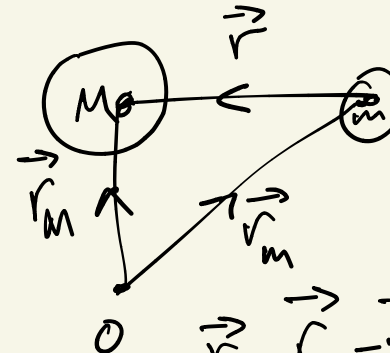
$$\vec{F}_{on\ m} = -\frac{GMm}{r^3}\vec{r} = m\vec{a}$$



$$\vec{r} = \vec{r}_M - \vec{r}_m$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\frac{GM}{r^3}\vec{r}$$

$$\frac{\Delta\vec{V}}{\Delta t} = -\frac{GM}{r^3}\vec{r}$$

$$\vec{V}_{new} = \vec{V}_{old} - \frac{GM}{r^3}\vec{r}\cdot\Delta t$$

$$\frac{d\vec{x}}{dt} = \vec{V}$$

$$\vec{x}_{new} = \vec{x}_{old} + \vec{V}_{old}\cdot\Delta t$$

$$\vec{X}_{new} = \vec{X}_{old} + \vec{v}\,\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

$$\ddot{x} = a \qquad \dot{x} = at + u \qquad x = ut + \frac{at^2}{2} + c$$

$$\vec{v}_{new} = \vec{v}_{old} + \frac{1}{2}(\vec{a}_{new} + \vec{a}_{old})\Delta t$$

Leapfrog:

$$a_i = A(x_i)$$

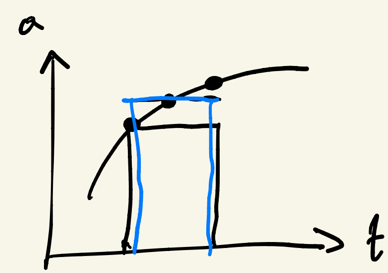
$$v_{i+\frac{1}{2}} = v_{i-\frac{1}{2}} + a_i \Delta t \rightarrow \boxed{\begin{aligned} x_{i+1} &= x_i + v_i \Delta t + \tfrac{1}{2} a_i \Delta t^2 \\ v_{i+1} &= v_i + \tfrac{1}{2}(a_i + a_{i+1}) \Delta t \end{aligned}}$$

$$x_{i+1} = x_i + v_{i+\frac{1}{2}} \Delta t$$

time symmetric
conserves energy

$$x_i = x_{i+1} - v_i \Delta t - \frac{1}{2} a_i \Delta t^2$$

$$v_i = v_{i+1} - \frac{1}{2}(\underline{a_i + a_{i+1}}) \Delta t$$



$$\int a\,dt = v$$