Day 2: dependent indefinites and negation

Takanobu Nakamura

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1   Summary of Day 1 lecture

- Dependent indefinites force a distributive reading.

(1)   a. Xeqatij **ox-ox**   wäy.
          we-eat three-three tortilla
          "We each ate three tortillas."   (Kaqchikel Mayan, Henderson, 2014)

      b. BOYS IX-arc-a read **one-arc-a** BOOK.
          "The boys read one book each."   (ASL, Kuhn, 2017)

- Dependent indefinites are incompatible with a singular argument.

(2)   a. * Xe'inchäp **ox-ox**   wäy.
            I-handle   three-three tortilla
          "I took (groups of) three tortillas."

                                      (Kaqchikel Mayan, Henderson, 2014)

      b. * JOHN-a READ **ONE-arc-a** BOOK.
          "John read one book (each time)."   (ASL, Kuhn, 2017)

- Dependent indefinites can occur under the scope of a quantifier.

(3)   a. Chikijujunal ri   tijoxela' xkiq'etej **ju-jun**   tz'i'.
          each   the students hugged   one-one dog
          "Each of the students hugged a dog."

                                      (Kaqchikel Mayan, Henderson, 2014)

      b. EACH-EACH-a PROFESSOR NOMINATE **ONE-redup-a** STUDENT.
          "Each professor nominated one student."   (ASL, Kuhn, 2017)

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- Henderson (2014); Kuhn (2017); Guha (2018) propose theories to account for these properties, each of which postulate a not-at-issue content for dependent indefinites.

  - They make different predictions for the behaviour of dependent indefinites under the scope of negation, which has not been studied well so far.

  - Today, I provide a case study from Turkish, and submit two claims.

  - (i) dependent indefinite takes scope just like ordinal indefinites, and

  - (ii) the co-variation condition of dependent indefinites is relevance-sensitive not-at-issue content.

2   Dependent indefinites in Turkish

- Turkish has a suffix "(ş)Ar," which attaches to cardinals.

  - The unit NUM-şAr shows several signature of dependent indefinites.

- First, they force a distributive reading when they occur below a plural argument as exemplified in (4).

(4)   Yedi çocuk üç-**er**   oyuncak seç-ti.
      seven child three-şer toy   choose-past
      "Seven children picked three toys each."

- If NUM-şAr occurs in a sentence without any plural argument or if no plural argument occurs above it, it is unacceptable as exemplified in (5a) and (5b).[1]

(5)   a. ?? Bir çocuk üç-**er**   oyuncak seç-ti.
             One child three-şer toy   choose-past
          "{A / one} child picked three toys each."

      b. ?? Yedi-**şer** çocuk   üç   oyuncak seç-ti.
             Seven-şer children three toy   choose-past
          (Intended) "Each group of seven children picked three toys."

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[1] However, one speaker accepted (5a) and (5b) and it may suggest that some speaker understand NUM-şAr in a similar way as Telugu dependent indefinites, which I briefly discuss in Day 3.

- Lastly, NUM-şAr can occur under the scope of distributive universal quantifier without being redundant as exemplified in (6).

(6) Her   çocuk üç-**er**   oyuncak seç-ti.
     Every child three-şer toy   choose-past
     "Every child picked three toys each."

  - These observations suggest that NUM-şAr in Turkish patterns with dependent indefinites in languages such as Kaqchikel Mayan and ASL.

- Now, I explore its behaviour under negation.

  - Negative examples are given in (7a) and (7b).[2]

(7) a. Bu yedi öğrenci üç-**er**   müze-ye   git-me-di.
        this seven student three-şer museum-dat visit-neg-past
        "These seven students did not visit three museums each."

     b. Her   öğrenci üç-**er**   müze-ye   git-me-di.
        Every student three-şer museum-dat visit-neg-past
        "Every student did not visit three museums each."

- Typical cases to use (7a) and (7b) are exemplified below.

(8) Context: seven students took a seminar of modern art. The lecturer told them that they should visit at least three museums in the city to see the general art style here. Now, it is the end of the semester and the lecture is asking about the students' visiting to museums.

     a. Scenario 1: four of the seven students visited three museums during the semester, but the other three just visited one or two museums.

                                                    → (7a) and (7b) are **true**

     b. Scenario 2: three of the seven students visited three museums, but the other four didn't visit any museums.   → (7a) and (7b) are **true**

- Atypical cases are exemplified below.

  - Here, none of the students did not visit three museums, which makes (7a) and (7b) sound much less natural than in Scenario 1 and 2.

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[2]I added "bu" (this) to the subject in (7a). This is to force a definite reading of the subject and eliminate possible readings in which the subject indefinite scopes under negation.

(9)   a.   Scenario 3: none of the seven students visited any museums.

$\rightarrow$ (7a) and (7b) are **true**

     b.   Scenario 4: all the seven students visited at least one museum, but none of them visited more than two museums.

