Online Appendix: Commitment Problem and International Investment Agreement under Firm Heterogeneity

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Appendix

A.1 Derivation details for third stage equilibrium

A.1.1 Price index

$$
\begin{aligned}
P_i^{1-\sigma} &= \int_{\omega\in\Omega_i} p_i(\omega)^{1-\sigma} d\omega \\
&= \int_{\varphi_{ii}}^{\infty} p_{ii}(\varphi)^{1-\sigma} M_i \frac{dG(\varphi)}{1-G(\varphi_{ii})} + \int_{\varphi_{ji}}^{\infty} \left(t_{ji}p_{ji}(\varphi)\right)^{1-\sigma} m_{ji}M_j \frac{dG(\varphi)}{1-G(\varphi_{ji})} \\
&= \left(\frac{t_{ii}\tau_{ii}w_i}{\rho}\right)^{1-\sigma} m_{ii}M_i \int_{\varphi_{ii}}^{\infty} \varphi^{\sigma-1} \frac{dG(\varphi)}{1-G(\varphi_{ii})} \\
&\quad + \left(\frac{t_{ji}\tau_{ji}w_j}{\rho}\right)^{1-\sigma} m_{ji}M_j \int_{\varphi_{ji}}^{\infty} \varphi^{\sigma-1} \frac{dG(\varphi)}{1-G(\varphi_{ji})} \\
&= \theta \sum_j m_{ji}M_j \left(\frac{\rho\varphi_{ji}}{t_{ji}\tau_{ji}w_j}\right)^{\sigma-1}
\end{aligned}
$$

where $\theta \coloneqq \frac{\beta}{\beta-(\sigma-1)}$. Notice that $\frac{1}{1-G(\varphi_{ji})} = {\varphi_{ji}}^{\beta}$ and $dG(\varphi) = \beta\varphi^{-\beta-1}$. The last equality follows from $\int_{\varphi_{ji}}^{\infty} \varphi^{\sigma-1} \frac{dG(\varphi)}{1-G(\varphi_{ji})} = \int_{\varphi_{ji}}^{\infty} \varphi^{\sigma-1}\beta\varphi^{-\beta-1}\varphi_{ji}^{\beta} = \frac{\beta}{\beta-(\sigma-1)}{\varphi_{ji}}^{\sigma-1}$.

A.1.2 Average revenue

$$
\begin{aligned}
\bar{r}_{ij} &= \int_{\varphi_{ij}}^{\infty} p_{ij}(\varphi) q_j(\varphi) \frac{dG(\varphi)}{1 - G(\varphi_{ii})} \\
&= E_j P_j^{\sigma-1} t_{ij}^{-\sigma} \left(\frac{\tau_{ij} w_i}{\rho} \right)^{1-\sigma} \int_{\varphi_{ij}}^{\infty} \varphi^{\sigma-1} \frac{1 - G(\varphi_{ij})}{1 - G(\varphi_{ii})} \frac{dG(\varphi)}{1 - G(\varphi_{ij})} \\
&= E_j P_j^{\sigma-1} t_{ij}^{-\sigma} \left(\frac{\tau_{ij} w_i}{\rho} \right)^{1-\sigma} m_{ij} \int_{\varphi_{ij}}^{\infty} \varphi^{\sigma-1} \frac{dG(\varphi)}{1 - G(\varphi_{ij})} \\
&= \theta E_j P_j^{\sigma-1} t_{ij}^{-\sigma} \left(\frac{\tau_{ij} w_i}{\rho} \right)^{1-\sigma} m_{ij} {\varphi_{ij}}^{\sigma-1} \\
&= \theta E_j P_j^{\sigma-1} t_{ij}^{-\sigma} \left(\frac{\tau_{ij} w_i}{\rho \varphi_{ij}^{*}} \right)^{1-\sigma} m_{ij} \mu_{ij} \\
&= \theta \sigma w_i m_{ij} \mu_{ij} f_{ij}
\end{aligned}
$$

The last equality follows from zero profit condition (7).

