Technical University of Denmark

Course name	Introduction to programming and data processing Programming and data processing (second programming language)
Course number	02631, 02632, 02633, 02634, 02692, 02694
Aids allowed	All Aids
Exam duration	2 hours
Weighting	All exercises have equal weight

Contents

Submission details

You must hand in your solution electronically:

1. You can upload your solutions individually on CodeJudge (dtu.codejudge.net/prog-aug16/assignment) under *Afleveringer/Exam*. When you hand in a solution on CodeJudge, the test example given in the assignment description will be run on your solution. If your solution passes this single test, it will appear as *Submitted*. This means that your solution passes on this single test example. You can upload to CodeJudge as many times as you like during the exam.

2. You must upload your solutions on CampusNet. Each assignment must be uploaded as one separate .py file, given the same name as the function in the assignment:

   (a) `confidence.py`
   (b) `weekday.py`
   (c) `symmetrize.py`
   (d) `voldif.py`
   (e) `stringcompare.py`

   The files must be handed in separately (*not* as a zip-file) and must have these exact filenames.

After the exam, your solutions will be automatically evaluated on CodeJudge on a range of different tests, to check that they work correctly in general. The assessment of you solution is based only on how many of the automated tests it passes.

- Make sure that your code follows the specifications exactly.
- Each solution shall not contain any additional code beyond the specified function.
- Remember, you can check if your solutions follow the specifications by uploading them to CodeJudge.
- Note that all vectors and matrices used as input or output must be numpy arrays.

Assignment A Confidence interval

When you have a number of noisy observations (represented by a vector x of decimal numbers) a simple confidence interval for the mean is given by the following expression:

$$m \pm 2\frac{s}{\sqrt{n}} \tag{1}$$

where m is the mean, s is the standard deviation, and n is the number of observations. We will use the following definitions:

$$m = \frac{\sum_{i=1}^{n} x_i}{n}, \qquad s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - m)^2}{n-1}}, \tag{2}$$

where x_i are the observations.

Problem definition

Create a function named `confidence` that takes a vector `x` as input and returns the lower and upper confidence interval bounds as a vector `conf` of length two. The first element of the vector must be the lower limit and the second element the upper limit.

Solution template

```
def confidence(x):
  #insert your code
  return conf
```

Input	
`x`	Observations (vector of decimal numbers).

Output	
`conf`	Lower and upper confidence bounds (vector of length 2).

Example

Consider the following input vector $x = [1,\ 2,\ 4,\ 3,\ 1]$. The mean and standard deviation can be computed as

$$m = \frac{1+2+4+3+1}{5} = 2.2, \qquad s = \sqrt{\frac{(1-2.2)^2 + (2-2.2)^2 + (4-2.2)^2 + (3-2.2)^2 + (1-2.2)^2}{5-1}} = 1.3038,$$

The confidence interval is thus given by

$$2.2 \pm 2 \cdot \frac{1.3038}{\sqrt{5}} \quad = \quad 2.2 \pm 1.1662$$

and the function should thus return the vector $[1.0338,\ 3.3662]$.

A

Assignment B Day of the week

The following formula can be used to compute the day of the week for any date:

$$w = \left(d + C + y + \left\lfloor \frac{y}{4} \right\rfloor\right) \bmod 7 \tag{3}$$

Input to the calculation are the date number $d \in \{1 \ldots 31\}$, the month number $m \in \{1 \ldots 12\}$, and the last two digits of the year $y \in \{0 \ldots 99\}$. C is the month code, which can be found from m using the table below. The notation $\lfloor \cdot \rfloor$ means rounding *down* to the nearest smaller integer (the floor function), and mod is the modulo operator (remainder after integer division).

Month (m)	1	2	3	4	5	6	7	8	9	10	11	12
Month code (C)	6	2	2	5	0	3	5	1	4	6	2	4

The result of the computation is the weekday code w which correponds to the following weekday names:

Weekday code (w)	0	1	2	3	4	5	6
Weekday name	`Sun`	`Mon`	`Tue`	`Wed`	`Thu`	`Fri`	`Sat`

Problem definition

Create a function named `weekday` that takes as input the date, month, and year numbers and returns the weekday name as a string written exacly as in the table above.

Solution template

```
def weekday(d, m, y):
  #insert your code
  return name
```

Input

`d`	Date number (integer, $1 \ldots 31$).
`m`	Month number (integer, $1 \ldots 12$).
`y`	Year number (integer, $0 \ldots 99$).

Output

`name`	Name of the weekday (string).

