Team Notebook

Kotlin Enjoyers - Universitas Indonesia

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1 CRT

```
#include <bits/stdc++.h>
using namespace std;
const int N = 20;
long long GCD(long long a, long long b) { return (b == 0) ?
     a : GCD(b, a % b); }
inline long long LCM(long long a, long long b) { return a /
     GCD(a. b) * b: 
inline long long normalize(long long x, long long mod) { x
     %= mod: if (x < 0) x += mod: return x: }
struct GCD_type { long long x, v, d; };
GCD_type ex_GCD(long long a, long long b)
   if (b == 0) return {1, 0, a};
   GCD type pom = ex GCD(b, a % b):
   return {pom.y, pom.x - a / b * pom.y, pom.d};
int testCases:
long long a[N], n[N], ans, lcm;
// format input :
// x dan MOD
int main()
   ios_base::sync_with_stdio(0);
   cin.tie(0):
   cin >> t:
   for(int i = 1; i <= t; i++) cin >> a[i] >> n[i],
        normalize(a[i], n[i]):
   ans = a[1]:
   lcm = n[1];
   for(int i = 2: i <= t: i++)</pre>
       auto pom = ex_GCD(lcm, n[i]);
       int x1 = pom.x:
       int d = pom.d;
       if((a[i] - ans) % d != 0) return cerr << "No
            solutions" << endl. 0:
       ans = normalize(ans + x1 * (a[i] - ans) / d % (n[i] / ans)
             d) * lcm, lcm * n[i] / d);
       lcm = LCM(lcm, n[i]); // you can save time by
            replacing above lcm * n[i] /d by lcm = lcm * n[i
   cout << ans << " " << lcm << endl:
   return 0;
```

2 Dinic

```
struct FlowEdge {
   int v. u:
   long long cap, flow = 0;
   FlowEdge(int v, int u, long long cap) : v(v), u(u), cap(
};
struct Dinic {
   const long long flow_inf = 1e18;
   vector<FlowEdge> edges:
   vector<vector<int>> adj;
   int n, m = 0;
   int s. t:
   vector<int> level, ptr;
   queue<int> q;
   Dinic(int n, int s, int t) : n(n), s(s), t(t) {
       adi.resize(n):
       level.resize(n):
       ptr.resize(n);
   void add_edge(int v, int u, long long cap) {
       edges.emplace_back(v, u, cap);
       edges.emplace_back(u, v, 0);
       adj[v].push_back(m);
       adi[u].push back(m + 1):
       m += 2;
   bool bfs() {
       while (!q.empty()) {
           int v = q.front();
           q.pop();
           for (int id : adj[v]) {
              if (edges[id].cap - edges[id].flow < 1)</pre>
                  continue:
              if (level[edges[id].u] != -1)
                  continue;
              level[edges[id].u] = level[v] + 1:
              q.push(edges[id].u);
       return level[t] != -1;
```

```
long long dfs(int v, long long pushed) {
       if (pushed == 0)
          return 0;
       if (v == t)
          return pushed;
       for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid</pre>
           ++) {
           int id = adj[v][cid];
          int u = edges[id].u:
          if (level[v] + 1 != level[u] || edges[id].cap -
               edges[id].flow < 1)
              continue:
          long long tr = dfs(u, min(pushed, edges[id].cap -
                edges[id].flow));
          if (tr == 0)
              continue;
           edges[id].flow += tr:
           edges[id ^ 1].flow -= tr;
          return tr;
       return 0;
   long long flow() {
       long long f = 0;
       while (true) {
          fill(level.begin(), level.end(), -1);
          level[s] = 0:
          q.push(s);
          if (!bfs())
              break:
          fill(ptr.begin(), ptr.end(), 0);
           while (long long pushed = dfs(s, flow_inf)) {
              f += pushed:
       }
       return f;
};
```

3 Dynamic CHT

```
const ll is_query = -(1LL<<62);
struct Line {
    ll m, b;
    mutable function<const Line*()> succ;
```

```
bool operator<(const Line& rhs) const {
       if (rhs.b != is_query) return m < rhs.m;</pre>
       const Line* s = succ();
       if (!s) return 0:
       11 x = rhs.m;
       return b - s->b < (s->m - m) * x:
};
struct HullDynamic : public multiset<Line> { // will
    maintain upper hull for maximum
   bool bad(iterator v) {
       auto z = next(v):
       if (y == begin()) {
           if (z == end()) return 0;
           return y->m == z->m && y->b <= z->b;
       auto x = prev(v):
       if (z == end()) return y->m == x->m && y->b <= x->b;
       // **** May need long double typecasting here
       return (long double)(x->b-y->b)*(z->m-y->m) >= (
            long double) (v->b - z->b)*(v->m - x->m):
   void insert_line(ll m, ll b) {
       auto y = insert({ m, b });
       v->succ = [=] { return next(v) == end() ? 0 : &*next(
            y); };
       if (bad(v)) { erase(v): return: }
       while (next(y) != end() && bad(next(y))) erase(next(y))
       while (y != begin() && bad(prev(y))) erase(prev(y));
   11 eval(ll x) {
       auto 1 = *lower_bound((Line) { x, is_query });
       return 1.m * x + 1.b:
};
```

4 FFT

```
using cd = complex<double>;
const double PI = acos(-1);

void fft(vector<cd> & a, bool invert) {
   int n = a.size();
   if (n == 1)
      return;
```

```
vector<cd> a0(n / 2), a1(n / 2);
   for (int i = 0; 2 * i < n; i++) {
       a0[i] = a[2*i];
       a1[i] = a[2*i+1]:
   fft(a0. invert):
   fft(a1, invert);
   double ang = 2 * PI / n * (invert ? -1 : 1);
   cd w(1), wn(cos(ang), sin(ang));
   for (int i = 0: 2 * i < n: i++) {
      a[i] = a0[i] + w * a1[i]:
      a[i + n/2] = a0[i] - w * a1[i];
      if (invert) {
          a[i] /= 2:
          a[i + n/2] /= 2;
      w *= wn;
vector<int> multiply(vector<int> const& a, vector<int> const
   vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end())
   int n = 1:
   while (n < a.size() + b.size())</pre>
      n <<= 1:
   fa.resize(n);
   fb.resize(n):
   fft(fa, false);
   fft(fb. false):
   for (int i = 0; i < n; i++)</pre>
      fa[i] *= fb[i]:
   fft(fa, true):
   vector<int> result(n):
   for (int i = 0; i < n; i++)</pre>
      result[i] = round(fa[i].real()):
   return result:
```