$\rightarrow$ (7a) and (7b) are **true**

- In the rest of this section, I discuss two aspects of dependent indefinites under negation, namely scope and co-variation.

2.1   Narrow scope dependent indefinites

- In this section, I discuss the narrow scope construal of the existential force of dependent indefinites under negation.

- All the three accounts assume that dependent indefinites introduce a new value to a dref.

- This takes place *in-situ* in Henderson (2014); Guha (2018), but it takes place at the QR landing site in Kuhn (2017).

- The former is compatible with $\forall > \neg > \exists$ scope order because the post-supposed test and the existential force of a dependent indefinite may take scope independently.

- However, the latter is not: **dependent indefinites have to outscope a quantifier** in Kuhn (2017).

- As we have seen, (7a) and (7b) have this reading as shown in (8) and (9).

- This poses a challenge to Kuhn (2017).

- Note that an intermediate scope reading is available in Turkish.

(10)   Scenario 5: seven students took a seminar of modern art. The lecturer told them that they should visit several museums in the city to see the general art style here. This city has seven museums and the students may visit any of them. Now, it is the end of the semester and the lecture is asking about the students' visiting to museums. Interestingly, all the seven students visited exactly four museums. $\rightarrow$ (7a) and (7b) are **true**

- This suggests that the existential force of a dependent indefinite may take scope independently of its co-variation condition.

• A test with discourse anaphora provides further support.

- If a dref obtains a new value under negation, anaphoric reference to this value is not possible.

(11)   Mary does not have a car. #It is red.

- Thus, if a dependent indefinite itself scopes under negation, its value cannot be picked up by a pronoun in later discourse.

• Indeed, discourse anaphora is possible only under the $\forall > \exists 3 > \neg$ reading.

- (12) shows that a pronoun may pick up the value of dref that is introduced with "üç-er müze" (three museums each) as long as it scopes over negation.

(12)   a. O   yedi öğrenci^{u_1} üç-er   müze-ye^{u_2} git-me-di.
            That seven student   three-şer museum-dat go-neg-past
            "Those seven students visited three museums each."

        b. Onlar-ı$_{u_2}$ beğen-m-iyor-lar.
            They-acc like-neg-pres-3pl
            " They don't like them."

- In this continuation, it is most coherent if the students did not visit some museums **because** they do not like the museums.

- i.e. the indefinite scopes over negation.

• In contrast, (13) shows the opposite of (12).

(13)   a. O   yedi öğrenci^{u_1} üç-er   müze-ye^{u_2} git-me-di.
            That seven student   three-şer museum-dat go-neg-past
            "Those seven students didn't visit three museums each."

        b. # (Ama) onlar-ı$_{u_2}$ çok   beğen-di-ler.
              (but)   they-acc much like-past-3pl
            "(But) They liked them very much."

- In this continuation, it is not coherent if there are three museums for each student such that they like them.

- Thus, the $\forall > \exists 3 > \neg$ should sounds contextually strange.

• Here, the $\forall > \neg > \exists 3$ reading is still compatible with a situation in which each student visited less than three museums and they each liked them.

- If the cardinality condition takes scope under negation while the existential import takes scope over negation, (13) should be perfect in this reading.

- The infelicity of (13) shows that such an option is not available.

- i.e. the infelicity of (13) is most likely to be because negation scopes over the indefinite, i.e. (11) and (13) are bad for the same reason.

- This argument bolsters the argument with the intermediate reading with a dependent indefinite, which poses a challenge to Kuhn (2017).

2.2 Co-variation

• In this section, I discuss co-variation with dependent indefinites.

- All the three accounts assume that dependent indefinites require co-variation.

• If a co-variation condition is evaluated above negation, e.g., it scopes over negation or it projects from the scope of negation, it runs into a problem:

- when a dependent indefinite takes scope under negation, the co-variation condition outside negation cannot access this value.

- This predicts that dependent indefinites are always infelicitous under negation, which is falsified with (8).

• Another option is to assume that a co-variation condition is evaluated under negation as an at-issue content.

- Then, absence of co-variation should suffice to verify (7a) and (7b).

- However, it does not seems to be the case.

(14)   Scenario 6 (accidental lack of co-variation): all the seven students independently visited the same three museums. $\rightarrow$ (7a) and (7b) are **false**

- Thus, absence of co-variation does not verify (7a) and (7b).

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- On this point, one may pursue for the possibility that dependent indefinites require **possible** co-variation, instead of **actual** co-variation.

  - To check this, I test cases in which even possible co-variation is not possible.

