# Cumulative questions and structured witnesses

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## 1 Introduction

- Questions with plural arguments allow *pair-list* responses (1) exemplifies.

(1) a. Who do the students like?
b. Ann likes Professor Jones and Ben likes Professor Smith.

- It is controversial whether this is semantically defined 'answer' or pragmatically enriched 'response'.

- Krifka (1992); Srivastav (1992) took the latter path, claiming that the denotation of (1a) is *cumulative*,

- i.e. (1a) asserts that there exists a correspondence between the students and those who they like, but underspecifies which correspondence holds.

- Sauerland (1998); Beck (2000); Beck and Sauerland (2000, a.o.) postulate the **-operator to derive cumulative readings.

- (2c) shows the denotation of (2b) if Prof. Jones and Prof. Smith are the ones that the students like.

(2) a. $[\![\text{R}^{**}]\!] = \lambda X \lambda Y \forall x \in X \rightarrow \exists y \in Y [\text{R}(x)(y)] \,\&\, \forall y \in Y \rightarrow \exists x \in X [\text{R}(x)(y)]$
b. [who $\lambda_{x_1}$ the students **(like) $x_1$ ]
c. $[\![(2b)]\!] = \{\forall x \in \{z : z \in [\![\text{the student}]\!]\} \rightarrow \exists y \in \{\text{Jones,Smith}\} [\text{R}(x)(y)] \,\&$
$\forall y \in \{\text{Jones,Smith}\} \rightarrow \exists x \in \{z : z \in [\![\text{the student}]\!]\} [\text{R}(x)(y)]\}$

- Thus, the denotation of (1a) is too weak to request pair-list answers.

- And yet, it is reasonable to assume that the addressee may offer more information than is required, e.g., clarification of who likes whom is over-informative, but still natural.

- I call it the *cumulation-and-elaboration* hypothesis, following Johnston (2023).

- Although this analysis is theoretically parsimonious, Johnston (2023) convincingly argues that this should not be the case.

- A crucial piece of data is that such a question can be asked even when the context already entails an existence of such correspondence.

(3) Scenario: The head coach of a basketball team had five jerseys made, numbered 1-5, for the five players on the team. Each player chose a jersey. The assistant coach knows all five players on the team, knows the numbers that were available, and believes that each of those players chose exactly one jerseys. However, the assistant coach was not present for the choosing, and so doesn't yet know which player selected which jersey.

   a. #Who are the players?

   b. Which numbers did the players pick? (Johnston, 2023)

- (3a) serves as a baseline: since the context already tells who the players are, it is infelicitous to ask it in this context.

- On this point, this context already supports a cumulative relation between the players and the numbers.

- Thus, if *wh*+a definite plural only induces a cumulative reading, (3b) should be infelicitous, just like (3a).

- However, this is not the case.

- Thus, Johnston (2023) argues that a pair-list answer should be semantically enabled in the denotation of (3b).

- He achieves it in a straightforward way: definite plurals may perform distributive quantification with a covert distributivity operator.

- This derives a pair-list reading on a par with a pair-list reading with distributive quantifiers.

- However, his analysis raises a problem with respect to difference between distributive quantifiers and definite plurals.

- Especially, Johnston (2023) himself mentions the number sensitivity of *wh* questions with definite plurals.

(4) Which **professors** do the students like?
  a. They like Professor Jones and Professor Smith.
  b. Ann likes Professor Jones, and Ben likes Professor Smith.

(Johnston, 2023)

(5) Which **professor** do the students like?
  a. They like Professor Jones.
  b. # Ann likes Professor Jones, and Ben likes Professor Smith.

(Johnston, 2023)

- In contrast, *wh*-questions with distributive quantifiers do not show such number-sensitivity.

(6) Which **professor** does every student like?
  a. Every student likes Professor Jones.
  b. Ann likes Professor Jones, and Ben likes Professor Smith.

(Johnston, 2023)

- I aim to maintain a fine distinction between distributive quantifiers and definite plurals while maintaining Johnston's (2023) argument against a simple cumulativity analysis of *wh*+a plural definite.

- As an alternative, I propose an analysis with a *structured witness set*: the speaker requests the witness of *wh*-variable including how its value corresponds to the values of other variables.

- The idea is that pair-list answers may arise from pluralities and quantification, which contribute to *wh*-dependencies via different paths.

- As a proof of the concept, I propose an implementation with *Dynamic Plural Logic* (DPlL) (van den Berg, 1996, *et seq*) a simple partition semantics.

- I briefly compare the resultant analysis with the recent analysis on *wh* plus definite plurals proposed in Xiang (2023).

- See the Appendices for its sub-clausally compositional implementation with *Dynamic Plural Inquisitive Semantics* (Dotlačil and Roelofsen, 2021; Roelofsen and Dotlačil, 2023).

## 2 Background

- In this section, I set up the background for the proposal account.

- I adopt a version of Dynamic Plural Logic (DPlL) (van den Berg, 1996), an extension of *Dynamic Predicate Logic* (Groenendijk and Stokhof, 1991) with **pluralities of variable assignments**.

- I call variables in the domain of assignment functions *discourse referents* (drefs) (Karttunen, 1969, *et seq*).

- An *information state* stores the information of which referents have been introduced to the conversational discourse.

- I model an information state with a variable assignment and represent it with small letters $g$, $h$, ...

- A *plural information state* is a set of information states and I represent it with capital letters $G$, $H$, ...

- The formal definition of dependencies is given in (7b) (van den Berg, 1996).

(7) a. $G_{u_n=d} = \{g : g \in G \,\&\, g(u_n) = d\}$ and $G(u_n) = \{x | g \in G \,\&\, g(u_n) = x\}$

b. In a plural information state $G$, $u_m$ is dependent on $u_n$ iff
$\exists d, e \in G(u_n)[G_{u_n=d}(u_m) \neq G_{u_n=e}(u_m)]$

- Consider Table 1: with respect to $u_1$, $u_2$ shows total co-variation, $u_3$ shows partial co-variation and $u_4$ shows no co-variation.

- By (7b), $u_2$ and $u_3$ are dependent on $u_1$, but $u_4$ is not.

| $H$ | $u_1$ | $u_2$ | $u_3$ | $u_4$ |
|---|---|---|---|---|
| $h_1$ | $a$ | $x_2$ | $x_3$ | $x_4$ |
| $h_2$ | $b$ | $y_2$ | $x_3$ | $x_4$ |
| $h_3$ | $c$ | $z_2$ | $z_3$ | $x_4$ |

Table 1: Dependency storage

- I adopt a definition of assignment extension that allows new values to be dependent to old values (Brasoveanu, 2008, a.o.).

- The basic assignment extension is defined in (9)

(8) $g[u_n]h \Leftrightarrow \forall u[u \neq u_n \rightarrow g(u) = h(u)]$

- Assignment extension on plural information states is defined as the point-wise generalisation of (8) as shown in (9).

