Scarab
SAT-based Constraint Programming System in Scala

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worked in cooperation with
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2014/09/05 (created)
2019/02/22 (revised)

Contents of Talk

SAT (Boolean satisfiability testing) Problems

- **SAT** is a problem of deciding whether a given Boolean formula is satisfiable or not.

- SAT was the first NP-complete problem [Cook, 1971] and is the most fundamental problem in Computer Science both theoretically and practically.

SAT instances are given in the Conjunctive Normal Form (CNF).

- A **CNF formula** is a conjunction of clauses.
- A **clause** is a disjunction of literals.
- A **literal** is either a Boolean variable or its negation.

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Example of an SAT instance (in CNF)

- Let $a, b, c \in \{True, False\}$ be Boolean variables.
- **Question**: the following CNF formula is satisfiable or not?

$$(a \vee b \vee c) \wedge$$
$$(\neg a \vee \neg b) \wedge$$
$$(\neg a \vee \neg c) \wedge$$
$$(\neg b \vee \neg c)$$

- **Answer**: Yes.
- There is an assignment $(a, b, c) = (True, False, False)$ satisfying all clauses.

SAT Solvers

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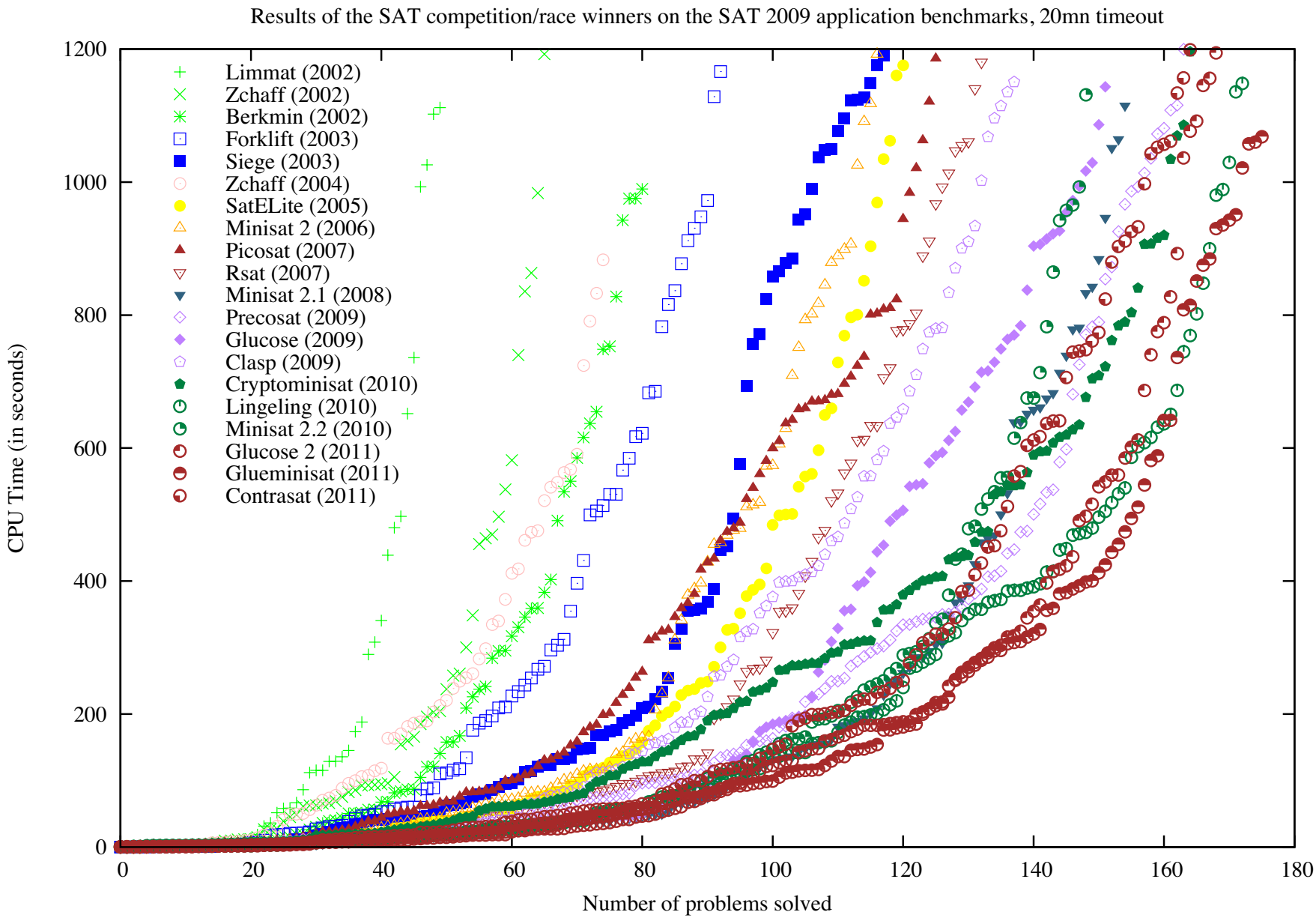
- Complete SAT solvers originated in **DPLL** [Davis et al., 1962].
- In particular, since around 2000, their performance is improved every year with techniques of Conflict Driven Clause Learning (CDCL), Non-chronological Backtracking, Rapid Restarts, and Variable Selection Heuristics etc.
- Modern SAT solvers can handle instances with more than 10^6 **variables and** 10^7 **clauses**.

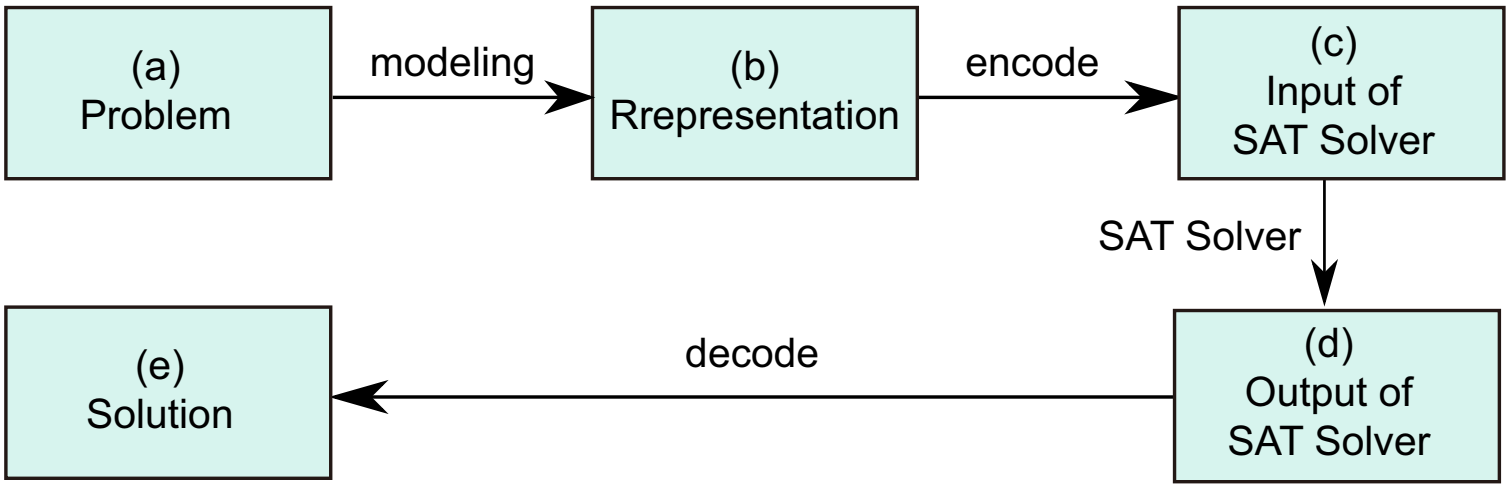
Progress of SAT Solvers (shown by [Simon 2011])



Cactus Plot shown by [Simon 2011]

Problem Solving using SAT Solvers

- Thanks to the remarkable progress of SAT solvers, **SAT-based Problem Solving** have been actively studied.



- **SAT-based Systems** are implementations of SAT-based problem solving.
- Many research topics in this field. Among them, the **importance of modeling and encoding** are re-recognized.
- Good modeling/encodings are developed considering the size of solver input and propagations in SAT solvers (and many many trial/errors are necessary!).

