussion on "different."

(41) # du-jon-**kore**-mee car-Te-boi poRe-che
two-CL-KORE-girl four-CL-book read-be.Pres.3rd
"Four books were read by two girls each." (Guha, 2018)

- If one fully assimilates a dependent indefinite with "same," sentences such as (41) should be acceptable.

- Now, the DRS of (30a) is composed as shown in (42)

(42) a. $[\![\text{three students saw } t_v]\!] =$
$[u_1|]; [|\text{students}\{u_1\}]; [|\text{three}\{u_1\}]; [|\text{saw}\{u_1\}\{v\}]$

b. $[\![\lambda v\,[\text{three students saw } t_v]]\!] =$
$\lambda v\,[u_1|]; [|\text{students}\{u_1\}]; [|\text{three}\{u_1\}]; [|\text{saw}\{u_1\}\{v\}]$

c. $[\![\text{two-two zebras}(\lambda v\,[\text{three students saw } t_v])]\!] =$
$[u_2|]; [|\text{zebras}\{u_2\}]; [u_1|]; [|\text{students}\{u_1\}]; [|\text{three}\{u_1\}]; [|\text{saw}\{u_1\}\{u_2\}];$
$[|\text{outside}(u_2/u_1) > 1]; [|\text{inside}(u_2/u_1) = 2]$

- Consider a possible output context in Table 7.

- Notice that each information state stores an atomic value in each dref in this PIS unlike the one in Table 4.

H	u_1	u_2
h_1	student_1	zebra_1
h_2	student_1	zebra_2
h_3	student_2	zebra_3
h_4	student_2	zebra_4
h_5	student_3	zebra_5
h_6	student_3	zebra_6

Table 7: Output for dependent indefinites

- Both the outside condition and the inside condition are met here:

- u_2 is dependent on u_1, and its value in each co-varying pair has 2 cardinality.

- Now, consider (30b): its DRS is composed as shown in (43).[23]

[23] Again, I ignore maximisation and the restrictor set for an expository reason.

(43) a. $[\![\text{each student saw } t_v]\!] =$
$[u_1|]; \delta_{u_1}([|\text{student}\{u_1\}]; [|\text{atom}\{u_1\}]; [|\text{saw}\{u_1\}\{v\}])$

b. $[\![\lambda v\,[\text{each student saw } t_v]]\!] =$
$\lambda v\,[u_1|]; \delta_{u_1}([|\text{student}\{u_1\}]; [|\text{atom}\{u_1\}]; [|\text{saw}\{u_1\}\{v\}])$

c. $[\![\text{two-two zebras}(\lambda v\,[\text{each student saw } t_v])]\!] =$
$[u_2|]; [|\text{zebras}\{u_2\}]; [u_1|]; \delta_{u_1}([|\text{student}\{u_1\}]; [|\text{atom}\{u_1\}]; [|\text{saw}\{u_1\}\{u_2\}]);$
$[|\text{outside}(u_2/u_1) > 1]; [|\text{inside}(u_2/u_1) = 2]$

- Here, QR of the dependent indefinite successfully make the dependent indefinite escape from the scope of δ.

- Thus, Kuhn (2017) can be thought of as a syntactic implementation of Henderson (2014) without post-suppositions.

- Consider the same output Table 7, assuming that there are three students.

- The outside condition is met because u_2 is dependent on u_1.

- The inside condition is also met because the value of u_2 in each co-varying pair has 2 cardinality.

- This result is in parallel with Henderson (2014): the relevant plurality/co-variation condition escapes from the scope of δ.

- The only difference is whether this escaping is achieved with semantic scope shifting or syntactic scope shifting.

- In Day 2, I come back to this difference in relation to the interaction between dependent indefinites and negation.

- Lastly, on a par with Henderson (2014), the analysis in Kuhn (2017) is compatible with insertion of a covert distributivity operator: QR of a dependent indefinite makes it escape from the scope of the covert δ.

3.3 Guha (2018): co-variation triggers distributive licensing

- She adopts the **double source view**, and **dependent plural extension**.

- However, her analysis is compatible with dependency-free plural extension unlike Henderson (2014) and Kuhn (2017).

