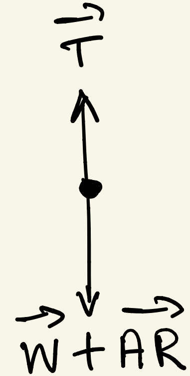
$$\vec{F} = m\ddot{\vec{x}}$$

$$\vec{T} + \vec{W} + \vec{AR} = m\ddot{\vec{x}}$$



$$\vec{W} = -mg\,\hat{\vec{y}}$$

$$\vec{AR} = -A|\dot{\vec{x}}^2|\,\hat{\dot{\vec{x}}}$$

$$\vec{T} = -\frac{\Delta m}{\Delta t}\vec{V_e} = -\frac{\Delta m}{\Delta t}|\vec{V_e}|\,\hat{\vec{x}}$$

$$\dot{\vec{V}} = \frac{\vec{F}}{m} \Rightarrow \frac{\Delta\vec{V}}{\Delta t} = \frac{\vec{F}}{m} \Rrightarrow \boxed{\vec{V}_{new} = \vec{V}_{old} + \frac{\vec{F}}{m}\Delta t}$$

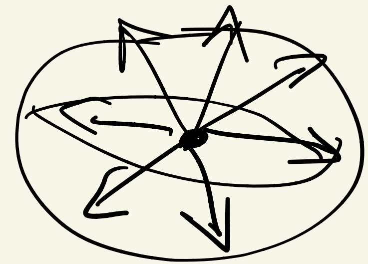
$$\dot{\vec{x}} = \vec{V} \Rightarrow \frac{\Delta\vec{x}}{\Delta t} = \vec{V} \Rrightarrow \boxed{\vec{X}_{new} = \vec{X}_{old} + \vec{V}\Delta t}$$

$$\vec{p}_{before} = m\vec{v}$$

$$\vec{p}_{after} = (m - \Delta m)(\vec{v} + \Delta\vec{V}) + \Delta m(\vec{v} - \vec{v}_{ejection})$$

$$\Delta\vec{p} = \cancel{m\vec{v}} + m\Delta\vec{V} - \cancel{\Delta m\vec{v}} - \underbrace{\Delta m\Delta\vec{V}}_{\text{ignore}} + \cancel{\Delta m\vec{v}} - \Delta m\vec{v}_e - \cancel{m\vec{v}}$$

$$\frac{1}{m}\frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{F}}{m} = \underbrace{\frac{\Delta\vec{V}}{\Delta t}}_{\vec{a}} - \frac{\Delta m}{\Delta t}\vec{v}_e \Rightarrow \vec{a} = \underbrace{\frac{\Delta m}{\Delta t}\vec{v}_e}_{-\frac{\vec{thrust}}{m}} + \frac{\vec{F}}{m}$$



$$\oint_S \vec{E}\cdot d\vec{S} = \frac{Q_{enc}}{\varepsilon_0}.$$

$$\vec{E} = \frac{Q_{enc}}{\varepsilon_0 \cdot 4\pi r^2}$$

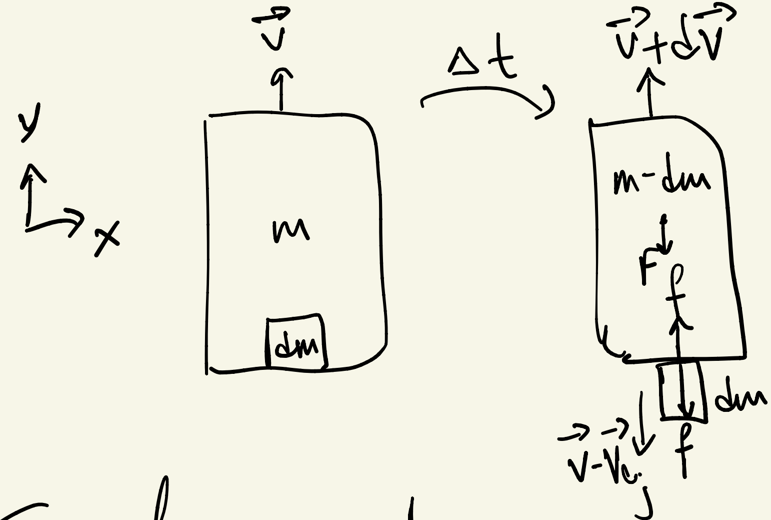
$$\vec{F} = q\vec{E} = \frac{Q_{enc}\, q}{4\pi\varepsilon_0 r^2}$$

$$\vec{F}_{AR} = -\frac{1}{2}\rho v^2 A C_D \hat{\vec{v}}$$

ρ = air density

$= \rho_0 e^{-y/H}$

$H = 8000\,m$



Impulse = force · time

= change in linear moment

Impulse on dm:

$$\underbrace{-\vec{f}\,dt} = dm(\vec{v}+d\vec{v}-\vec{v}_{ej})$$ (take $\vec{f}$ upwards hence need minus)

