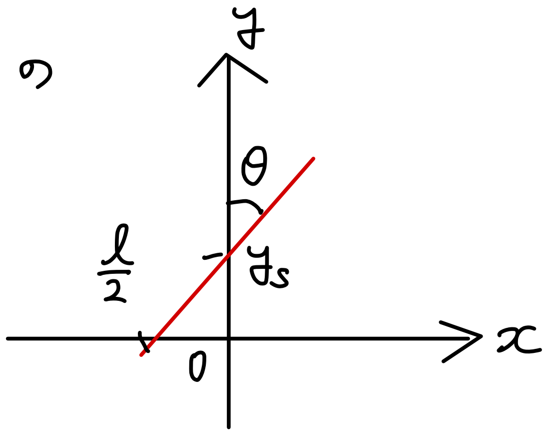
シミュレーション実習 中間レポート 262201036 石川慶太朗

問1

(1)

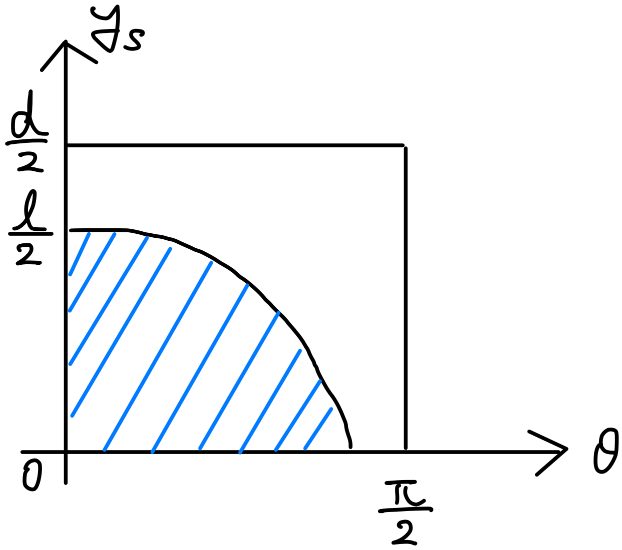
(y_s, θ) は $y_s : [0, \frac{d}{2}]$, $\theta : [0, \frac{\pi}{2}]$ の範囲でランダム.



x軸と交わるには、右上図から

$$y_s \leq \frac{l}{2}\cos\theta$$ であればよい.

(右下図青領域)



ランダムに点を1つ取る

⇔ 右下図の長方形内の点を取る.

