express ellipsoid in vtk format

Created	@May 18, 2022 2:01 AM
Tags	

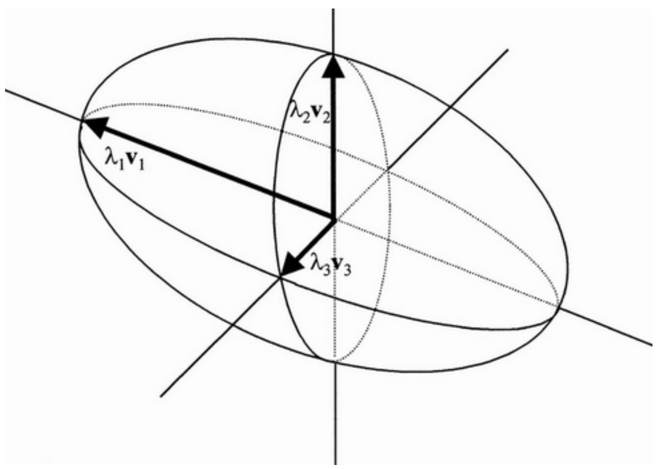
Ellipsoids are expressed in vtk format to visualize in Paraview

Parameters you need to visualize ellipsoid

- location of the ellipsoid center (rx, ry, rz)
- vector of the long axis (nx, ny, nz)

Diffusion tensor - expressing an ellipsoid using a tensor

Ellipsoids can be expressed by 3x3 tensor in Paraview, using eigenvalues and eigenvectors. Each column of the tensor is taken as the eigenvector and its norm as the eigenvalue.



The program defines eigenvectors and eigenvalues separately, then compose them later

$$\begin{bmatrix} v^1x & v^2x & v^3x \\ v^1y & v^2y & v^3y \\ v^1z & v^2z & v^3z \end{bmatrix} \begin{bmatrix} \lambda^1 \\ \lambda^2 \\ \lambda^3 \end{bmatrix}$$

Calculate eigenvectors from parameters nx, ny and nz.

- v^1, v^2, v^3 need to form orthogonal basis
- v^3 is predetermined by (nx, ny, nz).

Other two orthogonal vectors can be found using dot product and cross product

- $v^1 \cdot v^3 = 0$
- $v^1 \times v^3 = v^2$
- d ensures the norm of the vector is 1 for v^1, v^2, v^3
- After v^1, v^2, v^3 are found, scale them by the eigenvalues $\lambda^1, \lambda^2, \lambda^3$

$$\begin{bmatrix} {v^1}_x & {v^2}_x & {v^3}_x \\ {v^1}_y & {v^2}_y & {v^3}_y \\ {v^1}_z & {v^2}_z & {v^3}_z \end{bmatrix} = \begin{bmatrix} d \cdot nz & -d \cdot nx \cdot ny & nx \\ 0 & d \cdot (nx^2 - nz^2) & ny \\ -d \cdot nx & -d \cdot ny \cdot nz & nz \end{bmatrix}$$

d = $\frac{1}{\sqrt{nx^2+nz^2}}$

Change the size and shape of an ellipsoid through eigenvalues

There are two parameters responsible for the size and shape of an ellipsoids

- radius: scale determines the radius of an ellipsoid. radius = λ^2, λ^3
- ratio: ratio determines the shape of an ellipsoid ratio = $\frac{\lambda^1}{\lambda^2} = \frac{\lambda^1}{\lambda^3}$

vtk file format

<file_name>.vtk

```
# vtk DataFile Version 5.1
vtk output
ASCII
DATASET POLYDATA
POINTS <number of points> float
<X Y Z> location of a point
POINT_DATA <number of tensors>
TENSORS tensors double
<XX YX ZX XY YY ZY XZ YZ ZZ> eigenvectors/values of a point
SCALARS theta float 1
LOOKUP_TABLE default
<C> scalar value of a point
```

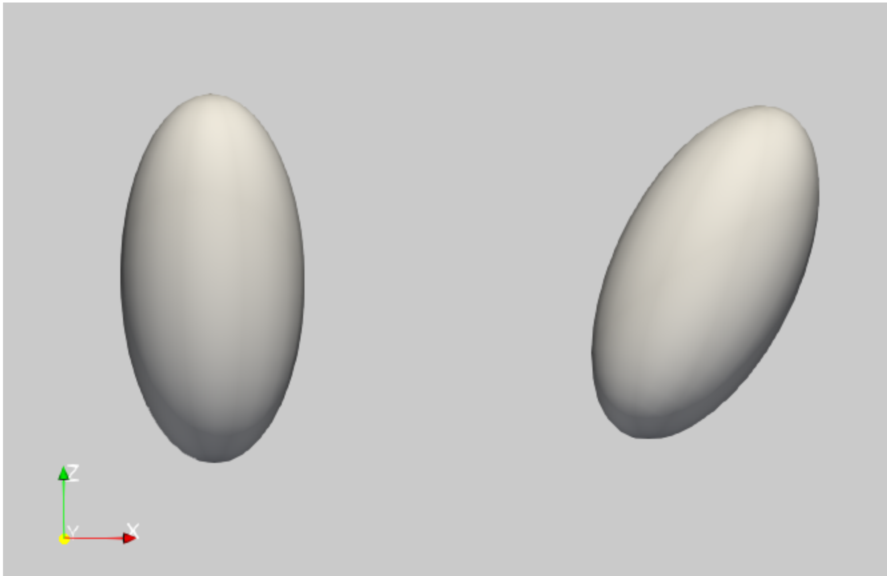
Note that

$$\begin{bmatrix} XX & YX & ZX \\ XY & YY & ZY \\ XZ & YZ & ZZ \end{bmatrix} = \begin{bmatrix} v^1{}_x & v^2{}_x & v^3{}_x \\ v^1{}_y & v^2{}_y & v^3{}_y \\ v^1{}_z & v^2{}_z & v^3{}_z \end{bmatrix} \begin{bmatrix} \lambda^1 \\ \lambda^2 \\ \lambda^3 \end{bmatrix}$$

example

```
# vtk DataFile Version 5.1
vtk output
ASCII
DATASET POLYDATA
POINTS 2 float
0 0 0
2 0 0
POINT_DATA 2
TENSORS tensors double
1 0 0 0 1 0 0 0 2
0.8944271909999159 0 -0.44721359549995787 -0.11180339887498944 0.9682458365518543 -0.2236067977499789 0.8660254037844386 0.4999999999999999
SCALARS phi float 1
LOOKUP_TABLE default
0
30
```

Resulting Ellipsoids


Z
Y
X