(9) $G[u]H \Leftrightarrow \forall g[g \in G \rightarrow \exists h[h \in H \,\&\, g[u]h]] \,\&\, \forall h[h \in H \rightarrow \exists g[g \in G \,\&\, g[u]h]]$

- It can produce both contexts in Table 2 and 3:

- the point-wise addition of new values is satisfied both context.

| $G$ | $u_1$ |
|---|---|
| $g_1$ | $x_1$ |
| $g_2$ | $x_2$ |

$\xrightarrow[G[u_2]H]{}$

| $H$ | $u_1$ | $u_2$ |
|---|---|---|
| $h_1$ | $x_1$ | $y_1$ |
| $h_2$ | $x_2$ | $y_2$ |

Table 2: Dependent

| $G$ | $u_1$ |
|---|---|
| $g_1$ | $x_1$ |
| $g_2$ | $x_2$ |

$\xrightarrow[G[u_2]H]{}$

| $H$ | $u_1$ | $u_2$ |
|---|---|---|
| $h_1$ | $x_1$ | $y_1$ |
| $h_2$ | $x_1$ | $y_2$ |
| $h_3$ | $x_2$ | $y_1$ |
| $h_4$ | $x_2$ | $y_2$ |

Table 3: Independent

- In this setting, I take a context as a set of *possibilities*, which are pairs of a possible world and a plural information state.

- A formula denotes a function from an input context to the output context.

- One may define the context in different ways, but it is easier to demonstrate the gist of my analysis with this assumption.

- Assignment extension is minimally refined as (10).

(10) $c[u] = \{\langle w, H\rangle : \exists G\,[\langle w, G\rangle \in c\,\&\,[\forall g \in G\,\exists h \in H\,[g[u]h]\,\&\,\forall h \in H\,\exists g \in G\,[g[u]h]]]\}$

- The sequencing operator ; signals dynamic conjunction.

(11) $c[\phi;\psi] = c[\phi][\psi]$

- Evaluation of lexical relations is distributive as default (Brasoveanu, 2008).

(12) $c[R(u_1,...,u_n)] = \{\langle w, G\rangle : \langle w, G\rangle \in c\,\&\,\forall g \in G[\langle g(u_1),...,g(u_n)\rangle \in I_w(R)]\}$

- The (non-)atomicity condition is defined collectively (Brasoveanu, 2008).[1]

(13) a. $\mathrm{Atom}(x) \Leftrightarrow \forall y\,[y \subseteq x \rightarrow y = x]$
b. $c[\mathrm{atom}(u)] = \{\langle w, G\rangle : \langle w, G\rangle \in c\,\&\,\mathrm{Atom}(G(u))\}$
c. $c[\text{non-atom}(u)] = \{\langle w, G\rangle : \langle w, G\rangle \in c\,\&\,\neg\mathrm{Atom}(G(u))\}$

- Lastly, the dynamic distributivity operator δ (van den Berg, 1996; Brasoveanu, 2008, a.o.) is defined in (14). $\delta_{u_n}$ evaluate $\phi$ with respect to each $G_{u_n=d}$.

(14) $c[\delta_{u_n}(\phi)] = \{\langle w, H\rangle : \exists G\,[\langle w, G\rangle \in c\,\&\,G(u_n) = H(u_n)\,\&$
$\forall d \in G(u_n)\,[G_{u_n=d} \in c\,\&\,H_{u_n=d} \in c[\phi]]]\}$

- At this point, I disregard sub-clausal compositionality, but see Appendix I.

- Now, I build a question semantics on it.

- I do not aim to propose an analysis that applies to various issues in questions in natural language semantics, but I aim to provide a proof of the concept of structured witness sets.

- As a crucial ingredient, I borrow the witness requesting operator $?u$ (Dotlačil and Roelofsen, 2021; Roelofsen and Dotlačil, 2023).

[1] At this point, I assume that the domain of variable assignments only contains atomic individuals, but see Appendix III for a motivation to include mareological plurals into the system.

- It raises a new issue regarding the value of $u$.[2]

(15) $c[?u] = \{s \subset c : \exists x_e \forall \langle w, G\rangle \in s \exists \langle w', G'\rangle \in c\, [w = w' \,\&\, \forall g' \in G'\, [g'(u) = x]]\}$

- Modifying Dotlačil and Roelofsen's (2021) paraphrase, $?u$ asks "Which individual $x$ has the properties ascribed to $u$?"

- The context is split into several sets of possibilities $s$ along with the value of $u$, e.g., a (downward closed) set of possibilities in which $x = a$ and another (downward closed) set of possibilities in which $x = b$.

- I call those (downward closed) sets of possibilities *inquisitive states*.

- Ignoring the syntax-semantics interface for now, I assume that the $?u$ operator comes from somewhere in the left-periphery Dotlačil and Roelofsen (2021); Roelofsen and Dotlačil (2023).[3]

- For now, I do not implement it under the setting of inquisitive semantics, i.e. a context is split into inquisitive states only after $?u$ is introduced.

- Still, some useful notions from inquisitive semantics can be defined on inquisitive states $s$.

- An *issue* $I$ is defined as a non-empty, downward closed set of information states, and an inquisitive state $s$ *resolve* an issue $I$ iff $s \in I$.

- Now, the question is how the $\delta$ operator contributes to pair-list answers and how *wh* plus a definite plural derives pair-list answers.

- At this point, the $?u$ operator is sensitive to the cumulative value of $u$ and thus it does not discriminate situations that involve the same values, but different correspondence.

[2] It is revised so that it fits in the current set-up. Its original definition is given below.

(i) $c[?u] = \{s \in c : \exists x_e \forall \langle w, G\rangle \in s \exists \langle w', G'\rangle \in \cup c\, [w = w' \,\&\, \forall g' \in G'\, [g'(u) = x]]\}$

I come back to the inquisitive setting in the end in Appendix I, but I work with a simpler system for now.

[3] However, see Appendix I, where I hard-wire the $?u*$ operator to the denotations of *wh*-expressions. Here, I differ from Dotlačil and Roelofsen (2021); Roelofsen and Dotlačil (2023). For the reason why I assume $?u$ comes from *wh*-expressions, see Appendix I. See Appendix II for how this modification affect an analysis of multiple *wh* under Dynamic Plural Inquisitive semantics.

- For example in (3), there can be only one set of possibilities in which $x$ is the sum of five numbers.

- However, the intuition is that (3b) can specify the exact correspondence between the numbers and the players which describes the true correspondence.

- To solve this, I refine the $?u$ operator so that it is not only sensitive to the value of $u$, but also to the dependencies that the value of $u$ participates with respect to other drefs.

# 3 Proposal: a structured witness set

- I propose a generalised version of the $?u$ operator that is sensitive to dependencies as defined in (35). Here, $?u*$ is dependent on another dref $u_n$.

(16) $c[?u*_{u_n}] = \{s \subset c : \exists f_{\langle e,e\rangle} \forall \langle w,G\rangle \in s \, \exists \langle w',G'\rangle \in c \, [w = w' \, \&$
$\forall d \in G(u_n) \, [f(d) = G_{u_n=d}(u)]]\}$

- (35) partitions the context based on a function $f$ and it is equivalent to (15) when $G$ is a singleton set of variable assignments.

- If $G$ is non-singleton, (35) distinguishes two possibilities that agree on the value of $G(u)$ but disagree on how its values are distributed across $G$.

- Thus, each $s \subset c$ expresses different dependencies regarding the values of $u$.

- There are two ways to make $G$ non-singleton: using a singular *wh*-expression under the scope of $\delta$ operator and using a plural *wh*-expression.