A.1.3 Production labor market clearing condition

$$
\begin{aligned}
L_i^p &= M_i \sum_j \int_{\varphi_{ij}}^{\infty} \frac{\tau_{ij} q_{ij}(\varphi)}{\varphi} \frac{dG(\varphi)}{1 - G(\varphi_{ii})} \\
&= M_i \sum_j m_{ij} E_j P_j^{\sigma-1} \tau_{ij}^{1-\sigma} \left(t_{ij} \frac{w_i}{\rho} \right)^{-\sigma} \int_{\varphi_{ij}}^{\infty} \varphi^{\sigma-1} \frac{dG(\varphi)}{1 - G(\varphi_{ij})} \\
&= M_i \sum_j m_{ij} E_j P_j^{\sigma-1} \tau_{ij}^{1-\sigma} \left(t_{ij} \frac{w_i}{\rho} \right)^{-\sigma} \theta {\varphi_{ij}}^{\sigma-1} \\
&= M_i \sum_j \left\{ \theta m_{ij} E_j P_j^{\sigma-1} t_{ij}^{-\sigma} \left(\frac{\tau_{ij} w_i}{\rho} \right)^{1-\sigma} {\varphi_{ij}}^{\sigma-1} \right\} \frac{\rho}{w_i} \\
&= M_i \sum_j \left\{ \theta \sigma w_i m_{ij} \mu_{ij} f_{ij} \right\} \frac{\rho}{w_i} \\
&= M_i \sum_j \theta (\sigma - 1) m_{ij} \mu_{ij} f_{ij}
\end{aligned}
$$

where the fifth equality follows from equation (8) and the last equality follows from $\rho = \frac{\sigma-1}{\sigma}$. Notice that terms inside the brace are the very average revenue.

A.1.4 Aggregate expenditure

$$
\begin{aligned}
E_i &= \sum_j t_{ji} \int_{\varphi_{ji}}^{\infty} r_{ji}(\varphi) M_j \frac{dG(\varphi)}{1 - G(\varphi_{jj})} \\
&= \sum_j t_{ji} M_j \bar{r}_{ji} \\
&= \theta\sigma \sum_j t_{ji} m_{ji} \mu_{ji} M_j w_j f_{ji} \\
&= \theta\sigma \sum_j t_{ji} m_{ij} \mu_{ij} M_i w_i f_{ij} \\
&= \theta\sigma M_i w_i \sum_j t_{ji} m_{ij} \mu_{ij} f_{ij}
\end{aligned}
$$

The fourth equality follows from trade balance condition (11).

A.1.5 Aggregate income

$$
\begin{aligned}
R_i &= w_i L_i + \int_{\varphi_{ji}}^{\infty} (t_{ji} - 1) r_{ji}(\varphi) M_j \frac{dG(\varphi)}{1 - G(\varphi_{jj})} + M_i \left[\bar{\pi}_{ii} + \bar{\pi}_{ij} - \frac{w_i f^e}{1 - G(\varphi_{ii})} \right] \\
&= w_i L_i + (t_{ji} - 1) M_j \bar{r}_{ji} + M_i \left[\bar{\pi}_{ii} + \bar{\pi}_{ij} - \frac{w_i f^e}{1 - G(\varphi_{ii})} \right] \\
&= w_i L_i + (t_{ji} - 1) M_j \bar{r}_{ji} + M_i \left[\frac{\bar{r}_{ii} + \bar{r}_{ij}}{\sigma} - w_i \sum_j m_{ij} f_{ij} - \frac{w_i f^e}{1 - G(\varphi_{ii})} \right] \\
&= w_i L_i^p + (t_{ji} - 1) M_j \bar{r}_{ji} + M_i \frac{\bar{r}_{ii} + \bar{r}_{ij}}{\sigma} \\
&= w_i L_i^p + (t_{ji} - 1) M_j \theta\sigma w_j m_{ji} \mu_{ji} f_{ji} \\
&\quad + M_i \left(\theta\sigma w_i m_{ii} \mu_{ii} f_{ii} + \theta\sigma w_i m_{ij} \mu_{ij} f_{ij} \right) / \sigma \\
&= w_i M_i \sum_j \theta(\sigma - 1) m_{ij} \mu_{ij} f_{ij} + (t_{ji} - 1) M_j \theta\sigma w_j m_{ji} \mu_{ji} f_{ji} \\
&\quad + M_i \left(\theta w_i m_{ii} \mu_{ii} f_{ii} + \theta w_i m_{ij} \mu_{ij} f_{ij} \right) \\
&= w_i M_i \sum_j \theta(\sigma - 1) m_{ij} \mu_{ij} f_{ij} + (t_{ji} - 1) M_i \theta\sigma w_i m_{ij} \mu_{ij} f_{ij} \\
&\quad + M_i \left(\theta w_i m_{ii} \mu_{ii} f_{ii} + \theta w_i m_{ij} \mu_{ij} f_{ij} \right) \\
&= w_i M_i \left[\theta(\sigma - 1) m_{ii} \mu_{ii} f_{ii} + \theta(\sigma - 1) m_{ij} \mu_{ij} f_{ij} \right. \\
&\quad \left. + (t_{ji} - 1) \theta\sigma m_{ij} \mu_{ij} f_{ij} + \theta m_{ii} \mu_{ii} f_{ii} + \theta m_{ij} \mu_{ij} f_{ij} \right] \\
&= w_i M_i \left[\theta\sigma m_{ii} \mu_{ii} f_{ii} + \theta\sigma t_{ji} m_{ji} \mu_{ji} f_{ji} \right] \\
&= \theta\sigma M_i w_i \sum_j t_{ji} m_{ij} \mu_{ij} f_{ij}
\end{aligned}
$$

The fourth equality follows from the definition of L_i^e and L_i^p. The sixth equality follows from production labor market clearing condition (10), and the seventh equality follows from trade balance condition (11).