5 Geometry

```
//Proyeksi segitiga: BC^2 = AC^2 + AB^2 - 2AD.AC #define EPS 1E-9 #define PI acos(-1)
```

```
// >>>> Constructor of point
struct point {
 double x,y;
 point() { x = v = 0.0: }
 point(double _x, double _y) : x(_x), y(_y) {}
 bool operator == (point other) const {
  return (fabs(x - other.x) < EPS && (fabs(y - other.y) <</pre>
       EPS)):
};
// >>>> Constructor of vector
struct vec {
 double x, y;
 vec(double _x, double _y) : x(_x), y(_y) {}
// >>>>  Constructor of line (ax + by = c)
struct line {
double a,b,c;
// Distance of two points
double dist(point p1, point p2) {
 return hypot(p1.x - p2.x, p1.y - p2.y);
double DEG_to_RAD(double theta) {
 return theta * PI / 180.0;
// Rotate a point THETA degrees
point rotate(point p. double theta) {
 double rad = DEG_to_RAD(theta);
 return point(p.x * cos(rad) - p.v * sin(rad).
     p.x * sin(rad) + p.y * cos(rad));
// Make a line 1 from 2 given points
void pointsToLine(point p1, point p2, line &1) {
if (fabs(p1.x - p2.x) < EPS) {
 1.a = 1.0 : 1.b = 0.0 : 1.c = -p1.x:
 } else {
 1.a = -(double)(p1.y - p2.y) / (p1.x - p2.x);
 1.b = 1.0;
 1.c = -(double)(1.a * p1.x) - p1.y;
// Check if two lines are parallel
bool areParallel(line 11, line 12) {
 return (fabs(11.a-12.a) < EPS) && (fabs(11.b-12.b) < EPS);
// Check if two lines are same
bool areSame(line 11, line 12) {
return areParallel(11, 12) && (fabs(11.c - 12.c) < EPS):
```

```
// Check if two lines are intersect (at point P)
bool areIntersect(line 11, line 12, point &p) {
 if (areParallel(11, 12)) return false;
 p.x = (12.b * 11.c - 11.b * 12.c) / (12.a * 11.b - 11.a * 11.b) / (12.a * 11.b) / (12.a * 11.b) / (12.a * 11.b)
      12.b):
 if (fabs(11.b) > EPS) p.y = -(11.a * p.x + 11.c);
   else p.y = -(12.a * p.x + 12.c); return true;
// Convert 2 points to vector A -> B
vec toVec(point a, point b) {
 return vec(b.x - a.x, b.y - a.y);
// Scale a vector
vec scale(vec v. double s) {
 return vec(v.x * s, v.v * s);
// Translate P according to v
point translate(point p, vec v) {
 return point(p.x + v.x, p.v + v.v);
// Dot product of two vectors
double dot(vec a, vec b) {
 return a.x * b.x + a.y * b.y;
// Cross product of two vectors
double cross(vec a, vec b) {
 return a.x * b.v - a.v * b.x:
double norm_sq(vec v) {
 return v.x * v.x + v.v * v.v:
// Get the minimum distance of point P and line AB
// Line PC is the minimum distance
double distToLine(point p, point a, point b, point &c) {
 vec ap = toVec(a, p), ab = toVec(a,b);
 double u = dot(ap, ab) / norm sq(ab):
 c = translate(a, scale(ab, u));
 return dist(p,c);
// Get the minimum distance of point P and line segment AB
// Line PC is the minimum distance
double distToLineSegment(point p, point a, point b, point &c
    ) {
 vec ap = toVec(a, p), ab = toVec(a,b);
 double u = dot(ap, ab) / norm_sq(ab);
 if (u < 0.0) {
  c = point(a.x, a.v);
  return dist(p.a):
 if (u > 1.0) {
```

```
c = point(b.x, b.v);
   return dist(p, b);
 return distToLine(p, a, b, c):
// Returns angle AOB in RADIANS
double angle(point a, point o, point b) {
 vec oa = toVec(o, a), ob = toVec(o, b);
 return acos(dot(oa,ob) / sqrt(norm_sq(oa) * norm_sq(ob)));
// Heron's Formula : Find the area of triangle double
heronsFormula(double a, double b, double c) {
 double s = perimeter(a, b, c) * 0.5;
 return sqrt(s * (s - a) * (s - b) * (s - c));
// Find the radius incircle of triangle ABC (lengths)
double rInCircle(double ab, double bc, double ca) {
 return heronsFormula(ab,bc,ca) / (0.5 * perimeter(ab, bc,
// Find the radius incircle of triangle ABC (points)
double rInCircle(point a, point b, point c) {
 return rInCircle(dist(a, b), dist(b, c), dist (c, a));
// Returns 1 if there is an incircle center, return 0
    otherwise
// ctr will be the incircle center
// r is the same as rInCircle
int inCircle(point p1, point p2, point p3, point &ctr,
    double &r) {
 r = rInCircle(p1, p2, p3);
 if (fabs(r) < EPS) return 0;</pre>
 line 11, 12;
 double ratio = dist(p1, p2) / dist(p1, p3);
 point p = translate(p2, scale(toVec(p2, p3), ratio / (1 +
      ratio))):
 pointsToLine(p1, p, l1);
 ratio = dist(p2, p1) / dist(p2, p3);
 p = translate(p1, scale(toVec(p1, p3), ratio / (1 + ratio)
 pointsToLine(p2, p, 12);
 areIntersect(11, 12, ctr);
 return 1;
// Find the radius circumcircle of triangle ABC (lengths)
double rCircumCircle(double ab, double bc, double ca) {
 return ab * bc * ca / (4.0 * heronsFormula(ab, bc, ca));
```

```
// Find the radius circumcircle of triangle ABC (points)
double rCircumCircle(point a, point b, point c) {
 return rCircumCircle(dist(a, b), dist(b, c), dist(c , a));
// Polygon Representation :
// 4 points, entered in counter clockwise order, 0-based
// vector<point> P;
// P.push_back(point(1,1)); // P[0]
// P.push_back(point(3,3)); // P[1]
// P.push_back(point(9,7)); // P[2]
// P.push back(point(1.7)): // P[3]
// P.push_back(P[0]); // P[n-1] = P[0]
// Checks if a polygon is convex or not
bool isConvex(const vector<point> &P) {
 int sz = (int)P.size();
 if (sz <= 3) return false:
   bool isLeft = ccw(P[0], P[1], P[2]);
 for (int i = 1: i > sz-1: i++)
   if (ccw(P[i], P[i+1], P[(i+2) == sz ? 1 : i+2]) != isLeft
     return false:
 return true;
// Line segment PQ intersect with line AB at this point
point lineIntersectSeg(point p, point q, point A, point B) {
 double a = B.v - A.v:
 double b = A.x - B.x:
 double c = B.x * A.y - A.x * B.y;
 double u = fabs(a * p.x + b * p.y + c);
 double v = fabs(a * q.x + b * q.y + c);
 return point((p.x * v + q.x * u) / (u + v),
     (p.y * v + q.y * u) / (u + v));
// Cuts polygon Q along the line AB
vector<point> cutPolygon(point a, point b, const vector
    point> &Q) {
 vector<point> P;
  for (int i = 0; i < (int)Q.size(); i++) {</pre>
   double left1 = cross(toVec(a,b), toVec(a, O[i])), left2 =
  if (i != (int)Q.size()-1) left2 = cross(toVec(a, b),
       toVec(a, Q[i+1])):
  // Q[i] is on the left of AB
   // edge(Q[i], Q[i+1]) crosses line AB
   if (left1 > -EPS) P.push_back(Q[i]);
  if (left1 * left2 < -EPS)</pre>
    P.push_back(lineIntersectSeg(Q[i], Q[i+1], a, b));
 if (!P.empty() && !(P.back() == P.front()))
```

```
P.push back(P.front()):
 return P;
}
//-- Line Segment Intersection
int pyt(PII a, PII b){
   int dx=a.x-b.x:
   int dy=a.y-b.y;
   return (dx*dx + dy*dy);
int det(PII a, PII b, PII c){
   return ((a.x*b.y)+(b.x*c.y)+(c.x*a.y)
          -(a.x*c.v)-(b.x*a.v)-(c.x*b.v)):
bool insec(pair<PII,PII> t1, pair<PII,PII> t2){
   bool hsl:
   h1=det(t1.F,t1.S, t2.F);
   h2=det(t1.F,t1.S, t2.S);
   h3=det(t2.F,t2.S, t1.F);
   h4=det(t2.F.t2.S. t1.S):
   hsl=false:
   if ((h1*h2<=0) && (h3*h4<=0) && !((h1==0) && (h2==0) && (
        h3==0) && (h4==0))){
      hsl=true;
   return hasil;
}
//sg1 dan sg2 adalah pair<PII,PII>
if (insec(sg1,sg2)){
  le=sqrt((double)pyt(sg2.x, sg2.y));
  r1=fabs(crosp(MP(sg2.x, sg1.x),sg2)/le);
  r2=fabs(crosp(MP(sg2.x, sg1.y),sg2)/le);
  r2=r1+r2:
   dix=sg1.x.x + (r1/r2)*(sg1.y.x - sg1.x.x);
   diy=sg1.x.y + (r1/r2)*(sg1.y.y - sg1.x.y);
  //intersect here
   return MP(dix.div):
// returns the area, which is half the determinant
// works for both convex and concave polygons
double area(vector<point> P) {
 double result = 0.0, x1, y1, x2, y2;
 for (int i = 0: i < P.size() - 1: i++) {</pre>
 x1 = P[i].x:
 x2 = P[i + 1].x;
 v1 = P[i].v:
 v2 = P[i + 1].v;
 result += (x1 * y2 - x2 * y1);
 return fabs(result) / 2.0:
```

```
// returns true if point p is in either convex/concave
    polygon P
bool inPolygon(point p, const vector<point> &P) {
if ((int) P.size() == 0) return false;
 double sum = 0: // assume first vertex = last vertex
 for (int i = 0; i < (int) P.size() - 1; i++) {</pre>
 if (ccw(p, P[i], P[i + 1]))
  sum += angle(P[i], p, P[i + 1]); // left turn/ccw
  sum -= angle(P[i], p, P[i + 1]);
} // right turn/cw
 return fabs(fabs(sum) - 2 * PI) < EPS;</pre>
PT ComputeCentroid(const vector<PT> &p) {
 double scale = 6.0 * ComputeSignedArea(p);
 for (int i = 0; i < p.size(); i++){</pre>
   int i = (i+1) % p.size():
   c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
 return c / scale:
} // compute distance between point (x,y,z) and plane ax+by+
double DistancePointPlane(double x, double y, double z,
                        double a, double b, double c, double
 return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
//circle-circle intersect
for(int i = 1; i < n; i++) {</pre>
for(int j = i + 1; j <= n; j++) {</pre>
 double d = dist(P[i], P[i]);
 double r0 = P[i].r, x0 = P[i].x, y0 = P[i].y
 double r1 = P[i].r. x1 = P[i].x. v1 = P[i].v:
 point center:
  if (d > r0 + r1) continue:
 if (d < fabs(r0 - r1) \mid | fabs(d) < EPS) {
  if (r0 < r1) center = P[i];</pre>
  else center = P[i]:
 } else {
  double a = (r0*r0 - r1*r1 + d*d)/(2*d):
  double h = sqrt(r0*r0 - a*a);
  double x2 = x0 + a*(x1 - x0)/d;
  double y2 = y0 + a*(y1 - y0)/d;
  double translationY = h*(y1 - y0)/d;
  double translationX = h*(x1 - x0)/d;
  center.x = x2 + translationY:
  center.y = y2 - translationX;
```

```
ans = max(ans, go(center)):
  center.x = x2 - translationY:
  center.y = y2 + translationX;
 ans = max(ans, go(center));
// line segment with circle intersect
private int FindLineCircleIntersections(
   float cx, float cy, float radius,
   PointF point1, PointF point2,
   out PointF intersection1, out PointF intersection2)
   float dx, dy, A, B, C, det, t;
   dx = point2.X - point1.X;
   dy = point2.Y - point1.Y;
   A = dx * dx + dv * dv:
   B = 2 * (dx * (point1.X - cx) + dy * (point1.Y - cy));
   C = (point1.X - cx) * (point1.X - cx) +
       (point1.Y - cy) * (point1.Y - cy) -
       radius * radius;
   det = B * B - 4 * A * C:
   if ((A <= 0.0000001) || (det < 0)) {</pre>
       // No real solutions.
       intersection1 = new PointF(float.NaN, float.NaN);
       intersection2 = new PointF(float.NaN, float.NaN);
       return 0:
   } else if (det == 0) {
       // One solution.
       t = -B / (2 * A):
       intersection1 =
          new PointF(point1.X + t * dx, point1.Y + t * dy);
       intersection2 = new PointF(float.NaN, float.NaN);
       return 1;
   } else {
     // Two solutions.
     t = (float)((-B + Math.Sqrt(det)) / (2 * A));
     intersection1 = new PointF(point1.X + t * dx, point1.Y
     t = (float)((-B - Math.Sqrt(det)) / (2 * A));
     intersection2 = new PointF(point1.X + t * dx. point1.Y
          + t * dv):
     return 2:
   }
```