- This explains the difference between *wh*-question with quantifiers and *wh*-questions with plural arguments in terms of number (in)sensitivity.

- Now, revisit the example (4).

- Henceforth, I use the superscript $^{u_n}$ for an assignment of a new value to $u_n$ and the subscript $_{u_n}$ for anaphoric reference to $u_n$.

(17) Which professors do the students like?

a. the students$^{u_1}$ like which$^{u_2}$ professors

b. $c[u_1$;student$(u_1)$;non-atom$(u_1)$; $u_2$,professor$(u_2)$;non-atom$(u_2)$;like$(u_1,u_2)$; $?u_2*_{u_1}]$

- (17b) partitions the context based on possible functional dependencies between the students and the professors.

- The output is a set of sets of world-PIS pairs that match not only in the values of $u_1$ and $u_2$, but also in the functional dependencies between them.

- For example, consider four possibilities $\langle w_1, G\rangle$, $\langle w_2, G'\rangle$, $\langle w_3, G''\rangle$ and $\langle w_4, G'''\rangle$.

| $\langle w_1, G\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g_1$ | Ann | Jones |
| $g_2$ | Ben | Smith |

| $\langle w_2, G'\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g'_1$ | Ben | Smith |
| $g'_2$ | Ann | Jones |

| $\langle w_3, G''\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g''_1$ | Ben | Jones |
| $g''_2$ | Ann | Smith |

| $\langle w_4, G'''\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g'''_1$ | Ann | Smith |
| $g'''_2$ | Ben | Smith |

Table 4: Dependencies stored in possibilities

- In this case, $c = \{s_1, s_2\}$.

- $s_1 = \{\langle w_1, G\rangle, \langle w_2, G'\rangle\}$ because they share $f_1 = \{\langle \text{Ann, Jones}\rangle, \langle \text{Ben, Smith}\rangle\}$.

- $s_2 = \{\langle w_3, G''\rangle\}$ with $f_2 = \{\langle \text{Ann, Smith}\rangle, \langle \text{Ben, Jones}\rangle\}$.

- Thus, (17b) semantically derives pair-list answers, i.e. $\{\langle \text{Ann, Jones}\rangle, \langle \text{Ben, Smith}\rangle\}$ and $\{\langle \text{Ann, Smith}\rangle, \langle \text{Ben, Jones}\rangle\}$ are two distinct answers.

- This means that (4) has pair list answers by virtue of plurality of *wh*-expression. Note that $\langle w_4, G'''\rangle$ is discarded through this update because the collective value of $u_2$ is singular and thus it violates the condition non-atom($u_2$).

- On this point, it is important that the $?u*_{u_n}$ operator itself does not perform distributive quantification.

- Consider the example (5): in this case, $[\![\text{which professor}]\!]$ only introduces a singular value to $u_2$ simply because it is singular.

- As a result, there is no possibility that $u_2$ co-varies with $u_1$, by definition.

(18) Which professor do the students like?

a. the students$^{u_1}$ like which$^{u_2}$ professor

b. $c[u_1;\text{student}(u_1);\text{non-atom}(u_1);u_2;\text{professor}(u_2);\text{atom}(u_2);\text{like}(u_1,u_2);?u_2{*}_{u_1}]$

- Consider the four possibilities in Table 4 again.

- The condition atom($u_2$) is only satisfied in $\langle w_4, G'''\rangle$ and the other three possibilities are discarded because they do not satisfy it.

- As a result only $s_4 = \{\langle w_4, G'''\rangle\}$ qualifies as an answer in this context.

- However, the situation is different if the subject is a quantifier because it introduces its own δ operator.

- Consider the example (6).

- For an explanatory sake, I simplify the content of dynamic generalised quantification (see van den Berg, 1996; Brasoveanu, 2007, for more details).

(19) Which professor does every student like?

a. every student$^{u_1}$ like which$^{u_2}$ professor

b. $c[u_1;\text{non-atom}(u_1);\delta_{u_1}(\text{student}(u_1);u_2;\text{professor}(u_2);\text{atom}(u_2);\text{like}(u_1,u_2));?u_2{*}_{u_1}]$

- In (19b), the value of $u_2$ is constrained to be singular just like it is in (18b).

- However, it is under the scope of δ in (19b).

- As a result, this atomicity constraint is evaluated with respect to each subset of information states, i.e. $G_{u_1=d}$, and $u_2$ may still have plural values under the sum of those subsets, i.e. $G$.

- Note that this 'neutralisation' of atomicity is reminiscent of DPlL-accounts of *quantificational subordination* (van den Berg, 1996, *et seq*).

- In Table 4, all the four possibilities are compatible with this requirement and thus all of $s_1$, $s_2$ and $s_3$ may resolve the issue raised by (6).

- This means that (19b) has pair-list answers by virtue of the δ operator.

- Note that $s_4 = \{\langle w_4, G'''\rangle\}$ is not discarded this case.

- It predicts that "Ann likes Prof. Smith and Ben likes Prof. Smith, too." is a legit answer to this question.

- Summing up, I proposed that the $?u*_{u_n}$ operator that is associated with its antecedent and it partitions the context based on a function from the value of its antecedent to the value of the *wh*-dref.

- This derives pair list readings when the context stores quantificational dependencies between two drefs, which can be achieved by a plural *wh* or a singular *wh* under the scope of δ.

- The most crucial aspect of this analysis is that one does not need the δ operator to derive a pair-list reading with *wh* plus definite plurals, which naturally accounts for its number sensitivity.

# 4 Comparison with Xiang (2023)

- So far, I have proposed that one may semantically derive pair-list answers with *wh* plus definite plurals without conflating it with *wh* plus quantifiers.

- Actually, Xiang (2023) has a similar criticism on Johnston (2023) and proposes a dependency-based analysis.

- Her analysis is couched in a static framework and dependency is established via the covert Resp operator (Gawron and Kehler, 2004).

- Here, $g$ is regarded as a pragmatically available sequencing function.

(20) $\text{Resp}_g = \lambda P \lambda x \forall n [1 \leq n \leq |g| \rightarrow [g(P)(n)](g(x)(n))]$
(The $n$-th part of the property $P$ holds for the $n$-th part of the individual $x$)

- She proposes that the Resp operator occurs between the trace of a definite plural and the predicate combined with the functional trace of *wh*.

(21) [ which numbers $\lambda i$ [ $\lambda w$ the players $\lambda j$ [ $x_j$ **Resp** picked$_w$-$f_i(x_j)$ ] ] ]

- Her analysis and my analysis differ in where dependencies come from: her analysis provides dependencies with a covert operator while my analysis rely on quantification over subsets of a plural information state.

- Here, I provide two reasons that favour the latter approach.

- First, it is not clear if the pair list readings of *wh* plus definite plurals should be analysed with the operator which is originally proposed for respective predication exemplified in (22).

(22) Tolstoy and Dostoyevsky wrote Anna Karenina and The Idiot, respectively. (Gawron and Kehler, 2004)

- If the order of elements is made clear in the context, non-conjoined plural arguments can also enter into this respective predication.

(23) The three best students received the three best scores, respectively. (Kubota and Levine, 2016)

- However, two definite plurals cannot enter into respective predication unless their cardinalities are specified.

(24) a. *The players picked the numbers, respectively.
b. The five players picked the five numbers, respectively.

