Relevance-sensitive co-variation inferences of dependent indefinites

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19th September 2024, Sinn und Bedeutung 29 at Sicily

1 Introduction

- In this talk, I discuss expressions called *dependent indefinites*, which triggers an **obligatory** distributive reading (Choe, 1987; Gil, 1982, 1995; Farkas, 1997; Oh, 2001; Balusu, 2006; Brasoveanu and Farkas, 2011; Henderson, 2014; Kuhn, 2017; Guha, 2018, among many others).[1,2]

(1) a. Xeqatij **ox-ox** wäy.
we-eat three-three tortilla
"We each ate three tortillas." (Kaqchikel Mayan, Henderson, 2014)

b. BOYS IX-arc-a read **one-arc-a** BOOK.
"The boys read one book each."
(American Sign Language, Kuhn, 2017)

[1]I would like to thank to Deniz Özyıldız, Yağmur Sağ, Beste Kamali and two other Turkish informants for data and discussion. All the errors and imprecision should be attributed to me.

[2]Several terms with slightly different coverage compete in the market, e.g., *distributive numerals*, *share-markers*. I adopt "dependent indefinite" to focus on the expressions that trigger an obligatory distributive reading of plurals but felicitously sit under the scope of a distributive quantifier. For example, the adnominal "each" in English may be classified as a distributive numeral or a share-marker, but it is unclear if it is a dependent indefinite (However, Szabolcsi, 2010, claims that many speakers accept a binominal "each" under the scope of "every" or sometimes even under the scope of the determiner "each."). Also, Henderson (2016) distinguishes *dependent existentials* and *dependent numerals*: the former is formed with an indefinite marker or the numeral "one," and the latter is formed with numerals other than "one." While this distinction is associated with several interesting difference between them, I mostly ignore this distinction in this talk.

- Dependent indefinites are incompatible with a singular argument.[3]

(2) a. * Xe'inchäp **ox-ox** wäy.
I-handle three-three tortilla
"I took (groups of) three tortillas."
(Kaqchikel Mayan, Henderson, 2014)

b. * JOHN-a READ **ONE-arc-a** BOOK.
"John read one book (each time).'
(American Sign Language, Kuhn, 2017)

- These observations may suggest that dependent indefinites themselves perform distributive quantification.

- Then, one would expect them to be infelicitous under the scope of another distributive quantifier, e.g., *Every student each bought two books.

- However, this is not the case.

(3) a. Chikijujunal ri tijoxela' xkiq'etej **ju-jun** tz'i'.
each the students hugged one-one dog
"Each of the students hugged a dog."
(Kaqchikel Mayan, Henderson, 2014)

b. EACH-EACH-a PROFESSOR NOMINATE **ONE-redup-a** STUDENT.
"Each professor nominated one student."
(American Sign Language, Kuhn, 2017)

- Henderson (2014); Kuhn (2017); Guha (2018) tackle this puzzle with a *co-variation* condition with delayed evaluation order, e.g., quantifier raising (QR) (Kuhn, 2017), *post-supposition* (Henderson, 2014; Guha, 2018), *higher-order GQ* (Charlow, 2017).

- The idea is that a dependent indefinite comes with the requirement that its value has to co-vary with the value of its 'licensor' (Farkas, 1997, *et seq*).

- Then, this requirement is evaluated outside the scope of a quantifier, avoiding co-variation failure.

[3]Dependent indefinites in some languages permit this, triggering a distributive reading with respect to events/occassions, e.g., Teligu reduplicative numerals (Balusu, 2006), Korean "ssik," (Oh, 2001) Japanese "zutsu" (Champollion, 2017; Nakamura, 2021), and so on.

- This talk aims to scrutinise this co-variation requirement through inspecting dependent indefinites occuring in sentences with negation.

- As a case study, I discuss dependent indefinites in Turkish.

2 Dependent indefinites in Turkish

- Turkish has a suffix "(ş)Ar," which attaches to cardinals.

- The unit NUM-şAr shows several signature of dependent indefinites.

- It forces a distributive reading when they occur below a plural argument:

(4) Yedi çocuk üç-**er** oyuncak seç-ti.
seven child three-şer toy choose-past
"Seven children picked three toys each."

- If it occurs in a sentence without any plural argument or if no plural argument occurs above it, it is unacceptable:[4]

(5) a. *Bir çocuk üç-**er** oyuncak seç-ti.
One child three-şer toy choose-past
"{A / one} child picked three toys each."

b. *Yedi-**şer** çocuk üç oyuncak seç-ti.
Seven-şer children three toy choose-past
(Intended) "Each group of seven children picked three toys."

- It can occur under the scope of distributive universal quantifier without being redundant:

(6) Her çocuk üç-**er** oyuncak seç-ti.
Every child three-şer toy choose-past
"Every child picked three toys each."

- These observations suggest that NUM-şAr in Turkish patterns with dependent indefinites in languages such as Kaqchikel Mayan and ASL.

[4]However, one speaker accepted (5a) and (5b). It may suggest that some speakers understand NUM-şAr in a similar way as Telugu dependent indefinites. I put aside this potential intra-linguistic variation today, though.