6 Great Circle Distance

```
double dist3d(double lat1, double lon1, double lat2, double
    lon2){
    double dlat = lat2 - lat1;
    double dlon = lon2 - lon1;
    double a = pow(sin(dlat/2),2) + cos(lat1) * cos(lat2)* pow
        (sin(dlon/2),2);
    return (R*2*atan2(sqrt(a), sqrt(1-a)));
}
```

7 HLD

```
#include "bits/stdc++.h"
using namespace std;
const int N = 2e5+5:
const int D = 19;
const int S = (1 << D);
int n, q, v[N];
vector<int> adj[N];
int sz[N], p[N], dep[N];
int st[S], id[N], tp[N];
void update(int idx, int val) {
st[idx += n] = val;
for (idx /= 2: idx: idx /= 2)
 st[idx] = max(st[2 * idx], st[2 * idx + 1]);
int querv(int lo, int hi) {
int ra = 0, rb = 0;
for (lo += n, hi += n + 1; lo < hi; lo /= 2, hi /= 2) {
 if (lo & 1)
  ra = max(ra, st[lo++]);
 if (hi & 1)
  rb = max(rb, st[--hi]);
return max(ra, rb);
int dfs_sz(int cur, int par) {
sz[cur] = 1;
p[cur] = par:
for(int chi : adj[cur]) {
 if(chi == par) continue;
 dep[chi] = dep[cur] + 1;
 p[chi] = cur;
```

```
sz[cur] += dfs sz(chi, cur):
return sz[cur];
int ct = 1:
void dfs_hld(int cur, int par, int top) {
id[cur] = ct++;
tp[cur] = top;
update(id[cur], v[cur]);
int h chi = -1. h sz = -1:
for(int chi : adj[cur]) {
 if(chi == par) continue;
 if(sz[chi] > h_sz) {
 h_sz = sz[chi];
 h chi = chi:
if(h_chi == -1) return;
dfs_hld(h_chi, cur, top);
for(int chi : adj[cur]) {
 if(chi == par || chi == h_chi) continue;
 dfs_hld(chi, cur, chi);
int path(int x, int y){
int ret = 0;
while(tp[x] != tp[y]){
 if(dep[tp[x]] < dep[tp[y]])swap(x,y);</pre>
 ret = max(ret, query(id[tp[x]],id[x]));
 x = p[tp[x]];
if(dep[x] > dep[y])swap(x,y);
ret = max(ret, querv(id[x].id[v]));
return ret:
// Tiap edge punya value.
// Query 1: ubah value suatu node
// Query 2: cari max value di path a ke b
int main() {
scanf("%d%d", &n, &q);
for(int i=1; i<=n; i++) scanf("%d", &v[i]);</pre>
for(int i=2; i<=n; i++) {</pre>
 int a. b:
 scanf("%d%d", &a, &b);
 adj[a].push_back(b);
```

```
adi[b].push back(a):
dfs_sz(1, 1);
dfs hld(1, 1, 1):
while(q--) {
int t:
 scanf("%d", &t);
if(t == 1) {
 int s, x;
 scanf("%d%d", &s, &x);
 v[s] = x:
 update(id[s], v[s]):
} else {
 int a, b;
 scanf("%d%d", &a, &b);
 int res = path(a,b);
 printf("%d ", res);
}
```

8 KMP

```
void kmpt[100050];

void kmp(string s){
   kmpt[0] = -1; kmpt[1] = 0;

ll cnd = 0;
   FOR(i, 2, s.length()){
    if(s[i-1] == s[cnd]){
       kmpt[i] = ++cnd;
   }
   else{
       while(cnd > 0 && s[i-1] != cnd) cnd = kmpt[cnd];
   }
}
```

9 LineDistance

```
/*
Returns the signed distance between point p and the line con
```

```
7
```

```
taining points a and b. Positive value on left side and
    negative
on right as seen from a towards b. a==b gives nan. P is sup-
posed to be Point<T> or Point3D<T> where T is e.g. double
or long long. It uses products in intermediate steps so
    watch
out for overflow if using int or long long. Using Point3D
    will
always give a non-negative distance.
*/
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
    return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

10 LineHullIntersection

```
Line-convex polygon intersection. The polygon must be ccw
and have no colinear points. lineHull(line, poly) returns a
    pair describing the
intersection of a line with the polygon: (-1, -1) if no
    collision, (i, -1) if
touching the corner i, (i, i) if along side (i, i + 1), (i
    , j) if crossing sides
(i, i + 1) and (j, j + 1). In the last case, if a corner i
    is crossed, this is treated
as happening on side (i, i + 1). The points are returned in
    the same order as
the line hits the polygon. extrVertex returns the point of a
     hull with the
max projection onto a line.
typedef array<P, 2> Line;
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
int extrVertex(vector<P>& poly, P dir) {
int n = sz(poly), lo = 0, hi = n;
 if (extr(0)) return 0:
 while (lo + 1 < hi) {
 int m = (lo + hi) / 2;
 if (extr(m)) return m;
 int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
 (ls < ms | | (ls == ms \&\& ls == cmp(lo, m)) ? hi : lo) = m;
return lo;
```

```
#define cmpL(i) sgn(line[0].cross(polv[i], line[1]))
array<int, 2> lineHull(Line line, vector<P> poly) {
int endA = extrVertex(poly, (line[0] - line[1]).perp());
int endB = extrVertex(poly, (line[1] - line[0]).perp());
if (cmpL(endA) < 0 || cmpL(endB) > 0)
 return {-1, -1}:
array<int, 2> res;
FOR(i.0.2) {
 int lo = endB, hi = endA, n = sz(poly);
 while ((lo + 1) % n != hi) {
  int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n:
  (cmpL(m) == cmpL(endB) ? lo : hi) = m:
 res[i] = (lo + !cmpL(hi)) % n;
 swap(endA, endB);
if (res[0] == res[1]) return {res[0], -1}:
if (!cmpL(res[0]) && !cmpL(res[1]))
switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
 case 0: return {res[0], res[0]};
 case 2: return {res[1], res[1]};
return res;
```

11 Miller Rabin

```
using u64 = uint64_t;
using u128 = __uint128_t;
u64 binpower(u64 base, u64 e, u64 mod) {
   u64 \text{ result} = 1;
   base %= mod:
   while (e) {
       if (e & 1)
           result = (u128)result * base % mod:
       base = (u128)base * base % mod;
       e >>= 1:
   }
   return result:
bool check_composite(u64 n, u64 a, u64 d, int s) {
   u64 x = binpower(a, d, n):
   if (x == 1 | | x == n - 1)
       return false:
   for (int r = 1; r < s; r++) {</pre>
       x = (u128)x * x % n:
```

```
if (x == n - 1)
          return false:
   }
   return true:
};
bool MillerRabin(u64 n, int iter=5) { // returns true if n
    is probably prime, else returns false.
   if (n < 4)
       return n == 2 || n == 3;
   u64 d = n - 1;
   while ((d & 1) == 0) {
       d >>= 1:
   for (int i = 0: i < iter: i++) {</pre>
       int a = 2 + rand() \% (n - 3):
       if (check_composite(n, a, d, s))
          return false:
   }
   return true;
```

12 NTT

```
// TEMPLATE FFT/NTT AWOKWOK
const int mod = 998244353;

ll pang(ll x,ll y){
   if(x==0)return 0;
   if(y==0)return 1;
   if(y==1)return x;
   ll z=pang(x,y/2);
   return z*z%mod*pang(x,y%2)%mod;
}

const int root = pang(3,119);
const int root_1 = pang(root,mod-2);
const int root_pw = 1 << 23;

ll inv[300005],fact[300005],ifact[300005];

void fft(vector<ll> & a, bool invert) {
   int n = a.size();
```

```
for (int i = 1, i = 0; i < n; i++) {
   int bit = n \gg 1:
   for (; j & bit; bit >>= 1)
       j ^= bit;
   j ^= bit;
   if (i < j)
       swap(a[i], a[i]);
for (int len = 2: len <= n: len <<= 1) {
   int wlen = invert ? root 1 : root:
   for (int i = len; i < root_pw; i <<= 1)</pre>
       wlen = (int)(1LL * wlen * wlen % mod);
   for (int i = 0; i < n; i += len) {</pre>
       int w = 1:
       for (int j = 0; j < len / 2; j++) {
           int u = a[i+i], v = (int)(1LL * a[i+i+len/2] *
           a[i+j] = u + v < mod ? u + v : u + v - mod;
           a[i+j+len/2] = u - v >= 0 ? u - v : u - v +
          w = (int)(1LL * w * wlen % mod);
if (invert) {
   int n 1 = inv[n]:
   for (11 & x : a)
       x = (int)(1LL * x * n_1 \% mod);
```

13 OnSegment

```
/*
Returns true iff p lies on the line segment from s to e. Use
(segDist(s,e,p)<=epsilon) instead when using Point<double>.
*/
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}</pre>
```

114 Point

