### Appendix D

This appendix is an independent analysis based on the framework presented in: A dynamic model of subjectivity integrating emotional gain, internal bias, and self-regulation  $\frac{\text{https:}}{\text{doi.org}} = \frac{10.1101}{\text{NXXXXXXX}}$ 

# Reconstructing ADHD through the $G-\mu$ -S Dynamic Model:

## A Framework Integrating Theory, Evidence, and Clinical Practice

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#### Abstract

This paper refines the  $G-\mu$ –S dynamic model, integrates recent computational and neuroscientific findings, and delineates clinical implications for Attention-Deficit/Hyperactivity Disorder (ADHD). By conceptualising ADHD as a dysregulation of a higher-order self-system that learns to modulate emotional gain (G) and internal bias  $(\mu)$ , the framework unifies diverse symptoms, individual differences, and treatment mechanisms within a coherent, testable architecture.

#### 0 Objective and Overview

We enhance the original draft along three axes:

- 1. Theoretical refinement of the  $G-\mu$ -S framework
- 2. Synthesis of cutting-edge evidence from computation and neuroscience
- 3. Concrete mapping of clinical and interventional implications

The paper is organised as follows:

- 1. Re-organisation of the  $G-\mu$ -S framework
- 2. Parameter-space interpretation of ADHD
- 3. Neural and computational substrates of G,  $\mu$ , and the Self-System (S)
- 4. Visualisation of dynamic instability and simulation insights
- 5. Individual differences and comorbidities via the SGBD extension
- 6. Intervention mapping and research predictions

#### 1 Key Features of the $G-\mu$ -S Framework

- Mind Topography Map The  $G-\mu$  plane comprises stable, bistable, and unstable regions actively navigated by the Self-System (S).
- Ideal Dynamic Equilibrium (IDE) At  $(G = 1, \mu = 0)$  the linear term disappears, yielding a state that is both stable and highly responsive to noise.

• Gain-dependent Effective Bias  $(G\mu)$  Because bias is multiplied by gain, changes in G amplify or attenuate  $\mu$  dynamically, distinguishing this model from prior approaches.

#### 2 Interpreting ADHD in Parameter Space

Table 1: Interpretation of ADHD core symptoms within the  $G-\mu$ -S parameter space.

Core Symptom	Emotional Gain $(G)$	Internal Bias $(\mu)$	State of $S$ & Dynamic Consequence
Inattention	Low $G$ weakens errordriven updating and stimulus selectivity.	$\mu$ fluctuates and fails to stabilise.	Context-adaptive retuning by S is delayed; even in a monostable zone convergence is slow, impairing sustained attention.
Hyperactivity / Impulsivity	High $G$ triggers over- reactions to minor errors.	$\mu$ rapidly collapses toward impulsive attractors.	Gain control by $S$ cannot keep pace; trajectories leap into high- $G$ regions, diverging or plunging toward a bias.
Emotional Dysregulation	High $G$ combined with strong negative $\mu$ .		$S$ 's objective $\mathcal{L}$ is shortened via delay discounting. IDE's restorative pull weakens, yielding chaotic transitions or bistable switching.

**Key Point.** ADHD primarily reflects a meta-learning deficit in S—either a reduced learning rate for its hyper-parameters ( $\theta_{\rm S}$ ) or optimisation of a short-sighted objective  $\mathcal{L}$ . Empirical evidence corroborates reward delay discounting and impaired neural gain.

#### 3 Neural and Computational Mechanisms

#### 3.1 Gain (G)

Dopamine and noradrenaline modulate the signal-to-noise ratio (SNR). Stimulants such as methylphenidate enhance prefrontal—striatal SNR, effectively nudging G toward 1. The neural-gain impairment hypothesis attributes behavioural variability to malfunctioning local gain regulation.

#### 3.2 Bias $(\mu)$

A latent vector shaped by development, memory, and belief. In ADHD,  $\mu$  oscillates between a positive component favouring immediate reward and a negative component driven by the anticipation of failure.

#### 3.3 Self-System (S)

Equivalent to Barkley's self-regulation construct: executive-function deficits signify weak control by S. Hierarchical Bayesian learning in S corresponds to the frontopolar–anterior-cingulate–basal-ganglia loop; in ADHD, a lowered learning rate  $(\eta_{\theta})$  and mis-estimated confidence slow convergence.

#### 4 Dynamic Instability and Clinical Observations

The bistable zone  $(G < 0, |\mu| > 0)$  reproduces abrupt switches between hyper-focus and distractibility reported by patients. Amplified noise  $(\epsilon)$  manifests as elevated reaction-time variability, which reliably normalises following treatment.

# 5 Modelling Individual Differences and Comorbidities with SGBD

Structural matrices  $\mathbf{M}_{\mathbf{G}}$  and  $\mathbf{M}_{\mu}$  capture variations in emotional-cognitive wiring. Persistent negative  $\mu$  in a specific dimension can simultaneously model comorbid anxiety or depression, e.g. a  $\mu < 0$  on the social-evaluation axis in social-anxiety comorbidity, while G remains typical.

#### 6 Therapeutic Intervention Mapping

Intervention	Model Target	Theoretical Effect	Expected Parameter Tra- jectory
Stimulants (MPH/AMP)	$G,\epsilon$	Elevate SNR $\rightarrow$ normalise $G$ toward 1, dampen noise.	Converge toward IDE from both high- and low- $G$ extremes.
CBT / Parent Training	$S\left(\boldsymbol{\theta}_{\mathrm{S}},\mathcal{L} ight)$	Support meta-learning; reappraise reward.	$\eta_{\theta} \uparrow$ ; $\mathcal{L}$ re-aligned to long-term goals.
Metacognitive Training / Mind- fulness	G,S	Detect and voluntarily modulate gain.	Reduced $G$ variability; increased IDE dwell time.
Digital Neurofeed- back	$\mu$ , $G$	Real-time bias visualisation.	$\mu \to 0$ , $G$ into optimal range.

Table 2: Mapping of the rapeutic interventions to the  $G-\mu$ -S model parameters.

**Key Point.** This mapping illustrates the framework's utility in mechanistically unifying diverse interventions. Pharmacological treatments can be seen as directly tuning low-level parameters  $(G, \epsilon)$ , while psychosocial therapies aim to reshape the higher-order Self-System (S), representing complementary approaches to regulating the same dynamic system.

#### **Future Research Directions**

- $\bullet$  Pupil-diameter variability as a biomarker of G and treatment responsiveness.
- Task-based estimation of  $\eta_{\theta}$  in hierarchical reinforcement-learning paradigms to quantify plasticity of S.
- Longitudinal neuro-model coupling to chart how interventions redirect Mind-Map trajectories.

### Conclusion

Reconceptualising ADHD as a dysregulation of the meta-learning Self-System (S) that governs emotional gain (G) and internal bias  $(\mu)$  coherently integrates computational, neuroscientific, and clinical findings. This framework:

- 1. Explains heterogeneous symptoms, individual differences, and comorbidities within a single dynamic model.
- 2. Treats pharmacological and psychosocial interventions as parameter-tuning processes.
- 3. Generates testable biomarkers and task predictions grounded in model dynamics.