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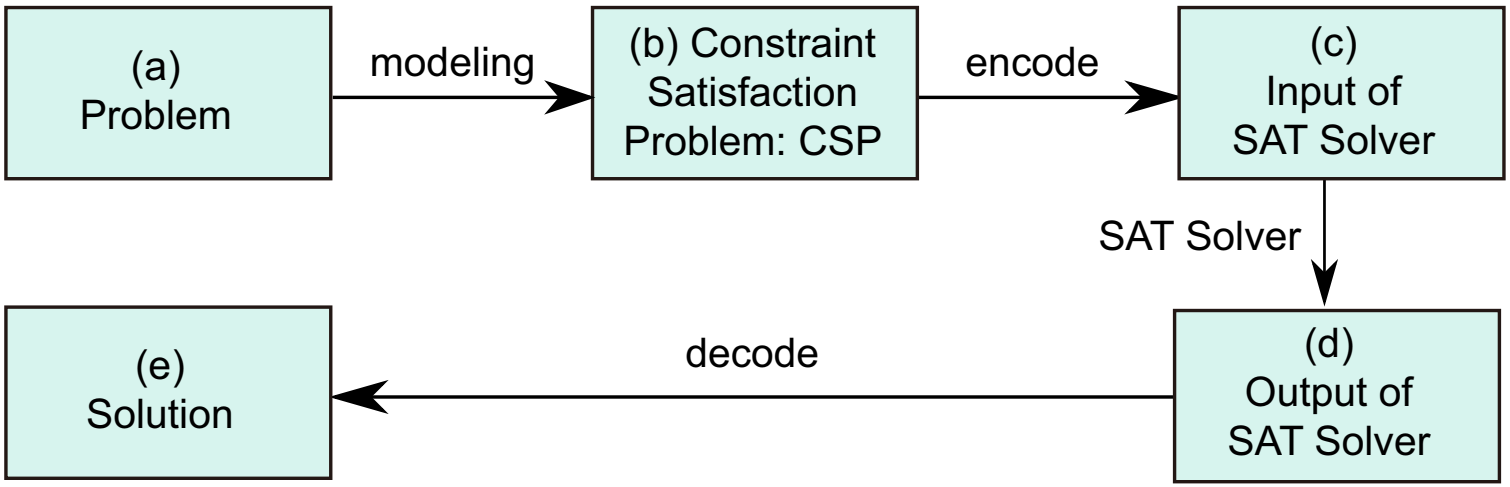

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SAT encodings

There have been several methods proposed to encode CSP into SAT.

- *Direct encoding* is the most widely used one [de Kleer, 1989].
- Other encodings:
  - *Multivalued encoding* [Selman et al., 1992]
  - *Support encoding* [Kasif, 1990]
  - *Log encoding* [Iwama and Miyazaki, 1994]
  - *Log-support encoding* [Gavanelli, 2007]

- **Order encoding** is a new encoding showing a good performance for a wide variety of problems [Tamura et al., 2006].
- It is shown that the order encoindg is the only encoding translating tractable CSP to tractable SAT [Petke and Jeavons, 2011].
  - It is first used to encode job-shop scheduling problems by [Crawford and Baker, 1994].
  - It succeeded to solve previously undecided problems in open-shop scheduling, job-shop scheduling, and two-dimensional strip packing.

Applications of SAT Technology

- **Planning** (SATPLAN, Blackbox) [Kautz and Selman, 1992]
- **Job-shop Scheduling** [Crawford and Baker, 1994]
- **Bounded Model Checking** [Biere et al., 1999]
- **Term Rewriting** (AProVE) [Giesl et al. 2004]
- **Constraint Satisfaction Problem**[Tamura et al., 2006]
  - **Sugar**, SAT-based CSP Solvr, which is the Winner of 2008 and 2009 CSP Solver Competitions in GLOBAL categories.
  - It adopts Order Encoding.
- Others
  - Test Case Generation,
  - Systems Biology,
  - Timetabling,
  - Packing,
  - Puzzle, and more!

Other News around SAT

- A **SAT solver Sat4j** implemented on Java has been integrated into **Eclipse** for managing plugins dependencies in their update manager.
- **Donald E. Knuth** gave an invited talk about SAT at the International Conference on Theory and Applications of Satisfiability Testing 2012.
  - SAT will be appeared in Volume 4b of **The Art Of Computer Programming**.

Reference to SAT

- Biere, A., Heule, M., van Maaren, H., and Walsh, T., editors (2009). Handbook of Satisfiability, volume 185 of Frontiers in Artificial Intelligence and Applications (FAIA). IOS Press.
- (in Japanese) Recent Advances in SAT Techniques, Journal of the Japan Society for Artificial Intelligence, Special Issue, 25(1), 2010.

Contents of Talk

1. What is SAT?
2. Scarab: SAT-based CP System in Scala
3. Designing Constraint Models in Scarab

Motivation

- Modern fast SAT solvers have promoted the development of **SAT-based systems** for various problems.
- For an intended problem, we usually need to develop a dedicated program that encodes it into SAT.
- It sometimes bothers focusing on **problem modeling** which plays an important role in the system development process.

In the following

- We introduce the **Scarab** system, which is a prototyping tool for developing SAT-based systems.
- Its features are also introduced through examples of **Graph Coloring**.

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Scarab is a prototyping tool for developing SAT-based Constraint Programming (CP) systems.
Its major design principle is to provide an expressive, efficient, customizable, and portable workbench for SAT-based system developers.

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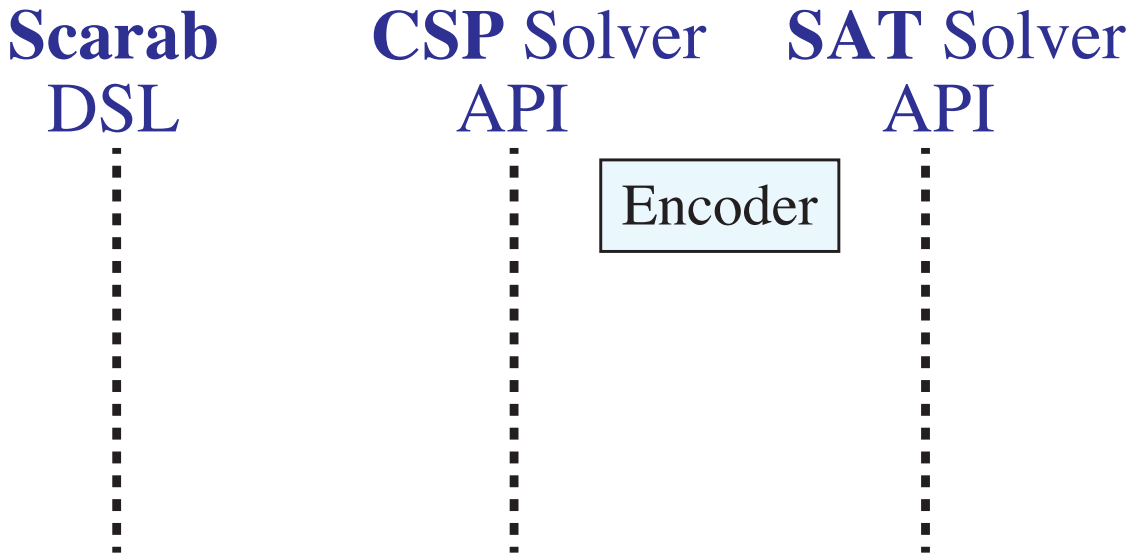
- It consists of the followings:
  1. Scarab DSL: Embedded DSL for Constraint Programming
  2. API of CSP Solver
  3. SAT encoding module
  4. API of SAT solvers



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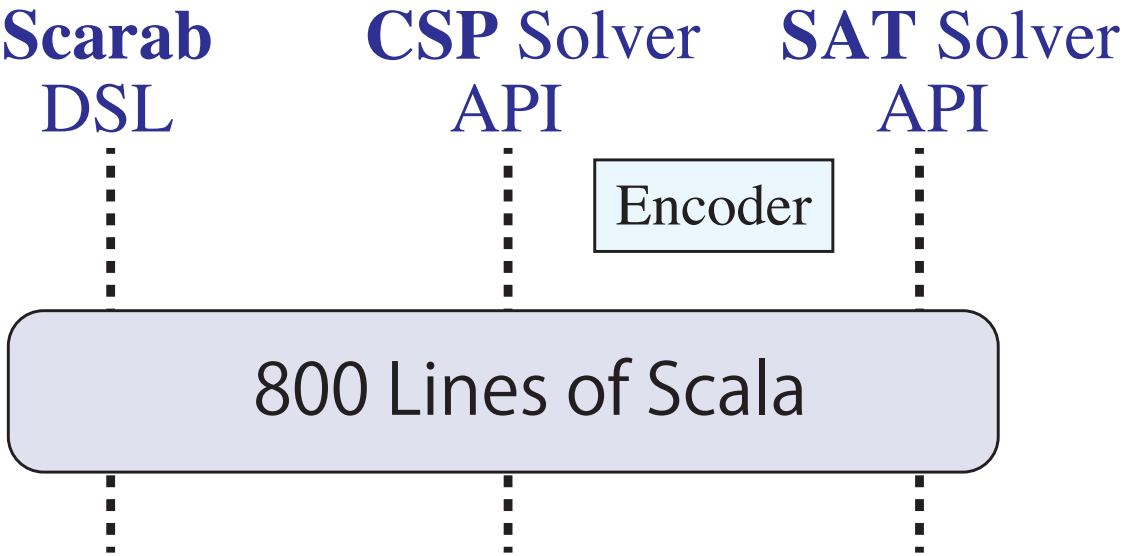