A.2 Derivation details for optimal tariff

A.2.1 Two types of closedness

$$\begin{aligned}
\alpha_i &:= \frac{M_i\bar{r}_{ii}}{M_i\bar{r}_{ii} + M_i\bar{r}_{ij}} \\
&= \frac{M_i\theta\sigma w_i m_{ii}\mu_{ii} f_{ii}}{M_i\theta\sigma w_i m_{ii}\mu_{ii} f_{ii} + M_i\theta\sigma w_i m_{ij}\mu_{ij} f_{ij}} \\
&= \frac{m_{ii}\mu_{ii} f_{ii}}{m_{ii}\mu_{ii} f_{ii} + m_{ij}\mu_{ij} f_{ij}} \\
&= \frac{1}{1 + m_{ij}\frac{\mu_{ij}}{\mu_{ii}}\frac{f_{ij}}{f_{ii}}} \\
\tilde{\alpha}_i &:= \frac{M_i\bar{r}_{ii}}{M_i\bar{r}_{ii} + t_{ji}M_j\bar{r}_{ji}} \\
&= \frac{M_i\theta\sigma w_i m_{ii}\mu_{ii} f_{ii}}{M_i\theta\sigma w_i m_{ii}\mu_{ii} f_{ii} + t_{ji}M_j\theta\sigma w_j m_{ji}\mu_{ji} f_{ji}} \\
&= \frac{M_i\theta\sigma w_i m_{ii}\mu_{ii} f_{ii}}{M_i\theta\sigma w_i m_{ii}\mu_{ii} f_{ii} + t_{ji}M_i\theta\sigma w_i m_{ij}\mu_{ij} f_{ij}} \\
&= \frac{m_{ii}\mu_{ii} f_{ii}}{m_{ii}\mu_{ii} f_{ii} + t_{ji}m_{ij}\mu_{ij} f_{ij}} \\
&= \frac{1}{1 + t_{ji}m_{ij}\frac{\mu_{ij}}{\mu_{ii}}\frac{f_{ij}}{f_{ii}}}
\end{aligned}$$

The third equality follows from trade balance condition (11).

A.2.2 Price index in changes

Notice that the price index (6) is rewritten as

$$P_i^{1-\sigma} = \theta M_i(\rho\varphi_{ii})^{\sigma-1}w_i^{1-\sigma} + \theta m_{ji}M_j\left(\frac{\rho\varphi_{ji}}{\tau_{ji}}\right)^{\sigma-1} t_{ji}^{1-\sigma}w_j^{1-\sigma}.$$

Totally differentiating the right-hand-side of this equation yields

$$\begin{aligned}
\text{(RHS)} &= (1-\sigma)\theta M_i(\rho\varphi_{ii})^{\sigma-1}w_i^{1-\sigma}w_i^{-1}dw_i \\
&\quad + (1-\sigma)\theta m_{ji}M_j\left(\frac{\rho\varphi_{ji}}{\tau_{ji}}\right)^{\sigma-1} t_{ji}^{1-\sigma}w_j^{1-\sigma}t_{ji}^{-1}dt_{ji} \\
&\quad + (1-\sigma)\theta m_{ji}M_j\left(\frac{\rho\varphi_{ji}}{\tau_{ji}}\right)^{\sigma-1} t_{ji}^{1-\sigma}w_j^{1-\sigma}w_j^{-1}dw_j \\
&= (1-\sigma)\theta\rho^{\sigma-1}\left[M_i{\varphi_{ii}}^{\sigma-1}w_i^{1-\sigma}\hat{w}_i + m_{ji}M_j{\varphi_{ji}}^{\sigma-1}\tau_{ji}^{1-\sigma}t_{ji}^{1-\sigma}w_j^{1-\sigma}(\hat{t}_{ji}+\hat{w}_j)\right].
\end{aligned}$$