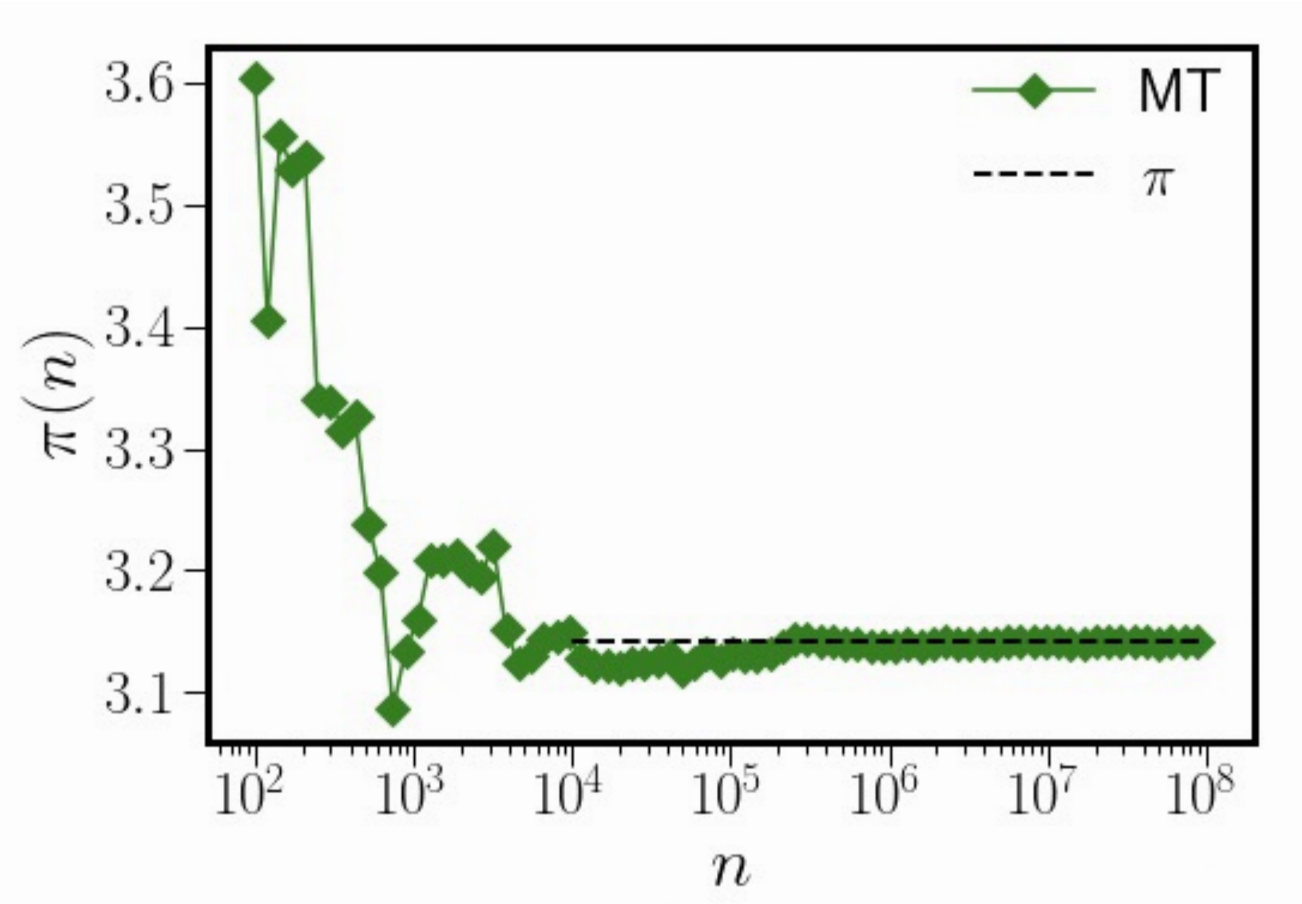
なので、

ランダムに取る全事象は $\frac{\pi d}{4}$

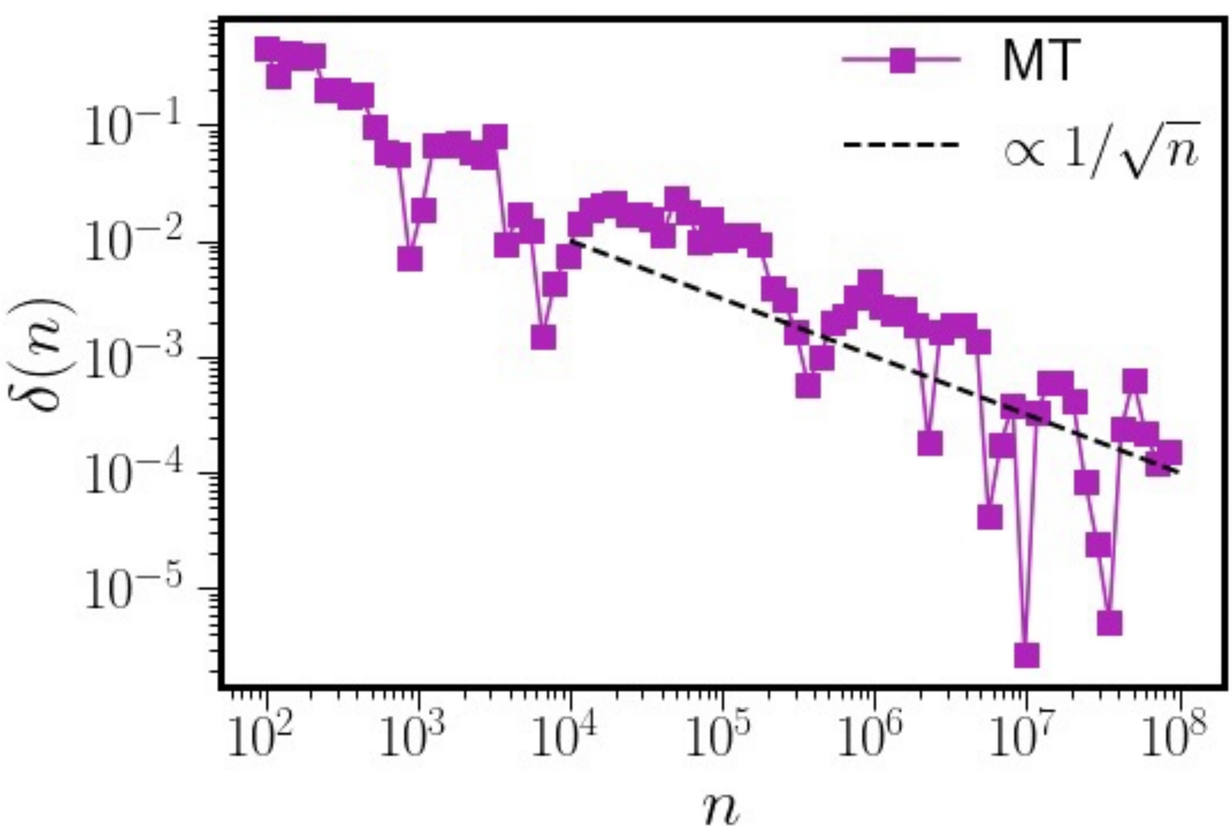
右下図青領域の面積は $\int_0^{\frac{\pi}{2}} \frac{l}{2}\cos\theta\, d\theta = \frac{l}{2}$ なので

求める確率は $\dfrac{\frac{l}{2}}{\frac{\pi d}{4}} = \underline{\underline{\dfrac{2l}{\pi d}}}$

(2)


MT
π
$\pi(n)$
3.6
3.5
3.4
3.3
3.2
3.1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
n

(3)


MT
$\propto 1/\sqrt{n}$
$\delta(n)$
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^2
10^3
10^4
10^5
10^6
10^7
10^8
n

問2 $m\ddot{x}(t) = -\zeta\dot{x}(t) - kx(t)$

(1) $x(t) = e^{\lambda t}$ と仮定して代入すると、 $m\lambda^2 + \zeta\lambda + k = 0$ …①

この二次方程式の判別式 $D = \zeta^2 - 4mk$ の値で運動が変わる.

(i) 減衰振動

$D = \zeta^2 - 4mk < 0$, i.e. $\zeta^2 < 4mk$ のとき

(ii) 過減衰