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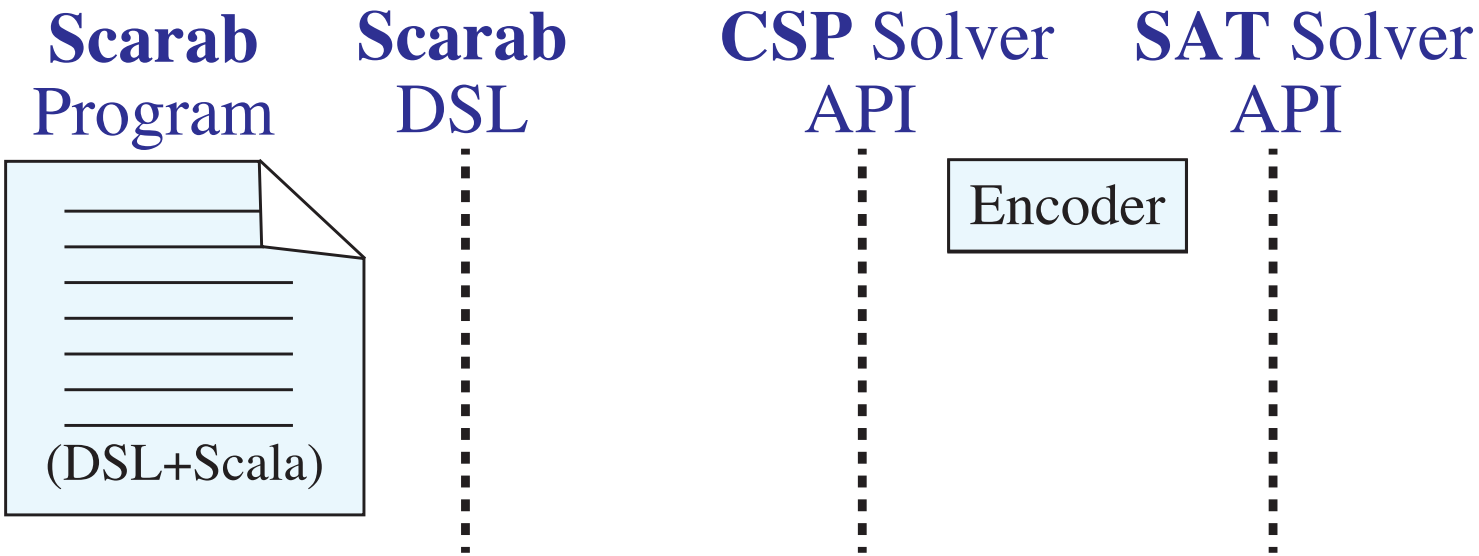

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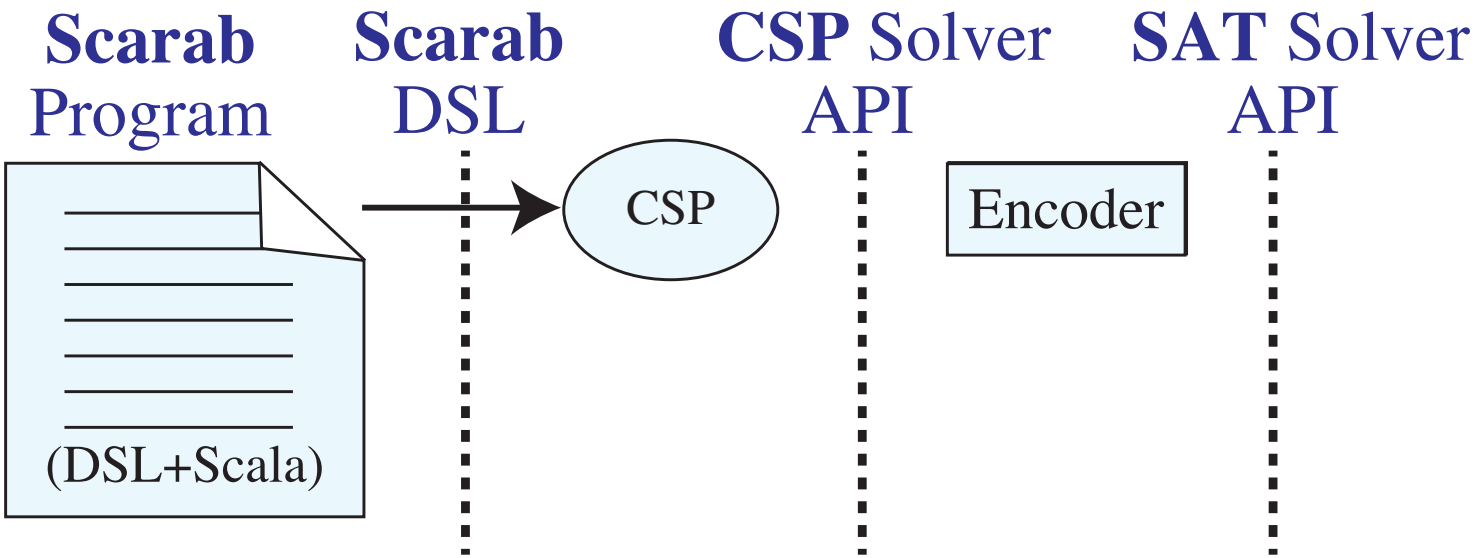

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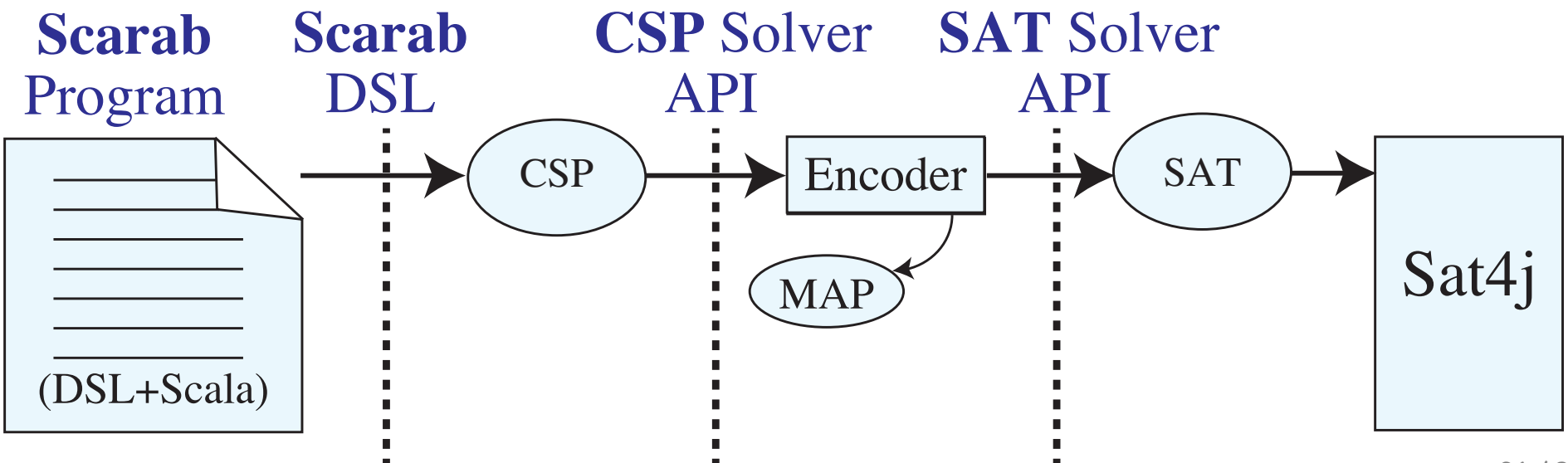

