Continuous Vertical Grain Sorting for Telemac & Sisyphe v6p2

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***Abstract*— Vertical grain sorting is one of the leading processes for many hydrodynamic and morphodynamic simulations. Like most hydraulic & morphological software packages Telemac & Sisyphe calculate sediment transport, sediment sorting and development of bed forms depending on the active layer of the bed. The empiric active layer thickness concept has been developed in 1971 by Hirano and expanded by Ribberink among others to fit the numerical models and demands of their time. With new high performance computers and the here presented continuous vertical grain sorting models with dynamically estimated active layer thickness it is now possible to overcome several limitations of this meanwhile 40 year old concept. Results of this model are compared to the classic Hirano-Ribberink implementations using measured data off 3 flume experiments performed at the Universities of Delft, Zürich and Illinois to validate and proof this concept.**

I. The Hirano-Ribberink Vertical Grain Sorting Model (HR-VSM)

A. Motivation

Many medium and long term hydrodynamic numerical simulations of rivers cannot be operated successfully without considering the flow-sediment interaction. Sediment layer thickness and grain size distributions influence the bed roughness and the flow field. Vice versa the flow field sorts sediments and develops bed forms. Telemac 2D/3D and Sisyphe enables interactive coupling of morphodynamic and hydrodynamic simulations and includes the state of the art grain sorting algorithm for numerical morphodynamic models: The Hirano (1971) concept with extensions of Ribberink (1987) and other research groups.

The basic idea of Hirano is the interaction of flow with a fully mixed top-most layer of the sediment, while the deeper sediment stratigraphy remains untouched. The thickness of this active layer describes the common depth of morphological processes in the riverbed per time step, which equals the maximum impact of the hydraulic shear forces. The empirical variable active layer thickness (ALT) is usually chosen as a multiple of the characteristic grain diameter and the mean height of bed forms. For numerical reasons it is the maximum depth that can be eroded in one time step. Below the active layer follows another empirical layer, the active stratum, which has no measurable equivalent in nature. It is used to refill or reduce the active layer to the predefined thickness after evolution calculations changed the active layer thickness.

The implementation in Telemac / Sisyphe v6p1 adds up to 7 more storage layers below these 2 layers. They keep different sediment mixtures until they are activated by erosive processes. Within 1 time step evolution only affects the active layer and the active stratum.

B. Limitations

This meanwhile 40 year old concept was developed as an averaging empirical approach at a time where the available computational performance was 1010 times less than in 2012. Saving computational resources by spatial averaging was necessary but comprehends several limitations:

- The number of discrete layers is limited to 9.
- The a priori chosen layer thickness depends on dune heights, grain roughness, depth of the rigid bed, mesh density and other parameters.
- The active layer + active stratum concept in fact mixes the second layer as well. See Fig. 2.
- Contrary requests of the active layer concept cannot be fulfilled in every case. E.g. the theoretical active layer filling is homogenous and cannot be subdivided. If the topmost layer is used to reproduce the stratigraphy, it cannot be used for the natural impact depth of the driving shear forces at the same time.

While the first two limitations could be easily removed, the last two require a new concept with less averaging effects on the bookkeeping.

II. The New Continuous Vertical Grain Sorting Model (C-VSM)

A. Divided in bookkeeping model and transport model

One of the main disadvantages of the Hirano method is the continuous mixing of the active layer and the active stratum after every time step, even with minor or without evolution.

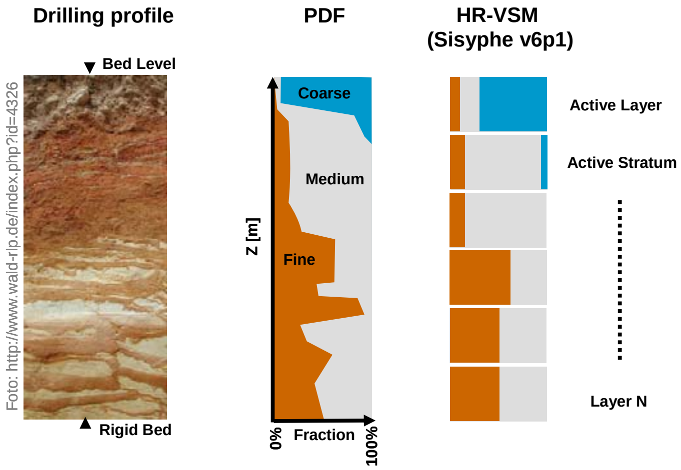

Figure 1. Legend for all following diagrams: Imagine the new continuous vertical grain size sorting model C-VSM as the drilling profiles probability density functions in polyline form, where grains are sorted from fine to coarse, left to right, resulting in 100% at any depth. The widespread Hirano / Ribberink layer sorting model HR-VSM is a simplified version.