$D = \zeta^2 - 4mk > 0$, i.e. $\zeta^2 > 4mk$ のとき

(iii) 臨界減衰

$D = \zeta^2 - 4mk = 0$, i.e. $\zeta^2 = 4mk$ のとき

(2)

$x = a\tilde{x}$, $t = t_0\tilde{t}$, $\dot{x} = v = \left(\frac{a}{t_0}\right)\tilde{v}$ とすると、

$\frac{d}{dt}v(t) = \frac{v(t+\Delta t) - v(t)}{\Delta t}$ より 離散化して

$$v(t+\Delta t) = v(t) - \frac{\zeta}{m}v(t)\Delta t - \frac{k}{m}x(t)\Delta t$$

無次元化して

$$\frac{a}{t_0}\tilde{v}(\tilde{t}+\Delta\tilde{t}) = \frac{a}{t_0}\left(1 - \frac{\zeta}{m}t_0\Delta\tilde{t}\right)\tilde{v}(\tilde{t}) - \frac{kat_0\Delta\tilde{t}}{m}\tilde{x}(\tilde{t})$$

$$\underline{\tilde{v}(\tilde{t}+\Delta\tilde{t}) = \left(1 - \frac{\zeta}{m}t_0\Delta\tilde{t}\right)\tilde{v}(\tilde{t}) - \frac{k}{m}t_0^2\Delta\tilde{t}\,\tilde{x}(\tilde{t})} \quad \cdots ②$$