  - In (15), co-variation is contextually ruled out, i.e. there are only three LOR films. Here, use of ɴᴜᴍ-şAr is judged infelicitous.[3]

(15) Scenario 7 (co-variation impossible): three students watched all the Lord of the Rings films.

    a. Her öğğrenci üç-er yüzüklerin efendisi filmi izle-me-di.
       every student three-şAr lord of the rings film watch-neg-past
       "Every student did not watch three LOR films each." → **infelicitous**

    b. Bu üc öğğrenci üç-er yüzüklerin efendisi filmi izle-me-di.
       this three student three-şAr lord of the rings film watch-neg-past
       "These three students did not watch three LOR films each."

                         → **infelicitous**

  - This suggests that dependent indefinites in Turkish require possible co-variation, but it tolerates accidental lack of co-variation.

- Guha (2018) provides a similar example in which a dependent indefinite in Bangla leads to infelicity without possibility of co-variation.

(16) a. Context: the number of Robi's dog is two.

    b. #o roj-i or-du-To-**kore**-kukur-ke bæRate nie
       he everyday-I his-two-CL-KORE-dog-Acc travel.impv take.pfv
       jae-na.
       go.Pres.3rd-Neg
       "He does not take two different dogs of his for a walk everyday."

                         (Guha, 2018)

  - See Guha (2018) for examples in which a dependent indefinite is embedded under a question and the antecedent of a conditional.

  - With these data, Guha (2018) argue that Bangla dependent indefinites encode the presupposed co-variation condition.

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[3]I deeply thank to Deniz Özyıldız for providing me this context. Note that one speaker judged the positive counterpart of (15b) true.

- However, (16b) is contextually consistent only if the indefinite itself takes the wide scope over negation.

- Thus, the above mentioned technical problem still remains:

- a dependent indefinite whose existential force scopes under negation is felicitous even if its value is not accessible above negation.

- All the three analyses seem problematic in this regard.

- They all impose the co-variation condition in the output PIS and thus it requires the actual co-variation.

- On this point, note that Guha (2018) argue that the co-variation condition makes reference to the common ground, i.e. it just requires that the common ground is compatible with co-variation.

- Her argument also points to the possible co-variation constraint, but I am not sure how her formulation of the $dif(u_m/u_n)$ test may globally constrain the context.

- In terms of scope of co-variation, Kuhn (2017) may circumvent the technical problem.

- However, it works as long as the existential force of a dependent indefinite always takes scope over negation, which is empirically challenged.

- The post-supposition-based theories work fine as long as negation discharges post-suppositions (Brasoveanu, 2013; Law, 2022).

- As long as a post-supposition under negation is evaluated *in-situ* and its violation leads to undefinedness, it is compatible with the observations above.

- In contrast, if post-suppositions project through negation, it faces the above mentioned technical problem: co-variation between a dref above negation and a dref under negation is trivially false.

2.3   Interim summary

- The existential force of NUM-şAr may take the intermediate scope between its licensor and negation.

- This is compatible with Henderson (2014); Guha (2018) because their post-supposed test and the existential force of a dependent indefinite may take scope independently.

- This poses a problem for Kuhn (2017) because an entire dependent indefinite has to be raised above its licensor, which takes scope over negation.

• The strength of co-variation condition poses a problem for all the three accounts: NUM-şAr tolerates an accidental lack of co-variation, but it still requires possible co-variation.

- All the three analyses make too strong predictions because they require an actual co-variation.

• There is also a technical problem concerning co-variation across negation:

- Since the existential force of a dependent indefinite may take scope under negation, the co-variation condition should not be evaluated above negation.

- This favours the approach in which post-suppositions are discharged under negation.

3 Context-sensitivity of co-variation inferences

• So far, I have argued that dependent indefinites in Turkish encode the possible co-variation condition.

- Interestingly but not unexpectedly, their co-variation condition seems to behave differently in different contexts.

- **Caveat: I have just found a critical flaw in the following argumentation, and welcome any suggestion to fix it.**

- Here, consider two examples that minimally differ in polarity.

(17)   a. Bu üç   öğrenci üç-**er**   müze-ye   git-ti.
          this three student three-şer museum-dat visit-past
          "These three students visited three museums each."

       b. Bu üç   öğrenci üç-**er**   müze-ye   git-**me**-di.
          this three student three-şer museum-dat visit-neg-past
          "These three students did not visit three museums each."

- Then, consider two contexts that differ in whether co-variation is relevant.

- The question is whether the editor or the PhD student is **truthfully speaking** to the professor in each of the following scenarios.

(18) Context 1 (co-variation **irrelevant**): three BA students are supervised by a PhD student in the department of art. The supervisor of the PhD student asked her to ask the three students to visit three museums and write an essay on them. A few weeks later, the BA students had a meeting with the PhD student. The PhD student told the professor that these three students {visited / did not visit} 3-er museums.
  a. Scenario 1 (**total co-variation**): they each visited a different set of three museums, i.e. nine museums are visited in total.
  b. Scenario 2 (**partial co-variation**): they each visited three museums, but they overlap in some museums. Between three and nine museums are visited in total.
  c. Scenario 3 (**no variation**): they each visited three museums, but they happened to visit the same three museums.

