Technical University of Denmark

Written exam, May 15, 2020.

Course name: Graph Theory.

Course number: 01227.

Aids allowed: All materials allowed by DTU including internet access.

Exam duration: 4 hours

Weighting: Question 1: 30% - Question 2: 24% - Question 3: 22% - Question 4: 12% - Question 5: 12%.

The weighting is only an approximative weighting. Your answers will be judged as a whole.

It is important that you justify your answers. An answer with no justification gives no credit.

You can answer the exam in either Danish or English or a mixture.

Question 1 Consider the transport network in Figure 1 where every edge has a capacity and an orientation.

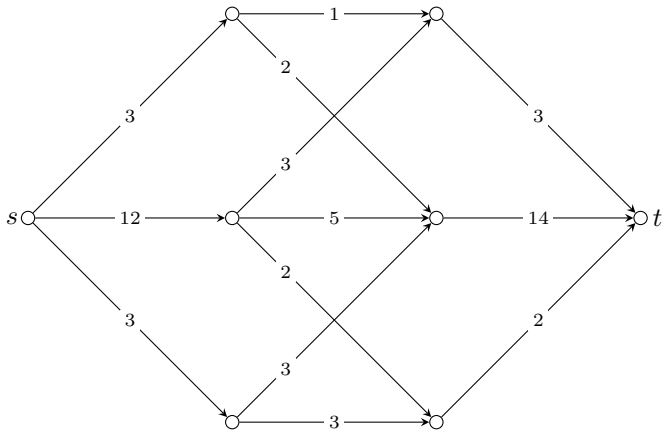

Figure 1: Transport network

(a) Find a maximal flow from s to t, and find a minimal cut. Find the value of the flow and the capacity of the cut.

(b) Find the critical edges. How many are there? Find the edges of optimum capacity. How many are there?

(c) How many integer maximal flows are there?

(d) How many minimum cuts are there?

(e) Which edges must have flow 0 in every maximal flow? Which edges may have flow 0 in some maximal flow?

Question 2 Consider the job assignment problem in Figure 2.

	j_1	j_2	j_3	j_4
w_1	3	8	4	2
w_2	1	3	2	5
w_3	5	8	3	2
w_4	4	4	1	3

Figure 2: Job assigment problem

(a) Which of the following labellings are admissible ("feasible" in Bondy and Murty)? There might be more than one. Draw the equality graph for the one(s) that is/are admissible. How many edges are there in a maximum matching in the equality graph?

	j_1	j_2	j_3	j_4	
w_1	3	8	4	2	9
w_2	1	3	2	5	9
w_3	5	8	3	2	7
w_4	4	4	1	3	4
	-2	1	-4	-4	

(a)

	j_1	j_2	j_3	j_4	
w_1	3	8	4	2	5
w_2	1	3	2	5	5
w_3	5	8	3	2	5
w_4	4	4	1	3	4
	0	3	0	0	

(b)

	j_1	j_2	j_3	j_4	
w_1	3	8	4	2	4
w_2	1	3	2	5	9
w_3	5	8	3	2	8
w_4	4	4	1	3	4
	5	3	4	5	

(c)

(b) Solve the job assignment problem in Figure 2. The answer should include an admissible labelling with minimum sum.

(c) Does there exist a 4 by 4 matrix with the following properties: all 16 entries are different, but all 24 job assignments have the same value?

Question 3 The numbers in Figure 3 are edge-lengths. The sum of these numbers is 101.

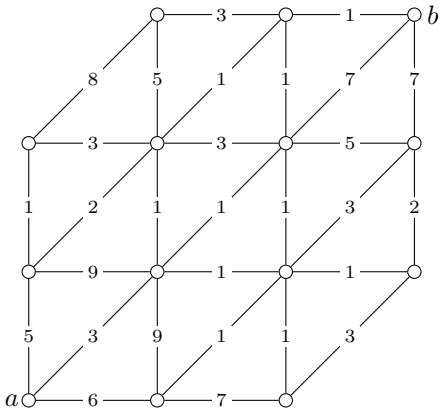

Figure 3: A graph with edge-lengths

(a) How long is the shortest walk which starts in a and ends in b and traverses every edge at least once?

(b) How long is the shortest walk which starts in a and ends in a, and traverses every edge at least once, and traverses each horizontal edge at least twice?

Question 4 A random walk starts in a in Figure 4. Find the probability that the walk gets to b before it gets to c.

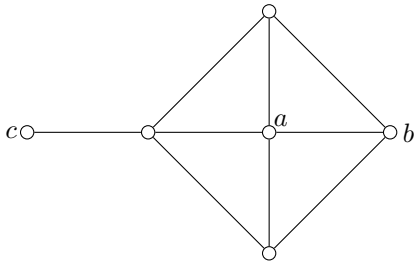

Figure 4: A random walk

Question 5 The graph in Figure 4 is now an electrical network where the edge ab has resistance x Ohm, and each other edge has resistance y Ohm (where x and y are positive real numbers).

Find the network determinant.