2.1 Scope of the existential import

- Negative examples are given in (7a) and (7b).[5]

(7) a. Bu yedi öğrenci üç-**er** müze-ye git-me-di.
this seven student three-şer museum-dat visit-neg-past
"These seven students did not visit three museums each."

b. Her öğrenci üç-**er** müze-ye git-me-di.
Every student three-şer museum-dat visit-neg-past
"Every student did not visit three museums each."

- The existential import of NUM-şAr may take scope below universal and above negation, i.e. an intermediate scope reading.[6]

(8) Context: seven students took a seminar of modern art. The lecturer told them that they should visit several museums in the city to see the general art style here. This city has seven museums and the students may visit any of them. Now, it is the end of the semester and the lecture is asking about the students' visiting to museums. Interestingly, all the seven students visited exactly four museums. → (7a) and (7b) are **true**

- The existential import of NUM-şAr may scope under negation as well.

(9) Context: seven students took a seminar of modern art. The lecturer told them that they should visit at least three museums in the city to see the general art style here. Now, it is the end of the semester and the lecture is asking about the students' visiting to museums.

a. Scenario 1: four of the seven students visited three museums during the semester, but the other three just visited one or two museums.
→ (7a) and (7b) are **true**

b. Scenario 2: three of the seven students visited three museums, but the other four didn't visit any museums. → (7a) and (7b) are **true**

- Scenario 3-4 in (7a) and (7b) sound much less natural than in Scenario 1 and 2: there is no reason to use the cardinal "üç" (three) in these cases.

[5] I added "bu" (this) to the subject in (7a). This is to force a definite reading of the subject and eliminate possible readings in which the subject indefinite scopes under negation.

[6] This does not seem generalisable across languages: Hofherr and Etxeberria (2017) observes that dependent indefinites in Basque may only take the narrow scope with respect to negation.

- Nonetheless, (7a) and (7b) are acceptable modulo pragmatic oddness.

(10) a. Scenario 3: none of the seven students visited any museums.
→ (7a) and (7b) are **true**

b. Scenario 4: all the seven students visited at least one museum, but none of them visited more than two museums.
→ (7a) and (7b) are **true**

- The relevant part of these observations is that the existential import of NUM-şAr may be evaluated in a negative environment.[7]

[7]Discourse anaphora provides further support for this. It has been observed that negation is an inherent barrier for anaphora, putting aside cases with disjunction, double negation and so on.

(i) Mary does not have a car. #It is red.

Thus, if the existential import of a NUM-şAr scopes under negation, its value cannot be picked up by a pronoun in later discourse. Indeed, discourse anaphora is possible only under the $\forall > \exists 3 > \neg$ reading. In (ii), it is most coherent if the students did not visit some museums **because** they do not like the museums i.e. the indefinite scopes over negation. Anaphora is fine in this case.

(ii) a. O yedi öğrenci^{x} üç-er müze-ye^{y} git-me-di.
That seven student three-şer museum-dat go-neg-past
"Those seven students did not visit three museums each."

b. Onlar-ı$_{y}$ beğen-m-iyor-lar.
They-acc like-neg-pres-3pl
" They don't like them."

In contrast, in (iii), it is not coherent if there are three museums for each student such that they like them, i.e. the $\forall > \exists 3 > \neg$ reading should sounds contextually strange. However, the $\forall > \neg > \exists 3$ reading is still compatible with a situation in which each student visited less than three museums and they each liked them. If one derives a 'pseudo' narrow scope reading in which the existential import scope over negation and the cardinality condition takes scope under negation, (iii) should be perfect in this reading. The infelicity of (iii) shows that such an option is not available.

(iii) a. = (iia)

b. # (Ama) onlar-ı$_{y}$ çok beğen-di-ler.
(but) they-acc much like-past-3pl
"(But) They liked them very much."

The intended reading can be expressed with an overt noun phrase + a relative clause, which suggests that (iii) is infelicitous for an anaphora related reason.

(iv) a. = (iia)

2.2 Co-variation in sentences with negation

- Now, I discuss co-variation in negated sentences.
- There is a technical consideration:
  - Take variables x and y such that x is bound by $\forall$ above $\neg$ and y is bound by $\exists$ under $\neg$, i.e. $\forall x \neg \exists y$.
  - Here, y may not co-vary with x because negation denies any assignment of a value on y and co-variation is impossible.
  - i.e. the co-variation condition is always false if it takes such x and y.
  - This predicts that either the narrow scope reading is always infelicitous or it is trivially true, i.e. the trivial falsity of the co-variation condition suffices to verify any given sentences with NUM-şAr and negation.
  - This is obviously too strong.
- Thus, one has to evaluate the co-variation condition under the scope of negation, in which it can access to the defined values of x and y.
  - If one takes the co-variation as an at-issue content, absence of co-variation should suffice to verify (7a) and (7b).
  - If one takes it as a not-at-issue content, absence of co-variation should make (7a) and (7b) sound illicit.
  - However, neither seems to be the case: as long as the students visited three museums each (7a) and (7b) are false despite the lack of co-variation.[8]

(11) Scenario 5 (accidental lack of co-variation): all the seven students independently visited the same three museums. → (7a) and (7b) are **false**

- Then, is co-variation simply ignored altogether in negated sentences?

b. (Ama) git-tik-leri müze-ler-i çok beğen-di-ler.
(But) go-nomin-poss3pl museum-pl-acc much like-past-3pl
"(But) they liked the museums they visited very much."