- As for the cardinality of plain indefinite, she adopts the condition *same*.[24]

(44) $[\![same(u)]\!] = \lambda G \lambda H\, [G = H \,\&\, \forall d, e \in H(u)\, [H_{u=d}(u) = H_{u=e}(u)]]$

- (44) requires u to have the same value across members of a PIS.

- Guha (2018) takes it as a post-supposition.

- A plain indefinite is defined with the combination of $\mathrm{Card}_2(u_n)$ and $same(u_n)$.

(45) $[\![\mathrm{two}]\!] = \lambda P_{\langle ET \rangle} \lambda Q_{\langle ET \rangle}\, [u_n|]; [|\, \mathrm{Card}_2(u_n)]; [|\, \overline{same(u_n)}]; [|\, P(u_n)]; [|\, Q(u_n)]$

- Note that it is stronger than what Henderson (2014) defines: $same(u)$ requires the values of u to be the same in every member of a PIS.

- Consider the cumulative reading of (46a), whose DRS is given in (46b).[25]

(46) a. Two girls^{u_1} read three books^{u_2}.

b. $[u_1|]; [|\, \mathrm{Card}_2(u_1)]; [|\, \overline{same(u_1)}]; [|\, \mathrm{girls}(u_1)]; [u_2|]; [|\, \mathrm{Card}_3(u_2)];$
$[|\, \overline{same(u_2)}]; [|\, \mathrm{books}(u_2)]; [|\, \mathrm{read}(u_1)(u_2)]$

- Since Guha (2018) adopts dependent plural extension, a PIS exemplified in Table 8 can be considered.

H	u_1	u_2
h_1	girl_1	book_1
h_2	girl_1	book_2
h_3	girl_2	book_3

Table 8: Cumulative dependency at the level of a PIS

- This kind of PISs is essential to license a dependent indefinite without δ.

- However, such a PIS violates $same(u_1)$ and $same(u_2)$ because the values of u_1 and u_2 vary across the PIS.

- To satisfy them, one may only consider PISs exemplified in Table 9.

H	u_1	u_2
h_1	$\text{girl}_1+\text{girl}_2$	$\text{book}_1+\text{book}_2+\text{book}_3$

Table 9: A 'flat' representation of cumulative dependency

- In this sense, her sameness condition 'supresses' dependencies just like dependency-free plural extension.

- I turn to dependent indefinites: a dependent indefinite encodes another post-supposition $dif(u_m/u_n)$.[26]

(47) $[\![dif(u_m/u_n)]\!] = \lambda G \lambda H\, [G = H \,\&\, \exists d, e \in H(u)\, [H_{u_n=d}(u_m) \neq H_{u_n=e}(u_m)]]$

- Here, u_m has to be dependent on u_n, i.e. a contribution similar to outside(u_m/u_n).

- A dependent indefinite is defined with this condition.

(48) $[\![\text{two-two}_{u_m}]\!] =$
$\lambda P_{\langle ET \rangle} \lambda Q_{\langle ET \rangle}\, [u_n|]; [|\text{Card}_2(u_n)]; [|\overline{dif(u_m/u_n)}]; [|P(u_n)]; [|Q(u_n)]$

- For Guha (2018), the difference condition is a primary trigger for insertion of the covert distributivity operator.[27]

- Take the following toy examples again.

(30b) Each student saw two-two zebras.

(30a) Three students saw two-two zebras.

- The DRS of (30b) is given in (49).[28]

[24] She defines the *relativised sameness* condition, which makes an indefinite selectively independent to another dref, but this is orthogonal to the discussion here.

[25] Guha (2018) adopts DPlL with Neo-Davidsonian composition. Since it is orthogonal to our purpose here, I disregard it for now.

[26] I adjuct the notation to assimilate it to Kuhn's (2017)

[27] While her implementation deals with it as a post-supposition, she takes it as presuppositional in a theory-neutral sense, i.e. it is a projective not-at-issue content. Brasoveanu and Farkas (2011) is similar to her theory in the sense that they take a dependent indefinite as a plain indefinite plus a dependency condition. They are non-commital to the status of the dependency condition (Brasoveanu and Farkas, 2011, Fn.9), though. I will come back to this point in Day 2 lecture.