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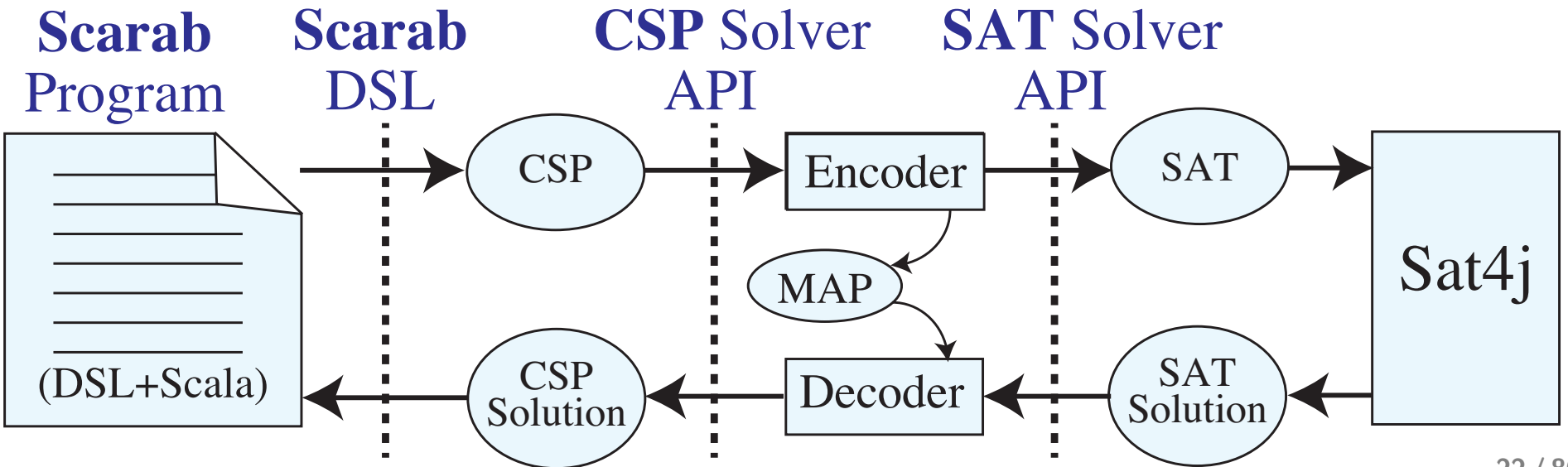
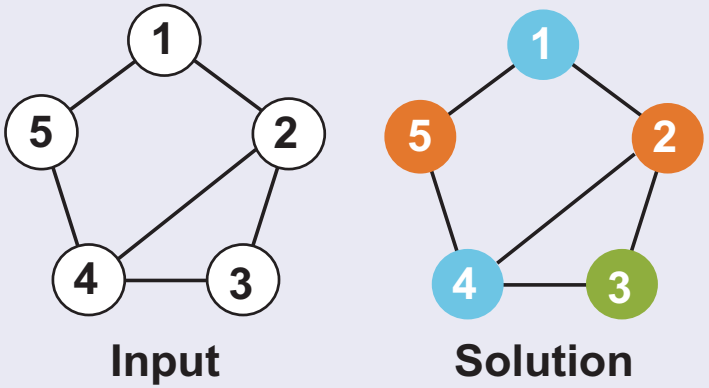


Example of Scarab Program: GCP.scala

Graph coloring problem (GCP) is a problem of finding a coloring of the nodes such that colors of adjacent nodes are different.



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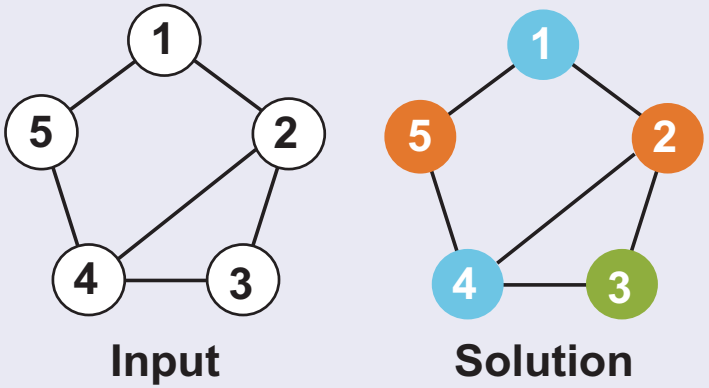



```
1:  import jp.kobe_u.scarab._ ; import dsl._
2:
3:  val nodes = Seq(1,2,3,4,5)
4:  val edges = Seq((1,2),(1,5),(2,3),(2,4),(3,4),(4,5))
5:  val colors = 3
6:  for (i <- nodes) int('n(i),1,colors)
7:  for ((i,j) <- edges)  add('n(i) !== 'n(j))
8:
9:  if (find) println(solution)
```

Imports

```
import jp.kobe_u.scarab._ ; import dsl._
```

- This line imports everything necessary and DSL methods provided by Scarab.

- `int(x, lb, ub)` method defines an integer variable.
- `add(c)` method defines a constraint.
- `find` method searches a solution.
- `solution` method returns the solution.
- etc.

Instance Structure

```
val nodes = Seq(1,2,3,4,5)
val edges = Seq((1,2),(1,5),(2,3),(2,4),(3,4),(4,5))
val colors = 3
```

- It defines the given set of nodes and edges as the sequence object in Scala.
- Available number of colors are defined as 3.

Defining CSP

```
for (i <- nodes) int('n(i),1,3)
```

- It adds an integer variable to the default CSP object by the **int** method.
- `'n` is a notation of symbols in Scala.
- They are automatically converted integer variable (`Var`) objects by an implicit conversion defined in Scarab.

```
for ((i,j) <- edges)  add('n(i) !== 'n(j))
```

- It adds constraints to the default CSP object.
- The following operators can be used to construct constraints:
  - logical operator: `&&`, `||`
  - comparison operator: `===`, `!==`, `<`, `<=`, `>=`, `>`
  - arithmetic operator: `+`, `-`

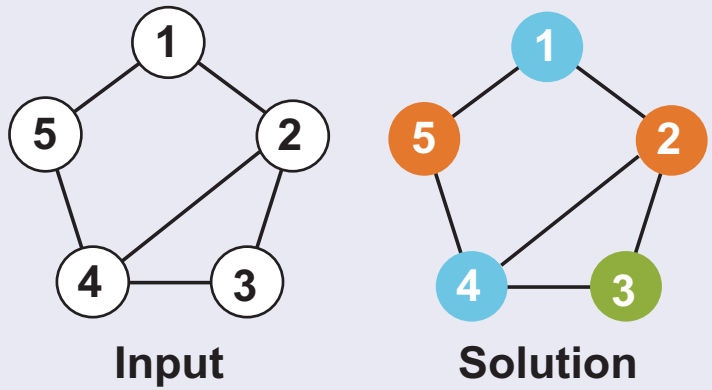
Solving CSP

```
if (find) println(solution)
```

- The **find** method encodes the CSP to SAT by order encoding, and call Sat4j to compute a solution.
- **solution** returns satisfiable assignment of the CSP.

We can do more in GCP?

Graph coloring problem (GCP) is a problem of finding a coloring of the nodes such that colors of adjacent nodes are different.



- How can we solve optimization version of GCP using Scarab?

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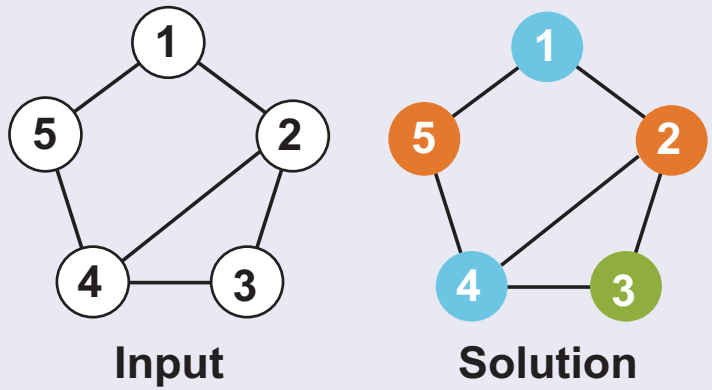

- How can we solve optimization version of GCP using Scarab?
- How can we adopt the change of constraints?

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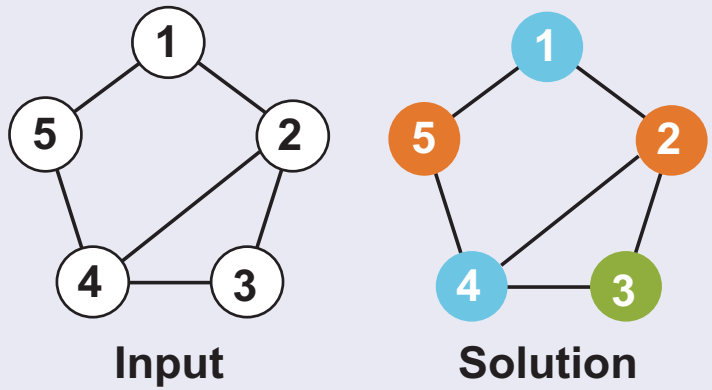

- How can we solve optimization version of GCP using Scarab?
- How can we adopt the change of constraints?
  - Let's consider bandwidth coloring problem!

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Pandiagonal Latin Square: $PLS(n)$

Place different n numbers into $n \times n$ matrix such that each number appears exactly once
for each row, column, diagonally down right, and diagonally up right.

2	3	5	1	4
5	1	4	2	3
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We can write five SAT-based PLS Solvers **within 35 lines**.

Name	Modeling	Encoding	Lines
AD1	alldiff	naive	17
AD2		with Perm. & P. H. Const.	31
BC1	Boolean	Pairwise	22
BC2	Cardinality	Totalizer [Bailleux '03]	35
BC3		Seq. Counter [Sinz '05]	27

Let's have a look their performance. Note that, in CSP Solver Comp. 2009, **NO CSP solver** (except Sugar) could solve $n > 8$.

alldiff Model

Pandiagonal Latin Square $PLS(5)$

x_{11}	x_{12}	x_{13}	x_{14}	x_{15}
x_{21}	x_{22}	x_{23}	x_{24}	x_{25}
x_{31}	x_{32}	x_{33}	x_{34}	x_{35}
x_{41}	x_{42}	x_{43}	x_{44}	x_{45}
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- $x_{ij} \in \{1, 2, 3, 4, 5\}$

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- alldiff in each row (5 rows)
- alldiff in each column (5 columns)
- alldiff in each pandiagonal (10 pandiagonals)
- $PLS(5)$ is satisfiable.