(19) Context 2 (co-variation **relevant**): three students are writers of a school biweekly newspaper in a university of fine art. A professor recommended their editor that the three students should visit 3-er museums so that they can write about as many museums as possible in this semester. A few weeks later, the students had a meeting with the editor. The editor told the professor that these three students {visited / did not visit} 3-er museums.
  a. Scenario 1 (**total co-variation**): they each visited a different set of three museums, i.e. nine museums are visited in total.
  b. Scenario 2 (**partial co-variation**): they each visited three museums, but they overlap in some museums. Between three and nine museums are visited in total.
  c. Scenario 3 (**no variation**): they each visited three museums, but they happened to visit the same three museums.

- The judgement does not seem super crisp, but judgments from five informants differ based on the relevance of co-variation.[4]

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[4]Interestingly but not surprisingly, a non-academic informant reported no contrast in co-variation across the contexts, and a logician informant reported the clear true/false contrast be-

- The judgements I have gathered at this point is summarised in Table 6.

- I boldface the part in which informants disagree.

	Scenario 1	Scenario 2	Scenario3
Context 1 Aff	true	true	true
Context 1 Neg	false	**false/?**	**false/?**
Context 2 Aff	true	**true/false/??**	**true/false/??**
Context 2 Neg	false	**true/false/??**	**true/false/??**

Table 1: Distribution of judgements on "-şer"

- I need to gather more data to find out a clear tendency here, but there are two things that should be emphasised.

- First, the difference in co-variation (almost) does not have an effect in Context 1, where variation is irrelevant.

- Second, the difference in co-variation matters in Context 2, where variation is relevant, but the pattern is chaotic.

- In both contexts, the possibility of co-variation is guaranteed, so this contrast would not be reduced to possible/actual distinction in co-variation.

- On the other hand, "farklı" (different) requires co-variation more actively.

- Note that it can even co-occur with "-şer."[5]

(20) a. Bu üç öğrenci üç(-**er**) **farklı** müze-ye git-ti.
        this three student three(-şer) different museum-dat visit-past
        "These three students visited three different museums (each.)"

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tween the two contexts, i.e. in Context 1, (17a) is true in all the scenarios while (17b) is false in all the scenarios, but in Context 2, (17a) is true in Scenario 1 while false in Scenario 2-3, and (17b) is false in Scenario 1 while true in Scenario 2-3. Other three informants are distributed somewhere in-between: they tend to be more sensitive to co-variation in Context 2 than in Context 1 while their judgement is not crisp, and also their judgement are sometimes not symmetric between the affirmative one and the negative one.

[5]An informant reported that they even prefer to put "-şer" than putting "farklı" (different) alone because its presence eliminates another possible reading of "different" with respect to an entity mentioned in the prior discourse, an *external reading*.

b. Bu üç öğrenci üç(-**er**) **farklı** müze-ye git-**me**-di.
   this three student three(-şer) different museum-dat visit-neg-past
   "These three students did not visit three different museums (each.)"

- The judgement I could obtain at this point is summarised in Table 2.

- Crucially, the contribution of "farklı" (different) does not vary depending on the relevance of co-variation *modulo* disagreement on whether it require total variation or partial variation

	Scenario 1	Scenario 2	Scenario3
Context 1 Aff	true	false/?	false
Context 1 Neg	false	true/?	true
Context 2 Aff	true	false/?	false
Context 2 Neg	false	true/?	true

Table 2: Distribution of judgements on "(-şer) farklı"

- The important observations are:

- "(-şer) farklı" does not lead to infelicity due to redundancy,

- co-variation does not matter unless it is contextually relevant.

- **However, this might mean that the semantic meaning of "-şer" only requires the possible co-variation, and the observed infelicity in Context 2 is due to contextual anomaly due to incongruence between the semantic content of (17a)/(17b) and the current issue in this context.**

- i.e. the observed effect in Context 2 is nothing to do with the co-variation condition, but it just means that (17a) and (17b) are not useful utterances to resolve the current issue.

- For this, one has to test the behaviour of plain numerals in the same contexts and scenarios, which I have not done unfortunately.

- If they give rise to a similar variation in judgements, the above observations do not tell anything about co-variation inferences.

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4   Co-variation as a relevance-sensitive inference

- The possible co-variation requirement suggests that the relevant test has to be a global constraint on the context.

  - i.e. the context contains at least one PIS with co-variation.

- I implement it with an *Update Semantics* or *Heimian* dynamic semantics.

  - A *possibility p* is a pair of a possible world and a PIS, i.e. $s = \langle w, G \rangle$.

  - A *context c* is a set of possibilities.

  - A formula ϕ denotes a **function** from a context to a context.

- I revise the type convention and model an assignment with type m.