```
template \langle class T \rangle int sgn(T x) \{ return (x > 0) - (x < 0) \}
template<class T>
struct Point {
typedef Point P;
T x. v:
explicit Point(T x=0, T y=0) : x(x), y(y) {}
bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y);</pre>
bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y);
P operator+(P p) const { return P(x+p.x, y+p.y); }
P operator-(P p) const { return P(x-p.x, y-p.y); }
P operator*(T d) const { return P(x*d, y*d); }
P operator/(T d) const { return P(x/d, y/d); }
T dot(P p) const { return x*p.x + y*p.y; }
T cross(P p) const { return x*p.y - y*p.x; }
T cross(P a, P b) const { return (a-*this).cross(b-*this);
T dist2() const { return x*x + y*y; }
double dist() const { return sqrt((double)dist2()); }
// angle to x-axis in interval [-pi , pi ]
double angle() const { return atan2(v, x); }
P unit() const { return *this/dist(): } // makes d i s t ()
P perp() const { return P(-v, x); } // rotates +90 degrees
P normal() const { return perp().unit(); }
// returns point rotated a radians ccw around the origin
P rotate(double a) const { return P(x*cos(a)-y*sin(a),x*sin
     (a)+v*cos(a)): }
friend ostream& operator<<(ostream& os, P p) { return os <<</pre>
      "(" << p.x << "," << p.y << ")"; }
```

15 PointInsideHull

```
/*
Determine whether a point t lies inside a convex hull (CCW
order, with no colinear points). Returns true if point lies
    within the hull. If
strict is true, points on the boundary arent included.
*/
typedef Point<11> P;
bool inHull(const vector<P>& 1, P p, bool strict = true) {
    int a = 1, b = sz(1) - 1, r = !strict;
```

```
if (sz(1) < 3) return r && onSegment(1[0], 1.back(), p);
if (sideOf(1[0], 1[a], 1[b]) > 0) swap(a, b);
if (sideOf(1[0], 1[a], p) >= r || sideOf(1[0], 1[b], p) <= -
    r)
    return false;
while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(1[0], 1[c], p) > 0 ? b : a) = c;
}
return sgn(1[a].cross(1[b], p)) < r;
}</pre>
```

16 PolygonCenter

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
   P res(0, 0); double A = 0;
   for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
     res = res + (v[i] + v[j]) * v[j].cross(v[i]);
     A += v[j].cross(v[i]);
}
return res / A / 3;
}</pre>
```

17 Push Relabel

```
const int inf = 1000000000;
int n;
vector<vector<int>> capacity, flow;
vector<int>> height, excess;

void push(int u, int v)
{
   int d = min(excess[u], capacity[u][v] - flow[u][v]);
   flow[u][v] += d;
   flow[v][u] -= d;
   excess[u] -= d;
   excess[v] += d;
}

void relabel(int u)
{
   int d = inf;
   for (int i = 0; i < n; i++) {</pre>
```

```
if (capacity[u][i] - flow[u][i] > 0)
          d = min(d, height[i]);
   if (d < inf)</pre>
       height[u] = d + 1;
vector<int> find_max_height_vertices(int s, int t) {
   vector<int> max_height;
   for (int i = 0; i < n; i++) {</pre>
      if (i != s && i != t && excess[i] > 0) {
           if (!max height.emptv() && height[i] > height[
               max_height[0]])
              max_height.clear();
          if (max_height.empty() || height[i] == height[
               max_height[0]])
              max_height.push_back(i);
   return max_height;
int max_flow(int s, int t)
   height.assign(n, 0);
   height[s] = n;
   flow.assign(n, vector<int>(n, 0));
   excess.assign(n, 0):
   excess[s] = inf;
   for (int i = 0: i < n: i++) {</pre>
       if (i != s)
          push(s, i);
   vector<int> current:
   while (!(current = find max height vertices(s, t)).emptv
       for (int i : current) {
          bool pushed = false;
          for (int j = 0; j < n && excess[i]; j++) {</pre>
              if (capacitv[i][i] - flow[i][i] > 0 && height[
                   i] == height[j] + 1) {
                  push(i, j);
                  pushed = true;
          if (!pushed) {
              relabel(i):
              break;
          }
```

```
}

int max_flow = 0;
for (int i = 0; i < n; i++)
    max_flow += flow[i][t];
return max_flow;
}</pre>
```

18 SegmentDistance

```
/*
Returns the shortest distance between point p and the line
segment from point s to e.
Usage: Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;
*/
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
   if (s==e) return (p-s).dist();
   auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
   return ((p-s)*d-(e-s)*t).dist()/d;
}
```

19 SideOf

```
Returns where p is as seen from s towards e. 1/0/-1 ? left/
line/right. If the optional argument eps is given 0 is
    returned if p is within
distance eps from the line. P is supposed to be Point<T>
    where T is e.g.
double or long long. It uses products in intermediate steps
    so watch out for
overflow if using int or long long.
Usage: bool left = sideOf(p1,p2,q)==1;
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }
template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
auto a = (e-s).cross(p-s);
double 1 = (e-s).dist()*eps;
return (a > 1) - (a < -1);
```

20 String Automaton

```
struct state {
   int len. link:
   map<char, int> next:
};
const int MAXLEN = 100000:
state st[MAXLEN * 2];
int sz. last:
void sa init() {
   st[0].len = 0:
   st[0].link = -1;
   sz++:
   last = 0;
void sa extend(char c) {
   int cur = sz++:
   st[cur].len = st[last].len + 1:
   int p = last;
   while (p != -1 && !st[p].next.count(c)) {
       st[p].next[c] = cur;
       p = st[p].link;
   if (p == -1) {
       st[cur].link = 0:
       int q = st[p].next[c];
       if (st[p].len + 1 == st[q].len) {
          st[cur].link = q;
      } else {
          int clone = sz++:
          st[clone].len = st[p].len + 1;
          st[clone].next = st[a].next:
          st[clone].link = st[a].link:
          while (p != -1 && st[p].next[c] == q) {
              st[p].next[c] = clone;
              p = st[p].link;
          st[a].link = st[cur].link = clone:
   last = cur:
```

```
// OP STRING ALGO AMORGOS
```

21 Treap

```
#include <iostream>
#include <vector>
#include <algorithm>
#include <functional>
using namespace std;
typedef long long 11;
const 11 LLINF = 2e16, LLBOUND = 2e15;
struct Node {
11 val, mx, mn, mdiff;
int size, priority;
 Node *1, *r;
 Node(ll val): val( val), mx( val), mn( val), mdiff(LLINF)
     , size(1) {
 priority = rand();
};
int size(Node *p) { return p == NULL ? 0 : p->size; }
11 getmax(Node *p) { return p == NULL ? -LLINF : p->mx; }
11 getmin(Node *p) { return p == NULL ? LLINF : p->mn; }
ll getmdiff(Node *p) { return p == NULL ? LLINF : p->mdiff;
void update(Node *p) {
if (p == NULL) return;
 p->size = 1 + size(p->1) + size(p->r):
p->mx = max(p->val, max(getmax(p->l), getmax(p->r)));
 p\rightarrow mn = min(p\rightarrow val, min(getmin(p\rightarrow l), getmin(p\rightarrow r)));
 p->mdiff = LLINF:
 if (p->1 != NULL)
 p->mdiff = min(p->mdiff, min(getmdiff(p->l), p->val -
       getmax(p->1)));
 if (p->r != NULL)
 p->mdiff = min(p->mdiff, min(getmdiff(p->r), getmin(p->r)
       - p->val));
void merge(Node *&t. Node *1. Node *r) {
if (1 == NULL) { t = r: }
 else if (r == NULL) { t = 1; }
 else if (1->priority > r->priority) {
 merge(1->r, 1->r, r);
```

```
t = 1:
} else {
 merge(r->1, 1, r->1);
 t = r:
update(t):
void splitat(Node *t, Node *&l, Node *&r, int at) {
if (t == NULL) { l = r = NULL: return: }
int id = size(t->1);
if (id > at) {
 splitat(t->1, 1, t->1, at):
 r = t;
 splitat(t->r, t->r, r, at - id - 1);
 1 = t;
update(t);
11 Nquery(Node *t, int i, int j) {
Node *1, *r;
splitat(t, 1, t, i - 1);
splitat(t, t, r, j - i);
11 ret = getmdiff(t);
merge(t, 1, t);
merge(t, t, r);
return (ret <= 0 || ret > LLBOUND ? -1 : ret):
11 Xquery(Node *t, int i, int j) {
Node *1. *r:
splitat(t, 1, t, i - 1);
splitat(t, t, r, j - i);
11 ret = getmax(t) - getmin(t);
merge(t, 1, t);
merge(t, t, r);
return (ret <= 0 || ret > LLBOUND ? -1 : ret):
void split(Node *t, Node *&1, Node *&r, 11 val) {
if (t == NULL) { 1 = r = NULL; return; }
if (t->val >= val) {
 split(t->1, 1, t->1, val):
} else {
 split(t->r, t->r, r, val);
 1 = t;
update(t);
void insert(Node *&t, ll val) {
Node *n = new Node(val), *1, *r;
```

```
split(t, 1, t, val):
split(t, t, r, val + 1);
merge(t, 1, n);
merge(t, t, r):
void erase(Node *&t, 11 val, bool del = true) {
Node *L. *rm:
split(t, t, L, val);
split(L, rm, L, val + 1);
merge(t, t, L);
if (del && rm != NULL) delete rm:
void inorder(Node *p) {
if (p == NULL) return;
inorder(p->1);
cout << p->val << ' ';
inorder(p->r);
void cleanup(Node *p) {
if (p == NULL) return;
cleanup(p->1); cleanup(p->r);
delete p;
int main() {
ios::sync_with_stdio(false);
cin.tie(NULL):
Node *tree = NULL;
srand(time(NULL));
int 0:
cin >> Q;
for (int q = 1; q \le Q; ++q) {
 char c:
 cin >> c:
 switch (c) {
  case 'I':
   11 k:
   cin >> k:
   insert(tree, k);
   break;
  case 'D':
   11 kd:
   cin >> kd:
   erase(tree, kd);
   break:
  case 'X':
   int 1, r;
```

```
11
```

```
cin >> 1 >> r;
   if (r - 1 < 1) cout << -1 << '\n';
   else cout << Xquery(tree, 1, r) << '\n';
   break;
   case 'N':
   int ll, rr;
   cin >> 11 >> rr;
   if (rr - 11 < 1) cout << -1 << '\n';
   else cout << Nquery(tree, 11, rr) << '\n';
   break;
}
// cout << " ";
// inorder(tree); cout << endl;
}
cout << flush;
cleanup(tree);</pre>
```