Boolean Cardinality Model

y_{11k}	y_{12k}	y_{13k}	y_{14k}	y_{15k}
y_{21k}	y_{22k}	y_{23k}	y_{24k}	y_{25k}
y_{31k}	y_{32k}	y_{33k}	y_{34k}	y_{35k}
y_{41k}	y_{42k}	y_{43k}	y_{44k}	y_{45k}
y_{51k}	y_{52k}	y_{53k}	y_{54k}	y_{55k}

- $y_{ijk} \in \{0, 1\}$ $\qquad$ $y_{ijk} = 1 \Leftrightarrow k$ is placed at (i, j)

Boolean Cardinality Model

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y_{41k}	y_{42k}	y_{43k}	y_{44k}	y_{45k}
y_{51k}	y_{52k}	y_{53k}	y_{54k}	y_{55k}

- $y_{ijk} \in \{0, 1\}$ $\quad$ $y_{ijk} = 1 \Leftrightarrow k$ is placed at (i, j)
- for each value (5 values)
  - for each row (5 rows) $\quad$ $y_{i1k} + y_{i2k} + y_{i3k} + y_{i4k} + y_{i5k} = 1$

Boolean Cardinality Model

y_{11k}	y_{12k}	y_{13k}	y_{14k}	y_{15k}
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  - for each pandiagonal (10 pandiagonals) $\quad y_{11k} + y_{22k} + y_{33k} + y_{44k} + y_{55k} = 1$

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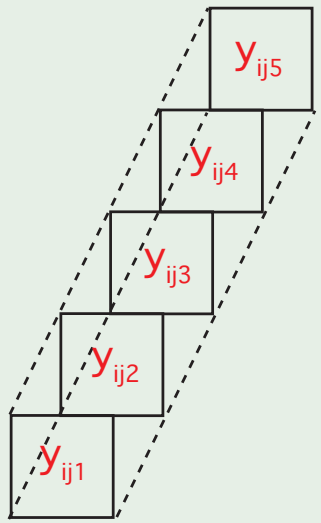

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  - for each pandiagonal (10 pandiagonals) $\quad y_{11k} + y_{22k} + y_{33k} + y_{44k} + y_{55k} = 1$
- for each (i, j) position (25 positions) $\quad y_{ij1} + y_{ij2} + y_{ij3} + y_{ij4} + y_{ij5} = 1$

Experiments

Comparison on Solving Pandiagonal Latin Square

To show the differences in performance, we compared the following 5 models.

1. AD1: naive alldiff
2. AD2: optimized alldiff
3. BC1: Pairwise
4. BC2: [Bailleux '03]
5. BC3: [Sinz '05]

Benchmark and Experimental Environment

- Benchmark: Pandiagonal Latin Square ($n = 7$ to $n = 16$)
- CPU: 2.93GHz, Mem: 2GB, Time Limit: 3600 seconds

Results (CPU Time in Seconds)

n	SAT/UNSAT	AD1	AD2	BC1	BC2	BC3
7	SAT	0.2	0.2	0.2	0.3	0.3
8	UNSAT	T.O.	0.5	0.3	0.3	0.3
9	UNSAT	T.O.	0.3	0.5	0.3	0.2
10	UNSAT	T.O.	0.4	1.0	0.3	0.3
11	SAT	0.3	0.3	2.3	0.5	0.4
12	UNSAT	T.O.	1.0	5.3	0.8	0.8
13	SAT	T.O.	0.5	T.O.	T.O.	T.O.
14	UNSAT	T.O.	9.7	32.4	8.2	6.8
15	UNSAT	T.O.	388.9	322.7	194.6	155.8
16	UNSAT	T.O.	457.1	546.6	300.7	414.8

- Only optimized version of alldiff model (AD2) solved all instances.
- Modeling and encoding have an important role in developing SAT-based systems. Just using SAT solvers is not enough!
- Scarab helps users to focus on them ;)

Experiments on Hamiltonian Cycle Problem

We evaluate the effectiveness of (1) CEGAR-HCP, (2) Native BC, (3) Implementation on Tightly Integrated System.
We also have (4) a comparison with other specialized methods.

Machine Spec and Benchmark

- CPU: Intel Xeon 2.93GHz, Memory: 4GB, Time Limit: 500 sec.
- **color04** (119 instances, #nodes: 11 to 10000), **knight** (11 instances, 8x8 to 100x100), **tsplib** (9 instances)

Systems Compared

- CEGAR-HCP (on Scarab)
  S4J-S (Seq. Counter), **S4J-N** (Native BC)

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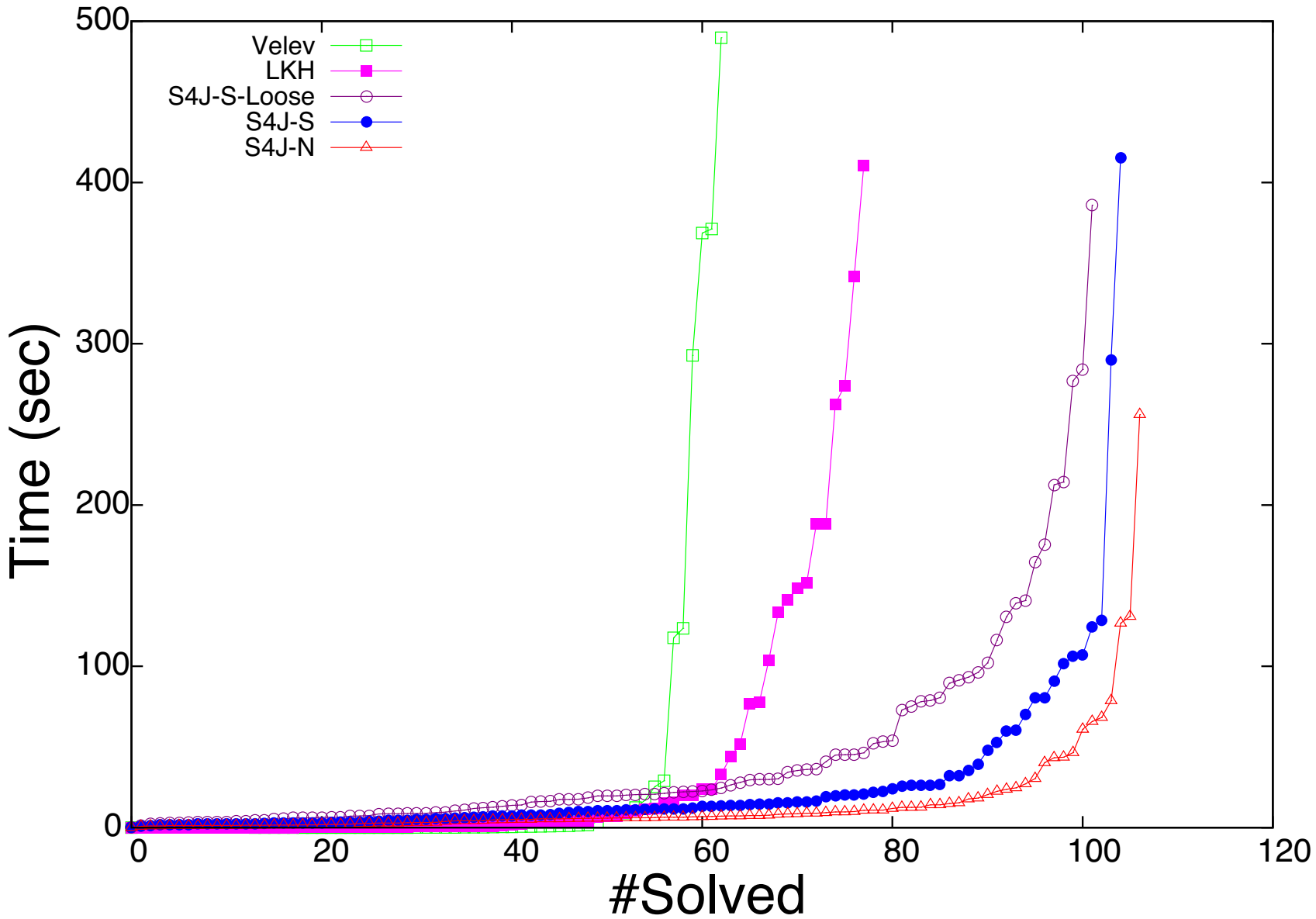
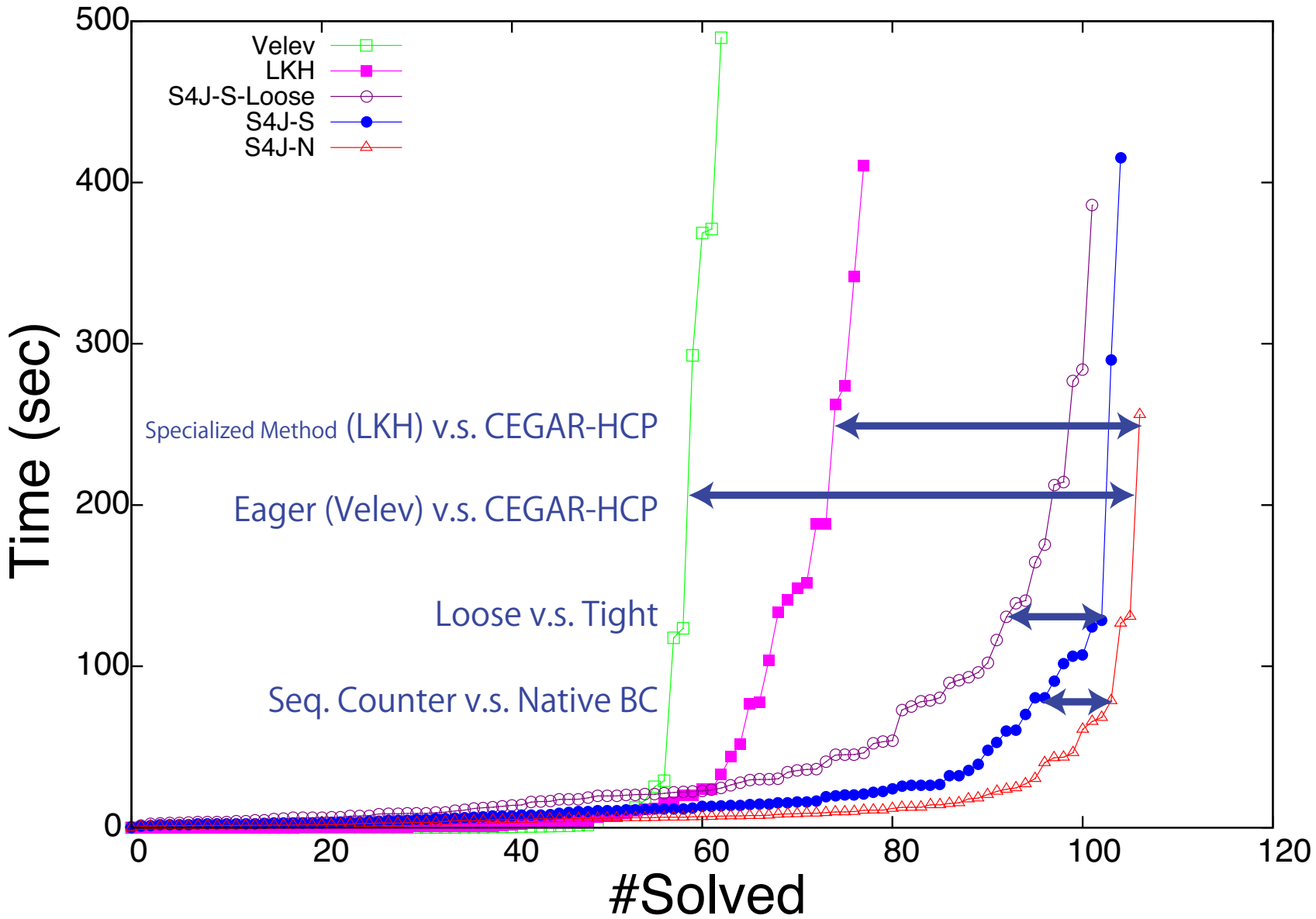
- CEGAR-HCP (on loosely integrated system) **S4J-S-Loose**