  - Thus, a PIS is type $\langle mt \rangle$ and a possibility is of type $w \times \langle mt \rangle$.

  - Now, s should be taken as an abbreviation, i.e. $s = w \times \langle mt \rangle$.

  - I refine T as an abbreviation of $\langle st, st \rangle$, a function from a context to a context.

- This revised abbreviation convention allows us to keep using the same subclausal composition.

  - In this system, the co-variation condition can be defined as a post-supposed constraint on the context.

(21) $[\![\mathrm{dep}(u_m/u_n)]\!] = \lambda c \, \{\langle w, G \rangle : \langle w, G \rangle \in c\}$
  a. defined iff $\exists \langle w, G \rangle \in c \, [\exists d, e \in G(u_n) \, [G_{u_n = d}(u_m) \neq H_{u_n = e}(u_m)]]$,
  b. undefined, otherwise.

  - This only requires that c is compatible with co-variation between u_n and u_m, and c may still contain possibilities in which u_n and u_m do not co-vary.

  - i.e. it tolerates accidental lack of co-variation.

- Since the value of u_m may be newly introduced, $\mathrm{dep}(u_m/u_n)$ has to be post-supposed, imposing a constraint on the output context.

  - For this, it is necessary that a post-supposition is evaluated under the scope of negation (Brasoveanu, 2013; Law, 2022), contra Guha (2018).

- Otherwise, it leads to trivial undefinedness if the value of u_m is introduced under negation.

- As long as violation of $\text{dep}(u_m/u_n)$ leads to undefinedness, *Strong Kleene* connectives let this project from negation.

- Thus, it still correctly predicts (16b) to be infelicitous.

• Then, does it capture the variable strength of co-variation inference?

- If one takes a perspective based on **Question Under Discussion** (QUD) (Roberts, 2012), the at-issueness and projection vary in regard to how relevant it is to the current QUD.

- a content p is at-issue and non-projective if it is relevant to the current QUD, i.e. p resolves the current QUD, and

- it is not-at-issue and projective if it is irrevant to the current QUD.

• This leads to the opposite result, though:

- the co-variation inference is evaluated as a not-at-issue content when it is relevant, and it seems to **disappear** when it is irrelevant.

• This behaviour of the co-variation inference seems analogous to **non-maximal readings** of definite plurals that is also sensitive to QUD (Malamud, 2012; Krifka, 1996; Križ, 2015, 2016; Križ and Spector, 2021, a.o.)[6]

(22) A: The windows are open.

    a. Scenario 1: A and B went on a trip. A storm is coming up. B asks whether the house will be safe. Only half the windows are closed.

$\rightarrow$ (22) is **adequate**

    b. Scenario 2: A and B hired painters to paint their house. They cannot start working until all the windows are open. A asks B whether the house is ready. Half of the windows are still closed.

$\rightarrow$ (22) is **inadequate**

(Malamud, 2012)

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[6]It is too simplistic to say that they are QUD-sensitive, and it is more accurate to say that they are sensitive to *Current Questions / Issues*. See Križ (2016) for discussion.

- Furthermore, addition of "all" removes this non-maximal reading (Brisson, 1998; Križ, 2016, a.o.).

  - The contribution of "farklı" to "-şer" might be taken as something analogous to this contribution of "all" to definite plurals.

  - I do not attempt to formalise its contribution today, though.

- In the rest of this lecture, I offer a proof of concept by emulating Križ's (2016) trivalent approach with the Heimian DPlL.

  - Križ (2015, 2016) propose that definite plurals involve **truth-value gap**, i.e. their truth conditions and falsity conditions are non-complementary.

  - Those conditions in the grey zone are **pragmatically** grouped up with truth or falsity conditions, depending on how the context is partitioned.

- To implement this, I adopt **Trilateral Heimian DPlL**.

  - a formula ϕ has three extensions:

  - *positive* $[\![\phi]\!]^{+}$, *negative* $[\![\phi]\!]^{-}$, and *gap* $[\![\phi]\!]^{\#}$.

- Negation is defined as (23).

(23) a. $c[\![\neg\,\phi]\!]^{+}=c[\![\phi]\!]^{-}$
  b. $c[\![\neg\,\phi]\!]^{-}=c[\![\phi]\!]^{+}$
  c. $c[\![\neg\,\phi]\!]^{\#}=c[\![\phi]\!]^{\#}$

- I adopt Strong Kleene connectives.