Totally differentiating the left-hand-side gives $(1-\sigma)P_i^{1-\sigma}\hat{P}_i$, so by dividing both side by $(1-\sigma)P_i^{1-\sigma}$, one can obtain

$$\begin{aligned}\hat{P}_i = &\frac{M_i {\varphi_{ii}}^{\sigma-1} w_i^{1-\sigma}}{M_i {\varphi_{ii}}^{\sigma-1} w_i^{1-\sigma} + m_{ji} M_j {\varphi_{ji}}^{\sigma-1} \tau_{ji}^{1-\sigma} t_{ji}^{1-\sigma} w_j^{1-\sigma}} \hat{w}_i \\ &+ \frac{m_{ji} M_j {\varphi_{ji}}^{\sigma-1} \tau_{ji}^{1-\sigma} t_{ji}^{1-\sigma} w_j^{1-\sigma}}{M_i {\varphi_{ii}}^{\sigma-1} w_i^{1-\sigma} + m_{ji} M_j {\varphi_{ji}}^{\sigma-1} \tau_{ji}^{1-\sigma} t_{ji}^{1-\sigma} w_j^{1-\sigma}} (\hat{t}_{ji} + \hat{w}_j).\end{aligned}$$

Notice that zero profit conditions for Home's entry and import are

$$\begin{aligned} E_H P_H^{\sigma-1} \left(\frac{\rho \varphi_{HH}^*}{w_H}\right)^{\sigma-1} &= \sigma w_H f_{HH} \\ E_H P_H^{\sigma-1} t_{FH}^{-\sigma} \left(\frac{\rho \varphi_{FH}^*}{\tau_{FH}}\right)^{\sigma-1} &= \sigma f_{FH}. \end{aligned}$$

Dividing both sides, one can obtain

$$\left(\frac{\varphi_{FH}^* w_H}{\varphi_{HH}^* \tau_{FH}}\right)^{\sigma-1} t_{FH}^{-\sigma} = \frac{1}{w_H}\frac{f_{FH}}{f_{HH}}. \tag{35}$$

Then, the coefficient of $\hat{w}_H$ in $\hat{P}_H$ becomes

$$\begin{aligned} &\frac{M_H {\varphi_{HH}}^{\sigma-1} w_H^{1-\sigma}}{M_H {\varphi_{HH}}^{\sigma-1} w_H^{1-\sigma} + m_{FH} M_F {\varphi_{FH}}^{\sigma-1} \tau_{FH}^{1-\sigma} t_{FH}^{1-\sigma} w_F^{1-\sigma}} \\ = &\frac{1}{1 + m_{FH}\frac{M_F}{M_H}\left(\frac{\varphi_{FH}}{\varphi_{HH}}\right)^{\sigma-1} w_H^{\sigma-1}\tau_{FH}^{1-\sigma} t_{FH}^{1-\sigma}} \end{aligned}$$

and the second term in the denominator is

$$\begin{aligned} & m_{FH}\frac{M_F}{M_H}\left(\frac{\varphi_{FH}}{\varphi_{HH}}\right)^{\sigma-1} w_H^{\sigma-1}\tau_{FH}^{1-\sigma} t_{FH}^{1-\sigma} \\ = & m_{FH}\frac{M_F}{M_H}\left(\frac{\varphi_{FH}/\varphi_{FH}^*}{\varphi_{HH}/\varphi_{HH}^*}\right)^{\sigma-1}\left(\frac{\varphi_{FH}^*}{\varphi_{HH}^*}\right)^{\sigma-1} w_H^{\sigma-1}\tau_{FH}^{1-\sigma} t_{FH}^{1-\sigma} \\ = & m_{FH}\frac{M_F}{M_H}\frac{\mu_{FH}}{\mu_{HH}}\left(\frac{\varphi_{FH}^* w_H}{\varphi_{HH}^* \tau_{FH}}\right)^{\sigma-1} t_{FH}^{-\sigma}\cdot t_{FH} \\ = & m_{FH}\frac{M_F}{M_H}\frac{\mu_{FH}}{\mu_{HH}}\frac{1}{w_H}\frac{f_{FH}}{f_{HH}} t_{FH} \\ = & \frac{m_{FH}\mu_{FH} M_F f_{FH}}{m_{HF}\mu_{HF} M_H w_H f_{HF}}\cdot\frac{m_{HF}\mu_{HF} f_{HF}}{\mu_{HH} f_{HH}} t_{FH} \\ = & m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}} t_{FH} \end{aligned}$$

where the third equality follows the ratio of Home's zero profit conditions (35). The fifth equality follows from the balanced trade. Substituting this, one can finally obtain