Cactus Plot (#Solved–CPU Time)



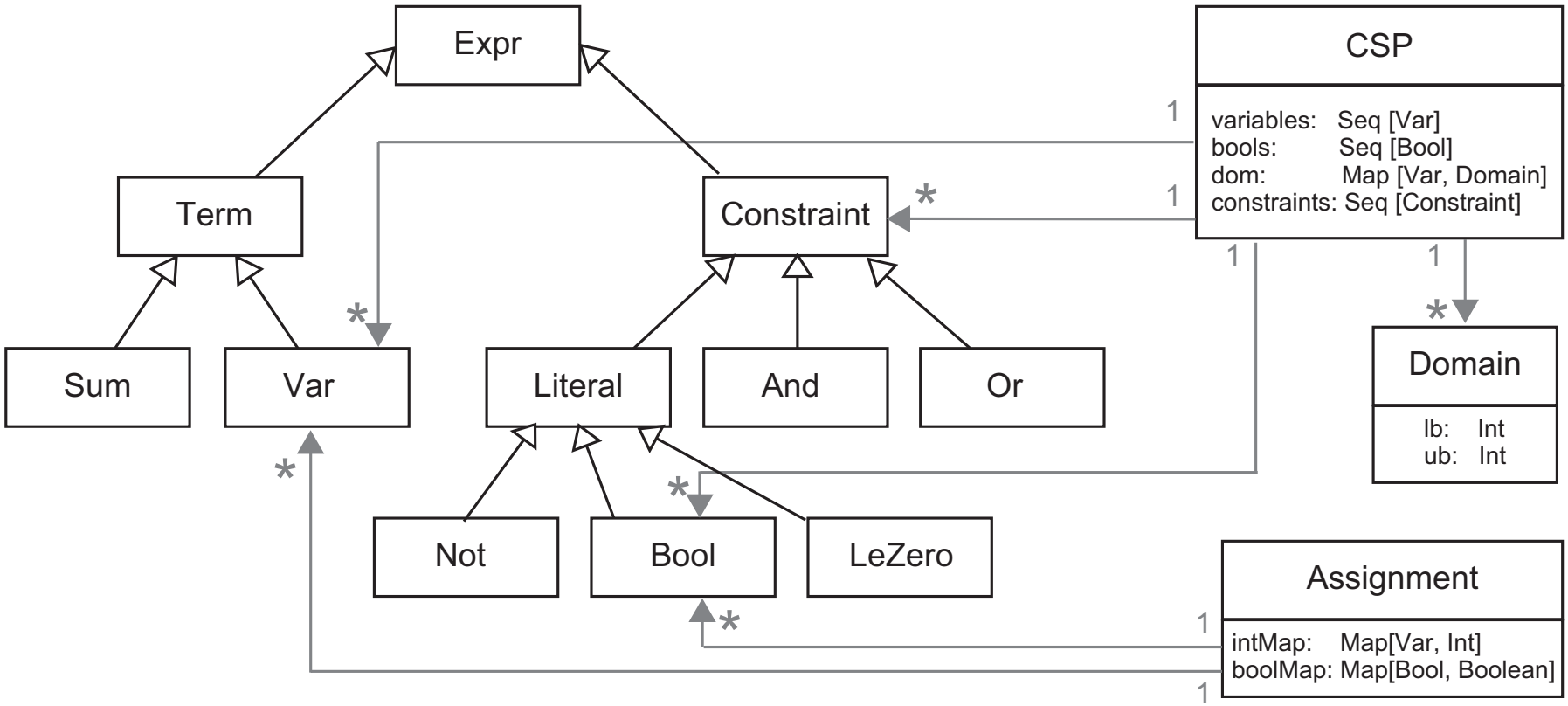
Cactus Plot (#Solved–CPU Time)



Features of Scarab

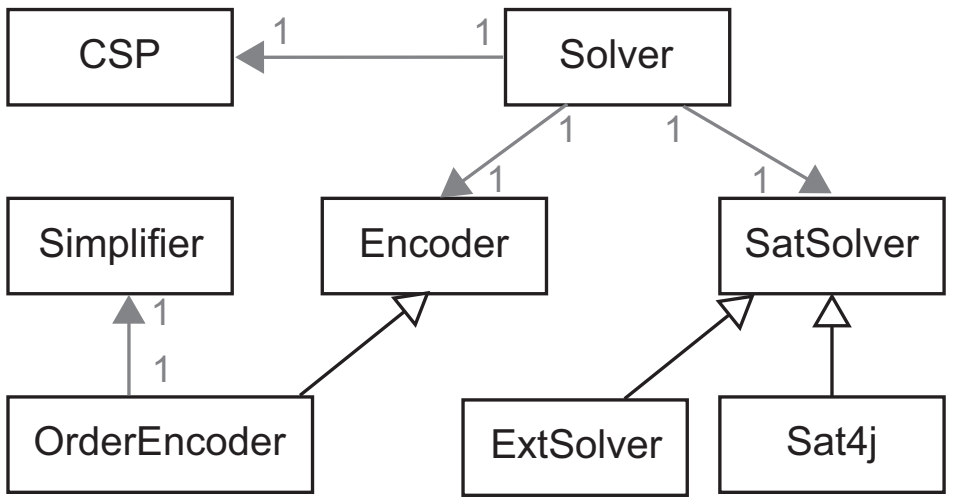
- **Efficiency**
  - Scarab is efficient in the sense that it uses an optimized version of the order encoding for encoding CSP into SAT.
- **Portability**
  - The combination of Scarab and Sat4j enables the development of portable applications on JVM (Java Virtual Machine).
- **Customizability**
  - Scarab is 800 lines long without comments.
  - Core of order encoding module is only 25 lines long.
  - It allows programmers to freely customize Scarab itself.
- **Availability of Advanced SAT Techniques**
  - Thanks to the tight integration to Sat4j, it is available to use several SAT techniques, *e.g.*, incremental SAT solving and native handling constraints.