- Furthermore, a plural *wh* and a definite plural cannot enter into respective predication, either.

(25) *Which numbers did the players pick, respectively?

- This is problematic for Xiang (2023) if the Resp operator is taken as a covert version of "respectively."

- One may still argue that the covert Resp operator is less constrained than the overt adverb "respectively."

- Then, let's suppose that the operator defined in (20) may occur in a *wh*-question with a plural definite and see if it derives the desired results.

- On this point, it is important to note that the property $P$ is under the scope of universal quantification in the Resp-based approach.

- This contrasts with the DPlL approach, in which the definite plural does not perform quantification.

- This leads to different predictions when an indefinite intervenes between *wh* and a definite plural.

- In general, co-varying interpretations of a singular indefinite is harder with non-quantificational plural arguments (see Dotlacil, 2010, for an experimental result in Dutch).

- Thus, the Resp-based approach predicts that the co-varying interpretation of a singular indefinite is readily available with *wh*-question with definite plurals and so as the covert distributivity approach in Johnston (2023).

- In contrast, the DPlL approach predicts that the co-varying interpretation of a singular indefinite is dispreferred.

- (26a) shows that the co-varying reading is hard or unavailable with *wh*-question plus definite plurals, which favours the DPlL approach.

(26) Which cities did the guides recommend to a customer?
   a. # Guide A recommended Amsterdam to Ann, Guide B recommended Utrecht to Bill and Guide C recommended Leiden to Chris
   b. All the guides recommended a city to Ann. Guide A recommended Amsterdam to Ann, Guide B recommended Utrecht to Ann and Guide C recommended Leiden to Ann.

- The intended reading becomes available with plural indefinites or singular indefinites under the scope of a quantifier.

(27) a. A: Which cities did the guides recommend to customers?
   b. B: Guide A recommended Amsterdam to Ann, Guide B recommended Utrecht to Bill and Guide C recommended Leiden to Chris

(28) a. A: Which city did every guide recommend to customers?
   b. B: Guide A recommended Amsterdam to Ann, Guide B recommended Utrecht to Bill and Guide C recommended Leiden to Chris

- Thus, Xiang (2023) makes a wrong prediction about co-variability of a singular indefinite occurring between the *wh* and the definite plural.

- This result echoes my main desiderata: pair-list answers of *wh* plus definite plurals do not involve distributive quantification.

- However, note that (26) cannot be asked in a context in which the speaker already know which cities were recommended.
  - For Johnston (2023); Xiang (2023), this means that insertion of a covert operator is banned in this context.
  - This cannot be due to the knowledge on *wh*-witnesses: in that case, these approaches lose their account on (3b).
  - This calls for an additional mechanism in my account as well.
- Note that my claim is that insertion of the Dist operator is a dispreferred option and **not** that insertion of the Dist is impossible.
  - Indeed, one of my informants reported that they accept
  - (i) co-variation of a singular indefinite under a definite plural, and
  - (ii) a pair-list answer to a singular *wh* with a definite plural.
  - This is exactly what Johnston (2023) should predict.
  - As long as insertion of Dist is 'costly', this is not a problem.
- However, there are cases in which Dist is rather preferred.
  - I thank to Patrick Elliot for discussions.

(29) The children are singing or dancing.
**True** if Child 1 and Child 2 are singing, and Child 3 is dancing.

  - Possibly, disjunction is the culprit?
  - e.g., the parse without Dist requires an additional epistemic uncertainty while one with Dist does not.
  - In any case, it should be further explored when insertion the Dist operator is motivated.
- As long as there are speakers whose judgements pattern with those reported in this talk, Johnston (2023) makes wrong predictions and the proposed analysis makes right predictions.

# 5 Conclusion

- I aimed to derive pair-list answers with cumulative readings without conflating questions with quantifiers and questions with definite plurals.

  - I proposed the structured witness requesting operator $?u*$.

  - $?u*$ is sensitive to quantificational dependencies that a *wh*-dref participates and derives pair-list readings of *wh* question with definite plurals without assuming that definite plurals perform distributive quantification.

- As a proof of the concept, I proposed a DPlL-based question semantics so that the $?u*$ operator can derive pair-list answers if the context entertain possibilities with a non-singleton set of variable assignments.

  - For this, one may either introduce the δ operator with a quantifier or assign a plural value to a dref with a plural argument.

  - Thus, definite plurals are still distributive without the δ operator in the sense that it introduces plurality of variable assignments by virtue of being plural.

  - Thus, it captures Johnston's (2023) intuition that *wh*+definite plural is not just cumulative, but distributive.

- I further compared it with Xiang (2023) and claim that her analysis runs into problems with the distribution of the Resp operator and "respectively," and the co-variability of a singular indefinite between *wh* and a definite plural.

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# Appendix I: An issue with subject-object asymmetry

- Johnston (2023) argues for a subject-object asymmetry in *wh*-questions with a definite plural.

- Precluding the possibility of cumulative questions with the context (3), Johnston (2023) shows that pair-list answers are unavailable with a subject *wh*.

(30) # Which players picked the numbers? (Johnston, 2023)

- In his account, this asymmetry follows because the definite plural does not c-command the trace of *wh* in (30).
- In my account, one may say that the assignment extension with ⟦the numbers⟧ takes place at the right of the assignment extension with ⟦which players⟧ and thus the *wh* fails to be associated with its antecedent.
- For this, it is necessary that $?u{*}_{u_n}$ operator is introduced with a *wh*-expression.
- Here, I differ from Dotlačil and Roelofsen (2021); Roelofsen and Dotlačil (2023), who assume that the $?u$ operator is introduced at the left-periphery.

- I implement a sub-clausally compositional version of my account in the style of Muskens (1996); Brasoveanu (2008); Dotlačil and Roelofsen (2021); Roelofsen and Dotlačil (2023).
  - I make the system state-based: a context is always a set of inquisitive states.

(31) Assignment extension:
$c[u] = \{s : \exists s' \in c\,[\forall\langle w,G\rangle \in s\,\exists\langle w',H\rangle \in s'\,[w = w' \,\&\, G[u]H] \,\&\, \forall\langle w,G\rangle \in s'\,\exists\langle w,H\rangle \in s\,[w = w' \,\&\, G[u]H]]\}$

(32) Evaluation of lexical relation:
$c[R(u_1,...,u_n)] = \{s :\ s \in c \,\&\, \forall\langle w,G\rangle \in s\,\forall g \in G[\langle g(u_1),...,g(u_n)\rangle \in I_w(R)]\}$

(33) (Non-)atomicity conditions
a. $\text{Atom}(x) \Leftrightarrow \forall y\,[y \subseteq x \rightarrow y = x]$
b. $c[\text{atom}(u)] = \{s :\ s \in c \,\&\, \forall\langle w,G\rangle\,[\text{Atom}(G(u))]\}$
c. $c[\text{non-atom}(u)] = \{s :\ s \in c \,\&\, \forall\langle w,G\rangle[\neg\text{Atom}(G(u))]\}$

(34) The δ operator:
$c[\delta_{u_n}(\phi)] = \{s : \exists s' \in c\,[\forall\langle w,G\rangle \in s'\,\exists\langle w',H\rangle \in s\,[w = w' \,\&\, G(u_n) = H(u_n) \,\&\, \forall d \in G(u_n)\,[\{\langle w,G_{u_n=d}\rangle\} \in c \,\&\, \{\langle w',H_{u_n=d}\rangle\} \in c[\phi]]]]]\}$

(35) The structured witness requesting operator (*to be revised*):
$c[?u{*}_{u_n}] = \{s \in c : \exists f_{\langle e,e\rangle}\,\forall\langle w,G\rangle \in s\,\exists\langle w',G'\rangle \in \cup c\,[w = w' \,\&\, \forall d \in G(u_n)\,[f(d) = G_{u_n=d}(u)]]]\}$

- I adopt four types: type *t* for truth values, type *w* for possible worlds, type *e* for individuals, type $\pi$ for drefs.