また、位置については

$$\tilde{x}(\tilde{t}+\Delta\tilde{t}) = \tilde{x}(\tilde{t}) + \tilde{v}(\tilde{t}+\Delta\tilde{t})\Delta\tilde{t} \quad \text{から}$$

$$\tilde{x}(\tilde{t}+\Delta\tilde{t}) = \tilde{x}(\tilde{t}) + \left(1-\frac{\zeta}{m}t_0\Delta\tilde{t}\right)\tilde{v}(\tilde{t})\Delta t - \frac{k}{m}t_0^2\Delta\tilde{t}^2\tilde{x}(\tilde{t})$$

$$= \left(1-\frac{k}{m}t_0^2\Delta\tilde{t}^2\right)\tilde{x}(\tilde{t}) + \left(1-\frac{\zeta}{m}t_0\Delta\tilde{t}\right)\tilde{v}(\tilde{t})\Delta t$$

(3) $t_d = \frac{m}{\zeta}$, $t_s = \sqrt{\frac{m}{k}}$ より、

$$\tilde{v}(\tilde{t}+\Delta\tilde{t}) = \left(1-\frac{t_0}{t_d}\Delta\tilde{t}\right)\tilde{v}(\tilde{t}) - \left(\frac{t_0}{t_s}\right)^2\Delta\tilde{t}\,\tilde{x}(\tilde{t})$$

(4) $t_s = t_0$ とすると

$$\tilde{v}(\tilde{t}+\Delta\tilde{t}) = \left(1-\frac{t_0}{t_d}\Delta\tilde{t}\right)\tilde{v}(\tilde{t}) - \Delta\tilde{t}\,\tilde{x}(\tilde{t})$$

また、初期条件は $x(0)=0$, $\dot{x}(0) = \frac{a}{t_0}$

$$\tilde{x}(\tilde{t}+\Delta\tilde{t}) = \left(1-\frac{k}{m}t_0^2\Delta\tilde{t}^2\right)\tilde{x}(\tilde{t}) + \left(1-\frac{\zeta}{m}t_0\Delta\tilde{t}\right)\tilde{v}(\tilde{t})\Delta\tilde{t}$$

$$= \left(1-\left(\frac{t_0}{t_s}\right)^2\Delta\tilde{t}^2\right)\tilde{x}(\tilde{t}) + \left(1-\frac{t_0}{t_d}\Delta\tilde{t}\right)\tilde{v}(\tilde{t})\Delta\tilde{t}$$

$$= \left(1-\Delta\tilde{t}^2\right)\tilde{x}(\tilde{t}) + \left(1-\frac{t_0}{t_d}\Delta\tilde{t}\right)\tilde{v}(\tilde{t})\Delta\tilde{t}$$

ここから $T^* = \frac{t_0}{t_d}$ がパラメータ. 運動が変わる境目を調べる.