[8] See Wohlmuth (2019) for an observation and argument for a similar point: (positive) sentences with Hungarian reduplicative numerals may not be rejected just because of lack of co-variation.

- Here, one may pursue the possibility that dependent indefinites require **possible** co-variation, instead of **actual** co-variation.

- To check this, I test cases in which even possible co-variation is not possible.

- In (12), co-variation is contextually ruled out, i.e. there are only three LOR films. Here, use of NUM-şAr is judged infelicitous.[9]

(12) Scenario 6 (co-variation impossible): three students watched all the Lord of the Rings films.

a. Her öğğrenci üç-er yüzüklerin efendisi filmi izle-me-di.
every student three-şAr lord of the rings film watch-neg-past
"Every student did not watch three LOR films each." → **infelicitous**

b. Bu üc öğğrenci üç-er yüzüklerin efendisi filmi izle-me-di.
this three student three-şAr lord of the rings film watch-neg-past
"These three students did not watch three LOR films each."
→ **infelicitous**

- This suggests that dependent indefinites in Turkish require possible co-variation, but it tolerates accidental lack of co-variation.[10]

- Guha (2018) provides a similar example in which a dependent indefinite in Bangla is judged illicit if the context is incompatible with co-variation.

(13) a. Context: the number of Robi's dog is two.

b. # o roj-i or-du-To-**kore**-kukur-ke bæRate nie
he everyday-I his-two-CL-KORE-dog-Acc travel.impv take.pfv
jae-na.
go.Pres.3rd-Neg
"He does not take two different dogs of his for a walk everyday."
(Guha, 2018)

- See Guha (2018) for examples in which a dependent indefinite is embedded under a question and the antecedent of a conditional.

[9] I deeply thank to Deniz Özyıldız for providing me this context. Note that one speaker judged the positive counterpart of (12b) true.

[10] Farkas (2021) leaves open the possibility that Hungarian reduplicative numerals only require possible co-variation.

- Guha (2018) concludes that Bangla dependent indefinites encode the presupposed co-variation condition.

- However, notice that this view just requires that the context is **compatible** with co-variation.[11]

- This is strange as a presupposition, considering presuppositions usually have to be contextually entailed.[12]

- It rather makes sense as an *anti-presupposition* i.e. absence of co-variation is not contextually entailed.

- Another possibility is that actual co-variation indeed arises as an inference, but this may disappear or cancelled in some contexts.

- I consider this possibility in the next section.

3 Context-sensitivity of co-variation inferences

- Here, I consider two examples that minimally differ in polarity.

(14) a. Bu üç öğrenci üç-**er** müze-ye git-ti.
this three student three-şer museum-dat visit-past
"These three students visited three museums each."

b. Bu üç öğrenci üç-**er** müze-ye git-**me**-di.
this three student three-şer museum-dat visit-neg-past
"These three students did not visit three museums each."

- Then, consider two contexts that differ in whether co-variation is relevant.

- I asked my informants whether the editor or the PhD student is truthfully speaking to the professor in each of the following scenarios.

[11] I am not sure if her implementation ensures this, though. She defines co-variation condition as a post-supposition in a dynamic setting *a la* Henderson (2014), but his definition applies a post-suppositional test *point-wise* to each output (plural) information state. This would lead to actual co-variation requirement.

[12] I thank to Yasu Sudo (p.c.) to pointing it out.

(15) Context 1 (co-variation **irrelevant**): three BA students are supervised by a PhD student in the department of art. The supervisor of the PhD student asked her to ask the three students to visit three-er museums and write an essay on them. A few weeks later, the BA students had a meeting with the PhD student. The PhD student told the professor that these three students {visited / did not visit} 3-er museums.

a. Scenario 1 (**total co-variation**): they each visited a different set of three museums, i.e. nine museums are visited in total.

b. Scenario 2 (**partial co-variation**): they each visited three museums, but they overlap in some museums. Between three and nine museums are visited in total.

c. Scenario 3 (**no variation**): they each visited three museums, but they happened to visit the same three museums.

(16) Context 2 (co-variation **relevant**): three students are writers of a school biweekly newspaper in a university of fine art. A professor recommended their editor that the three students should visit 3-er museums so that they can write about as many museums as possible in this semester. A few weeks later, the students had a meeting with the editor. The editor told the professor that these three students {visited / did not visit} 3-er museums.

a. Scenario 1 (**total co-variation**): they each visited a different set of three museums, i.e. nine museums are visited in total.

b. Scenario 2 (**partial co-variation**): they each visited three museums, but they overlap in some museums. Between three and nine museums are visited in total.

c. Scenario 3 (**no variation**): they each visited three museums, but they happened to visit the same three museums.

- The judgement does not seem super crisp, but judgments from five informants differ based on the relevance of co-variation.[13]

[13] Interestingly but not surprisingly, a non-academic informant reported no contrast in co-variation across the contexts, and a logician informant reported the clear true/false contrast between the two contexts, i.e. in Context 1, (14a) is true in all the scenarios while (14b) is false in all the scenarios, but in Context 2, (14a) is true in Scenario 1 while false in Scenario 2-3, and (14b) is false in Scenario 1 while true in Scenario 2-3. Other three informants are distributed somewhere in-between: they tend to be more sensitive to co-variation in Context 2 than in Context 1 while their judgement is not crisp, and also their judgement are sometimes not symmetric between the affirmative one and the negative one.