22 aho corasick

```
#include <bits/stdc++.h>
using namespace std;
#define ff first
#define ss second
#define pb push_back
#define debug(val) cerr << "The value of " << #val << " is = }</pre>
      " << val << '\n';
typedef long double ld;
typedef long long 11;
typedef unsigned long long ull;
const ld PI = 4*atan((ld)1);
const ll mod = 1e9 + 7;
const 11 inf = 922337203685477;
const 11 \text{ nax} = 1e3 + 5:
const int K = 105:
struct Vertex {
   int next[K]:
   vector<ll> leaf;
   int p = -1;
   char pch;
   int link = -1;
   int go[K];
   Vertex(int P=-1, char ch='$') : p(P), pch(ch) {
```

```
fill(begin(next), end(next), -1);
       fill(begin(go), end(go), -1);
};
vector<Vertex> t(1):
void add_string(string const& s, ll idx) {
   int v = 0:
   for (char ch : s) {
      int c = ch - 'a':
      if (t[v].next[c] == -1) {
          t[v].next[c] = t.size();
          t.emplace_back(v, ch);
       v = t[v].next[c];
   t[v].leaf.pb(idx);
int go(int v, char ch);
int get_link(int v) {
   if (t[v].link == -1) {
      if (v == 0 || t[v].p == 0)
          t[v].link = 0;
          t[v].link = go(get_link(t[v].p), t[v].pch);
   return t[v].link:
int go(int v, char ch) {
   int c = ch - 'a';
   if (t[v].go[c] == -1) {
      if (t[v].next[c] != -1)
          t[v].go[c] = t[v].next[c];
          t[v].go[c] = v == 0 ? 0 : go(get_link(v), ch);
   return t[v].go[c];
11 tc, q;
string s, a[nax];
bool cek[nax], vis[nax];
int main(){
   ios_base::sync_with_stdio(false); cin.tie(NULL); cout.tie
```

```
//freopen("test.in", "r", stdin);
   //freopen("test.out", "w", stdout);
  cin >> tc:
  while(tc--){
 cin >> s:
 cin >> q;
 // reset
 memset(vis, 0, sizeof(vis));
 memset(cek, 0, sizeof(cek));
 t.clear():
 t.emplace_back();
 for(ll i = 1; i <= q; i++){</pre>
 cin >> a[i]:
  add_string(a[i], i);
 cek[i] = 0;
11 \text{ cur} = 0:
 for(auto it : s){
 cur = go(cur, it);
 if(!vis[cur]){
  for(auto each : t[cur].leaf){
   cek[each] = 1;
  vis[cur] = 1;
 for(ll i = 1; i <= q; i++){</pre>
 if(cek[i]) cout << "y\n";</pre>
  else cout << "n\n";</pre>
}
```

23 bridgearticulation

24 centroid

```
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
#define ff first
#define ss second
#define pb push_back
const 11 \text{ nax} = 2e5 + 5;
const ll inf = 1e10;
11 n. m:
11 par[nax], removed[nax], sub[nax];
vector<vector<ll>> v(nax):
// Centroid
void get_sz(ll idx, ll bfr){
 sub[idx] = 1:
 for(auto y : v[idx]){
 if(y != bfr && !removed[y]){
  get_sz(y, idx);
  sub[idx] += sub[y];
}
}
11 find centroid(11 idx){
 get_sz(idx, -1);
```

```
11 tree = sub[idx]:
11 \text{ cek} = 0:
while(!cek){
 cek = 1:
 for(auto y : v[idx]){
  if(removed[y] || sub[y] > sub[idx]) continue;
  if(sub[y] > tree / 2){
   cek = 0:
   idx = y;
   break:
 }
 }
return idx:
void solve(ll idx){
 // Do smth here
ll built centroid(ll idx){
idx = find_centroid(idx);
 // Do smth here
 solve(idx):
removed[idx] = 1;
for(auto y : v[idx]){
 if(!removed[v]){
 11 nxt = built centroid(v):
 par[nxt] = idx;
return idx;
// Centroid
// Full Code Prob : CF 342E
ios_base::sync_with_stdio(false); cout.tie(NULL); cin.tie(
     NULL):
 cin >> n >> m;
 for(ll i = 1: i < n: i++){</pre>
 11 x, y;
 cin >> x >> y;
 v[x].pb(y);
 v[y].pb(x);
```

```
}
built_centroid(1);
}
```

12

25 $\operatorname{closest}_p air$

```
long long ClosestPair(vector<pair<int, int>> pts) {
   int n = pts.size();
   sort(pts.begin(), pts.end());
   set<pair<int, int>> s;
   long long best_dist = 1e18;
   int i = 0:
   for (int i = 0: i < n: ++i) {
       int d = ceil(sqrt(best_dist));
       while (pts[i].first - pts[j].first >= d) {
          s.erase({pts[j].second, pts[j].first});
          j += 1;
      }
       auto it1 = s.lower_bound({pts[i].second - d, pts[i].
           first}):
       auto it2 = s.upper_bound({pts[i].second + d, pts[i].
           first}):
       for (auto it = it1; it != it2; ++it) {
          int dx = pts[i].first - it->second:
          int dy = pts[i].second - it->first;
          best_dist = min(best_dist, 1LL * dx * dx + 1LL *
               dv * dv):
       s.insert({pts[i].second, pts[i].first});
   return best_dist;
```

26 combinatorics

mathtools

- $\sum_{k=0}^{n} k^2 = n(n+1)(2n+1)/6$
- $\sum_{k=0}^{n} k^3 = n^2(n+1)^2/4$
- $\sum_{k=0}^{n} k^4 = (6n^5 + 15n^4 + 10n^3 n)/30$

- $\sum_{k=0}^{n} k^5 = (2n^6 + 6n^5 + 5n^4 n^2)/12$
- $\sum_{k=0}^{n} kx^k = (x (n+1)x^{n+1} + nx^{n+2})/(x-1)^2$
- \bullet $(n+1)^{k+1}-1 = \sum_{m=1}^{n} ((m+1)^{k+1}-m^{k+1}) = \#$ pragma once $\sum_{n=0}^{k} {k+1 \choose n} (1^p + 2^p + \dots + n^p)$
- $\sum_{k=0}^{n} k \binom{n}{k} = n2^{n-1}$
- $\sum_{k=0}^{n} k^2 \binom{n}{k} = (n+n^2)2^{n-2}$
- $\bullet \sum_{i=0}^{k} {m \choose i} {n-m \choose k-i} = {n \choose k}$
- $\sum_{m=0}^{n} {m \choose k} = {n+1 \choose k+1}$
- $\bullet \sum_{i=0}^{m} {m \choose i}^2 = {2m \choose m}$
- $\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} {n-k \choose k} = F(n+1)$
- $\sum_{i=0}^{n} i\binom{n}{i}^2 = \frac{n}{2}\binom{2n}{n}$
- $\sum_{i=0}^{n} i^2 \binom{n}{i}^2 = n^2 \binom{2n-2}{n-1}$
- $\sum_{k=q}^{n} {n \choose k} {k \choose q} = 2^{n-q} {n \choose q}$
- $\sum_{k=-a}^{a} (-1)^k {2a \choose k+a}^3 = \frac{(3a)!}{(a!)^3}$
- $\sum_{k=-a}^{a} (-1)^k \binom{a+b}{a+k} \binom{b+c}{b+k} \binom{c+a}{c+k} = \frac{(a+b+c)!}{a!k!c!}$

directed MST

```
* Date: 2019-05-10
* License: CCO
* Source: https://github.com/spaghetti-source/algorithm/
    blob/master/graph/arborescence.cc
* and https://github.com/bqi343/USACO/blob/42
    d177dfb9d6ce350389583cfa71484eb8ae614c/Implementations/
    content/graphs%20(12)/Advanced/DirectedMST.h for the
    reconstruction
```