  - Conjunction is defined as (24).[7]

(24) a. $c[\![\phi;\psi]\!]^{+}=c[\![\phi]\!]^{+}[\![\phi]\!]^{+}$
  b. $c[\![\phi;\psi]\!]^{-}=$
    $c[\![\phi]\!]^{-}[\![\psi]\!]^{+}\cup c[\![\phi]\!]^{-}[\![\psi]\!]^{-}\cup c[\![\phi]\!]^{-}[\![\psi]\!]^{\#}\cup c[\![\phi]\!]^{+}[\![\psi]\!]^{-}\cup c[\![\phi]\!]^{\#}[\![\psi]\!]^{-}$.
  c. $c[\![\phi;\psi]\!]^{\#}=c[\![\phi]\!]^{+}[\![\psi]\!]^{\#}\cup c[\![\phi]\!]^{\#}[\![\psi]\!]^{+}\cup c[\![\phi]\!]^{\#}[\![\psi]\!]^{\#}$.

- In this setting, I define the denotations of "*n*-şer" as (25).

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[7]I ignore disjunction and material implication in this lecture. Note that disjunction is defined as the mirror image of conjunction.

(25)   a. $c[\![n\ \text{şer}(u_m/u_n)]\!]^+ = \{\langle w', H\rangle : \exists \langle w, G\rangle \in c\,[w = w'\,\&\,G[u_m]H\,\&\,\exists d, e \in H(u_n)\,[H_{u_n=d}(u_m) \neq H_{u_n=e}(u_m)]\,\&\,\forall d \in H(u_n)\,[|H_{u_n=d}(u_m)| = n]]\}$

   b. $c[\![n\ \text{şer}(u_m/u_n)]\!]^- = \{\langle w', H\rangle : \exists \langle w, G\rangle \in c\,[w = w'\,\&\,G[u_m]H\,\&\,\exists d, e \in H(u_n)\,[H_{u_n=d}(u_m) \neq H_{u_n=e}(y)]\,\&\,\exists d \in H(u_n)\,[|H_{u_n=d}(u_m)| \neq n]]\}$

   c. $c[\![n\ \text{şer}(u_m/u_n)]\!]^{\#} = \{\langle w', H\rangle : \exists \langle w, G\rangle \in c\,[w = w'\,\&\,G[u_m]H\,\&\,\forall d, e \in H(u_n)\,[H_{u_n=d}(u_m) = H_{u_n=e}(u_m)]]\}$

- The possibilities that fall in $[\![\phi]\!]^{\#}$ may be grouped up with $[\![\phi]\!]^+$, depending on the context.

  - I adopt Križ's (2016) *Weak Maxim of Quality* refined in this setting.

(26)   a. **Weak Maxim of Quality**: Say only sentences which you believe to be *true enough*.

   b. A sentence S is *true enough* in a possibility $\langle w, K\rangle$ with respect to an issue I iff there is a possibility $\langle w', H\rangle$ such that $\langle w', H\rangle \in [\![S]\!]^+$, and w and w' are in the same cell of I.

  - One may dynamicise I by taking it as a set of sets of possibilities (e.g., Roelofsen and Dotlačil, 2023), but it is not necessary for now.

(27)   A sentence S is *true enough* in a possibility $\langle w, K\rangle$ with respect to an issue I iff there is a possibility $\langle w', H\rangle$ such that $\langle w', H\rangle \in [\![S]\!]^+$, and $\langle w, G\rangle$ and $\langle w', H\rangle$ are in the same cell of I.

- Now, revisit the core data.

  - The DRS of (17a) is given in (28).

(17a)   Bu üç   öğrenci^{u_1} üç-**er**$^{u_2}_{u_1}$ müze-ye   git-ti.
        this three student   three-şer museum-dat visit-past
        "These three students visited three museums each."

(28) $[u_1|];[\,|\,\text{students}(u_1)];[\,|\,u_1 = 3];[\,|\,3\ \text{şer}(u_2/u_1)];[\,|\,\text{museums}(u_2)];[\,|\,\text{visited}(u_1)(u_2)]$

- The positive extension of (28) is essentially the same as Kuhn (2017) except that the dependent indefinite does not undergo QR.

  - It only stores possibilities in which u_2 is dependent on u_1.

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- Possibilities in which u_1 and u_2 are independent fall into the gap extension.[8]

- The DRSs of (17b) is given in (29).

(17b) Bu üç öğrenci^{u_1} üç-**er**$^{u_2}_{u_1}$ müze-ye git-**me**-di.
this three student   three-şer museum-dat visit-neg-past
"These three students did not visit three museums each."

(29) $[u_1|]; [|\text{students}(u_1)]; [|u_1 = 3]; \neg([|3 \text{ şer}(u_2/u_1)]; [|\text{museums}(u_2)]; [|\text{visited}(u_1)(u_2)])$

- Since $[|3 \text{ şer}(u_2/u_1)]$ is evaluated under negation, the positive extension of (17b) considers the negative extension of $[|3 \text{ şer}(u_2/u_1)]$.

- Recall that its negative extension stores co-variation between u_1 and u_2 just like its positive extension.

- Accordingly, the co-variation requirement itself is evaluated under the scope of negation while its infelicity projects through Strong Kleene connectives.[9]

- This avoids the issue of co-variation between a dref above negation and another dref below negation: the requirement itself scopes below negation and thus both values are accessible when it is evaluated.