- The judgements I have gathered at this point is summarised in Table 8.

	Scenario 1	Scenario 2	Scenario3
Context 1 Aff	true	true	true
Context 1 Neg	false	**false/?**	**false/?**
Context 2 Aff	true	**true/false/??**	**true/false/??**
Context 2 Neg	false	**true/false/??**	**true/false/??**

Table 1: Distribution of judgements on "-şer"

- There are two things that should be emphasised.
- First, the difference in co-variation (almost) does not have an effect in Context 1, where variation is irrelevant.
- Second, the difference in co-variation matters in Context 2, where variation is relevant, but the pattern is chaotic.
- In both contexts, the possibility of co-variation is guaranteed, so this contrast would not be reduced to possible/actual distinction in co-variation.
- On the other hand, "farklı" (different) requires co-variation more actively.
- Note that it can even co-occur with "-şer."[14]

(17) a. Bu üç öğrenci üç(-**er**) **farklı** müze-ye git-ti.
this three student three(-şer) different museum-dat visit-past
"These three students visited three different museums (each.)"

b. Bu üç öğrenci üç(-**er**) **farklı** müze-ye git-**me**-di.
this three student three(-şer) different museum-dat visit-neg-past
"These three students did not visit three different museums (each.)"

- The judgement I could obtain at this point is summarised in Table 2.

[14] An informant reported that they even prefer to put "-şer" than putting "farklı" (different) alone because its presence eliminates another possible reading of "different" with respect to an entity mentioned in the prior discourse, an *external reading*.

- Crucially, the contribution of "farklı" (different) does not vary depending on the relevance of co-variation.

- Note that there is disagreement on whether it require total variation or partial variation.

	Scenario 1	Scenario 2	Scenario3
Context 1 Aff	true	false/?	false
Context 1 Neg	false	true/?	true
Context 2 Aff	true	false/?	false
Context 2 Neg	false	true/?	true

Table 2: Distribution of judgements on "(-şer) farklı"

- The important observations are:

- "(-şer) farklı" does not lead to infelicity due to redundancy,

- co-variation does not matter unless it is contextually relevant.

4 Possible ways to model the co-variation inference

4.1 Implicature

- Henderson (2014) proposes that dependent indefinites signal **discourse plurality** (Brasoveanu, 2008), i.e. plurality of assignments, while plain numerals signal discourse singularity.

- Discourse plurality entails co-variation.

- One may say that this discourse plurality arises as an implicature[15]:

- dependent indefinites underspecify discourse plurality and plain indefinites asymmetrically entail them.

[15] cf. Balusu (2006), which proposes an event-based analysis of Telugu reduplicative numerals and conjectures that an event plurality condition is an implicature.

- It works well for positive cases:

- co-variation inference is generated if it is relevant to the issue but not if it is irrelevant.

- However, it does not for negative cases:

- dependent indefinites become stronger than plain numerals under negation and it wrongly predicts that the co-variation inference disappears.

- In addition, it is not trivial how plain indefinites compete with dependent indefinites.

- It works fine for a plural licensor if one adopts Kuhn's (2017) definitions.[16]

- However, it does not in cases with a quantificational licensor: the licensor itself already enables co-variation with plain indefinites.

4.2 Anti-presupposition

- Another possibility is that the co-variation inference is an anti-presupposition.

- For this, one has to adopt a more committed view on plain numerals:

- plain numerals presuppose/post-suppose discourse singularity (Guha, 2018).[17]

- Then, the competition between a plain numeral and a dependent indefinite results in a presuppositional discourse-plurality implicature.

- In this approach, no asymmetry between positive and negative is predicted.

- However, I am not sure about relevance.

[16] His definition of plain numerals allows one to derive a non-co-varying distributive reading without the covert distributivity operator. Then, if one combines it with his definition of discourse plurality as plurality of dependencies held in an information state, plain numerals and dependent indefinites minimally differ in terms of plurality of co-varying pair, and their competition leads to generation of the co-variation inference.

[17] The idea that plain numerals do not introduce an additional layer of plurality is shared among Henderson (2014); Kuhn (2017); Guha (2018), and Guha (2018) is the one who explicitly models it as a post-supposition. Her reason for this is due to specificity, which goes beyond this presentation today.

- If anti-presuppositions are not calculated when the corresponding presuppositions are irrelevant, this may account for disappearance of the co-variation inference in Context 1.

• As for the competition, this view also faces difficulty in cases with a quantificational licensor:

- If one follows Guha's (2018) analysis of plain numerals in Bangla, it post-supposes non-variation.

- Then, one may say dependent indefinites 'anti-post-suppose' co-variation.

• However, this wrongly predicts that plain numerals may not co-vary even if they dit under the scope of the distributivity operator.

- One may say that plain indefinites are ambiguous between non-varying one and co-varying one, and dependent indefinites only compete with the first one.

- I have not encountered a convincing case for this ambiguity yet, though.

