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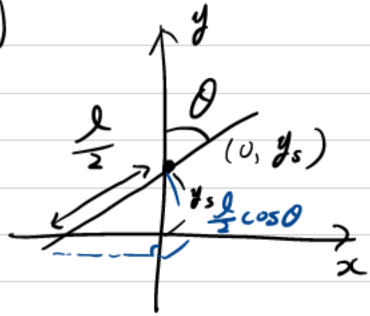
第1問

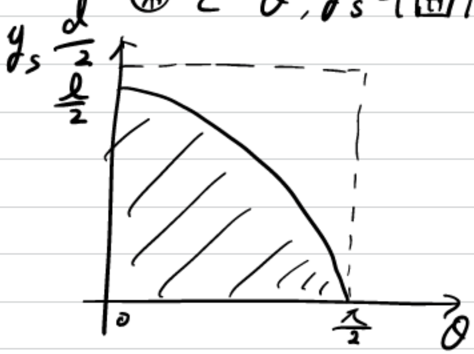
(1)



棒が直線に交わるパラメータ y_s, θ の条件は

$$y_s \le \frac{l}{2}\cos\theta \quad \cdots (*)$$

(*)を θ, y_s 平面に図示すると以下のようになる。



$$\left(\begin{array}{l} 0 < \theta < \frac{\pi}{2} \\ 0 < y_s < \frac{d}{2} \end{array} \right)$$

$$\frac{l}{2} < \frac{d}{2}$$

$$f(\theta, y_s) = \begin{cases} 1 & \left(y_s \le \frac{l}{2}\cos\theta \right) \\ 0 & \left(y_s > \frac{l}{2}\cos\theta \right) \end{cases}$$

と定義すると棒が直線に交わる確率 p は

$$p = \frac{\int_0^{\frac{\pi}{2}} \int_0^{\frac{d}{2}} f(\theta, y_s)\, dy_s\, d\theta}{\int_0^{\frac{\pi}{2}} \int_0^{\frac{d}{2}} dy_s\, d\theta} = \frac{\int_0^{\frac{\pi}{2}} \frac{l}{2}\cos\theta\, d\theta}{\frac{\pi}{2} \times \frac{d}{2}}$$

$$= \frac{4}{\pi d} \times \frac{l}{2} = \underline{\frac{2l}{\pi d}} \quad //$$

(2)

$$m\ddot{x}(t) = -\zeta\dot{x}(t) - kx(t) \quad \cdots (*)$$

$x(t) = e^{\lambda t}$ とおく

$$(m\lambda^2 + \zeta\lambda + k)e^{\lambda t} = 0$$

特性方程式

$m\lambda^2 + \zeta\lambda + k = 0$ を得る

これを解いて

$$\lambda = \frac{-\zeta \pm \sqrt{\zeta^2 - 4mk}}{2m}$$

$$\lambda_1 = -\frac{\zeta}{2m} + \frac{1}{2m}\sqrt{\zeta^2 - 4mk}$$

$$\lambda_2 = -\frac{\zeta}{2m} - \frac{1}{2m}\sqrt{\zeta^2 - 4mk}$$ とおく

(i) $\zeta^2 - 4mk < 0$ のとき　　$\omega = \sqrt{4mk - \zeta^2}\, i$ とおくと

$$\begin{aligned} x(t) &= C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \\ &= e^{-\frac{\zeta}{2m}t}\left(C_1 e^{\frac{\omega}{2m}it} + C_2 e^{-\frac{\omega}{2m}it}\right) \\ &= e^{-\frac{\zeta}{2m}t}\left(A_1 \cos\left(\frac{\omega}{2m}t\right) + A_2 \sin\left(\frac{\omega}{2m}t\right)\right) \end{aligned}$$

減衰振動　　(A_1, A_2 は任意定数)

(ii) $\zeta^2 - 4mk > 0$ のとき　　$\eta = \sqrt{\zeta^2 - 4mk}$ とおくと

λ_1, λ_2 は実数、このとき

$$\begin{aligned} x(t) &= C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \\ &= e^{-\frac{\zeta}{2m}t}\left(C_1 e^{\frac{\eta}{2m}t} + C_2 e^{-\frac{\eta}{2m}t}\right) \end{aligned}$$

過減衰　　(C_1, C_2 は任意定数)

(iii) $\gamma^2-4mk=0$ のとき

$$x_1(t)=e^{-\frac{\gamma}{2m}t}$$
$$x_2(t)=te^{-\frac{\gamma}{2m}t}$$ が解

一般解は
$$x(t)=(C_1+C_2t)e^{-\frac{\gamma}{2m}t}$$ (C_1, C_2 は任意定数)

