Technical University of Denmark

Written exam, May 18, 2021.

Course name: Graph Theory.

Course number: 01227.

Aids allowed: All materials allowed by DTU including internet access.

Exam duration: 4 hours

Weighting: Question 1: 30% - Question 2: 24% - Question 3: 22% - Question 4: 12% - Question 5: 12%.

The weighting is only an approximative weighting. Your answers will be judged as a whole.

It is important that you justify your answers. An answer with no justification gives no credit.

You can answer the exam in either Danish or English or a mixture.

Question 1

Figure 1 shows a transport network where every edge has a capacity and an orientation.

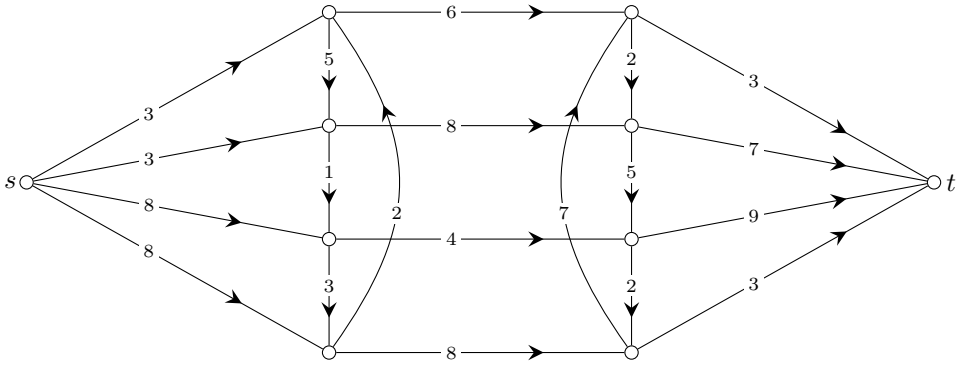

Figure 1.

(a) Find a maximum flow from s to t, and find a minimum cut. Find the value of the flow and the capacity of the cut. Is there more than one minimum cut? (The answer should be justified.)

(b) Find the critical edges. How many are there? Find the edges of optimum capacity. How many are there? (The answer should be justified.)

(c) How many maximum integer flows are there? How many minimum cuts are there? (The answer should be justified.)

Question 2

Consider a school with 5 teachers and 10 classes. Figure 2 shows how many hours each teacher must teach each class during a week. We wish to minimize the number of hours we need to use the school. We assume that the number of classrooms is unlimited.

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
t_1	3	3	3	3	3	3	3	3	3	3
t_2	3	3	0	0	3	0	3	0	3	0
t_3	3	3	0	3	3	0	3	3	0	3
t_4	3	3	0	3	0	3	0	3	3	3
t_5	4	4	4	4	0	4	4	4	4	0

Figure 2.

(a) How many hours must the school be used during a week? What is the number of classrooms needed for this?

(b) Now we add the additional condition that teachers t_3 and t_4 are never allowed to teach at the same time. How many hours must the school be used during a week? What is the number of classrooms needed for this?

(You need not show the schedule. But, you must justify your answer by a graph argument.)

Question 3

(a) How many dominos can be cut out of the board on Figure 3? Show the dominos, for example as a matching in the dual graph.

(b) What is the smallest number of squares in Figure 3, that has to be cut out in order to destroy all the dominos on the board. Show those squares.

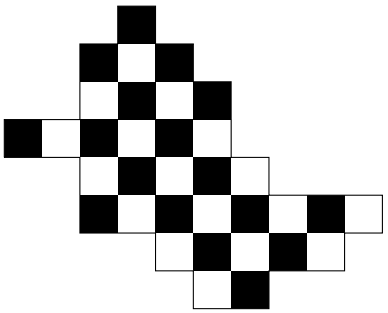

Figure 3.

Question 4

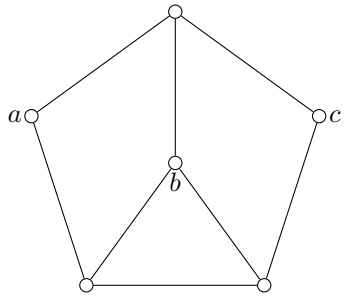

Figure 4.

A random walk starts in a in Figure 4. Find the probability that the walk gets to b before it gets to c.

Question 5

(a) Does there exist a graph which is a electrical network with 12 resistors such that the network determinant is

$$(R_1 + R_2 + R_3)(R_4 + R_5 + R_6 + R_7)(R_8 + R_9 + R_{10} + R_{11} + R_{12})?$$

(b) Are there two non-isomorphic graphs with that network determinant?

(c) Can a graph with that network determinant be 3-edge-connected?