- This co-variation requirement should still be evaluated above a distributive universal quantifier (recall Day 1 lecture.)

- This is compatible with the view that post-suppositions project from the scope of distributive quantification while they are discharged under the scope of negation (Brasoveanu, 2013; Law, 2022).

- This is different from Guha (2018), but her argument for projection of co-variation inferences is still preserved thanks to this setting with Strong Kleene connectives and trilateralism.[10]

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[8]To be precise, $c[\![n \text{ şer}(u_m/u_n)]\!]$ should probably require $u_m > 1$ in addition. Otherwise, it predicts that NUM-şer with a singular argument is licensed in some contexts.

[9]In this context, $[|\text{museum}(u_1)]$ and $[|\text{visited}(u_2)]$ are forced to be true. Thus, the definedness of the resultant value solely relies on the definedness of the value of $[|3 \text{ şer}(u_2/u_1)]$. I do not have anything to say about when $[|\text{museum}(u_1)]$ or $[|\text{visited}(u_2)]$ is false. Prediction is that (17b) is false regardless of co-variation between u_1 and u_2 in such cases. I leave examination of it for now.

[10]The same thing can be done with Trivalent DPlL, but I put aside this option for now.

- Thus, this trivalent/trilateral analysis of the possible co-variation condition may serve as a solution to the technical problem with negation and indefinite, independently of the point with relevance-sensitivity.

• Let me discuss relevance-sensitivity in light of this trilateral setting.

- Context 1 and the three scenarios are repeated below.

(18) Context 1 (co-variation **irrelevant**): three BA students are supervised by a PhD student in the department of art. The supervisor of the PhD student asked her to ask the three students to visit three museums and write an essay on them. A few weeks later, the BA students had a meeting with the PhD student. The PhD student told the professor that these three students {visited / did not visit} 3-er museums.

 a. Scenario 1 (**total co-variation**): they each visited a different set of three museums, i.e. nine museums are visited in total.
 b. Scenario 2 (**partial co-variation**): they each visited three museums, but they overlap in some museums. Between three and nine museums are visited in total.
 c. Scenario 3 (**no variation**): they each visited three museums, but they happened to visit the same three museums.

- (18) concerns the number of museums per student: did the students all visit at least three museums?

(30) Non-dynamic representation of *I*:
$I = \{\{w_1 : \text{All the three students visited three museums in } w_1\},$
$\{w_2 : \text{not all the three students visited three museums in } w_2\}\}$

(31) Dynamic representation of *I*:
$I = \{[\![\text{All the three students visited three museums}]\!](c),$
$[\![\neg(\text{All the three students visited three museums})]\!](c)\}$

• All the three scenarios, including Scenario 3, belong to the *yes*-partition, i.e. all the students visited three museums, and get grouped up with (25a).

- At the same time, the possibilities that describe Scenario 3 belong to the gap extension of (17a) and (17b).

- For example, consider a world in which all the three students visited the same three museums.

(32) I_w(visited)=
    {⟨student$_1$, museum$_{1-3}$⟩, ⟨student$_2$, museum$_{1-3}$⟩, ⟨student$_3$, museum$_{1-3}$⟩}

  - Here, $w \in \{w_1 :$ All the three students visited three museums$\}$.

- This world is compatible with the two PISs exemplified below.

  - Here, ⟨w,G⟩, ⟨w,G'⟩ $\in$ ⟦All the three students visited three museums⟧(c).[11]

G	u_1	u_2
g_1	student$_1$	museum$_1$
g_2	student$_1$	museum$_2$
g_3	student$_1$	museum$_3$
g_4	student$_2$	museum$_1$
g_5	student$_2$	museum$_2$
g_6	student$_2$	museum$_3$
g_7	student$_3$	museum$_1$
g_8	student$_3$	museum$_2$
g_9	student$_3$	museum$_3$

Table 3: A PIS without variation

G'	u_1	u_2
g'_1	student$_1$	museum$_1$+museum$_2$+museum$_3$
g'_2	student$_2$	museum$_1$+museum$_2$+museum$_3$
g'_3	student$_3$	museum$_1$+museum$_2$+museum$_3$

Table 4: A PIS without variation

- Notice that u_2 is independent of u_2 in these possibilities.

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[11]I leave the audiences to confirm that this holds.

- Thus, $\langle w,G \rangle, \langle w,G' \rangle \in [[(17a)]]^\#$ and $\langle w,G \rangle, \langle w,G' \rangle \in [[(17b)]]^\#$

- However, since w (or $\langle w,G \rangle$, $\langle w,G' \rangle$) belongs to the *yes* partition, (17a) is **true enough** in $\langle w,G \rangle$ and $\langle w,G' \rangle$ with respect to the issue I in Context 1.