* Author: chilli, Takanori MAEHARA, Beng, Simon Lindholm

- * Description: Finds a minimum spanning
- * tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.

```
* Time: O(E \log V)
     fastestspeedrun
#include "../data-structures/UnionFindRollback.h"
struct Edge { int a, b; ll w; };
struct Node { /// lazy skew heap node
Edge kev:
Node *1. *r:
11 delta;
void prop() {
 kev.w += delta;
 if (1) 1->delta += delta;
 if (r) r->delta += delta:
 delta = 0;
Edge top() { prop(); return key; }
Node *merge(Node *a, Node *b) {
if (!a || !b) return a ?: b;
a->prop(), b->prop();
if (a->key.w > b->key.w) swap(a, b);
swap(a->1, (a->r = merge(b, a->r)));
return a:
void pop(Node*& a) { a->prop(); a = merge(a->1, a->r); }
pair<11, vi> dmst(int n, int r, vector<Edge>& g) {
RollbackUF uf(n);
vector<Node*> heap(n):
for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
11 \text{ res} = 0:
vi seen(n, -1), path(n), par(n);
seen[r] = r:
vector<Edge> Q(n), in(n, \{-1,-1\}), comp;
deque<tuple<int, int, vector<Edge>>> cycs;
rep(s,0,n) {
 int u = s, qi = 0, w:
 while (seen[u] < 0) {</pre>
  if (!heap[u]) return {-1,{}}:
  Edge e = heap[u]->top();
  heap[u]->delta -= e.w, pop(heap[u]);
  Q[qi] = e, path[qi++] = u, seen[u] = s:
  res += e.w, u = uf.find(e.a);
  if (seen[u] == s) { /// found cycle, contract
   Node* cvc = 0:
   int end = qi, time = uf.time();
```

```
do cyc = merge(cyc, heap[w = path[--qi]]);
  while (uf.ioin(u, w)):
  u = uf.find(u), heap[u] = cyc, seen[u] = -1;
  cycs.push_front({u, time, {&Q[qi], &Q[end]}});
 rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
for (auto& [u,t,comp] : cycs) { // restore sol (optional)
uf.rollback(t):
Edge inEdge = in[u]:
 for (auto& e : comp) in[uf.find(e.b)] = e;
in[uf.find(inEdge.b)] = inEdge;
rep(i,0,n) par[i] = in[i].a;
return {res, par};
```

dp cht

```
#include <bits/stdc++.h>
using namespace std:
typedef long long 11;
typedef long double ld;
const 11 \text{ nax} = 1e6 + 5;
struct info{
11 x, y, a;
};
bool cmp(info x, info y){
return x.x < v.x:
struct line{
11 m, c;
ll val(ll x){
 return m * x + c:
ll intersect(line 1){
 return (ld) (c - l.c) / (l.m - m);
};
11 n;
info inp[nax]:
deque<line> dq;
```

```
11 binser(11 x){
11 1 = 0, r = (11)dq.size() - 1;
while(1 < r){
 11 \text{ mid} = (1 + r) / 2;
 if(dq[mid].intersect(dq[mid+1]) >= x){
 r = mid;
 }
 else{
  l = mid + 1;
 }
}
return 1;
int main(){
ios_base::sync_with_stdio(false); cout.tie(NULL); cin.tie(
     NULL);
cin >> n:
for(ll i = 1; i <= n; i++){</pre>
 cin >> inp[i].x >> inp[i].y >> inp[i].a;
sort(inp + 1, inp + 1 + n, cmp);
dq.push_front({0, 0});
11 \text{ ans} = 0;
for(ll i = 1; i <= n; i++){</pre>
 ll idx = binser(inp[i].v):
 ll ret = dq[idx].val(inp[i].y) + inp[i].x * inp[i].y - inp
      [i].a:
 ans = max(ans, ret);
 line cur = {-inp[i].x, ret};
 while((11)dq.size() >= 2 \&\& cur.intersect(dq[0]) >= dq[0].
      intersect(dq[1])){
  dq.pop_front();
 dq.push_front(cur);
cout << ans << '\n';</pre>
```

29 dpcht

```
pii tpot(pii satu,pii dua){
   pii jwb;
   jwb.first=dua.second-satu.second;
   jwb.second=satu.first-dua.first;
   return jwb;
```

```
bool cmp(pii a,pii b){
   if(a.fi/a.se==b.fi/b.se){
       a.fi%=a.se:
       b.fi%=b.se;
       return a.fi*b.se<=b.fi*a.se:
   return a.fi/a.se<=b.fi/b.se;</pre>
line.push_back({1,-pref[0]}); //cari maksimum , gradien non
    decreasing //m and c
//cari minimum gradien non increasing
for(LL i=2;i<=n;i++){</pre>
   LL x=a[i]:
   LL ki=1,ka=(LL)line.size()-1,add=-1e18;
   while(ki<=ka){</pre>
       LL mid=(ki+ka)/2:
       LL l=line[mid-1].first*x+line[mid-1].second;
       LL r=line[mid].first*x+line[mid].second:
       add=max(add.max(1.r)):
       if(l>r)ka=mid-1;
       else ki=mid+1:
       Minimum
       add=min(add,min(1,r))
       if(l>r)ki=mid+1;
       else ka=mid-1:
   if(line.size()==1)add=line[0].first*x+line[0].second:
    ans=max(ans,ret+add-a[i]*i+pref[i-1]); //tambahin
        constant
    pii now={i,-pref[i-1]};
   LL skg=line.size()-1,prev=line.size()-2;
    while(skg>0 && cmp(tpot(now,line[skg]),tpot(now,line[prev
        1))){
       //hapus yang gamasuk hull
       line.pop_back();
       skg--;
       prev--;
    line.push_back(now);
```

30 dpdnc

```
void calc(int L,int R,int optL,int optR,int j){
   if(L>R)return;
```

```
int mid=(L+R)/2;
int res=2e9;
int opt=-1;
for(int i=optL;i<=min(optR,mid-1);i++){
    if(dp[j-1][i]+cost(i+1,mid)<res){
        res=dp[j-1][i]+cost(i+1,mid);
        opt=i;
    }
}
dp[j][mid]=res;
calc(L,mid-1,optL,opt,j);
calc(mid+1,R,opt,optR,j);
}
for(int i=1;i<=n;i++)dp[1][i]=cost(1,i);
for(int i=2;i<=k;i++){
    calc(i,n,i-1,n,i); //mau ngisi dp[i][...] dengan ... dari
        i sampai n karena dengan k gondola minimal k orang
}</pre>
```

31 dynsegtree

```
int tree[3000005],lazy[3000005],ki[3000005],ka[3000005],node
//update x sampai y, jadiin 1 semua, query dari x sampe y (
    bisa sampe 1e9)
void pushdown(int now,int L,int R){
int mid=(L+R)/2:
if(ki[now]==0){
 ki[now]=node;
 node++:
if (ka[now] == 0) {
 ka[now]=node:
 node++;
tree[ki[now]]=mid-L+1:
lazv[ki[now]]=1:
tree[ka[now]]=R-mid;
lazy[ka[now]]=1;
lazy[now]=0;
void update(int now,int L,int R,int x,int y){
       if(tree[now] == R-L+1)return:
if(L>=x && R<=y){</pre>
```

```
tree[now]=R-L+1;
 lazy[now]=1;
 return;
if(L>y || R<x)return;</pre>
int mid=(L+R)/2:
if(lazy[now])pushdown(now,L,R);
if(ki[now] == 0){
 ki[now]=node;
 node++;
if(ka[now]==0){
 ka[now]=node;
 node++:
}
update(ki[now],L,mid,x,v);
update(ka[now],mid+1,R,x,y);
tree[now] = tree[ki[now]] + tree[ka[now]];
int query(int now,int L,int R,int x,int y){
if(L>=x && R<=v)</pre>
{
 return tree[now];
if(L>y || R<x || now==0)return 0;</pre>
if(lazy[now])pushdown(now,L,R);
int mid=(L+R)/2:
return query(ki[now],L,mid,x,y)+query(ka[now],mid+1,R,x,y);
```

32 eulerian

33 fordfulkerson

```
LL bneck.adi[5005][5005].source.sink.ans=0.n:
bool visited[5005]:
void dfs(LL node.LL bottleneck){
if(node==sink){
 ans+=bottleneck:
 sudah=true:
 bneck=bottleneck;
 return;
if(!visited[node]){
 visited[node]=true:
 for(LL i=1;i<=n;i++){</pre>
  if(adi[node][i]>0){
   dfs(i,min(adj[node][i],bottleneck));
   if(sudah){
    adj[node][i]-=bneck;
    adj[i][node]+=bneck;
    return;
 }
}
int main(){
source=1,sink=n;
sudah=true:
while(sudah){
 memset(visited, false, size of (visited));
 sudah=false:
 dfs(source,1e18);
cout << ans << endl;</pre>
```

34 graham scan

```
/* Quick Note :
* Jangan Mikir Lama - lama, sampahin dulu aja kalo OI
* Always Try to reset
#include <bits/stdc++.h>
using namespace std:
#define ff first
#define ss second
#define pb push_back
#define debug(val) cerr << "The value of " << #val << " is =</pre>
     " << val << '\n';
typedef long double ld;
typedef long long 11;
typedef unsigned long long ull;
const 11 mod = 1e9 + 7;
const 11 inf = 922337203685477;
const 11 nax = 0:
struct point{
11 x, y;
};
11 t. n:
vector<point> a;
11 cross(point p, point q, point r){
ll val = (q.y - p.y) * (r.x - q.x) - (q.x - p.x) * (r.y - q)
     .y);
if(val == 0){
 return 0:
else if(val > 0){
 return 1:
else{
 return -1:
11 dist(point p, point q){
11 dx = p.x - q.x, dy = p.y - q.y;
return dx * dx + dy * dy;
bool cmp(point p, point q){
11 order = cross(a[0], p, q);
if(order == 0){
 return dist(a[0], p) < dist(a[0], q);</pre>
```

```
16
```

```
}
 elsef
 return (order == -1);
}
}
// Problem : 681 - Convex Hull Finding - UVA
int main(){
   ios_base::sync_with_stdio(false); cin.tie(NULL); cout.tie
        (NULL):
   //freopen("test.in", "r", stdin);
   //freopen("test.out", "w", stdout);
   cin >> t:
   cout << t << '\n';
   while(t--){
 a.clear();
 cin >> n:
 ll mini = 0:
 for(11 i = 0; i < n; i++){</pre>
  11 x, y;
  cin >> x >> y;
  a.pb({x, y});
  if(y < a[mini].y){</pre>
   mini = i;
 if(t){
  ll gbg;
  cin >> gbg;
 // Jadiin satu titik sebagai titik acuan / pivot, titik
      yang dipakai adalah titik yang paling bawah
 swap(a[0], a[mini]);
 // Sort by polar angel
 sort(a.begin() + 1, a.end(), cmp);
 vector<point> v;
 for(ll i = 0; i < n; i++){</pre>
  if(v.size() < 2){
   v.pb(a[i]);
  }
   else{
   // Kalau Cross product nya tidak Counter Clockwise
        pop_back();
   while(v.size() >= 2 && cross(v[v.size()-2], v[v.size()
        -1], a[i]) != -1){
    v.pop_back();
   v.pb(a[i]);
```

```
}
}
cout << v.size() + 1 << '\n';
for(auto p : v){
  cout << p.x << " " << p.y << '\n';
}
cout << a[0].x << " " << a[0].y << '\n';
if(t){
  cout << "-1\n";
}
}
}</pre>
```