臨界減衰

まとめると
(i) $\gamma^2-4mk<0$ のとき 減衰振動
(ii) $\gamma^2-4mk>0$ のとき 過減衰
(iii) $\gamma^2-4mk=0$ のとき 臨界減衰

(2)

$$m\ddot{x}(t)=-\gamma\dot{x}(t)-kx(t)$$

$$\begin{cases} \cdot\ x=a\tilde{x} \\ \cdot\ t=t_0\tilde{t} \\ \cdot\ \dot{x}=\frac{a}{t_0}\tilde{v} \end{cases}$$ とおくと

$$m\frac{\frac{a}{t_0}\tilde{v}(\tilde{t}+\Delta\tilde{t})-\frac{a}{t_0}\tilde{v}(\tilde{t})}{t_0\Delta\tilde{t}}=-\gamma\frac{a}{t_0}\tilde{v}(\tilde{t})-ka\tilde{x}(\tilde{t})$$

$$\frac{m}{t_0\Delta\tilde{t}}\left(\tilde{v}(\tilde{t}+\Delta\tilde{t})-\tilde{v}(\tilde{t})\right)=-\gamma\tilde{v}(\tilde{t})-kt_0\tilde{x}(\tilde{t})$$

$$\tilde{v}(\tilde{t}+\Delta\tilde{t})=\tilde{v}(\tilde{t})-\frac{\gamma}{m}t_0\Delta\tilde{t}\,\tilde{v}(\tilde{t})-\frac{k}{m}t_0^2\Delta\tilde{t}\,\tilde{x}(\tilde{t})$$
$$=\left(1-\frac{\gamma}{m}t_0\Delta\tilde{t}\right)\tilde{v}(\tilde{t})-\frac{k}{m}t_0^2\Delta\tilde{t}\,\tilde{x}(\tilde{t})$$

…ⓒ

位置に関しては

$$\hat{x}(\hat{t}+\Delta\hat{t}) = \hat{x}(\hat{t}) + \hat{v}(\hat{t}+\Delta\hat{t})\Delta\hat{t} \quad \cdots ②$$

①,② をまとめて

$$\hat{v}(\hat{t}+\Delta\hat{t}) = \left(1-\frac{g}{m}t_0\Delta\hat{t}\right)\hat{v}(\hat{t}) - \frac{k}{m}t_0^2\Delta\hat{t}\,\hat{x}(\hat{t})$$

$$\hat{x}(\hat{t}+\Delta\hat{t}) = \hat{x}(\hat{t}) + \hat{v}(\hat{t}+\Delta\hat{t})\Delta\hat{t}$$

(3) 式①について $t_d = \frac{m}{g}$, $t_s = \sqrt{\frac{m}{k}}$ とおくと

$$\hat{v}(\hat{t}+\Delta\hat{t}) = \left(1-\frac{1}{t_d}\cdot t_0\Delta\hat{t}\right)\hat{v}(\hat{t}) - \frac{1}{t_s^2}t_0^2\Delta\hat{t}\,\hat{x}(\hat{t})$$

$$= \left(1-\frac{t_0}{t_d}\Delta\hat{t}\right)\hat{v}(\hat{t}) - \left(\frac{t_0}{t_s}\right)^2\Delta\hat{t}\,\hat{x}(\hat{t})$$

(4) $g = \frac{m}{t_d}$, $k = \frac{m}{t_s^2}$

これらから、

$$g^2 - 4mk = m^2\left(\frac{1}{t_d^2} - 4\frac{1}{t_s^2}\right)$$

(i) 減衰振動の条件は

$$g^2 - 4mk = m^2\left(\frac{1}{t_d^2} - 4\frac{1}{t_s^2}\right) < 0$$

$$\left(\frac{t_0}{t_d}\right)^2 < 4\left(\frac{t_0}{t_s}\right)^2$$

$\frac{t_0}{t_d} > 0$, $\frac{t_0}{t_s} > 0$ より

$$\frac{t_0}{t_d} < 2\frac{t_0}{t_s}$$ であればよい

(< 2)

(ii) 過減衰の条件は

$$\frac{t_0}{t_d} > 2\frac{t_0}{t_s} \; (=2)$$

(iii) 臨界減衰の条件は

$$\frac{t_0}{t_d} = 2\frac{t_0}{t_s} \; (=2)$$