$$
\begin{aligned}
\hat{P}_H &= \frac{M_H {\varphi_{HH}}^{\sigma-1} w_H^{1-\sigma}}{M_H {\varphi_{HH}}^{\sigma-1} w_H^{1-\sigma} + m_{FH} M_F {\varphi_{FH}}^{\sigma-1} \tau_{FH}^{1-\sigma} t_{FH}^{1-\sigma} w_F^{1-\sigma}} \hat{w}_H \\
&\quad + \frac{m_{FH} M_F {\varphi_{FH}}^{\sigma-1} \tau_{FH}^{1-\sigma} t_{FH}^{1-\sigma} w_F^{1-\sigma}}{M_H {\varphi_{HH}}^{\sigma-1} w_H^{1-\sigma} + m_{FH} M_F {\varphi_{FH}}^{\sigma-1} \tau_{FH}^{1-\sigma} t_{FH}^{1-\sigma} w_F^{1-\sigma}} (\hat{t}_{FH} + \hat{w}_F) \\
&= \frac{1}{1 + t_{FH} m_{HF} \frac{\mu_{HF}}{\mu_{HH}} \frac{f_{HF}}{f_{HH}}} \hat{w}_H + \frac{t_{FH} m_{HF} \frac{\mu_{HF}}{\mu_{HH}} \frac{f_{HF}}{f_{HH}}}{1 + t_{FH} m_{HF} \frac{\mu_{HF}}{\mu_{HH}} \frac{f_{HF}}{f_{HH}}} (\hat{t}_{FH} + \hat{w}_F) \\
&= \tilde{\alpha}_H \hat{w}_H + (1 - \tilde{\alpha}_H)(\hat{t}_{FH} + \hat{w}_F) \\
&= \tilde{\alpha}_H \hat{w}_H + (1 - \tilde{\alpha}_H)\hat{t}_{FH}.
\end{aligned}
$$

The last equality follows from the choice of numeraire ($\hat{w}_F = 0$). By the same discussion, one can obtain $\hat{P}_F = \tilde{\alpha}_F \hat{w}_F + (1 - \tilde{\alpha}_F)(\hat{t}_{HF} + \hat{w}_H) = (1 - \tilde{\alpha}_F)(\hat{t}_{HF} + \hat{w}_H)$.

A.2.3 Aggregate expenditure in changes

Aggregate expenditure (12) is rewritten as

$$E_H = \theta \sigma M_H w_H [\mu_{HH} f_{HH} + t_{FH} \mu_{HF} f_{HF}].$$

Totally differentiating both sides, one can obtain

$$
\begin{aligned}
dE_H = {} & \theta \sigma M_H [\mu_{HH} f_{HH} + t_{FH} \mu_{HF} f_{HF}] dw_H \\
& + \theta \sigma M_H w_H f_{HH} d\mu_{HH} + \theta \sigma M_H w_H m_{HF} \mu_{HF} f_{HF} dt_{FH} \\
& + \theta \sigma M_H w_H t_{FH} m_{HF} f_{HF} d\mu_{HF}.
\end{aligned}
$$

This implies

$$
\begin{aligned}
\hat{E}_H &= \hat{w}_H + \frac{\mu_{HH} f_{HH} \hat{\mu}_{HH} + t_{FH} m_{HF} \mu_{HF} f_{HF} (\hat{t}_{FH} + \hat{\mu}_{HF})}{\mu_{HH} f_{HH} + t_{FH} m_{HF} \mu_{HF} f_{HF}} \\
&= \hat{w}_H + \frac{1}{1 + t_{FH} m_{HF} \frac{\mu_{HF}}{\mu_{HH}} \frac{f_{HF}}{f_{HH}}} \hat{\mu}_{HH} + \frac{t_{FH} m_{HF} \frac{\mu_{HF}}{\mu_{HH}} \frac{f_{HF}}{f_{HH}}}{1 + t_{FH} m_{HF} \frac{\mu_{HF}}{\mu_{HH}} \frac{f_{HF}}{f_{HH}}} (\hat{t}_{FH} + \hat{\mu}_{HF}) \\
&= \hat{w}_H + \tilde{\alpha}_H \hat{\mu}_{HH} + (1 - \tilde{\alpha}_H)(\hat{t}_{FH} + \hat{\mu}_{HF}).
\end{aligned}
$$

By the same discussion, and using $\hat{w}_F = 0$, one gets $\hat{E}_F = \tilde{\alpha}_F \hat{\mu}_{FF} + (1 - \tilde{\alpha}_F)(\hat{t}_{HF} + \hat{\mu}_{FH})$.