Impulse on rocket:

$$\vec{F}dt + \underbrace{\vec{f}dt} = m(\vec{v}+d\vec{v}) - m\vec{v} = m\,d\vec{v}$$

$$F\,dt - dm(\vec{v} - \vec{v}_{ej}) = m\,d\vec{v}$$

$$F - \frac{dm}{dt}(\vec{v} - \vec{v}_{ej}) = m\frac{d\vec{v}}{dt}$$

$$\vec{g} = -g\hat{\vec{y}}$$

$-\frac{1}{2}\rho v^2 A C_D \hat{\vec{v}}$ $\quad \rho$ = air density $= \rho_0 e^{-y/H}$ $\quad H = 8000\,m$

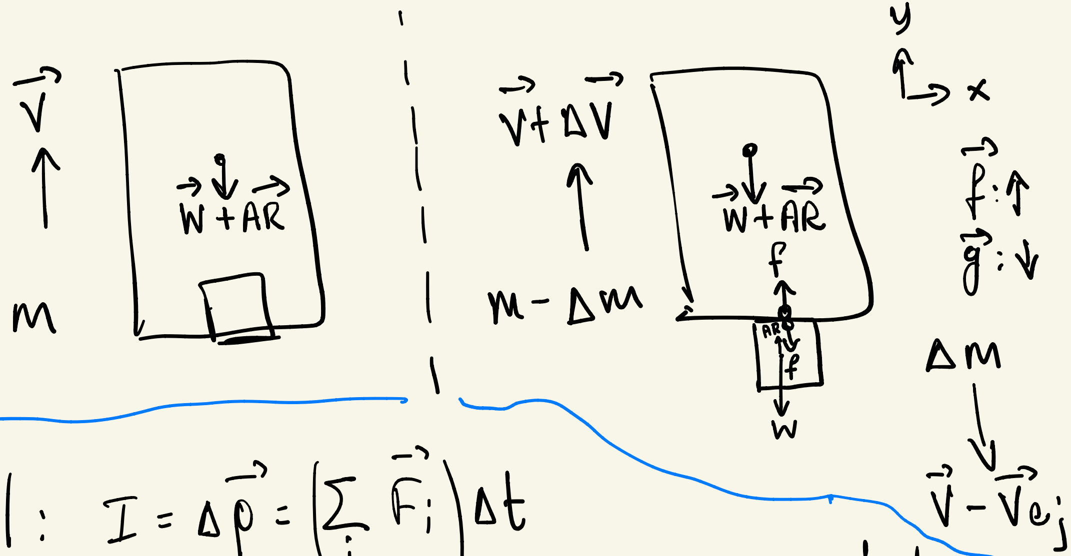
$$\Downarrow$$

$$\frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt} = \sum_i F_i = F + \vec{v}_{ej}\frac{dm}{dt} = \underbrace{m\vec{g}}_{\text{weight}} \underbrace{- \frac{1}{2}\rho_0 e^{-y/H} v^2 A C_D \hat{\vec{v}}}_{AR} \underbrace{+ \vec{v}_{ej}\frac{dm}{dt}}_{\text{thrust}}$$

$$\Delta v = \frac{\Delta t}{m}\left(m\vec{g} - \frac{1}{2}\rho_0 e^{-y/H} v^2 A C_D \hat{\vec{v}} \underbrace{- \frac{dm}{dt}\vec{v}}_{+L\vec{v}} + \underbrace{\vec{v}_{ej}\frac{dm}{dt}}_{-L\vec{v}_{ej}}\right) \quad (1)$$

$$\Delta x = v\,\Delta t \quad (2)$$

$$\frac{dm}{dt} = -L \Rightarrow \Delta m = -L\Delta t \quad (3)$$



$$\vec{F}_{AR} = -\frac{1}{2}\rho v^2 A C_D \hat{\vec{v}}$$

ρ = air density

$= \rho_0 e^{-y/H}$

$H = 8000\,m$

A = cross-sectional area

fuel: $I = \Delta\vec{p} = \left(\sum_i \vec{F}_i\right)\Delta t$

$$\Delta m(\vec{v} - \vec{v}_{e_j} - \vec{v}) = -\vec{f}\Delta t + \Delta m \overset{\text{neglect}}{\cancel{\vec{g}}}\Delta t - \frac{1}{2}\rho \overset{\text{neglect}}{\cancel{|(\vec{v} - \vec{v}_{e_j})|^2 A}} C_D (\widehat{\vec{v} - \vec{v}_{e_j}})\Delta t$$

$$-\Delta m \vec{v}_{e_j} = -\vec{f}\Delta t$$

R: $$(m - \Delta m)(\vec{v} + \Delta\vec{v}) - m\vec{v} = m\vec{g}\Delta t - \frac{1}{2}\rho|\vec{v}|^2 A C_D \hat{\vec{v}}\Delta t + \vec{f}\Delta t$$