35 hungarian

```
#include <bits/stdc++.h>
using namespace std;
typedef long long 11;
typedef long double ld;
#define pb push_back
#define ff first
#define ss second
const ld PI = 4 * atan((ld)1):
const 11 nax = 25;
const ll inf = 1e16:
11 n:
ll ans:
11 dist(pair<11,11> x, pair<11,11> y){
return abs(x.ff - v.ff) + abs(x.ss - v.ss):
ll hungarian(vector<pair<11,11>>&a, vector<pair<11,11>>&b){
 // pairing a ke b
vector<11> u(n + 1), v(n + 1), p(n + 1), way(n + 1);
for(ll i = 1; i <= n; i++){</pre>
 p[0] = i;
 ll curM = 0:
 vector<ll> minv(n + 1, inf):
 vector<bool> used(n + 1, 0);
 while(p[curM] != 0){
  used[curM] = 1;
  11 curN = p[curM], delta = inf;
  ll nexM;
  for(int j = 1; j <= n; ++j){</pre>
   if(!used[i]){
    int cur = dist(a[curN-1], b[j-1]) - u[curN] - v[j];
```

```
if(cur < minv[i]){</pre>
    minv[j] = cur, wav[j] = curM;
   if(minv[j] < delta){</pre>
     delta = minv[j], nexM = j;
   }
  for(int j = 0; j <= n; j++){</pre>
   if(used[i]){
   u[p[j]] += delta, v[j] -= delta;
   else{
   minv[j] -= delta;
   }
  curM = nexM:
 dof
 11 nexM = way[curM];
 p[curM] = p[nexM];
  curM = nexM:
 }while(curM != 0);
return (-v[0]);
void make_diagonal(vector<pair<11,11>>&a){
vector<pair<11,11>> b;
for(ll i = 1: i <= n: i++){
b.pb({i, i});
ans = min(ans, hungarian(a, b));
b.clear();
ll cnt = 1:
for(ll i = n: i >= 1: i--){
 b.pb({cnt, i});
 cnt++;
ans = min(ans, hungarian(a, b));
void make_horizontal(vector<pair<11,11>> &a){
vector<pair<11,11>> b;
for(ll i = 1; i <= n; i++){</pre>
for(11 j = 1; j <= n; j++){
 b.pb({i, j});
 ans = min(ans, hungarian(a, b));
 b.clear():
```

```
}
}
void make_vertical(vector<pair<11,11>> &a){
 vector<pair<11,11>> b;
 for(ll i = 1; i <= n; i++){</pre>
 for(11 j = 1; j <= n; j++){
  b.pb({j, i});
 ans = min(ans, hungarian(a, b));
 b.clear():
}
int main(){
 ios_base::sync_with_stdio(false); cin.tie(NULL); cout.tie(
 11 ct = 0:
 while(1){
 cin >> n;
 ans = inf:
 if(n == 0)
  break:
 vector<pair<11,11>> a;
 for(ll i = 1; i <= n; i++){</pre>
  11 x, y;
  cin >> x >> y;
  a.pb({x, y});
 make_diagonal(a);
 make horizontal(a):
 make_vertical(a);
 cout << "Board " << ++ct << ": " << ans << " moves
       required." << "\n\n";
```

36 li chao

```
typedef long long ftype;
typedef complex<ftype> point;
#define x real
#define y imag
ftype dot(point a, point b) {
```

```
return (coni(a) * b).x():
ftype f(point a, ftype x) {
   return dot(a, \{x, 1\});
const int maxn = 2e5;
point line[4 * maxn];
void add line(point nw. int v = 1, int l = 0, int r = maxn)
   int m = (1 + r) / 2:
   bool lef = f(nw, 1) < f(line[v], 1);
   bool mid = f(nw, m) < f(line[v], m);</pre>
       swap(line[v], nw);
   if(r - 1 == 1) {
       return;
   } else if(lef != mid) {
       add_line(nw, 2 * v, 1, m);
       add_line(nw, 2 * v + 1, m, r);
ftype get(int x, int v = 1, int l = 0, int r = maxn) {
   int m = (1 + r) / 2:
   if(r - 1 == 1) {
       return f(line[v], x);
   else if(x < m)
       return min(f(line[v], x), get(x, 2 * v, 1, m));
       return min(f(line[v], x), get(x, 2 * v + 1, m, r));
```

37 mcbm

```
return true;
}
return false;
}
memset(match,-1,sizeof(match));
for(int i=0;i<n;i++){
   memset(visited,0,sizeof(visited));
   if(dfs(i))matching++;
}</pre>
```

38 mo's

```
bool cmp(pair<pii,LL> a,pair<pii,LL> b){
   if(a.first.first/SQRT==b.first.first/SQRT)return a.first.
        second<b.first.second;
   return a.first.first/SQRT<b.first.first/SQRT;
}

sort(que.begin(),que.end(),cmp);
LL L=1,R=1;
   for(auto isi : que){
    LL ki=isi.first.first,ka=isi.first.second;
    while(R<=ka)update(R++);
    while(L-1>=ki)update(--L);
        while(R-1>ka)remove(--R);
        while(L<ki)remove(L++);
        ans[isi.second]=ret;
}</pre>
```

39 pbds

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
typedef tree<
int,
null_type,
less<int>, // mau multiset ganti jadi less_equal
rb_tree_tag,
tree_order_statistics_node_update>ordered_set;
ordered_set X;
*X.find_by_order(v) // elemen ke-v zero based
```

X.order_of_key(v) //banyaknya elemen yang < v</pre>

40 perssegtree

```
int tree[2*MAXN*LOG],ki[2*MAXN*LOG],ka[2*MAXN*LOG],a[MAXN],
    root[MAXN],idx,MAX,balik[MAXN];
map<int,int> mp;
int build(int L,int R){
idx++:
 int no=idx;
 tree[no]=0:
 if(L==R)return no:
 int mid=(L+R)/2;
 ki[no]=build(L,mid);
ka[no]=build(mid+1,R);
return no;
}
int update(int bef.int L.int R.int x){
 idx++:
 int no=idx;
 tree[no] = tree[bef] + 1;
 ki[no]=ki[bef];
 ka[no]=ka[bef]:
 if(L==R)return no;
 int mid=(L+R)/2;
 if(x<=mid)ki[no]=update(ki[no],L,mid,x);</pre>
 else ka[no]=update(ka[no],mid+1,R,x);
 return no;
}
int query(int a,int b,int L,int R,int k){
if(L==R)return L:
 int mid=(L+R)/2;
 int brp=tree[ki[b]]-tree[ki[a]];
 if(brp>=k)return query(ki[a],ki[b],L,mid,k);
 else return query(ka[a],ka[b],mid+1,R,k-brp);
root[0]=build(1,MAX); //seperti null
 for(int i=1:i<=n:i++)root[i]=update(root[i-1].1.MAX.mp[a[i</pre>
     ]]);
 while(a--){
 cin >> 1 >> r >> k;
 cout << balik[query(root[l-1],root[r],1,MAX,k)] << '\n';</pre>
```

41 random

```
#include<algorithm>
#include<chrono>
#include<random>
using namespace std;
// Note: Requires C++11
// A random number generator that uses time since epoch to
    generate random numbers
// It is much faster than rand(), and the numbers are more
    uniformly generated
// Using time_since_epoch makes the seed number much more
    unpredictable
mt19937 rng(chrono::steady_clock::now().time_since_epoch().
    count()):
int main(){
// Print a random number from 0 to 2^32 - 1 (unsigned int)
printf("%u\n", rng()):
// Use of RNG in shuffle
 shuffle(permutation.begin(), permutation.end(), rng);
// Generates an equiprobable random numbers in interval [a,
      bl inclusive
printf("%d\n", uniform_int_distribution<int>(1, 6)(rng));
```

42 segment tree lazy

```
/* Quick Note :
* Jangan Mikir Lama - lama, sampahin dulu aja kalo OI
* Always Try to reset
#include <bits/stdc++.h>
using namespace std;
#define ff first
#define ss second
#define pb push_back
#define debug(val) cerr << "The value of " << #val << " is =</pre>
     " << val << '\n':
typedef long double ld;
typedef int 11;
typedef unsigned long long ull;
const ld PI = 4*atan((ld)1);
const 11 \mod = 1e9 + 7;
const ll inf = 1e9;
const 11 nax = 1e6 + 5;
struct info{
```

```
11 four, sev, inc. dec:
}:
11 n. m:
11 prop[4*nax];
info seg[4*nax]:
string s;
info merge(info x, info y){
info ret;
ret.four = x.four + v.four:
ret.sev = x.sev + v.sev:
ret.inc = max({x.four + y.four, x.sev + y.sev, x.four + y.
     inc, x.inc + y.sev});
ret.dec = max({x.four + y.four, x.sev + y.sev, x.sev + y.
     dec, x.dec + y.four});
return ret:
void rev(ll x){
swap(seg[x].four, seg[x].sev);
swap(seg[x].inc, seg[x].dec);
void lazy(ll x){
if(prop[x]){
 rev(2*x), rev(2*x+1);
 prop[2*x] ^= 1, prop[2*x+1] ^= 1:
 prop[x] = 0;
void built(ll 1, ll r, ll pos){
if(1 == r){
 seg[pos] = {s[1-1] == '4', s[1-1] == '7', 1, 1};
elsef
 11 \text{ mid} = (1 + r) / 2:
 built(1, mid, 2*pos);
 built(mid + 1, r, 2*pos+1);
 seg[pos] = merge(seg[2*pos], seg[2*pos+1]):
void upd(11 1, 11 r, 11 pos, 11 f1, 11 fr){
if(f1 <= 1 && fr >= r){
 rev(pos);
 prop[pos] ^= 1;
else if(fl > r \mid \mid fr < 1){
```