Example

Consider the date 21 August 2016, which is represented by the input $d = 21$, $m = 8$, $y = 16$. Looking up $m = 8$ in the month code table yields $C = 1$. The weekday code can be computed as:

$$w = \left(21 + 1 + 16 + \left\lfloor \frac{16}{4} \right\rfloor\right) \bmod 7 = (21 + 1 + 16 + 4) \bmod 7 = 42 \bmod 7 = 0 \tag{4}$$

The name of the weekday can then be found by looking up the weekday code in the table, and the string `Sun` is thus the final result.

Assignment C — Matrix symmetrization

In linear algebra, a symmetric matrix is a square matrix that is equal to its transpose. Given an arbitrary square matrix x, a symmetric matrix y can be constructed as follows:

For *each entry* (i, j) *in the matrix.*
  If $i = j$
    Set $y_{i,j} = x_{i,j}$.
  else
    Set $y_{i,j} = x_{i,j} + x_{j,i}$.

Problem definition

Create a function named `symmetrize` that takes a quadratic matrix x as input and returns a symmetrized matrix y computed according to the algorithm above.

Solution template

```
def symmetrize(x):
  #insert your code
  return y
```

Input	
`x`	Matrix to be symmetrized (quadratix matrix).

Output	
`y`	Symmetrized matrix (symmetric matrix).

Example

Consider the following input matrix:

$$x = \begin{bmatrix} 1.2 & 2.3 & 3.4 \\ 4.5 & 5.6 & 6.7 \\ 7.8 & 8.9 & 10.0 \end{bmatrix}.$$

According to the algorithm, the output matrix should then be:

$$y = \begin{bmatrix} 1.2 & 6.8 & 11.2 \\ 6.8 & 5.6 & 15.6 \\ 11.2 & 15.6 & 10.0 \end{bmatrix}.$$

C

Assignment D Volume difference

A hypersphere is a generalization of a circle (2-d) and a sphere (3-d) to n-dimensinoal space. The volume of a hypersphere is given by:

$$V_s = \frac{\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2}+1)} R^n, \tag{5}$$

where n is the dimension of the space, R is the radius of the hypersphere, and $\Gamma(\cdot)$ is the gamma function which is implemented in Python as the function `math.gamma`.

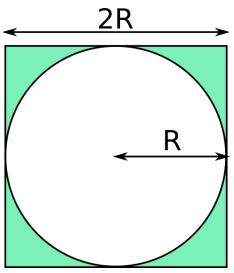
A hypershpere with radius R will fit inside a hyper-cube with side length $2R$. The volume of such a hyper-cube is given by:

$$V_c = (2R)^n. \tag{6}$$

The difference between the two volumes is given by

$$V_d = V_c - V_s, \tag{7}$$

and is illustrated by the colored areas in the figure below, which shows the case in the 2-dimensional setting.



Problem definition

Create a function named `voldif` that takes the radius R and the dimensionality n as input, and returns the difference between the volumes of the hypercube and hypersphere, V_d.

Solution template

```
def voldif(R, n):
  #insert your code
  return Vd
```

Input	
`R`	Radius (non-negative decimal number).
`n`	Dimensionality (positive integer).

Output	
`Vd`	Difference between volume of hypercube and hypersphere (decimal number).

Example

Consider a radius $R = 5$ and $n = 2$ dimensions. The volumes (which are actually areas in the 2-dimensional case) can be computed as:

$$V_s = \frac{\pi^{\frac{2}{2}}}{\Gamma(\frac{2}{2}+1)} 5^2 \approx 78.54, \qquad V_c = (2 \cdot 5)^2 = 100,$$

and the difference, which is the final result, is given by

$$V_d = 100 - 78.54 = 21.46\,.$$

Assignment E String comparison

A measure of dis-similarity between strings can for example be used to compare two strings from different data sources. In this exercise we will work with a simple measure defined as the number of different letters `a`–`z` that occur in one and only one of the two strings. If, for example, one string contains two `a`'s and the other contains none, this counts as a difference of 1, and if one string contains two `a`'s and the other contains one `a`, this does not count as a difference since both strings contain the letter `a`. You may assume that the inputs contain only lower case characters `a`–`z`.

Problem definition

Create a function named `stringcompare` that takes as input two strings and returns their dis-similarity as defined above.

Solution template

```
def stringcompare(string1, string2):
  #insert your code
  return disSimilarity
```

Input	
`string1`, `string2`	Strings to be compared (string).

Output	
`disSimilarity`	Dis-similarity measure (integer).

Example

Consider comparing the two strings `aardvark` and `artwork`. The letters `a`, `r`, and `k` occur in both strings and can be ignored. The two letters `d` and `v` occur only in the first string, and the three letters `t`, `w`, and `o` occur only in the second string. Thus the dis-similarity is $2 + 3 = 5$, and the function must return the number 5.

E