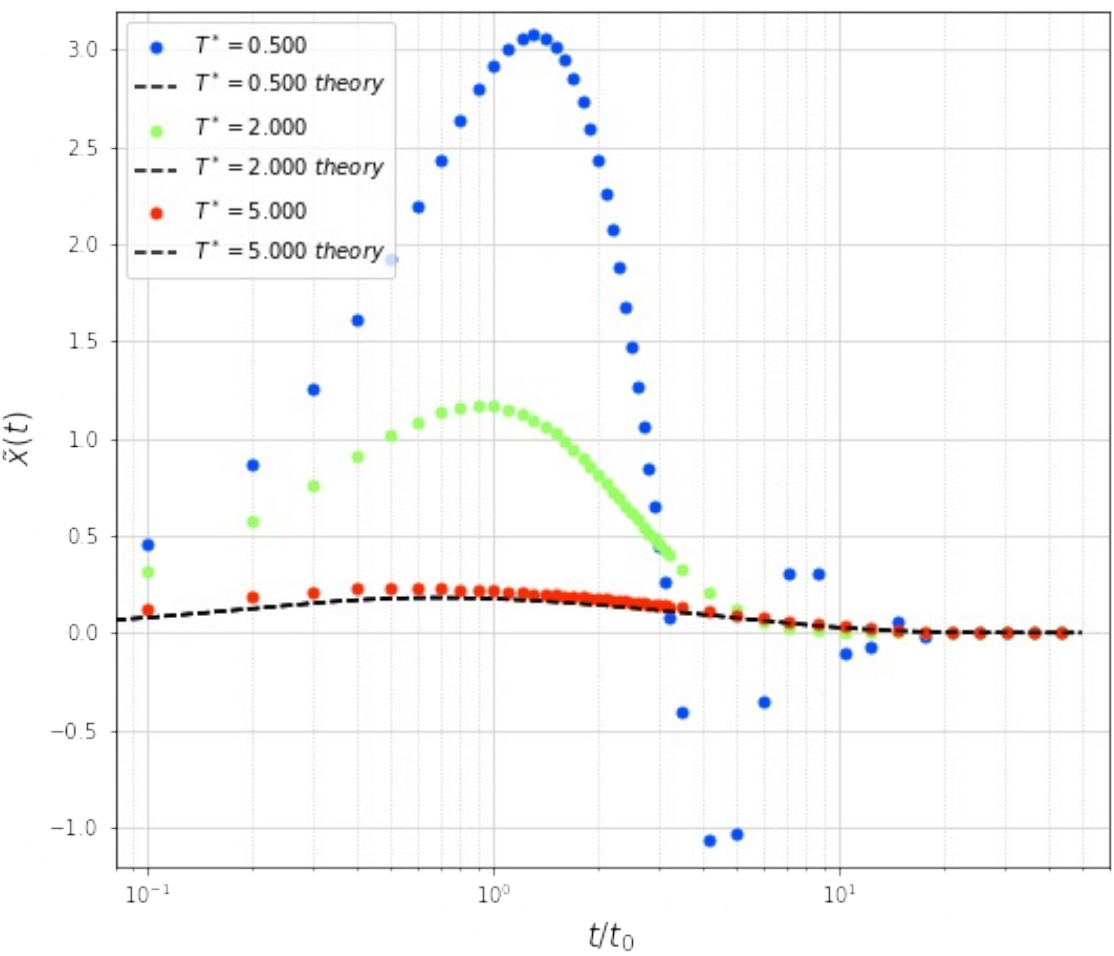
(1)より $D = \zeta^2 - 4mk$ の正負だったので、

(3)より $D = \frac{m^2}{t_d^2} - 4m\frac{m}{t_s^2} = m^2\left(\frac{1}{t_d}+\frac{2}{t_s}\right)\left(\frac{1}{t_d}-\frac{2}{t_s}\right)$

$$= m^2\left(\frac{1}{t_d}+\frac{2}{t_0}\right)\boxed{\left(\frac{1}{t_d}-\frac{2}{t_0}\right)}$$

この正負は青枠で決まる.

i.e. $t_0 - 2t_d$ の正負 $\Leftrightarrow$ $\underbrace{\frac{t_0}{t_d}}_{T^*} - 2$ の正負 ($\because t > 0$)



減衰振動と臨界減衰の理論推定値もplotする予定でしたが
計算ミスで上手く一致しませんでした. 今後の課題とします.