A.2.4 Zero profit productivity, production labor market clearing, and trade balance in changes

Recall that zero profit productivity from country j to market i is given by

$$E_i P_i^{\sigma-1} t_{ji}^{-\sigma} \left(\frac{\rho \varphi_{ji}^*}{\tau_{ji} w_j} \right)^{\sigma-1} = \sigma w_j f_{ji}. \tag{7}$$

Totally differentiating this yields

$$\begin{aligned} & \hat{E}_i + (\sigma-1)\hat{P}_i - \sigma\hat{t}_{ji} + (\sigma-1)\hat{\varphi}^*_{ji} - (\sigma-1)\hat{w}_j = \hat{w}_j \\ \Rightarrow \quad & \hat{E}_i + (\sigma-1)\hat{P}_i - \sigma\hat{t}_{ji} - \hat{\mu}_{ji} - \sigma\hat{w}_j = 0 \end{aligned}$$

$\hat{\mu}_{ji} = (1-\sigma)\hat{\varphi}^*_{ji}$ is used to substitute out φ^*_{ji}. Substituting H, F into i, j and using $\hat{t}_{HH} = \hat{t}_{FF} = \hat{w}_F = 0$, one obtains the following four equations;

$$\hat{\mu}_{HH} = \hat{E}_H + (\sigma-1)\hat{P}_H - \sigma\hat{w}_H \tag{20}$$

$$\hat{\mu}_{HF} = \hat{E}_F + (\sigma-1)\hat{P}_F - \sigma\hat{t}_{HF} - \sigma\hat{w}_H \tag{21}$$

$$\hat{\mu}_{FF} = \hat{E}_F + (\sigma-1)\hat{P}_F \tag{22}$$

$$\hat{\mu}_{FH} = \hat{E}_H + (\sigma-1)\hat{P}_H - \sigma\hat{t}_{FH}. \tag{23}$$

Also, production labor market clearing conditions (10) and trade balance condition (11) in changes are obtained by the total differentiation as

$$\alpha_H\hat{\mu}_{HH} + (1-\alpha_H)\hat{\mu}_{HF} = 0 \tag{24}$$

$$\alpha_F\hat{\mu}_{FF} + (1-\alpha_F)\hat{\mu}_{FH} = 0 \tag{25}$$

$$\hat{w}_H + \hat{\mu}_{HF} = \hat{\mu}_{FH}. \tag{26}$$

Differentiated production labor market clearing conditions imply that an increase in the aggregate revenue (and labor input) of exporters, $(1-\alpha_H)\hat{\mu}_{HF}$, requires the same amount of decline of the aggregate revenue of domestic suppliers, $\alpha_H\hat{\mu}_{HH}$. Also, the trade balance condition requires that the change in import (summarized by $\hat{\mu}_{FH}$) requires the change in export with the same direction (summarized by $\hat{\mu}_{FH}$ and $\hat{w}_H$).

To obtain the first equation (24), notice that production labor market clearing condition (10) for Home is rewritten as

$$L^p_H = M_H\theta(\sigma-1)[\mu_{HH}f_{HH} + m_{HF}\mu_{HF}f_{HF}].$$

Recall that L^p_H is constant in the second and third stages. Totally differentiating both sides, one can obtain

$$\begin{aligned} 0 &= M_H\theta(\sigma-1)[f_{HH}d\mu_{HH} + m_{HF}f_{HF}d\mu_{HF}] \\ 0 &= \mu_{HH}f_{HH}\hat{\mu}_{HH} + m_{HF}\mu_{HF}f_{HF}\hat{\mu}_{HF} \\ 0 &= \hat{\mu}_{HH} + m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}}\hat{\mu}_{HF} \\ &= \hat{\mu}_{HH} + \frac{1-\alpha_H}{\alpha_H}\hat{\mu}_{HF} \\ 0 &= \alpha_H\hat{\mu}_{HH} + (1-\alpha_H)\hat{\mu}_{HF}. \end{aligned}$$

Also, one can obtain (25) from the production labor market clearing condition of Foreign. Finally, recall that the trade balance condition is given by

$$m_{HF}\mu_{HF}M_H w_H f_{HF} = m_{FH}\mu_{FH}M_F w_F f_{FH}. \tag{11}$$

Substituting $w_F = 1$ and totally differentiating both sides gives

$$\begin{aligned}
& m_{HF}\mu_{HF}M_H f_{HF} dw_H + m_{HF}M_H w_H f_{HF} d\mu_{HF} = m_{FH}M_F f_{FH} d\mu_{FH} \\
\Rightarrow\quad & m_{HF}\mu_{HF}M_H w_H f_{HF}(\hat{w}_H + \hat{\mu}_{HF}) = m_{FH}\mu_{FH}M_F f_{FH}\hat{\mu}_{FH} \\
\Rightarrow\quad & \hat{w}_H + \hat{\mu}_{HF} = \hat{\mu}_{FH}.
\end{aligned}$$

Thus, one can derive equation (26).