- For the same reason, (17b) is false in $\langle w,G \rangle$ and $\langle w,G' \rangle$ with respect to the issue I in Context 1.

- This derives the pattern of judgement in Context 1 repeated below: co-variation does not affect the truth of (17a) and (17b).

	Scenario 1	Scenario 2	Scenario3
Context 1 Aff	true	true	true
Context 1 Neg	false	**false/?**	**false/?**

Table 5: Distribution of judgements in Context 2

- In contrast, Context 2 and the three scenarios are repeated below.

(19) Context 2 (co-variation **relevant**): three students are writers of a school biweekly newspaper in a university of fine art. A professor recommended their editor that the three students should visit 3-er museums so that they can write about as many museums as possible in this semester. A few weeks later, the students had a meeting with the editor. The editor told the professor that these three students {visited / did not visit} 3-er museums.

  a. Scenario 1 (**total co-variation**): they each visited a different set of three museums, i.e. nine museums are visited in total.

  b. Scenario 2 (**partial co-variation**): they each visited three museums, but they overlap in some museums. Between three and nine museums are visited in total.

  c. Scenario 3 (**no variation**): they each visited three museums, but they happened to visit the same three museums.

- (18) concerns the total number of museums: did the students maximise the number of museums they visited?

- More precisely, this question seems to involve vagueness, i.e. whether the students visited more than n museums, where n is the contextual standard.

- I suspect that the noise in the result reflects this uncontrolled vagueness, and this test shall be done again after removing it.

• The issue in Context 2 is informally given as follows.

(33)   Non-dynamic representation of I:
       $I = \{\{w_1 : \text{The three students visited more than or equal to } n \text{ museums in } w_1\}, \{w_2 : \text{The three students visited less than } n \text{ museums in } w_2\}\}$

(34)   Dynamic representation of I:
       $I = \{[\![\text{The three students visited more than or equal to } n \text{ museums}]\!](c),$
       $[\![\neg(\text{The three students visited less than } n \text{ museums})]\!](c)\}$

• Scenario 1 clearly belongs to the *yes*-partition because it hits the maximum number of museums they can visit, but it is unclear for the other two.

- Recall the PISs given in Table 3 and Table 4.

- In these PISs, u_2 only stores 3 museums.

- For speakers whose n is larger than 3, the prediction is that (17a) and (17b) are infelicitous in Scenario 3.

• However, this line of thoughts predicts a contrast between Scenario 2-3.

- None of the informants I consulted with has reported a clear contrast between these scenarios, though.[12]

- The pattern of judgements in Context 2 is repeated below.

	Scenario 1	Scenario 2	Scenario3
Context 2 Aff	true	**true/false/??**	**true/false/??**
Context 2 Neg	false	**true/false/??**	**true/false/??**

Table 6: Distribution of judgements on "-şer"

• Thus, it does not seem to be a matter of co-variation, but rather a matter of congruence between an utterance with a possible co-varying numeral and a vague question about quantity.

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[12]One informant has reported that in Context 2, the speaker is more likely to use (17b) in Scenario 3 than in Scenario 2. This is expected if this is a matter of congruence and the current question here is vague.

5 Conclusion

- The intermediate scope construal of NUM-şAr is available and it suggests that its co-variation condition and its existential force may take scope separately.

 - This favours an approach with post-suppositions.[13]

- NUM-şAr only requires possible co-variation, i.e. it suffices if the context is compatible with a PIS with co-variation.

 - For this, the co-variation condition should be a test on the entire context rather than a constraint on particular PISs.

 - If one defines it as a post-supposed global definedness condition on the output context, it predicts the projection of co-variation condition without running into a technical problem.

 - For this, one has to say that the co-variation post-supposition is discharged under negation, but its violation leads to undefinedness.

- Then, I examined whether the co-variation inference is sensitive to the QUD.

 - However, the data obtained at this point is noisy, and the counter-analysis in terms of a general congruence seems to work better.

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[13]One may consider the *higher-order GQ* analysis of post-suppositions, i.e. a dependent indefinite has *polymorphic* type so that its co-variation/plurality condition takes the widest scope. However, this still needs an additional mechanism to let its existential force take its scope independently. Charlow (2014) offers a theory in which an indefinite takes its scope as *state-sensitive, non-determinate side effect*. As long as a post-supposition takes its scope as a canonical scope-taker and an indefinite takes its scope as a side effect, one may tease apart their scope-taking. However, one still has to make sure that the co-variation post-supposition takes scope **below** negation because it runs into a problem when the existential force of a dependent indefinite takes scope under negation. Since I have been concerned with a plural licensor in this lecture, one has to apply the same test to the cases with a distributive licensor. If the same relevance-sensitivity is observed, it poses a challenge to a polymorphic type approach: the co-variation condition has to take scope over a distributive quantifier, but below negation.

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