- A variable assignment is modelled as a function of type $\langle\pi, e\rangle$ and a set of variable assignments is type $\langle\langle\pi, e\rangle, t\rangle$.
- A plural information state $G$ is a set of variable assignment, possibility $p$ is a pair of a possible world and a plural information state, and an inquisitive state $s$ is a set of possibilities.
- A context $c$ is a set of inquisitive states.

• Following Dotlačil and Roelofsen (2021), I abbreviate type $\langle\langle\pi, e\rangle, t\rangle$ with $m$ and notate the type of a possibility with $\langle w \times m\rangle$.

- Inquisitive states are defined with type $\langle w \times m, t\rangle$
- I use the *meta-type* $T$ as an abbreviation of $\langle\langle\langle w \times m, t\rangle, t\rangle, \langle\langle w \times m, t\rangle, t\rangle\rangle$, which is a function from a context to a context.

• Based on these type conventions, I further abbreviate dynamic existential quantification and evaluation of lexical relations.

(36) a. $\exists u = \lambda c_{\langle\langle w\times m,t\rangle,t\rangle} \lambda s_{\langle w\times m,t\rangle}\, s \in c[u]$
b. $\mathrm{R}\{u_1\}...\{u_n\} = \lambda c_{\langle\langle w\times m,t\rangle,t\rangle} \lambda s_{\langle w\times m,t\rangle}\, s \in c[\mathrm{R}(u_1)...(u_n)]$

• Now, the Heim and Kratzer-style sub-clausal compositionality can be emulated by replacing $t$ in their translations with $T$.

- Some examples are given below.[4]

[4]It is obviously an oversimplification to assume that the non-atomicity condition comes from plural nouns. For example, “I just have one.” is a good answer to “how many child**ren** do you have?” This does not follow if the non-atomicity condition comes from plural nouns. Thus, it is more plausible that “which” has number distinction and the non-atomicity condition comes from its plural version. Furthermore, there is a case that suggests that it is due to a pragmatic competition: plural “which” gives rise to co-called **partial plurality inference**, i.e. the value of *wh*-dref is plural in at least one pair. I thank to Patrick Elliot for the data and discussion.

(i) a. Which professors does every student like?
b. Johannes likes Ad and Hans, Deborah likes Klaus and Hans, Matthew likes **Nathan**.

Partial plurality inferences have been observed for bare plurals and also for plural pronouns.

(ii) a. Exactly one of these coats has **pockets**.
b. Every passenger of this flight lost their **suitcases**. (Sudo, 2023)

(37) a. $[\![\text{professor}]\!] = \lambda v_\pi\, [\text{atom}\{u\}\,\&\,[\text{professor}\{v\}]$

b. $[\![\text{professors}]\!] = \lambda v_\pi\, [\text{non-atom}\{u\}\,\&\,[\text{professor}\{v\}]$

c. $[\![\text{like}]\!] = \lambda R_{\langle \pi T, T\rangle}\, \lambda v_\pi\, R(\lambda v'\, [\text{like}\{v'\}\{v\}])$

d. $[\![\text{a}]\!] = \lambda P_{\langle \pi T\rangle}\, Q_{\langle \pi T\rangle}\, [\exists u\,\&\,P\{u\}\,\&\,Q\{u\}]$

e. $[\![\text{it}_{u_n}]\!] = \lambda P_{\langle \pi T\rangle}\, [P\{u_n\}]$

- Roelofsen and Dotlačil (2023) define *wh*-expressions analogously to indefinites and assume that $?u$ comes from the Foc head in the left-periphery.[5]

(38) a. $[\![\text{who}]\!] = \lambda Q_{\langle \pi T\rangle}\, [\exists u\,\&\,\text{Person}(u)\,\&\,Q(u)]$

b. $[\![\text{which}]\!] = \lambda P_{\langle \pi T\rangle}\, Q_{\langle \pi T\rangle}\, [\exists u\,\&\,P(u)\,\&\,Q(u)]$

(39) $[\![C_{u_1,\ldots,u_n}]\!] = \lambda p_T\, p; ?u_1 \ldots u_n$

- I hard-wire the $?u*$ operator to *wh*-expressions.

(40) a. $[\![\text{who}_{u_n}]\!] = \lambda Q_{\langle \pi T\rangle}\, [\exists u\,\&\,\text{Person}(u)\,\&\,Q(u)\,\&\,?u*_{u_n}]$

b. $[\![\text{which}_{u_n}]\!] = \lambda P_{\langle \pi T\rangle}\, Q_{\langle \pi T\rangle}\, [\exists u\,\&\,P(u)\,\&\,Q(u)\,\&\,?u*_{u_n}]$

- I put aside the issue of *wh*-movement and assume that resolution of the dependent variable in $?u*$ takes place before *wh*-movement.

- Now, I am ready to account for the infelicity of (30).

- (41b) shows the sentential denotation of the felicitous case with the object *wh* and (42b) shows one for the infelicitous case with the subject *wh*.

(41) a. Which numbers did the players pick?

b. $\exists u_1\,\&\,\text{non-atom}(u_1)\,\&\,\text{players}(u_1)\,\&\,\exists u_2\,\&\,\text{non-atom}(u_2)\,\&\,\text{numbers}(u_2)\,\&\,?u_1*_{u_2}$

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(iii) Context: There are ten PhD students in this department. This semester, seven of them wrote exactly one paper, while the other three students wrote more than one paper. They all submitted their papers to a journal.

a. Every PhD student$^{u_1}$ wrote (some) papers$^{u_2}$ in this semester.

b. They$_{u_1}$ each submitted $\{$*it / **them**$\}_{u_2}$ to a journal. (Nakamura, under edition)

Partiality follows if plurality inference is due to pragmatic competition (cf. Sudo, 2023) and (i) calls for competition mechanism in the plural dynamic inquisitive setting.

[5] I put aside the issue of the issue-cancelling operator, the presuppositional closure operator and the maximality operator.

(42) a. # Which players picked the numbers?

b. $\exists u_1 \,\&\, \text{non-atom}(u_1) \,\&\, \text{players}(u_1) \,\&\, ?u_1 *_{u_2} \,\&\, \exists u_2 \,\&\, \text{non-atom}(u_2) \,\&\, \text{numbers}(u_2)$

- In (41b), $?u_1 * u_2$ occurs on the left of $\exists u_1$ and this association is fine.

- In contrast, in (42b), $?u_1 * u_2$ occurs on the right of $\exists u_1$ and this association is not possible, i.e. the value of $u_1$ has not been introduced yet when $?u_1 * u_2$ is evaluated in this sequence of dynamic conjunction.