4.3 Gappy approach

• The other option is to introduce trivalence to model a truth-value gap, and let relevance handle it (Križ, 2015, 2016, *et seq*).

- This approach *prima facie* predicts symmetry between positive cases and negative cases.

- Also, it predicts the correct correlation between presence/absence of co-variation inference and its relevance in the context.

5 A proof of concept

• I adopt *Plural Compositional DRT* (Brasoveanu, 2007, 2008), which is the combination of *Compositional DRT* (Muskens, 1996) and *Dynamic Plural Logic* (van den Berg, 1996).

- Dependencies among variables are modelled with a **set of variable assignments**. I call it a *plural information state* (PIS) G, H,... (Brasoveanu, 2008).

- I illustrate a PIS as a matrix as exemplified in Table 3.

H	u_1	u_2	u_3	...
h_1	Alex	Bede	Chris	...
h_2	Dan	Elin	Fausto	...
h_3	Greg	Isobel	Hideki	...
...	...	...	...	...

Table 3: A plural information state

- I model singular individuals with singleton sets, e.g., $a = \{a\}$, and plural individuals with non-singleton sets, e.g., $a + b = \{a, b\}$.

  - One may obtain plural individuals by summing up the values spread across a PIS, e.g., $H(u_1) = \{\{\text{Alex}\},\{\text{Dan}\},\{\text{Greg}\}\}$ and $\cup H(u_1) = \{\text{Alex,Dan,Greg}\}$.

(18) **Value projection**:
$G(u_n) = \{x : g \in G \,\&\, g(u_n) = x\}$

  - One may obtain subsets of a plural assignments along with particular values of a variable u_n as given in (19).

(19) **Subset assignments**:
$G_{u_n=d} = \{g : g \in G \,\&\, g(u_n) = d\}$

- With (19), *dependencies* are defined in terms of co-variation as given in (20): u_m is dependent on u_n iff their values co-vary (van den Berg, 1996).

(20) **(Quantificational) Dependency**:
In a plural information state G, u_m is dependent on u_n iff
$\exists d, e \in G(u_n)[G_{u_n=d}(u_m) \neq G_{u_n=e}(u_m)]$

  - For example, u_2 is dependent on u_1 in I and J, but not in K in Table 4.

I	u_1	u_2
i_1	x_1	y_1
i_2	x_2	y_2

/

J	u_1	u_2
j_1	x_1	y_1
j_2	x_1	y_2
j_3	x_2	y_2

/

K	u_1	u_2
k_1	x_1	y_1
k_2	x_1	y_2
k_3	x_2	y_1
k_4	x_2	y_2

Table 4: Dependency

- Addition of new values to a variable is defined as a point-wise generalisation of assignment extension (Brasoveanu, 2008).[18]

(21) a. **Assignment Extension**:
$g[u_n]h = \forall u\,[u \neq u_n \rightarrow g(u) = h(u)]$

b. **Plural assignment extension**:
$G[u]H \Leftrightarrow \forall g \in G\,\exists h \in H\,[g[u]h]] \,\&\, \forall h \in H\,\exists g \in G\,[g[u]h]$

- The possible co-variation requirement suggests that the relevant test has to be a global constraint on the context.

- i.e. the context contains at least one PIS with co-variation.

- I implement it with an *Update Semantics* or *Heimian* dynamic semantics.

- A *possibility* p is a pair of a possible world and a PIS, i.e. $s = \langle w, G\rangle$.

- A *context* c is a set of possibilities.

- A formula ϕ denotes a **function** from a context to a context.

- I adopt four basic types: t for truth values, e for individuals, w for worlds, π for *registers*, which model dynamic variables (Muskens, 1996).

- I define a variable assignment as a function from D_π to D_e.[19]

[18] His definition may allow non-distributive plurals to introduce new dependencies while the definition in van den Berg (1996) does not. This difference is vital for the controversy of whether a dependent indefinite is distributive by itself or requires a distributive licensor. However, this does not matter for the main point of this talk.

[19] Here, I diverge from Muskens (1996) and Brasoveanu (2008). The former postulates type s entity called *states* and uses it to model assignments. The latter also take assignments as primitive,

- Following the idea of *meta-type* convention (Brasoveanu, 2008), I adopt the following abbreviations:

  - m abbreviates variable assignments.

  - i.e. a PIS is type $\langle mt \rangle$ and a possibility is of type $w \times \langle mt \rangle$.

  - s abbreviates a possibility, i.e. $s = w \times \langle mt \rangle$.

  - T abbreviates an update function of type $\langle st, st \rangle$.

  - This abbreviation convention enables one to emulates Heim and Kratzer style sub-clausal composition in the dynamic setting.

- I model relevance based on an issue I given in the context.[20]

  - The guiding intuition comes from **non-maximal readings** of definite plurals (Krifka, 1996; Malamud, 2012; Križ, 2015, 2016; Križ and Spector, 2021, a.o.).

(22) A: The windows are open.

  a. Scenario 1: A and B went on a trip. A storm is coming up. B asks whether the house will be safe. Only half the windows are closed.

  → (22) is **adequate**

  b. Scenario 2: A and B hired painters to paint their house. They cannot start working until all the windows are open. A asks B whether the house is ready. Half of the windows are still closed.