[28] The condition Atom(u_1) can be replaced with the evaluation atomicity condition $u_1 = 1$ in Henderson (2014).

(49) $[u_1|]; \delta_{u_1}([|\,\text{student}(u_1)]; [|\,\text{Atom}(u_1)]; [u_2|]; [|\,\text{Card}_2(u_2)]; \overline{[|\,dif(u_m/u_n)]};$
$[|\,\text{zebras}(u_2)]; [|\,\text{saw}(u_1)(u_2)])$
$= [u_1|]; \delta_{u_1}([|\,\text{student}(u_1)]; [|\,\text{Atom}(u_1)]; [u_2|]; [|\,\text{Card}_2(u_2)]; [|\,\text{zebras}(u_2)];$
$[|\,\text{saw}(u_1)(u_2)]); \overline{[|\,dif(u_2/u_1)]}$ (post-supposition)

- For this case, it essentially works the same as Henderson (2014).
- Consider a sample PIS given in 10.

H	u_1	u_2
h_1	student_1	$\text{zebra}_1+\text{zebra}_2$
h_2	student_2	$\text{zebra}_3+\text{zebra}_4$
h_3	student_3	$\text{zebra}_5+\text{zebra}_6$

Table 10: Output for dependent indefinites

- $\text{Card}_2(u_2)$ requires each member of H to store two atomic zabras.
- Here, the difference condition is satisfied.

- The uniqueness of Guha (2018) can be found in her treatment of (30a).
  - This calls for a more committed entry of a covert distributivity operator.[29]

(50) a. $[\![u_n \leq u_m]\!] = \lambda G \lambda H\,[G = H \,\&\, H(u_n) \subseteq H(U_m)]$
b. $[\![\text{Dist}]\!] = \lambda P_{\langle ET\rangle}\, \lambda v_E\, [u|]; [|\,u \leq v]; [|\,\text{Card}_1(u)]; \delta_u(P(u))$

- (50b) 'reintroduce' the value of v that is free from $same(v)$.
- The DRS of (30a) is given in (51).

(51) $[u_1|]; [|\,\text{Card}_3(u_1)]; [|\,\overline{same(u_1)]}; [|\,\text{students}(u_1)]; [u_2|]; [|\,u_2 \leq u_1]; [|\,\text{Card}_1(u_2)];$
$\delta_{u_2}([u_3|]; [|\,\text{Card}_2(u_3)]; [|\,\overline{dif(u_3)_{u_2}}]; |\,\text{zebras}(u_3)]; [|\,\text{see}(u_2)(u_3)])$
$= [u_1|]; [|\,\text{Card}_3(u_1)]; [|\,\overline{same(u_1)}]; [|\,\text{students}(u_1)]; [u_2|]; [|\,u_2 \leq u_1]; [|\,\text{Card}_1(u_2)];$
$\delta_{u_2}([u_3|]; [|\,\text{Card}_2(u_3)]; |\,\text{zebras}(u_3)]; [|\,\text{see}(u_2)(u_3)]); \overline{[|\,dif(u_3)_{u_2}]}$ (post-supposition)

[29]The original definition in Guha (2018) applies dynamic maximisation on u, but I disregard this for an expository sake.

- Consider a sample PIS given in Table 11.
- Here, u_1 is constrained by $same(u_1)$ and does not provide dependencies necessary for the dependent indefinite.
- However, the covert Dist introduces a new dref u_2 that is free from the sameness condition, and may provide the relevant dependency for $dif(u_3)$.

H	u_1	u_2	u_3
h_1	$\text{student}_1+\text{student}_2+\text{student}_3$	student_1	$\text{zebra}_1+\text{zebra}_2$
h_2	$\text{student}_1+\text{student}_2+\text{student}_3$	student_2	$\text{zebra}_3+\text{zebra}_4$
h_3	$\text{student}_1+\text{student}_2+\text{student}_3$	student_3	$\text{zebra}_5+\text{zebra}_6$

Table 11: Output for dependent indefinites

- If Dist is not present, there is no way to satisfy $dif(u_3/u_1)$.