$$= m\Delta\vec{v} - \underline{\Delta m \vec{v}}$$

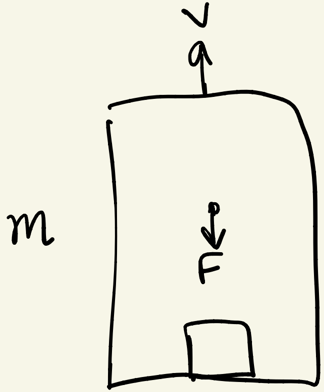
$$m\Delta\vec{v} - \Delta m\vec{v} = \vec{F}\Delta t + \Delta m\vec{v}_{ej}$$

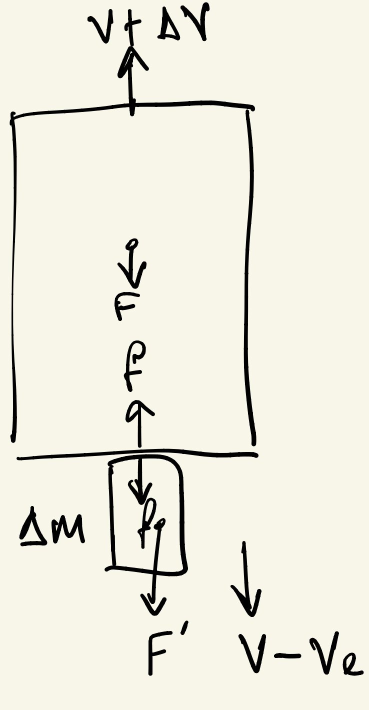
$$m\frac{\Delta\vec{v}}{\Delta t} = \vec{F} + \frac{\Delta m}{\Delta t}(\vec{v}_{ej} + \vec{v})$$

$$v' = v - v_{ej}$$

$$v + \Delta v - v + v_{ej}$$

$$\Delta v + v_{ej}$$





$$\Delta m: \quad \Delta m\,(v - v_{ej} - v) = -f\Delta t - \cancel{f'\Delta t}$$

$$R: \quad (m-\Delta m)(v+\Delta v) - mv = -F\Delta t + f\Delta t$$

$$\Delta m\, v_{ej} = f\Delta t$$

$$m\Delta v - \Delta m\, v = -F\Delta t + \Delta m\, v_{ej}$$

$$m\frac{\Delta v}{\Delta t} - \frac{\Delta m}{\Delta t} v = -F + \frac{\Delta m}{\Delta t} v_{ej}$$

$$\frac{d(m\vec{v})}{dt} = \vec{F} + \frac{dm}{dt}\vec{v}_{ej}$$

$$\frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt} = \vec{F} + \frac{dm}{dt}\vec{v}_{ej}$$

$$m\frac{d\vec{v}}{dt} = \vec{F} + \frac{dm}{dt}(\vec{v}_{ej} - \vec{v})$$

$$m\Delta\vec{v} = \vec{F}\Delta t + \Delta m\,\vec{v}_{ej} - \Delta m\,\vec{v}$$

$$m\Delta\vec{v} + \Delta m\,\vec{v} = \vec{F}\Delta t + \Delta m\,\vec{v}_{ej}$$

Final:

$$m_{Rocket\ after} = m_{Rocket\ before} - \Delta M$$

$$\frac{\Delta m_{Rocket}}{\Delta t} = -\frac{\Delta M}{\Delta t}$$

$$\vec{p}_{Rocket\ after} - \vec{p}_{Rocket\ before} = -\Delta M(\vec{V}_R + \vec{V}_{Ej}) + \vec{F}\Delta t$$

$$\frac{\Delta p_{Rocket}}{\Delta t} = -\frac{\Delta M}{\Delta t}(\vec{V}_R + \vec{V}_{Ej}) = \frac{\Delta M_R}{\Delta t}(\vec{V}_R + \vec{V}_{Ej})$$

$$\frac{\Delta(m_R \vec{V}_R)}{\Delta t} = \frac{\Delta M_R}{\Delta t}\vec{V}_R + m_R \frac{\Delta \vec{V}_R}{\Delta t}$$

$$\cancel{\frac{\Delta m_R}{\Delta t}\vec{V}_R} + m_R\frac{\Delta \vec{V}_R}{\Delta t} = \cancel{\frac{\Delta m_R}{\Delta t}\vec{V}_R} + \frac{\Delta m_R}{\Delta t}\vec{V}_{Ej} + \vec{F}$$

$$m_R\frac{\Delta \vec{V}_R}{\Delta t} = \frac{\Delta m_R}{\Delta t}\vec{V}_{Ej} + \vec{F}$$

$$= -L\,|\vec{V}_{ej}|\,(-\hat{\vec{V}}) + \vec{F}$$