以下、理論解.

$$\ddot{x}(t)+\frac{1}{t_d}\dot{x}(t)+\frac{1}{t_s^2}x(t)=0$$

$x(t)=e^{\lambda t}$ と仮定すれば

$$\lambda^2+\frac{1}{t_d}\lambda+\frac{1}{t_s^2}=0 \quad \therefore \lambda_\pm=-\frac{1}{2t_d}\pm\frac{\sqrt{t_s^2-4t_d^2}}{2t_dt_s}$$

以降、$T^*=\frac{t_s}{t_d}$, $t_s=t_0$ などとする。

(i) $t_s-2t_d<0$ i.e. $T^*<2$ のとき

$$\lambda'_\pm=-\frac{1}{2t_d}\pm i\frac{1}{2t_0}\sqrt{4-T^{*2}}$$

$$x(t)=Ae^{-\frac{1}{2t_d}t}e^{+i\frac{1}{2t_0}\sqrt{4-T^{*2}}t}+Be^{-\frac{1}{2t_d}t}e^{-i\frac{1}{2t_0}\sqrt{4-T^{*2}}t}$$

初期条件 $x(0)=A+B=0$ より

$$x(t)=Ae^{-\frac{1}{2t_d}t}\left(e^{+i\frac{1}{2t_0}\sqrt{4-T^{*2}}t}-e^{-i\frac{1}{2t_0}\sqrt{4-T^{*2}}t}\right)$$

$$=Ae^{-\frac{1}{2t_d}t}\cdot 2i\sin\frac{\sqrt{4-T^{*2}}}{2t_0}t$$

また、

$$\dot{x}(0)=2Ai\frac{\sqrt{4-T^{*2}}}{2t_0}=\frac{a}{t_0} \quad \therefore A=-i\frac{a}{\sqrt{4-T^{*2}}}$$

$$\therefore \tilde{x}(\tilde{t})=\frac{2}{\sqrt{4-T^{*2}}}e^{-\frac{T^*}{2}\tilde{t}}\cdot\sin\frac{\sqrt{4-T^{*2}}}{2}\tilde{t}$$

(ii) $t_s - 2t_d = 0$ i.e. $T^* = 2$ のとき

$$x(t) = (A + Bt)e^{\lambda t} \qquad \left(\lambda = -\frac{1}{2t_d}\right)$$

初期条件 $x(0) = A = 0$

$\dot{x}(0) = B = \frac{a}{t_0}$ より

$$x(t) = a\tilde{t}e^{-\tilde{t}}$$

無次元化して, $\tilde{x}(\tilde{t}) = \tilde{t}e^{-\tilde{t}}$

(iii) $t_s - 2t_d > 0$ i.e. $T^* > 2$ のとき

$$x(t) = Ae^{\lambda_+ t} + Be^{\lambda_- t}$$

初期条件 $x(0) = A + B = 0$

$\dot{x}(0) = A(\lambda_+ - \lambda_-) = \frac{a}{t_0}$ $\therefore A = \frac{a}{\sqrt{T^{*2} - 4}}$ より

$$x(t) = \frac{a}{\sqrt{T^{*2}-4}}\left(e^{\lambda_+ t} - e^{\lambda_- t}\right) \qquad \left(\lambda_\pm t = \left(-T^* \pm \sqrt{T^{*2}-4}\right)\frac{\tilde{t}}{2}\right)$$

無次元化して,

$$\tilde{x}(\tilde{t}) = \frac{1}{\sqrt{T^{*2}-4}}\left(e^{\lambda_+ t} - e^{\lambda_- t}\right) \qquad \left(\lambda_\pm t = \left(-T^* \pm \sqrt{T^{*2}-4}\right)\frac{\tilde{t}}{2}\right)$$

以上より

(i) 減衰振動

$$\tilde{x}(\tilde{t}) = \frac{2}{\sqrt{4-T^{*2}}} e^{-\frac{T^*}{2}\tilde{t}} \cdot \sin\frac{\sqrt{4-T^{*2}}}{2}\tilde{t}$$

(ii) 過減衰

$$\tilde{x}(\tilde{t}) = \tilde{t}e^{-\tilde{t}}$$

(iii) 臨界減衰

$$\tilde{x}(\tilde{t}) = \frac{1}{\sqrt{T^{*2}-4}}\left(e^{\lambda_+ t} - e^{\lambda_- t}\right) \quad \left(\lambda_\pm t = \left(-T^* \pm \sqrt{T^{*2}-4}\right)\frac{\tilde{t}}{2}\right)$$