H	u_1	u_3
h_1	student_{1-3}	zebra_{1-6}

Table 12: Output for dependent indefinites

- Therefore, Guha (2018) predicts that a plural argument licenses a dependent indefinite only when a covert Dist is inserted.
- On this point, note that a theory with dependency-free plural extension makes essentially the same prediction.
- (30a) without a covert δ may only produce PISs such as Table 13.
- Here, $dif(u_2/u_1)$ is not satisfied because u_2 is not dependent on u_1.
- i.e. dependency is introduced only when δ is present.

H	u_1	u_2
h_1	student$_1$	zebra$_1$+zebra$_2$
h_2	student$_1$	zebra$_3$+zebra$_4$
h_3	student$_1$	zebra$_5$+zebra$_6$
h_4	student$_2$	zebra$_1$+zebra$_2$
h_5	student$_2$	zebra$_3$+zebra$_4$
h_6	student$_2$	zebra$_5$+zebra$_6$
h_7	student$_3$	zebra$_1$+zebra$_2$
h_8	student$_3$	zebra$_3$+zebra$_4$
h_9	student$_3$	zebra$_5$+zebra$_6$

Table 13: Output for dependent indefinites

4 Bridges to the upcoming lectures

- Among many possible choice points, I take up two issues:

- One concerns the status of post-suppositions, which I will discuss in Day 2.

- The other concerns the status of their licensors, i.e. plural licensors vs. distributive licensors, which I will discuss in Day 3.

- In this final section, I briefly discuss them in light of the three analyses.

4.1 Post-supposition

- Henderson (2014); Guha (2018) postulate a plurality condition and a co-variation condition that take scope via post-supposition.

- Theoretically speaking, however, the status of post-supposition has not been as clear as other not-at-issue contents.

- Especially, there seems to be no consensus about which operator let a post-supposition project.

- For example, the δ operator in Henderson (2014) is defined so that it passes through post-suppositions under its scope to the output context.

- However, Brasoveanu (2013) defines the δ operator so that post-suppositions are **discharged** under its scope, i.e. δ is a 'post-supposition filter'.

- I do not discuss their motivations here, but let me note that if post-suppositions project or not depends on the definition of the δ operator.[30]

- Crucially for Day 2 lecture, there seems to be a disagreement on whether a post-supposition may project from the scope of negation.

- It is crucial for Guha (2018) that a post-supposition projects from negation but it is crucial for Law (2022) that it **does not**.

4.2 The type of licrnsors

- While Guha (2018) needs *Dist* to license a dependent indefinite, Henderson (2014); Kuhn (2017) do not.

- Kuhn (2017) argues that it is empirically correct that *Dist* is absent in licensing of dependent indefinites.

- He places his argument on two assumptions.[31,32]

- (i) covert distributivity operators occur over the VP, and (ii) both conjuncts in a coordinate structure must take scope at the same level.

- Then, if one VP contains an ordinal indefinite and the other VP contains a dependent indefinite, it provides a test case for covert distributivity operator.

[30] Brasoveanu (2013) proposes that the cardinality of (non-increasing) modified numerals are post-positional. Then, the following data suggests that their post-supposed cardinality condition has to be evaluated under the scope of δ: the cardinality of "exactly five" has to be evaluated last under the scope of δ, but not after "a cake."

(i) Every student δ(ate exactly five cupcakes) and I drank a coke.

However, see Charlow (2017) for an argument that one does not need post-supposition nor any scoping mechanism for the behaviour of (non-increasing) modified numerals.

[31] The first assumption is challenged by Dotlačil (2010): he argues that a covert distributive operator is not at the VP level, but it should be attached to each argument that co-varies with a plural argument that is distributively interpreted. At the same time, however, it is not clear how the Dist-licensor approach can be implemented in his analysis.

[32] See Guha (2018) for a critique on this argument of Kuhn (2017).

- If a dependent indefinite has to sit under the scope of a covert distributivity operator, the ordinary indefinite should co-vary with the subject.

- If not, it should be possible that the ordinal indefinite do not co-vary with the subject.

- Kuhn (2017) shows that the result favours an approach without covert distributivity operator.

