

## Appendix D

This appendix is an independent analysis based on the framework presented in:

*A dynamic model of subjectivity integrating emotional gain, internal bias, and self-regulation*

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# Reconstructing ADHD through the $G$ - $\mu$ - $S$ Dynamic Model: A Framework Integrating Theory, Evidence, and Clinical Practice

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## Abstract

This paper refines the  $G$ - $\mu$ - $S$  dynamic model, integrates recent computational and neuroscientific findings, and delineates clinical implications for Attention-Deficit/Hyperactivity Disorder (ADHD). By conceptualising ADHD as a dysregulation of a higher-order self-system that learns to modulate emotional gain ( $G$ ) and internal bias ( $\mu$ ), the framework unifies diverse symptoms, individual differences, and treatment mechanisms within a coherent, testable architecture.

## 0 Objective and Overview

We enhance the original draft along three axes:

1. Theoretical refinement of the  $G$ - $\mu$ - $S$  framework
2. Synthesis of cutting-edge evidence from computation and neuroscience
3. Concrete mapping of clinical and interventional implications

The paper is organised as follows:

1. Re-organisation of the  $G$ - $\mu$ - $S$  framework
2. Parameter-space interpretation of ADHD
3. Neural and computational substrates of  $G$ ,  $\mu$ , and the Self-System ( $S$ )
4. Visualisation of dynamic instability and simulation insights
5. Individual differences and comorbidities via the SGBD extension
6. Intervention mapping and research predictions

## 1 Key Features of the $G$ - $\mu$ - $S$ Framework

- **Mind Topography Map** The  $G$ - $\mu$  plane comprises stable, bistable, and unstable regions actively navigated by the Self-System ( $S$ ).
- **Ideal Dynamic Equilibrium (IDE)** At ( $G = 1, \mu = 0$ ) the linear term disappears, yielding a state that is both stable and highly responsive to noise.

- **Gain-dependent Effective Bias ( $G\mu$ )** Because bias is multiplied by gain, changes in  $G$  amplify or attenuate  $\mu$  dynamically, distinguishing this model from prior approaches.

## 2 Interpreting ADHD in Parameter Space

Table 1: Interpretation of ADHD core symptoms within the  $G$ – $\mu$ – $S$  parameter space.

| Core Symptom                       | Emotional Gain ( $G$ )  | Internal Bias ( $\mu$ )                              | State of $S$ & Dynamic Consequence   |
|------------------------------------|---|--|--|
| <b>Inattention</b>                 | Low $G$ weakens error-driven updating and stimulus selectivity. | $\mu$ fluctuates and fails to stabilise.             | Context-adaptive retuning by $S$ is delayed; even in a monostable zone convergence is slow, impairing sustained attention.                             |
| <b>Hyperactivity / Impulsivity</b> | High $G$ triggers over-reactions to minor errors.               | $\mu$ rapidly collapses toward impulsive attractors. | Gain control by $S$ cannot keep pace; trajectories leap into high- $G$ regions, diverging or plunging toward a bias.                                   |
| <b>Emotional Dysregulation</b>     | High $G$ combined with strong negative $\mu$ .                  |  | $S$ 's objective $\mathcal{L}$ is shortened via delay discounting. IDE's restorative pull weakens, yielding chaotic transitions or bistable switching. |

**Key Point.** ADHD primarily reflects a meta-learning deficit in  $S$ —either a reduced learning rate for its hyper-parameters ( $\theta_S$ ) or optimisation of a short-sighted objective  $\mathcal{L}$ . Empirical evidence corroborates reward delay discounting and impaired neural gain.

## 3 Neural and Computational Mechanisms

### 3.1 Gain ( $G$ )

Dopamine and noradrenaline modulate the signal-to-noise ratio (SNR). Stimulants such as methylphenidate enhance prefrontal–striatal SNR, effectively nudging  $G$  toward 1. The neural-gain impairment hypothesis attributes behavioural variability to malfunctioning local gain regulation.

### 3.2 Bias ( $\mu$ )

A latent vector shaped by development, memory, and belief. In ADHD,  $\mu$  oscillates between a positive component favouring immediate reward and a negative component driven by the anticipation of failure.

### 3.3 Self-System ( $S$ )

Equivalent to Barkley's self-regulation construct: executive-function deficits signify weak control by  $S$ . Hierarchical Bayesian learning in  $S$  corresponds to the frontopolar–anterior-cingulate–basal-ganglia loop; in ADHD, a lowered learning rate ( $\eta_\theta$ ) and mis-estimated confidence slow convergence.

## 4 Dynamic Instability and Clinical Observations

The bistable zone ( $G < 0, |\mu| > 0$ ) reproduces abrupt switches between hyper-focus and distractibility reported by patients. Amplified noise ( $\epsilon$ ) manifests as elevated reaction-time variability, which reliably normalises following treatment.

## 5 Modelling Individual Differences and Comorbidities with SGBD

Structural matrices  $\mathbf{M}_G$  and  $\mathbf{M}_\mu$  capture variations in emotional-cognitive wiring. Persistent negative  $\mu$  in a specific dimension can simultaneously model comorbid anxiety or depression, e.g. a  $\mu < 0$  on the social-evaluation axis in social-anxiety comorbidity, while  $G$  remains typical.

## 6 Therapeutic Intervention Mapping

Table 2: Mapping of therapeutic interventions to the G- $\mu$ -S model parameters.

| Intervention                                | Model Target                | Theoretical Effect  | Expected Parameter Trajectory   |
|---|-----------------------------|---|---|
| <b>Stimulants (MPH/AMP)</b>                 | $G, \epsilon$               | Elevate SNR $\rightarrow$ normalise $G$ toward 1, dampen noise. | Converge toward IDE from both high- and low- $G$ extremes.            |
| <b>CBT / Parent Training</b>                | $S (\theta_S, \mathcal{L})$ | Support meta-learning; re-appraise reward.                      | $\eta_\theta \uparrow$ ; $\mathcal{L}$ re-aligned to long-term goals. |
| <b>Metacognitive Training / Mindfulness</b> | $G, S$                      | Detect and voluntarily modulate gain.                           | Reduced $G$ variability; increased IDE dwell time.                    |
| <b>Digital Neurofeedback</b>                | $\mu, G$                    | Real-time bias visualisation.                                   | $\mu \rightarrow 0$ , $G$ into optimal range.                         |

**Key Point.** This mapping illustrates the framework’s utility in mechanistically unifying diverse interventions. Pharmacological treatments can be seen as directly tuning low-level parameters ( $G, \epsilon$ ), while psychosocial therapies aim to reshape the higher-order Self-System ( $S$ ), representing complementary approaches to regulating the same dynamic system.

## Future Research Directions

- Pupil-diameter variability as a biomarker of  $G$  and treatment responsiveness.
- Task-based estimation of  $\eta_\theta$  in hierarchical reinforcement-learning paradigms to quantify plasticity of  $S$ .
- Longitudinal neuro-model coupling to chart how interventions redirect Mind-Map trajectories.

## Conclusion

Reconceptualising ADHD as a dysregulation of the meta-learning Self-System ( $S$ ) that governs emotional gain ( $G$ ) and internal bias ( $\mu$ ) coherently integrates computational, neuroscientific, and clinical findings. This framework:

1. Explains heterogeneous symptoms, individual differences, and comorbidities within a single dynamic model.
2. Treats pharmacological and psychosocial interventions as parameter-tuning processes.
3. Generates testable biomarkers and task predictions grounded in model dynamics.