  → (22) is **inadequate**

  (Malamud, 2012)

- I emulate Križ's (2016) trivalent approach with the Heimian DPlL.

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and defines a 'lifted' version of dynamic variables so that a dynamic variable takes a state and returns an individual. The setting I adopt is closer to (Dotlačil and Roelofsen, 2021; Roelofsen and Dotlačil, 2023), which takes dynamic variables as primitives and model assignments as functions. Nothing in this talk hinges on this choice.

[20] The idea is quite similar to textbfQuestion Under Discussion (QUD) (Roberts, 2012). However, see Križ (2016) for discussion in which he argues that the issue I relevant for computing the maximality inference of definite plurals is not always identical to the immediate QUD in the discourse.

- Križ (2015, 2016) propose that definite plurals involve **truth-value gap**, i.e. their truth conditions and falsity conditions are non-complementary.

- Those conditions in the grey zone are **pragmatically** grouped up with truth or falsity conditions, depending on how the context is partitioned.

• To implement this, I adopt **Trilateral Heimian DPlL**.

- a formula ϕ has three extensions:

- *positive* $[\![\phi]\!]^+$, *negative* $[\![\phi]\!]^-$, and *gap* $[\![\phi]\!]^\#$.

• Negation is defined as (23).

(23) a. $c[\![\neg\ \phi]\!]^+ = c[\![\phi]\!]^-$
b. $c[\![\neg\ \phi]\!]^- = c[\![\phi]\!]^+$
c. $c[\![\neg\ \phi]\!]^\# = c[\![\phi]\!]^\#$

• I adopt Strong Kleene connectives.

- Conjunction is defined as (24).[21]

(24) a. $c[\![\phi;\psi]\!]^+ = c[\![\phi]\!]^+[\![\phi]\!]^+$
b. $c[\![\phi;\psi]\!]^- =$
$c[\![\phi]\!]^-[\![\psi]\!]^+ \cup c[\![\phi]\!]^-[\![\psi]\!]^- \cup c[\![\phi]\!]^-[\![\psi]\!]^\# \cup c[\![\phi]\!]^+[\![\psi]\!]^- \cup c[\![\phi]\!]^\#[\![\psi]\!]^-$.
c. $c[\![\phi;\psi]\!]^\# = c[\![\phi]\!]^+[\![\psi]\!]^\# \cup c[\![\phi]\!]^\#[\![\psi]\!]^+ \cup c[\![\phi]\!]^\#[\![\psi]\!]^\#$.

• In this setting, I define the denotations of "*n*-şer" as (25).

(25) a. $c[\![n\ \text{şer}(u_m/u_n)]\!]^+ = \{\langle w', H\rangle : \exists \langle w, G\rangle \in c\,[w = w' \,\&\, G[u_m]H \,\&\, \exists d, e \in H(u_n)\,[H_{u_n=d}(u_m) \neq H_{u_n=e}(u_m)] \,\&\, \forall d \in H(u_n)\,[|H_{u_n=d}(u_m)| = n]]$
b. $c[\![n\ \text{şer}(u_m/u_n)]\!]^- = \{\langle w', H\rangle : \exists \langle w, G\rangle \in c\,[w = w' \,\&\, G[u_m]H \,\&\, \exists d, e \in H(u_n)\,[H_{u_n=d}(u_m) \neq H_{u_n=e}(y)] \,\&\, \exists d \in H(u_n)\,[|H_{u_n=d}(u_m)| \neq n]]$
c. $c[\![n\ \text{şer}(u_m/u_n)]\!]^\# = \{\langle w', H\rangle : \exists \langle w, G\rangle \in c\,[w = w' \,\&\, G[u_m]H \,\&\, \forall d, e \in H(u_n)\,[H_{u_n=d}(u_m) = H_{u_n=e}(u_m)]]$

• The possibilities that fall in $[\![\phi]\!]^\#$ may be grouped up with $[\![\phi]\!]^+$, depending on the context.

[21] I ignore disjunction and material implication in this talk.

- I adopt Križ's (2016) *Weak Maxim of Quality* refined in this setting.

(26) a. **Weak Maxim of Quality**: Say only sentences which you believe to be *true enough*.

b. A sentence S is *true enough* in a possibility $\langle w, K\rangle$ with respect to an issue I iff there is a possibility $\langle w', H\rangle$ such that $\langle w', H\rangle \in [\![S]\!]^+$, and w and w' are in the same cell of I.

- One may dynamicise I by taking it as a set of sets of possibilities (e.g., Roelofsen and Dotlačil, 2023), but it is not necessary for now.

(27) A sentence S is *true enough* in a possibility $\langle w, K\rangle$ with respect to an issue I iff there is a possibility $\langle w', H\rangle$ such that $\langle w', H\rangle \in [\![S]\!]^+$, and $\langle w, G\rangle$ and $\langle w', H\rangle$ are in the same cell of I.

- Now, revisit the core data.

- The DRS of (14a) is given in (28).

(14a) Bu üç öğrenci^{u_1} üç-**er**$^{u_2}_{u_1}$ müze-ye git-ti.
this three student three-şer museum-dat visit-past
"These three students visited three museums each."