- Thus, the subject-object asymmetry is reduced to the usual co-indexing constraint: $?u*$ can only be associated with a dref occuring on its left.

- One problem is that the $?u*$ operator is evaluated under the scope of $\delta$ operator in the revised system in cases such as (6).

- If the $?u*$ operator is evaluated under the scope of $\delta$, it raises an issue with respect to each subsets the original plural information state.

- One possible solution calls for the revised definition of the $?u*$ operator in the next Appendix.

# Appendix II: multiple *wh*

- Dotlačil and Roelofsen (2019); Roelofsen and Dotlačil (2023) defines a generalised version of $?u$. When $n = 1$, (43) is equivalent to (15).

(43) $c[?u_1...u_{n-1}, u_n] = \{s \subset c : \exists f_{(e^{n-1},e)} \forall \langle w, G\rangle \in s \, \exists \langle w', G'\rangle \in c \, [w = w' \,\&\, \forall g \in G \, [g(u_n) = f(g(u_1), ...g(u_{n-1}))]]\}$

- In my analysis, I hard-wire the $?u*$ operator to *wh*-expressions and thus the virtue of the generalised operator $?u_1...u_{n-1}, u_n$ operator is lost:

- Thus, I need an alternative way to analyse multiple *wh*-questions.

- I do not go into so much details, but I aim to emulate the generalised $?u_1...u_n$ operator with a sequence of $?u*$ operators.

- I assume that a *wh*-expression can be dependent on (an)other *wh*-expression(s) in cases of multiple *wh*-questions and a *wh*-expression is allowed to be dependent on itself, in which case the $?u*$ operator is equivalent to $?u$ operator.

- Then, I revise the definition of the $?u*$ operator so that it is sensitive to the issues already raised in the input context.

(44) $c[?u*_{u_n}] = \{s \in c : \exists f_{\langle e,e \rangle} \forall \langle w, G \rangle \in s \exists s' \in c\, [s \geq s' \,\&\, \exists \langle w', G' \rangle \in s' \,[w = w' \,\&\, \forall d \in G(u_n)\, [f(d) = G_{u_1 = d}(u)]]]\}$

- It differs from the previous definition in that there has to be a possibility $\langle w, G' \rangle$ in $s'$ such that $s$ is an *extension* of it.

(45)
a. Given information states $g$, $g'$, $g'$ is an extension of $g$ iff $g \subset g'$
b. Given plural information states $G, G'$, $G'$ is an extension of $G$, i.e. $G' \geq G$, iff every $g' \in G'$ is an extension of some $g \in G$ and every $g \in G$ is extended by some $g' \in G$
c. Given possibilities $\langle w, G \rangle$, $\langle w', G' \rangle$, $\langle w', G' \rangle$ is an extension of $\langle w, G \rangle$, i.e. $\langle w', G' \rangle \geq \langle w, G \rangle$, iff $w' = w$ and $G' \geq G$.
d. Given inquisitive states $s$, $s'$, $s'$ is an extension of $s$, i.e. $s' \geq s$, iff every possibility in $s'$ is an extension of some possibility in $s'$.

- All the information available in $s$ is also available in its extension $s'$.

- The idea is that $?u*$ can be placed one after another as long as the outer one does not break the partition already established by the inner one.

- If the input context does not contain any unresolved issue, this difference is inert, i.e. $c$ is a singleton set of an inquisitive state $s'$ and any inquisitive state $s$ in the output context is trivially an extension of it.

- If the input context that already contains an issue, (44) has to preserve it.[6]

- Let me compare a sequence of $?u*$ and $?u_1 ... u_n$. (46) shows a simplified example with the generalised $?u$ operator.

(46) $c[?u_1, u_2] = \{s \in c : \exists f_{(e,e)} \forall \langle w, G \rangle \in s \exists \langle w', G' \rangle \in \cup c\, [w = w' \,\&\, \forall g \in G\, [g(u_2) = f(g(u_1))]]]\}$

[6]If conjoined *wh*-questions such as (i) may have pair-list answers, it independently motivates this issue-preserving $?u$.

(i) Who$^{u_1}$ ate lunch? And what$^{u_2}$ did they$_{u_1}$ eat?

- It raises an issue with a functional witness that takes the values of $u_1$ as its domain and the values of $u_2$ as its range.

- (47) shows a simplified example with a sequence of $?u*$.

(47) $c[?u_1 *_{u_1}, ?u_2 *_{u_1}] = c[?u_1 *_{u_1}][?u_2 *_{u_1}]$

a. $c[?u_1 *_{u_1}] = c' = \{s \in c : \exists f_{\langle e,e\rangle} \forall \langle w, G\rangle \in s \exists s' \in c\, [s \subseteq s' \,\&\, \exists \langle w', G'\rangle \in s'\, [w = w' \,\&\, \forall d \in G(u_1)\, [f(d) = G_{u_1=d}(u_1)]]]\}$
$= \{s \in c : \exists f_{\langle e,e\rangle} \forall \langle w, G\rangle \in s \exists \langle w', G'\rangle \in \cup c\, [w = w' \,\&\, \forall d \in G(u_1)\, [f(d) = G_{u_1=d}(u_1)]]\}$

b. $c'[?u_2 *_{u_1}] = \{s \in c' : \exists f_{\langle e,e\rangle} \forall \langle w, G\rangle \in s \exists s' \in c'\, [s \subseteq s' \,\&\, \exists \langle w', G'\rangle \in s'\, [w = w' \,\&\, \forall d \in G(u_1)\, [f(d) = G_{u_1=d}(u_2)]]]\}$

- In a context without an issue, $?u_1 * u_1$ split $\cup c$ into a set of inquisitive states based on the identify function from the value of $u_1$.

- Thus, it has the same effect as $?u_1$. The resultant context $c'$ is a set of inquisitive states each of which assigns a different value on $u_1$.

- Then, $?u_2 *_{u_1}$ further updates $c'$. Here, the inquisitive states in $c'$ are further split along the value of $u_2$ relative to the value of $u_1$ in each inquisitive state.

- Take (48) as a concrete example.

(48) Which students$^{u_1}$ likes which professors$^{u_2}$?

- The $?u_1 * u_1$ operator first raises an issue with respect to the value of $u_1$.

| $\langle w_1, G\rangle$ | $u_1$ |
|---|---|
| $g_1$ | Ann |
| $g_2$ | Ben |

| $\langle w_3, G'\rangle$ | $u_1$ |
|---|---|
| $g'_1$ | Chris |
| $g'_2$ | Dan |

| $\langle w_2, G''\rangle$ | $u_1$ |
|---|---|
| $g''_1$ | Ben |
| $g''_2$ | Ann |

| $\langle w_4, G'''\rangle$ | $u_1$ |
|---|---|
| $g'''_1$ | Dan |
| $g'''_2$ | Chris |

Table 5: Raising issue with the first witness requesting operator

- The resultant context $c'$ is $\{s_1, s_2\}$ where $s_1 = \{\langle w_1, G\rangle, \langle w_2, G''\rangle\}$ and $s_2 = \{\langle w_3, G'\rangle, \langle w_4, G'''\rangle\}$.