Class Diagrams



Class Diagrams for CSPs

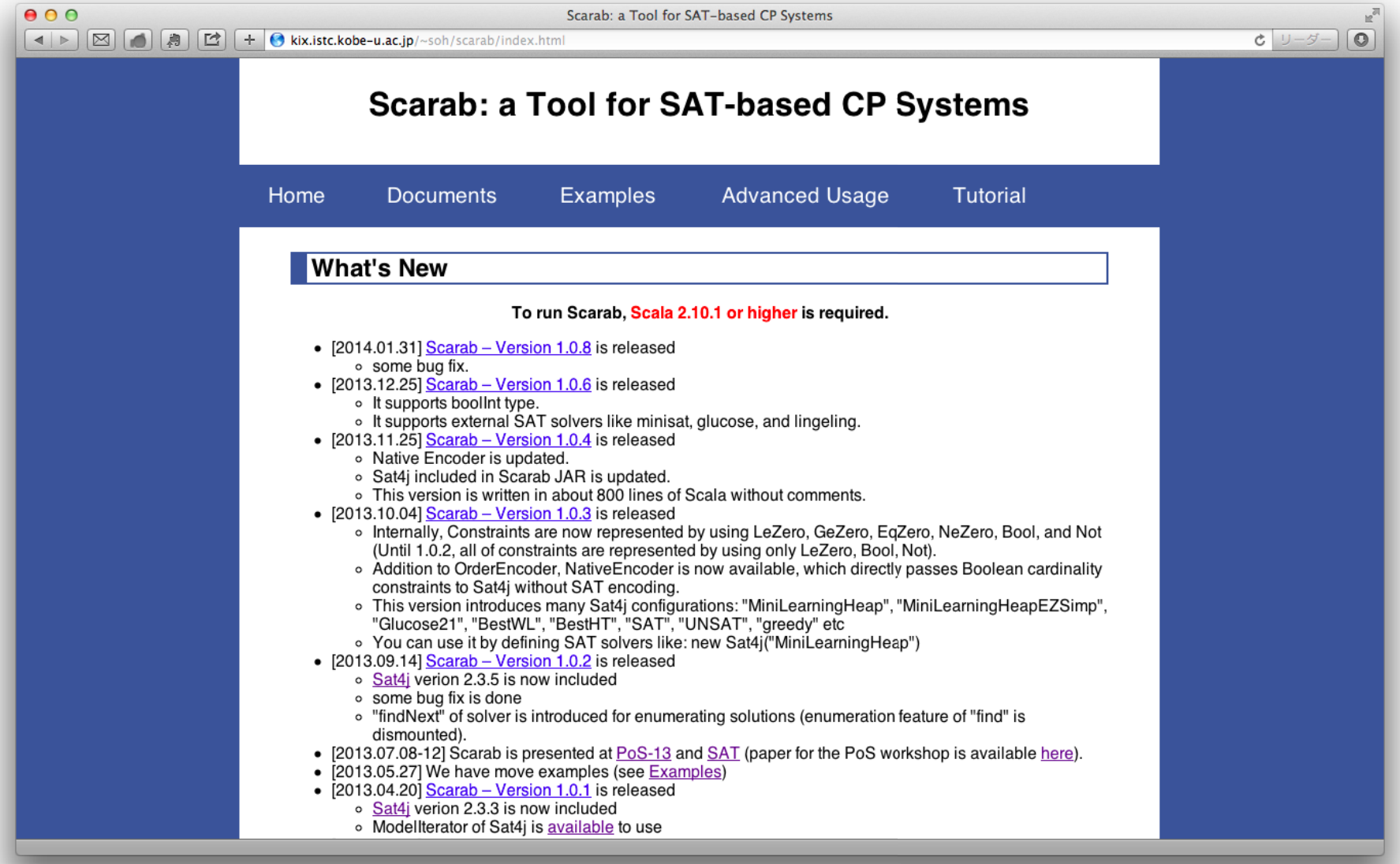
Class Diagrams



Class Diagrams for Solvers

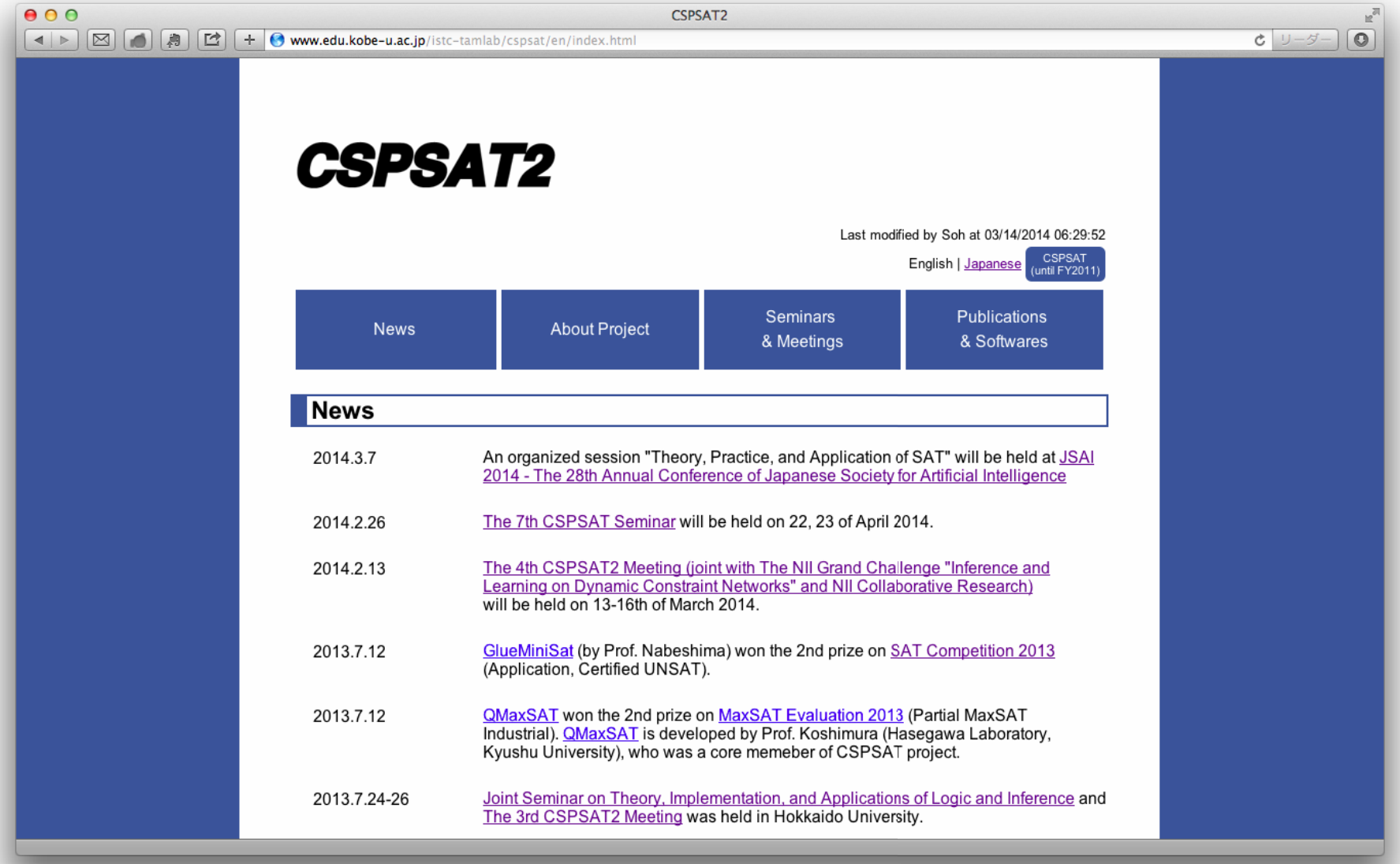
Web Page for Scarab

http://kix.istc.kobe-u.ac.jp/~soh/scarab/



Web Page for CSPSAT2

http://www.edu.kobe-u.ac.jp/istc-tamlab/cspsat/en/



Conclusion

- Introducing Architecture and Features of Scarab
- Using Scarab, we can write various constraint models without developing dedicated encoders, which allows us to focus on problem modeling and encoding.
- **Future Work**
  - Introducing more features from Sat4j
  - Sat4j has various functions of finding MUS, optimization, solution enumeration, handling natively cardinality and pseudo-Boolean constraints.
- URL of Scarab `http://kix.istc.kobe-u.ac.jp/~soh/scarab/`

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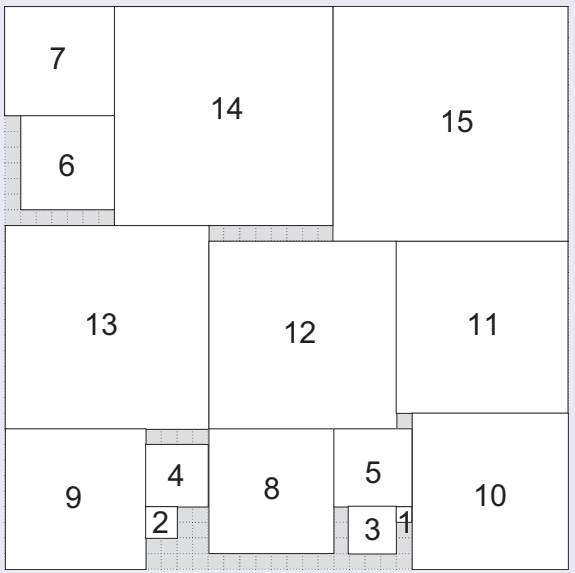
Example: Square Packing

- **Square Packing** $SP(n, s)$ is a problem of packing a set of squares of sizes 1×1 to $n \times n$ into an enclosing square of size $s \times s$ without overlapping.