(28) $[u_1|]; [|\text{students}(u_1)]; [|u_1 = 3]; [|3\ \text{şer}(u_2/u_1)]; [|\text{museums}(u_2)]; [|\text{visited}(u_1)(u_2)]$

- The positive extension of (28) is essentially the same as Kuhn (2017) except that the dependent indefinite does not undergo QR.

- It only stores possibilities in which u_2 is dependent on u_1.

- Possibilities in which u_1 and u_2 are independent fall into the gap extension.[22]

- The dynamic representation of (14b) is given in (29).

- I omit the composition for reasons of time.

(14b) Bu üç öğrenci^{u_1} üç-**er**$^{u_2}_{u_1}$ müze-ye git-**me**-di.
this three student three-şer museum-dat visit-neg-past
"These three students did not visit three museums each."

[22] To be precise, $c[\![n\ \text{şer}(u_m/u_n)]\!]$ should probably require $u_m > 1$ in addition. Otherwise, it predicts that NUM-şer with a singular argument is licensed in some contexts.

(29) $[u_1|];[|\text{students}(u_1)];[|u_1 = 3];\neg([|3\ \text{şer}(u_2/u_1)];[|\text{museums}(u_2)];[|\text{visited}(u_1)(u_2)])$

- $[|3\ \text{şer}(u_2/u_1)]$ is evaluated under negation here.
  - Accordingly, the positive extension of (14b) considers the negative extension of $[|3\ \text{şer}(u_2/u_1)]$.
  - Recall that its negative extension stores co-variation between u_1 and u_2 just like its positive extension.
- Accordingly, the co-variation requirement itself is evaluated under the scope of negation while its infelicity projects through Strong Kleene connectives.[23]
  - This avoids the issue of co-variation between a variable above negation and another variable below negation: the requirement itself scopes below negation and thus both values are accessible when it is evaluated.
- This co-variation requirement should still be evaluated above a distributive universal quantifier.
  - This is compatible with the view that post-suppositions project from the scope of distributive quantification while they are discharged under the scope of negation (Brasoveanu, 2013; Law, 2022).
  - This is different from Guha (2018), but her argument for projection of co-variation inferences is still preserved thanks to this setting with Strong Kleene connectives and trilateralism.[24]
  - Thus, this trivalent/trilateral analysis of the possible co-variation condition may serve as a solution to the technical problem with negation and indefinite, independently of the point with relevance-sensitivity.
- Let me discuss relevance-sensitivity in light of this trilateral setting.
  - Context 1 and the three scenarios are repeated below.

[23] In this context, $[|\text{museum}(u_1)]$ and $[|\text{visited}(u_2)]$ are forced to be true. Thus, the definedness of the resultant value solely relies on the definedness of the value of $[|3\ \text{şer}(u_2/u_1)]$. I do not have anything to say about when $[|\text{museum}(u_1)]$ or $[|\text{visited}(u_2)]$ is false. Prediction is that (14b) is false regardless of co-variation between u_1 and u_2 in such cases. I leave examination of it for now.

[24] The same thing can be done with Trivalent DPlL, but I put aside this option for now.

(15) Context 1 (co-variation **irrelevant**): three BA students are supervised by a PhD student in the department of art. The supervisor of the PhD student asked her to ask the three students to visit three museums and write an essay on them. A few weeks later, the BA students had a meeting with the PhD student. The PhD student told the professor that these three students {visited / did not visit} 3-er museums.

a. Scenario 1 (**total co-variation**): they each visited a different set of three museums, i.e. nine museums are visited in total.

b. Scenario 2 (**partial co-variation**): they each visited three museums, but they overlap in some museums. Between three and nine museums are visited in total.

c. Scenario 3 (**no variation**): they each visited three museums, but they happened to visit the same three museums.

- (15) concerns the number of museums per student: did the students all visit at least three museums?

(30) Non-dynamic representation of I:
$I = \{\{w_1 : \text{All the three students visited three museums in } w_1\},$
$\{w_2 : \text{not all the three students visited three museums in } w_2\}\}$

(31) Dynamic representation of I:
$I = \{[\![\text{All the three students visited three museums}]\!](c),$
$[\![\neg(\text{All the three students visited three museums})]\!](c)\}$

- All the three scenarios, including Scenario 3, belong to the *yes*-partition, i.e. all the students visited three museums, and get grouped up with (25a).

- At the same time, the possibilities that describe Scenario 3 belong to the gap extension of (14a) and (14b).

- For example, consider a world in which all the three students visited the same three museums.

(32) $I_w(\text{visited}) =$
$\{\langle \text{student}_1, \text{museum}_{1-3}\rangle, \langle \text{student}_2, \text{museum}_{1-3}\rangle, \langle \text{student}_3, \text{museum}_{1-3}\rangle\}$

- Here, $w \in \{w_1 : \text{All the three students visited three museums}\}$.

- This world is compatible with the two PISs exemplified below.

- Here, $\langle w, G\rangle, \langle w, G'\rangle \in [\![\text{All the three students visited three museums}]\!](c)$.[25]

[25] I leave the audiences to confirm that this holds.