- Then, the $?u_2 * u_1$ operator raises an issue, preserving this partition of the logical space with respect to the value of $u_1$.

| $\langle w_1, G\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g_1$ | Ann | Jones |
| $g_2$ | Ben | Smith |

| $\langle w_3, G'\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g'_1$ | Chris | Freeman |
| $g'_2$ | Dan | McGregor |

| $\langle w_2, G''\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g''_1$ | Ben | Jones |
| $g''_2$ | Ann | Smith |

| $\langle w_4, G'''\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g'''_1$ | Dan | Freeman |
| $g'''_2$ | Chris | McGregor |

Table 6: Raising issue with the second witness requesting operator

- The resultant context $c''$ is $\{s'_1, s'_2, s'_3, s'_4\}$ where $s'_1 = \{\langle w_1, G\rangle\}$, $s'_2 = \{\langle w_2, G''\rangle\}$, $s'_3 = \{\langle w_3, G'\rangle\}$ and $s'_4 = \{\langle w_4, G'''\rangle\}$.
- Crucially, $s'_1$ and $s'_2$ are extensions of $s_1$ and $s'_3$ and $s'_4$ are extensions of $s_2$.
- Each state stores a different function from the value of $u_1$ to the value of $u_2$.

- In this way, pair-list readings with multiple *wh* can be analysed with a sequence of $?u*$ operators without a generalised $?u_1 ... u_n$ operator.

- Here is indeed a case that motivates an issue-preserving version of $?u$ independently of dependency-sensitivity.
- One may answer conjoined single-*wh* questions with a pair-list answer.

(49) Sentence-internal multiple *wh*

a. Who$^{u_1}$ ate what$^{u_2}$?

b. Tom ate bitterballen, Émile ate boterham and Giorgio ate pizza.

(50) Cross-sentential multiple *wh*

a. Who$^{u_1}$ ate lunch in the cafe? and what$^{u_2}$ did **they**$_{u_1}$ eat?

b. Tom ate bitterballen, Émile ate boterham and Giorgio ate pizza.

- Just one generalised $?u$ has difficulty in deriving pair-list answer to a conjunction of two single-*wh* question.
- However, an issue-preserving $?u*$ can derive pair-list answers with (50a).

- This system can be enriched with the maximality operator defined in Dotlačil and Roelofsen (2021); Roelofsen and Dotlačil (2023) so that it can deal with mention-all and mention-some readings, including *partial mention-some* readings.

- However, much more work is necessary to check if this approach can cover the entire empirical landscape of multiple *wh*-questions.
- As it goes beyond the main purpose of this paper, I leave it for future work.[7]

- The revised definition of the $?u*$ operator opens the possibility to analyse (6) without assuming post-supposition.

[7] In principle, it is possible to let the second occurrence of $?u*$ operator be dependent on itself and obtain a sequence of $?u_1*_{u_1}$ and $?u_2*_{u_2}$. In this case, the second witness request is done with the identity function from the value of $u_2$ to itself, i.e. it just splits each states in $c'$ with respect to the global value of $u_2$. As a result, it does not distinguish inquisitive states that agree with the collective values of $u_1$ and $u_2$ but disagree with the dependencies between them. In the case of Table 6, $s_1$ and $s_2$ are not extended because $\langle w_1, G' \rangle$ and $\langle w_2, G'' \rangle$ agree with the global values of $u_1$ and $u_2$, so as $\langle w_1, G' \rangle$ and $\langle w_2, G'' \rangle$. Accordingly, it makes a prediction that "Ann and Ben like Prof. Jones and Prof. Smith" and "Chris and Dan like Prof. Freeman and Prof. McGregor." are legitimate answers for "Which students like which professors?" At this point, I do not have empirical data on English yet on this point. Abe (2017) reports that Japanese multiple *wh*-questions with the plural marker "-tachi" allows such a cumulative answer, which I also agree: the following question can be answered with "these man saw these women."

(i) Dono-otoko-tachi-ga dono-onna-tachi-ni attanodesu-ka?
which-man-pl-nom which-woman-pl-dat meet-ka
"Which men saw which women?" (Abe, 2017)

If this type of cumulative answers qualifies as a complete answer to a possible parse of questions with multiple *wh*, instead of a partial answer to their only one possible parse, then the current analysis makes a right prediction for Japanese multiple *wh*-questions, unless "tachi-" plays a role other than ensuring plurality to derive a cumulative answer. If it turns out that English multiple *wh*-questions do not permit cumulative answers, then this analysis has to be modified to analyse English data. If non-left-most occurrences of $?u*$ operator have to be dependent on a dref other than itself, this problem disappears, but it is quite *ad hoc*. Alternatively, one may posit a pragmatic principle that requires the speaker to choose the strongest interpretation among different co-indexing possibilities.

- Recall the simple context given in Table 4.

- Table 7 shows an issue raising with respect to $G_{u_1=Ann}$.

| $\langle w_1, G_{u_1=A}\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g_1$ | Ann | Jones |

| $\langle w_2, G'_{u_1=A}\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g'_2$ | Ann | Jones |

| $\langle w_3, G''_{u_1=A}\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g''_2$ | Ann | Smith |

| $\langle w_4, G'''_{u_1=A}\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g'''_1$ | Ann | Smith |

Table 7: Issue raising with one subset from different plural information states

- The resultant context is $\{s_1, s_2\}$ where $s_1 = \{\langle w_1, G_{u_1=Ann}\rangle, \langle w_2, G'_{u_1=Ann}\rangle\}$ and $s_2 = \{\langle w_3, G''_{u_1=Ann}\rangle, \langle w_4, G''''_{u_1=Ann}\rangle\}$.

- Table 8 shows an issue raising with respect to $G_{u_1=Ben}$.

| $\langle w_1, G_{u_1=B}\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g_2$ | Ben | Smith |

| $\langle w_2, G'_{u_1=B}\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g'_1$ | Ben | Smith |

| $\langle w_3, G''_{u_1=B}\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g''_1$ | Ben | Jones |

| $\langle w_4, G'''_{u_1=B}\rangle$ | $u_1$ | $u_2$ |
|---|---|---|
| $g'''_2$ | Ben | Smith |

Table 8: Issue raising with another subset from different plural information states

- The resultant context is $\{s'_1, s'_2\}$ where $s'_1 = \{\langle w_1, G_{u_1=Ben}\rangle, \langle w_2, G'_{u_1=Ben}\rangle, \langle w_4, G'''_{u_1=Ben}\rangle\}$ and $s_2 = \{\langle w_3, G''_{u_1=Ben}\rangle\}$.

- On this point, if the issue-raising with respect to $G_{u_1=Ben}$ preserves the issue raised with respect to $G_{u_1=Ann}$, then the resultant context $c'$ is $\{s_1, s_2, s_3\}$ where $s_1 = \{\langle w_1, G\rangle, \langle w_2, G'\rangle\}$ $s_2 = \{\langle w_3, G''\rangle\}$ and $s_3 = \{\langle w_4, G'''\rangle\}$.

- This can be achieved if the distributive update with $\delta$ is performed asymmetrically in the style of Bumford (2015).

- I leave the precise implementation for future research.

## Appendix III: Cardinals and mereological plurals

- Johnston (2023) shows that the cardinal modifier within the *wh*-restrictor is distributively evaluated if it semantically induces pair-list readings and non-distributively evaluated if it is parsed as a cumulative question.