A.2.5 Effect of wage on relative revenues

Assume Foreign tariff as given. This means $\hat{t}_{HF} = 0$. By subtracting both sides of Home's export zero profit productivity equation (21) and that of Foreign's domestic entry (22), one can obtain

$$\hat{\mu}_{FF} = \hat{\mu}_{HF} + \sigma\hat{w}_H. \tag{36}$$

Substituting trade balance condition (26) into it yields

$$\begin{aligned}
\hat{\mu}_{FF} &= (\hat{\mu}_{FH} - \hat{w}_H) + \sigma\hat{w}_H \\
&= (\sigma - 1)\hat{w}_H + \hat{\mu}_{FH}. \qquad (37)
\end{aligned}$$

By solving this and production labor market clearing condition of Foreign (25) simultaneously, one can obtain

$$\hat{\mu}_{FH} = -\alpha_F(\sigma - 1)\hat{w}_H \tag{38}$$

$$\hat{\mu}_{FF} = (1 - \alpha_F)(\sigma - 1)\hat{w}_H. \tag{39}$$

By substituting $\hat{\mu}_{FH}$ into the trade balance condition (26), one can get

$$\begin{aligned}
\hat{\mu}_{HF} &= \hat{\mu}_{FH} - \hat{w}_H \\
&= -(\alpha_F(\sigma - 1) + 1)\hat{w}_H. \qquad (40)
\end{aligned}$$

Substituting this into production labor market clearing condition of Home yields

$$\begin{aligned}
\hat{\mu}_{HH} &= -\frac{1 - \alpha_H}{\alpha_H}\hat{\mu}_{HF} \\
&= \frac{1 - \alpha_H}{\alpha_H}(\alpha_F(\sigma - 1) + 1)\hat{w}_H. \qquad (41)
\end{aligned}$$

Equations (41) and (39) imply that an increase in Home's wage raises the sales (revenue) relative to the zero-profit firm in the domestic market (μ_{HH} and μ_{FF} rise) for firms in both countries. On the other hand, (40) and (38) imply that an increase in Home's wage declines the relative sales in the foreign market (μ_{HF} and μ_{FH} drop) for firms in both countries.

A.2.6 Effect of tax on wage

Recall that the zero profit productivity equation for Home's domestic supply in changes is given by

$$\hat{\mu}_{HH} = \hat{E}_H + (\sigma - 1)\hat{P}_H - \sigma \hat{w}_H. \tag{20}$$

By substituting Home's price index in changes (16) and the relationship between w_H and $\hat{\mu}_{HH}$ (41) into this equation, one can obtain

$$\begin{aligned}
\hat{E}_H &= -(\sigma - 1)(\tilde{\alpha}_H \hat{w}_H + (1 - \tilde{\alpha}_H)\hat{t}_{FH}) + \frac{1 - \alpha_H}{\alpha_H}(\alpha_F(\sigma - 1) + 1)\hat{w}_H + \sigma \hat{w}_H \\
&= -(\sigma - 1)(1 - \tilde{\alpha}_H)\hat{t}_{FH} \\
&\quad - \left((\sigma - 1)\tilde{\alpha}_H - \frac{1 - \alpha_H}{\alpha_H}(\alpha_F(\sigma - 1) + 1) - \sigma\right)\hat{w}_H.
\end{aligned} \tag{42}$$

Whereas, substituting (40) and (41) into the aggregate expenditure in changes (18) yields

$$\begin{aligned}
\hat{E}_H &= \hat{w}_H + \tilde{\alpha}_H \hat{\mu}_{HH} + (1 - \tilde{\alpha}_H)(\hat{t}_{FH} + \hat{\mu}_{HF}) \\
&= \hat{w}_H + \tilde{\alpha}_H \frac{1 - \alpha_H}{\alpha_H}(\alpha_F(\sigma - 1) + 1)\hat{w}_H + (1 - \tilde{\alpha}_H)\hat{t}_{FH} \\
&\quad - (1 - \tilde{\alpha}_H)(\alpha_F(\sigma - 1) + 1)\hat{w}_H \\
&= (1 - \tilde{\alpha}_H)\hat{t}_{FH} + \left(1 + (\alpha_F(\sigma - 1) + 1)\left(\frac{\tilde{\alpha}_H}{\alpha_H} - 1\right)\right)\hat{w}_H.
\end{aligned} \tag{43}$$

Comparing two equations (42) and (43) and rearranging terms yields the relationship between change in tax rate and change in wage

$$\hat{w}_H = \sigma \left[\frac{\alpha_H}{(\sigma - 1)(\alpha_H + \alpha_F) + 1}\right]\hat{t}_{FH}. \tag{44}$$

Since terms in the square bracket are strictly positive, an increase in Home's tax rate necessarily increases Home's (relative) wage.