- (52) and (53) have a reading in which the students order two appetizers in total and they each order one main dish.

(52) A diákok két elöételt és **egy-egy** föételt rendeltek.
The students two appetizers and one-one main dish ordered
"The students ordered two appetizers, and n main dishes (n = the # of students.)" (Kuhn, 2017, Hungarian)

(53) Manavarkkal thankalai kaga oru-oru appetizer o irenDu desserts share-panna
students themselves for one-one appetizer and two desserts share-do
order pannagu.
order did
"The students ordered one appetizer each for themselves and two desserts to share." (Kuhn, 2017, Tamil (Chennai dialect))

- However, Kuhn (2017) also shows that the same test with an overt distributive quantifier poses a problem.

(54) Minden diák két elöételt és **egy-egy** föételt rendeltek.
Every student two appetizers and one-one main dish ordered
"Every student ordered two appetizers, and n main dishes (n = the # of students.)" (Kuhn, 2017, Hungarian)

- He reports that (54) permits the narrow scope reading of the plain indefinite in the first conjunct.

- Since he assumes that the dependent indefinite in the second conjunct takes the wide scope over the subject, this reading is unexpected.

- Kuhn (2017) admits that this calls for a split-scope mechanism.[33]

[33] Another case for a split-scope involves in-scope binding.

- In contrast, Guha (2018) argues that *Dist* is obligatory present in licensing of dependent indefinites.

\- She makes her argument with cumulative readings between co-arguments.

- First, the indirect object may be cumulatively related with the subject.

(55) a. **Scenario**: There are three small scale companies A, B and C. A has ten employees B and C each have twenty employees. In June each of the companies spent one lakh rupees as salary and thus the cumulative salary expenditure for the three companies were three lakh rupees.

b. Jun maš-e tin-Te-company pɔncaš-jon-kɔrmocari-ke
June month-Loc 3-CL-company 50-CL-employee-Dat
tin-lokkho-Taka-maine die-che.
3-lakh-rupee-salary give.pfv-be.Pres.3rd
"In the month of June, three companies paid fifty employees three lakh rupees." → **true** in this scenario (Guha, 2018)

\- If a dependent indefinite is licensed with dependencies introduced with plurals, it may retain this cumulative relation between S and IO.

- However, once the object is replaced with a dependent indefinite, this reading is lost.

(56) a. **Scenario**: There are three small scale companies A, B and C. A has ten employees and B and C each have twenty employees. In June each of the companies spent one lakh rupees as salary.

b. Jun maš-e tin-Te-company pɔncaš-jon-kɔrmocari-ke
June month-Loc 3-CL-company 50-CL-employee-Dat
tin-lokkho-Taka-**kore**-maine die-che.
3-lakh-rupee-KORE-salary give.pfv-be.Pres.3rd
"In the month of June, three companies paid fifty employees one lakh rupee each." → **false** in this scenario (Guha, 2018)

(i) Minden rendezö benevezte két-két filmjét.
every director entered two-two his-films
"Every director entered two of his films." (Kuhn, 2017, owed to an anonymous reviwer in SALT)

If the QR of the dependent indefinite raises "his films," too, it predicts that "his" may not be bound by "every director." Nevertheless, such binding is possible in (i).

- This lack of a cumulative reading on the indirect object is puzzling if a dependent indefinite can be licensed with non-distributive dependencies.

- This is one argument for the Dist-licensor approach.

- In Day 3, I aim to discuss it further through observing the interaction between dependent indefinites and discourse anaphora.

5 Day 1 summary

- Dependent indefinites pose a puzzle concerning distributivity and quantificational dependencies.

- I have reviewed two possible approaches to this puzzle:

- The Dist-licensor approach assumes that they are not distributive themselves but have to be licensed under distributive operators.

- The concord approach assumes that they are distributive themselves but they perform concord with an additional distributive operator.

- As their concrete incarnations, I reviewed Henderson (2014); Kuhn (2017); Guha (2018).

- I took up two questions, namely:

- how does their plurality/co-variation condition scope over its distributive licensor?

- are they only licensed with a distributive licensor?

- In the following lectures, I tackle these questions through inquiring how dependent indefinites interact with negation and distributivity.

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