- First, (51) is a context in which the questioner already knows that there are exactly ten numbers.

- It excludes the possibility that (51a) is parsed as a cumulative question.

- In this case, the cardinal modifier "two" is distributively evaluated.

(51) Context: A basketball team's head coach had ten jerseys made, numbered 1-10. From these, each of the team's five players chose exactly two jerseys, an even-numbered jersey for home games and an odd-numbered jersey for away games. The assistant coach (the questioner) knows all five players, knows the numbers that were available, and believes that each player chose exactly two jerseys.

a. Which **two** numbers did the players pick?

b. Ann picked 1 and 10, Ben picked 2 and 3, Chris picked 4 and 7, Dan picked 5 and 6, Emma picked 8 and 9.

- On the other hand, (52) is a context in which the questioner does not know about the identity of ten numbers.

- This context allows a cumulative question.

- In this case, the cardinal modifier "ten" is collectively evaluated.

(52) Context: A basketball team's head coach got out a large crate of jerseys with various numbers. From these, each of the team's five players chose exactly two jerseys, an even-numbered jersey for home games and an odd-numbered jersey for away games. The assistant coach (the questioner) knows all five players and believes that each player chose exactly two jerseys, but has no information about what jersey numbers were available.

a. Which **ten** numbers did the players pick?

b. i. They picked numbers 1-10.

ii. Ann picked 1 and 10, Ben picked 2 and 3, Chris picked 4 and 7, Dan picked 5 and 6, Emma picked 8 and 9.

- I modify the proposed system to account for this interpretive difference.
  - First, I let the domain of assignment function include mereological plurals.
  - I choose to model atomic individuals as a singleton set of individual and non-atomic individuals as a non-singleton set of individuals.
  - Second, I take evaluation of lexical relation to be cumulative (Dotlačil and Roelofsen, 2021; Roelofsen and Dotlačil, 2023).

(53) a. $c[R(u_1,...,u_n)] = \{s : s \in c \,\&\, \forall\langle w,G\rangle \in s\,[\langle G(u_1),...,G(u_n)\rangle \in I_w(*R)]\}$
b. i. $R \subseteq *R$
ii. If $\langle a_1,...,a_n\rangle \in *R$ and $\langle b_1,...,b_n\rangle \in *R$, then $\langle a_1+b_1,...,a_n+b_n\rangle \in *R$.
iii. Nothing else is in $*R$.

  - Dotlačil and Roelofsen (2021); Roelofsen and Dotlačil (2023) adopt both assumptions and these just make the proposed system much closer to theirs.

- One may now consider a plural information state such as one in Table 9.

| $H$ | $u_1$ | $u_2$ |
|---|---|---|
| $h_1$ | $x_1+x_2$ | $y_1$ |
| $h_2$ | $x_3+x_4$ | $y_2$ |
| $h_3$ | $x_5+x_6$ | $y_3$ |

Table 9: A plural information state with mereological plurals

  - This setting is independently motivated by (54): an answer to plural *wh* plus definite plural does not need to consist of one-to-one pairs.

(54) a. Which professors do the students like?
b. Johannes likes **Ad and Hans**, and Deborah likes Klaus.
(Patrick Elliot p.c.)

  - With mereological plurals, $f$ is a function from $*D$ to $*D$

- On the top of it, I assume that the restrictor of *wh*-expression is under the scope of $\delta$ operator when it is marked plural.

(55) a. $[\![\text{which}^{SG}_{u_n}]\!] = \lambda \mathrm{P}_{\langle \pi T \rangle}\, \mathrm{Q}_{\langle \pi T \rangle}\, [\exists u \,\&\, \mathrm{P}(u) \,\&\, \mathrm{atom}(u) \,\&\, \mathrm{Q}(u) \,\&\, ?u{*}_{u_n}]$

b. $[\![\text{which}^{PL}_{u_n}]\!] = \lambda \mathrm{P}_{\langle \pi T \rangle}\, \mathrm{Q}_{\langle \pi T \rangle}\, [\exists u \,\&\, \delta_u(\mathrm{P}(u)) \,\&\, \text{non-atom}(u) \,\&\, \mathrm{Q}(u) \,\&\, ?u{*}_{u_n}]$

- See Brasoveanu (2011) for an analogous use of the distributivity operator for "different" with plural noun phrases.

- The cardinality condition is defined as a collective condition, analogously to the (non-)atomicity condition.

(56) $c[\mathrm{two}(u)] = \{s : s \in c \,\&\, \forall \langle w, G \rangle\, [|G(u)| = 2]$

- Now, (57) is the proposed denotation of "which two numbers."

(57) $[\![\text{which}^{PL}_{u_n} \text{ two numbers}]\!] =$
$\mathrm{Q}_{\langle \pi T \rangle}\, [\exists u \,\&\, \delta_u(\mathrm{two}(u) \,\&\, \mathrm{number}(u)) \,\&\, \text{non-atom}(u) \,\&\, \mathrm{Q}(u) \,\&\, ?u{*}_{u_n}]$

- Recall that $\delta_u$ split a plural information state into subsets of information state each of which agree on the value of $u$.

- Since the domain of assignment functions contain mereological plurals, such values may be mereologically plural.

- For example, $H_{u_1=d} = \{h_1, h_2, h_3\}$ in Table 9.

- In this case, the cardinality condition is satisfied in each of these three because they each store a mareological plural that consists of two individuals.

- This derives the distriburive reading of the cardinal modifier.

- At the same time, one may put the maximal sum of individuals in a singleton set of information state as exemplified in Table 10.

| $H$ | $u_1$ | $u_2$ |
|---|---|---|
| $h_1$ | $x_1 + x_2 + ... + x_{10}$ | $y_1 + ... + y_5$ |

Table 10: A 'flat' plural information state with mereological plurals

- Here, the contribution of $\delta_{u_1}$ is trivialised: $H_{u_1=d} = \{h_1\}$, i.e. it looks like being collectively evaluated.

- On this point, it is important that evaluation of lexical relation is cumulative.
  - Although there is no quantificational dependency between $u_1$ and $u_2$, they are still cumulatively related.
  - Essentially, this emulates the original cumulation analysis in (Krifka, 1992; Srivastav, 1992): $*R(x_1 + ... + x_{10})(y_1 + ... + y_5)$ is true iff there is at least one way to pair $x_1 + ... + x_{10}$ and $y_1 + ... + y_5$ so that each pair is included in $I_w(R)$.
  - This is on a par with Johnston (2023), who preserves the possibility of a genuine cumulative parse of *wh* plus definite plurals and adds another parse with the covert distributivity operator.
  - Seeming pair-list responses are due to pragmatic follow-up.
- One disappealing point is that I have to say that plural *wh*-expressions perform distributive quantification only over its restrictor and this permit quantification over non-atoms when it contains a cardinal modifier.[8]

[8] One may alternatively adopt an inquisitive version of Minor's (2022) dynamic plural semantics, in which assignment extension does not introduce a new dependency and the cardinality condition is distriutivelty evaluated. See Minor (2022) for the motivation of this system based on his discussion on *dependent plurals*. However, one still needs to insert an operator to make the restrictor distriburive in his system. Thus, this distributivity of *wh*-restrictor has to be assumed with this alternative as well.