A.2.7 Optimal tax rate

From (27), $\hat{E}_H - \hat{P}_H = 0$ means that the following condition holds.

$$\begin{aligned}
&(1 - \tilde{\alpha}_H) + (\alpha_F(\sigma - 1) + 1)\left(\frac{\tilde{\alpha}_H}{\alpha_H} - 1\right) = 0 \\
\Rightarrow\ &(\alpha_F(\sigma - 1) + 1)^{-1} = \left(1 - \frac{\tilde{\alpha}_H}{\alpha_H}\right)(1 - \tilde{\alpha}_H)^{-1}
\end{aligned} \tag{45}$$

Here, the right-hand-side can be simplified as

$$\begin{aligned}
&\left(1-\frac{\tilde{\alpha}_H}{\alpha_H}\right)(1-\tilde{\alpha}_H)^{-1}\\
&=\left(1-\frac{1+m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}}}{1+t_{FH}m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}}}\right)\frac{1+t_{FH}m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}}}{t_{FH}m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}}}\\
&=\frac{t_{FH}m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}}-m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}}}{1+t_{FH}m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}}}\cdot\frac{1+t_{FH}m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}}}{t_{FH}m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}}}\\
&=\frac{t_{FH}m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}}-m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}}}{t_{FH}m_{HF}\frac{\mu_{HF}}{\mu_{HH}}\frac{f_{HF}}{f_{HH}}}\\
&=1-\frac{1}{t_{FH}}.
\end{aligned}$$

Finally, substituting this result into (45), the optimal tax rate that satisfies $\hat{E}_H-\hat{P}_H=0$ is given by

$$\begin{aligned}
\frac{1}{t^*_{FH}}&=1-\frac{1}{\alpha_F(\sigma-1)+1}\\
&=\frac{\alpha_F(\sigma-1)}{\alpha_F(\sigma-1)+1}\\
\Rightarrow t^*_{FH}&=\frac{\alpha_F(\sigma-1)+1}{\alpha_F(\sigma-1)}\\
&=1+\frac{1}{\alpha_F(\sigma-1)}\\
&=\frac{\sigma+\frac{1-\alpha_F}{\alpha_F}}{\sigma-1}.
\end{aligned}$$

A.2.8 Effect of Home's tax on Foreign's welfare

From Foreign's expenditure in changes (19) and the price index in changes (17), one can obtain the effect of the change in relative wage on the welfare of Foreign as

$$\begin{aligned}
&\hat{E}_F-\hat{P}_F\\
&=[\tilde{\alpha}_F\hat{\mu}_{FF}+(1-\tilde{\alpha}_F)\hat{\mu}_{FH}]-[(1-\tilde{\alpha}_F)\hat{w}_H]\\
&=\tilde{\alpha}_F(1-\alpha_F)(\sigma-1)\hat{w}_H-(1-\tilde{\alpha}_F)\alpha_F(\sigma-1)\hat{w}_H-(1-\tilde{\alpha}_F)\hat{w}_H\\
&=(\tilde{\alpha}_F-\alpha_F)(\sigma-1)\hat{w}_H-(1-\tilde{\alpha}_F)\hat{w}_H\\
&=-\left[\sigma(\alpha_F-\tilde{\alpha}_F)+(1-\alpha_F)\right]\hat{w}_H.
\end{aligned}$$

This must be negative since $\alpha_F\geq\tilde{\alpha}_F$.

A.3 Derivation details for entire equilibrium

A.3.1 Free entry condition

According to equation (9), the average profit from one market with $\mu_{ij} = 1$ is given by

$$\bar{\pi}_{ij} = (\theta - 1) w_i m_{ij} f_{ij}.$$

Therefore, the average profit discounted by the probability of successful entry is

$$(1 - G(\varphi_{ii}))(\bar{\pi}_{ii} + \bar{\pi}_{ij}) = \varphi_{ii}{}^{-\beta} w_i (\theta - 1)(f_{ii} + m_{ij} f_{ij}).$$

Since this must be equal to the research cost $w_i f_e$, one can obtain the condition (31).

A.3.2 Labor market clearing condition

By summing up the labor requirement for entry process (5) and production labor market clearing condition (10) and substituting $\mu_{ij} = 1$, one can obtain

$$\begin{aligned}
L_i &= M_i \left[\frac{f_e}{1 - G(\varphi_{ii})} + \sum_j m_{ij} f_{ij} + \sum_j \theta(\sigma - 1) m_{ij} f_{ij} \right] \\
&= M_i \left[(\theta - 1) \sum_j m_{ij} f_{ij} + \sum_j m_{ij} f_{ij} + \sum_j \theta(\sigma - 1) m_{ij} f_{ij} \right] \\
&= \theta \sigma M_i \sum_j m_{ij} f_{ij}
\end{aligned}$$

where the second equality follows the free entry condition. Rearranging terms and substituting the free entry condition, one can get the condition (32).