G	u_1	u_2
g_1	student_1	museum_1
g_2	student_1	museum_2
g_3	student_1	museum_3
g_4	student_2	museum_1
g_5	student_2	museum_2
g_6	student_2	museum_3
g_7	student_3	museum_1
g_8	student_3	museum_2
g_9	student_3	museum_3

Table 5: A PIS without variation

G'	u_1	u_2
g'_1	student_1	$\text{museum}_1+\text{museum}_2+\text{museum}_3$
g'_2	student_2	$\text{museum}_1+\text{museum}_2+\text{museum}_3$
g'_3	student_3	$\text{museum}_1+\text{museum}_2+\text{museum}_3$

Table 6: A PIS without variation

- Notice that u_2 is independent of u_2 in these possibilities.

- Thus, $\langle w,G\rangle$, $\langle w,G'\rangle \in [\![(14a)]\!]^{\#}$ and $\langle w,G\rangle$, $\langle w,G'\rangle \in [\![(14b)]\!]^{\#}$

- However, since w (or $\langle w,G\rangle$, $\langle w,G'\rangle$) belongs to the *yes* partition, (14a) is **true enough** in $\langle w,G\rangle$ and $\langle w,G'\rangle$ with respect to the issue I in Context 1.

- For the same reason, (14b) is false in $\langle w,G\rangle$ and $\langle w,G'\rangle$ with respect to the issue I in Context 1.

- This derives the pattern of judgement in Context 1 repeated below: co-variation does not affect the truth of (14a) and (14b).

	Scenario 1	Scenario 2	Scenario3
Context 1 Aff	true	true	true
Context 1 Neg	false	**false/?**	**false/?**

Table 7: Distribution of judgements in Context 2

- In contrast, Context 2 and the three scenarios are repeated below.

(16) Context 2 (co-variation **relevant**): three students are writers of a school biweekly newspaper in a university of fine art. A professor recommended their editor that the three students should visit 3-er museums so that they can write about as many museums as possible in this semester. A few weeks later, the students had a meeting with the editor. The editor told the professor that these three students {visited / did not visit} 3-er museums.

a. Scenario 1 (**total co-variation**): they each visited a different set of three museums, i.e. nine museums are visited in total.

b. Scenario 2 (**partial co-variation**): they each visited three museums, but they overlap in some museums. Between three and nine museums are visited in total.

c. Scenario 3 (**no variation**): they each visited three museums, but they happened to visit the same three museums.

- (15) concerns the total number of museums: did the students maximise the number of museums they visited?

- The issue in Context 2 is informally given as follows.

(33) Non-dynamic representation of I:
$I = \{\{w_1 : \text{The three students visited different sets of three museums in } w_1\}, \{w_2 : \text{The three students did not visit different sets of three museums in } w_2\}\}$

(34) Dynamic representation of I:
$I = \{[\![\text{The three students visited different sets of three museums}]\!](c),$
$[\![\neg(\text{The three students did not visit different sets of three museums})]\!](c)\}$

- Scenario 1 clearly belongs to the *yes*-partition because it hits the maximum number of museums they can visit, but the other two do not.

- Thus, the infelicity in the positive case arising in Scenario 2 and 3 is a matter of congruence.

• On the other hand, Scenario 3 clearly belongs to the *no*-partition.

- Thus, it should be the clearest case in which (14b) is judged true.

- However, it is not.

• If non-co-varying case is semantically classified in the gappy zone, this pattern follows.

• The pattern of judgements in Context 2 is repeated below.

	Scenario 1	Scenario 2	Scenario3
Context 2 Aff	true	**true/false/??**	**true/false/??**
Context 2 Neg	false	**true/false/??**	**true/false/??**

Table 8: Distribution of judgements on "-şer"

• However, note that none of the informants I consulted with has reported a clear contrast between Scenario 2 and 3.

- Some informants have reported that in Context 2, the speaker is more likely to use (14b) in Scenario 3 than in Scenario 2.

- This might suggest that the question I am postulating here involves vagueness, and this is a confound that generates the variability in the informants' judgement.

• This is the matter that should be removed in the next step of this project.

- Quite informally, though, Scenario 3 is the clearest case in which (14a) is false and (14b) is true.

- Thus, its strangeness should be attributed to something other than congruence.

• Another possibility is that NUM-şAr actually requires total variation.

- This is a strong requirement, but this makes the same prediction in the context where co-variation is irrelevant.

- Then, this predicts no contrast between Scenario 2 and 3.

- I need to check it with a better context in which partial co-variation matters.

6 Conclusion

- NUM-şAr only requires possible co-variation, i.e. it suffices if the context is compatible with a PIS with co-variation.

- For this, the co-variation condition should be a test on the entire context rather than a constraint on particular PISs.

- This condition is too weak as a presupposition.

- Furthermore, I claimed that the presence/absence of actual co-variation inference is context-dependent.

- An implicature approach may predict this, but this wrongly predicts that even a possible co-variation inference disappears under negation.

- It could be an anti-presupposition if a plain numeral presupposes absence of variation.

- However, a simplistic implementation of it predicts that plain indefinites may not tolerate co-variation under the distributivity operator.

- A gappy approach may predict dieappearance of actual co-variation inferences when co-variation is irrelevant.

- I offered a proof of concept with Trilateral Heimian DPlL.

- However, the data obtained at this point is still noisy, and there are potential confound I should test.

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