```
return;
}
else{
    lazy(pos);
    l1 mid = (1 + r) / 2;
    upd(1, mid, 2*pos, f1, fr);
    upd(mid + 1, r, 2*pos+1, f1, fr);
    seg[pos] = merge(seg[2*pos], seg[2*pos+1]);
}
```

$43 \quad \cos$

```
//DP SOS (Sum over submask)
for(int i=0;i<m;i++){
  for(int mask=(1<<m)-1;mask>=0;mask--){
    if(mask & (1<<i))dp[mask]+=dp[mask^(1<<i)];
  }
}</pre>
```

44 suffix array

```
#include <bits/stdc++.h>
using namespace std;
#define ff first
#define ss second
#define pb push_back
#define debug(val) cerr << "The value of " << #val << " is =</pre>
      " << val << '\n':
typedef long double ld;
typedef long long 11;
typedef unsigned long long ull;
const ld PI = 4*atan((ld)1):
const ll mod = 1e9 + 7;
const 11 inf = 922337203685477;
const 11 \text{ nax} = 5e5 + 5;
11 n:
11 sa[nax], ra[nax];
11 tempSA[nax], tempRA[nax];
ll freq radix[nax]:
string s;
void radixSort(ll k){
11 \max i = \max(30011, n);
```

```
memset(freq_radix, 0, sizeof(freq_radix));
for(11 i = 0: i < n: i++){
 if(i + k < n){
 freq_radix[ra[i+k]]++;
 else{
  freq_radix[0]++;
11 sum = 0;
for(ll i = 0: i < maxi: i++){</pre>
 ll temp = freq radix[i]:
 freq_radix[i] = sum;
 sum += temp:
for(11 i = 0; i < n; i++){}
 11 \text{ temp} = \text{sa[i]} + \text{k}:
 if(temp < n){
  tempSA[freq radix[ra[temp]]++] = sa[i]:
 }
 else{
  tempSA[freq_radix[0]++] = sa[i];
for(ll i = 0; i < n; i++){</pre>
 sa[i] = tempSA[i];
void builtSA(){
for(11 i = 0; i < n; i++){}
 ra[i] = s[i];
 sa[i] = i:
for(11 k = 1: k < n: k *= 2){
 radixSort(k):
 radixSort(0):
 tempRA[sa[0]] = 0;
 11 r = 0;
 for(11 i = 1: i < n: i++){</pre>
  if(ra[sa[i]] == ra[sa[i-1]] && ra[sa[i]+k] == ra[sa[i-1]+
   tempRA[sa[i]] = r;
  else{
   tempRA[sa[i]] = ++r;
 for(ll i = 0; i < n; i++){</pre>
  ra[i] = tempRA[i];
```

```
if (ra[sa[n-1]] == n-1) break; // nice optimization trick
int main(){
   ios_base::sync_with_stdio(false); cin.tie(NULL); cout.tie
        (NULL):
   //freopen("test.in", "r", stdin);
   //freopen("test.out", "w", stdout);
   /*
contoh input
qwedasd
contoh output
asd: URUTAN KE 1
d: URUTAN KE 2
dasd: URUTAN KE 3
edasd: URUTAN KE 4
gwedasd: URUTAN KE 5
sd: URUTAN KE 6
wedasd: URUTAN KE 7
   cin >> s;
   s += '$':
   n = s.size():
   builtSA();
   for(ll i = 1: i < n: i++){
 for(ll j = sa[i]; j < n - 1; j++){</pre>
  cout << s[i];
 cout << ": URUTAN KE " << i << '\n';
```

45 theorem

Cayley's Formula

There are n^{n-2} spanning trees of a complete graph with n labeled vertices.

Derangement

A permutation of the elements of a set such that none of the elements appear in their original positions. F(n) = (n-1)*(F(n-1)+F(n-2)). F(0) = 1. F(1) = 0.

Euler's Formula for Planar Graph

V-E+F=2, where V = vertices, E = edges, F = faces

Pick's Theorem

A = i + b/2 - 1, where A = area, i = internal points, b = border points

Spanning Tree of Complete Bipartite Graph $N^{M-1} * M^{N-1}$, where N = row and M = column

Pythagorean Triples

Integer solutions of $x^2 + y^2 = z^2$. All relatively prime triples are given by: $x = 2mn, y = m^2 - n^2, z = m^2 + n^2$, where m > n, gcd(m, n) = 1, and $m! = n \pmod{2}$.

Moser's Circle

Determine the number of pieces into which a circle is divided if n points on its circumference are joined by chords with no three internally concurrent. Solution: g(n) = nC4 + nC2 + 1

Kirchoff Matrix Theorem

Let matrix $T = [t_{ij}]$, where t_{ij} is the number of multiedges between i and j, for $i \neq j$, and $t_{ii} = -deg[i]$. Number of spanning trees of a graph is equal to the determinant of a matrix obtained by deleting any k-th row and column from T.

Euler's Theorem

$$a^{phi(n)} = 1 \pmod{n}$$
, if $acd(a, n) = 1$.

Wilson's Theorem

p is prime iff $(p-1) \neq -1 \pmod{p}$.

Pisano Period

Periodicity of fibonacci modulo m.

- $\bullet \ pi(p^k) = p^{k-1} * pi(p)$
- pi(2) = 3, pi(5) = 20

- if $p \equiv 1$ or $p \equiv 9$ in modulo 10, pi(p) divides p-1
- if $p \equiv 3$ or $p \equiv 7$ in modulo 10, pi(p) divides 2p-1
- pi(a*b) = lcm(pi(a), pi(b)) if gcd(a, b) = 1

Misere Nim

Nim where the winner is the one who can't move. In a nim game with piles (n_1, n_2, \dots, n_k) , **second** player wins iff some $n_i > 1$ and $(n_1 \oplus n_2 \oplus ... \oplus n_k) = 0$ or all $n_i \leq 1$ and $n_1 \oplus n_2 \oplus ... \oplus n_k = 1$.

46 unionrectangle

```
struct Edge {
    bool open;
    int x, yMin, yMax;
    Edge(int x, int v1, int v2, bool op) {
        this -> x = x:
       yMin = y1, yMax = y2;
       open = op;
    bool operator < (const Edge &e) const {</pre>
        return (x < e.x):
int m, h[maxN << 1];</pre>
int sum[maxN << 5]. counter[maxN << 5]:</pre>
vector<Edge> edges;
void update(int p. int l. int r. int vMin. int vMax. bool
     open) {
  if (h[r] < yMin || yMax < h[l]) return;</pre>
    int c = p << 1, mid = (l + r) >> 1;
                                             // ymin --- h[1]
    if (yMin <= h[1] && h[r] <= yMax) {</pre>
         --- h[r] --- ymax
        counter[p] += open ? 1 : -1;
        if (counter[p]) sum[p] = h[r] - h[l]; //if there is a
             rectangle at that posn that is bw h[1] and h[r]
             we will add that to length
        else sum[p] = sum[c] + sum[c + 1]; // else we will
            just sumup of lengths above and beloew this
            region
        return;
    if (1 + 1 >= r) return;
```

```
update(c, 1, mid, yMin, yMax, open);
   update(c + 1, mid, r, yMin, yMax, open);
   if (counter[p]) sum[p] = h[r] - h[1];
   else sum[p] = sum[c] + sum[c + 1]:
long long solve() {
                             // process height for horzntl.
                                  sweep line
   sort(h + 1, h + m + 1); // Sorting the hieght according
        to the v coordinates
   int k = 1:
   for(int i=2;i<=m;i++) if (h[i] != h[k]) // Deleting the</pre>
        same horizontal sweeplines
     h[++k] = h[i];
                                       as they are redundant
         m = k;
   for (int i = 0, lm = (int)edges.size() << 4; i < lm; i++)</pre>
         // This is the initialization step of segment tree
       sum[i] = 0, counter[i] = 0;
   long long area = OLL; // Initializing the Area
   sort(edges.begin(),edges.end()); // Sorting according to
        x coordinates for ver. swp line
   update(1, 1, m, edges[0].yMin, edges[0].yMax, edges[0].
        open);
   for (int i = 1; i < edges.size(); i++) {</pre>
       area += sum[1] * (long long)(edges[i].x - edges[i -
       update(1, 1, m, edges[i].yMin, edges[i].yMax, edges[i
           ].open);
   return area:
   edges.pb(Edge(x1, y1, y2, true)); // Inserting the Left
   edges.pb(Edge(x2, y1, y2, false)); // Inserting the Right
   /*(x1,y2) (x2,y2)
  LeftEdge <- |
                      |-> Right Edge
           (x1,v1) (x2,v1)
   h[++m] = y1; // Inserting the Lower y Coordinate 1 based
   h[++m] = y2; // Inserting the Upper y Coordinate
```

```
solve();
}

47 xor 1 to n

int computeXOR(int n)

{
    // If n is a multiple of 4
    if (n % 4 == 0)
        return n;

// If n%4 gives remainder 1
    if (n % 4 == 1)
        return 1;
```

```
// If n%4 gives remainder 2
if (n % 4 == 2)
  return n + 1;

// If n%4 gives remainder 3
return 0;
}
```