Example of $SP(15, 36)$



- **Optimum soluiton of** $SP(n, s)$ is the smallest size of the enclosing square having a feasible packing.

Non-overlapping Constraint Model for $SP(n, s)$

Integer variables

- $x_i \in \{0, \ldots, s - i\}$ and $y_i \in \{0, \ldots, s - i\}$
- Each pair $(x_i,\ y_i)$ represents the lower left coordinates of the square i.

Non-overlapping Constraint ($1 \leq i < j \leq n$)

$$(x_i + i \leq x_j) \vee (x_j + j \leq x_i) \vee (y_i + i \leq y_j) \vee (y_j + j \leq y_i)$$

Decremental Seach

Scarab Program for $SP(n, s)$

```
for (i <- 1 to n) { int('x(i),0,s-i) ; int('y(i),0,s-i) }
for (i <- 1 to n; j <- i+1 to n)
  add(('x(i) + i <= 'x(j)) || ('x(j) + j <= 'x(i)) ||
      ('y(i) + i <= 'y(j)) || ('y(j) + j <= 'y(i)))
```

Searching an Optimum Solution

```
val lb = n; var ub = s; int('m, lb, ub)
for (i <- 1 to n)
  add(('x(i)+i <= 'm) && ('y(i)+i <= 'm))

// Incremental solving
while (lb <= ub && find('m <= ub)) { // using an assumption.
  add('m <= ub)
  ub = solution.intMap('m) - 1
}
```

Bisection Search

Bisection Search

```
var lb = n; var ub = s; commit

while (lb < ub) {
  var size = (lb + ub) / 2
  for (i <- 1 to n)
    add(('x(i)+i<=size)&&('y(i)+i<=size))
  if (find) {
    ub = size
    commit // commit current constraints
  } else {
    lb = size + 1
    rollback // rollback to the last commit point
  }
}
```

Advanced Solving Techniques using Sat4j

- Thanks to the **tight integration to Sat4j**, Scarab provides the functions: Incremental solving and CSP solving with assumptions.
- We explain it using the following program.

```
1:  int('x, 1, 3)
2:  int('y, 1, 3)
3:  add('x === 'y)
4:  find                  // first call of find
5:  add('x !== 3)
6:  find                  // second call of find
7:
8:  find('y === 3)        // with assumption y = 3
9:  find('x === 1)        // with assumption x = 1
```

Incremental SAT Solving

```
int('x, 1, 3)
int('y, 1, 3)
add('x === 'y)
find                // first call of find
add('x !== 3)
find                // second call of find
```

- In the first call of **find** method, the whole CSP is encoded and generated SAT clauses are added to Sat4j, then it computes a solution.
- In the second call of **find** method, only the extra constraint $x \neq 3$ is encoded and added to Sat4j, then it computes a solution.
- The learned clauses obtained by the first **find** are kept at the second call.

CSP Solving under Assumption

```
find('y === 3)   // with assumption y = 3
find('x === 1)   // with assumption x = 1
```

- **find(assumption: Constraint)** method provides CSP solving under assumption given by the specified constraint.
- The constraint of assumption should be encoded to a conjunction of literals (otherwise an exception is raised).
- Then, the literals are passed to Sat4j, then it computes a solution under assumption.
- We can utilize those techniques for optimization and enumeration problems.

Scarab Program for alldiff Model

```
 1:  import jp.kobe_u.scarab._ ; import dsl._
 2:
 3:  val n = args(0).toInt
 4:
 5:  for (i <- 1 to n; j <- 1 to n)  int('x(i,j),1,n)
 6:  for (i <- 1 to n) {
 7:   add(alldiff((1 to n).map(j => 'x(i,j))))
 8:   add(alldiff((1 to n).map(j => 'x(j,i))))
 9:   add(alldiff((1 to n).map(j => 'x(j,(i+j-1)%n+1))))
10:   add(alldiff((1 to n).map(j => 'x(j,(i+(j-1)*(n-1))%n+1))))
11:  }
12:
13:  if (find)  println(solution)
```

Encoding alldiff

- In Scarab, all we have to do for implementing global constraints is just decomposing them into simple arithmetic constraints [Bessiere et al. '09].

In the case of alldiff($a_1, \ldots, a_n$),

It is decomposed into pairwise not-equal constraints

$$\bigwedge_{1 \leq i < j \leq n} (a_i \neq a_j)$$

.

- This (naive) alldiff is enough to just have a feasible constraint model for $PLS(n)$.
- But, one probably want to improve this :)

Extra Constraints for alldiff$(a_1, \ldots, a_n)$

- In Pandiagonal Latin Square $PLS(n)$, all integer variables $a_1, \ldots, a_n$ have the same domain $\{1, \ldots, n\}$.
- Then, we can add the following extra constraints.
- **Permutation constraints**:

$$\bigwedge_{i=1}^{n} \bigvee_{j=1}^{n} (a_j = i)$$

  - It represents that one of $a_1, \ldots, a_n$ must be assigned to i.
- **Pigeon hole constraint**:

$$\neg \bigwedge_{i=1}^{n} (a_i < n) \wedge \neg \bigwedge_{i=1}^{n} (a_i > 1)$$

  - It represents that mutually different n variables cannot be assigned within the interval of the size $n - 1$.

alldiff (naive)

```
def alldiff(xs: Seq[Var]) =
  And(for (Seq(x, y) <- xs.combinations(2))
      yield x !== y)
```

alldiff (optimized)

```
def alldiff(xs: Seq[Var]) = {
  val lb = for (x <- xs) yield csp.dom(x).lb
  val ub = for (x <- xs) yield csp.dom(x).ub
  // pigeon hole
  val ph =
    And(Or(for (x <- xs) yield !(x < lb.min+xs.size-1)),
        Or(for (x <- xs) yield !(x > ub.max-xs.size+1)))
  // permutation
  def perm =
    And(for (num <- lb.min to ub.max)
        yield Or(for (x <- xs) yield x === num))
  val extra = if (ub.max-lb.min+1 == xs.size) And(ph,perm)
              else ph

  And(And(for (Seq(x, y) <- xs.combinations(2))
          yield x !== y),extra)
}
```

Scarab Program for Boolean Cardinality Model

```
 1:  import jp.kobe_u.scarab._ ; import dsl._
 2:
 3:  for (i <- 1 to n; j <- 1 to n; num <- 1 to n)
 4:    int('y(i,j,num),0,1)
 5:
 6:  for (num <- 1 to n) {
 7:    for (i <- 1 to n) {
 8:     add(BC((1 to n).map(j => 'y(i,j,num)))===1)
 9:     add(BC((1 to n).map(j => 'y(j,i,num)))===1)
10:     add(BC((1 to n).map(j => 'y(j,(i+j-1)%n+1,num))) === 1)
11:     add(BC((1 to n).map(j => 'y(j,(i+(j-1)*(n-1))%n+1,num))) === 1)
12:   }
13:  }
14:
15:  for (i <- 1 to n; j <- 1 to n)
16:    add(BC((1 to n).map(k => 'y(i,j,k))) === 1)
17:
18:  if (find)  println(solution)
```

SAT Encoding of Boolean Cardinality in Scarab

- There are several ways for encoding Boolean cardinality.
- In Scarab, we can easily write the following encoding methods by defining your own BC methods.
  - Pairwise
  - Totalizer [Bailleux '03]
  - Sequential Counter [Sinz '05]
- In total, **3 variants of Boolean cardinality model** are obtained.
  - BC1: Pairwise (implemented by 2 lines)
  - BC2: Totalizer [Bailleux '03] (implemented by 15 lines)
  - BC3: Sequential Counter [Sinz '05] (implemented by 7 lines)
- Good point to use Scarab is that we can test those models **without writing dedicated programs**.