This is done to numerically refill or reduce the active layer content with active stratum content.

- For example to restore the full theoretical thickness of the active layer after evolution
- or even when just a dynamic active layer thickness formula resizes the thickness based on turbulence shuffled flow conditions.

But it is not necessarily a process that happens in nature. Figure 2 shows the dynamic formula effect to the ALT, due to turbulence influenced shear stress variability in between 2 time steps (ALT i = +/- 0.000001 * ALTi-1), but with a long term equilibrium state (E∞ ~ 0). After 1 000 000 refill and reduction cycles for an alignment of the active layer and active stratum each time, the content of both layers is almost fully mixed, without any change in bed level. Even though this is mathematically correct it leads to wrong vertical grain sorting. Hirano demanded morphological activity ends below the active layer, but here the mixing reaches one layer deeper.

The solution is to separate the grain storage model from the evolution calculation model. This means that 2 datasets are used for the grain sorting. Dataset 1 is a storage model that keeps the information about the vertical grain sorting, without regard to any layer boundary, like the drilling profile, with as much resolution as possible. Dataset 2 is again the Hirano active layer, newly filled with the actual average grain mixture

of the equivalent top sections of the Dataset 1. It is newly averaged at each time step and used for evolution calculations. This works not different than the original Hirano concept. But now any evolution is not shifted to the active stratum, but to the fine resolution storage model, which gets an addition or subtraction of volumes. Exchange goes from top to bottom as deep as necessary to get the needed volumes. It is not anymore a shifting of fractions over the full strength of a theoretical layer.

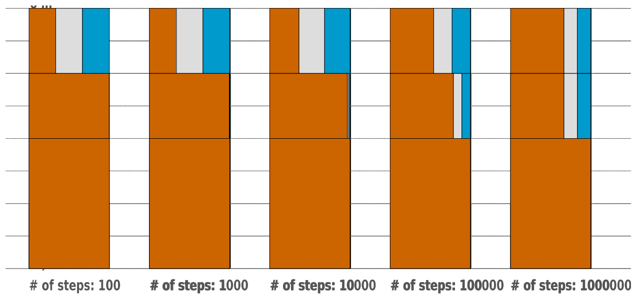

Figure 2. Development of an academic sorting profile in HR-VSM (Sisyphe v6p0). For a channel with equilibrium transport, but short term evolution of 0,0001% of the active layer thickness.

B. *The new book keeping model*

We decided to add a depth dependent book keeping model for each grain size fraction with unlimited numerical resolution for each node of the 2D morphological model. As the transport model remains unchanged and does not directly interact with the bookkeeping model, both are kept in separate software modules without knowing the existence of each other.

As shown in Figure 1 a drilling core is the physical equivalent to the storage model. The numerical implementation is a set of depth dependent probability density functions (PDF) for each grain size fraction. The sum of all grain fraction PDFs is always 100% at any depth. These PDFs are stored as polylines. The number of line sections is theoretically unlimited. For visualization the grain size fractions of each profile are always drawn additive from fine (left) to coarse (right) (Figure 1 is the legend to all other figures). In contrast to the classic Hirano-Ribberink layer model there are no theoretical limitations to the discretization of thicknesses except the capabilities of the hardware.

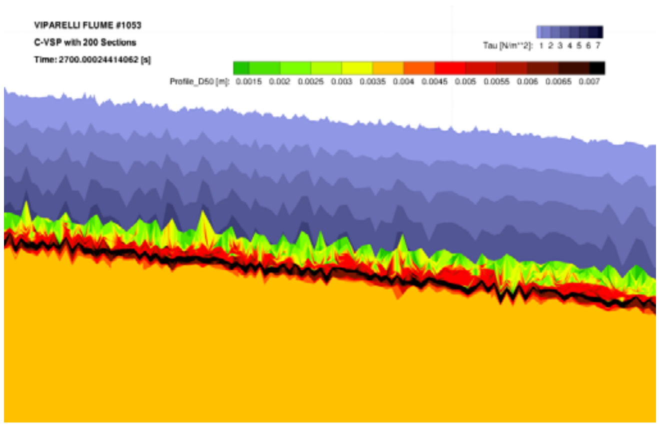

Figure 3. In case of immobile coarse fractions within the active layer, the active layer itself can develop a sub ordinary stratigraphy which is relevant for the development of bed forms and armoring effects. This figure shows the mean grain diameter d50 (green to red) in the middle of a laboratory flume and the corresponding hydraulic impact, shown as shear stress (blue). Fine grains separate from the mixture and move on top of the coarse ones, which stay behind as an immobile under layer.

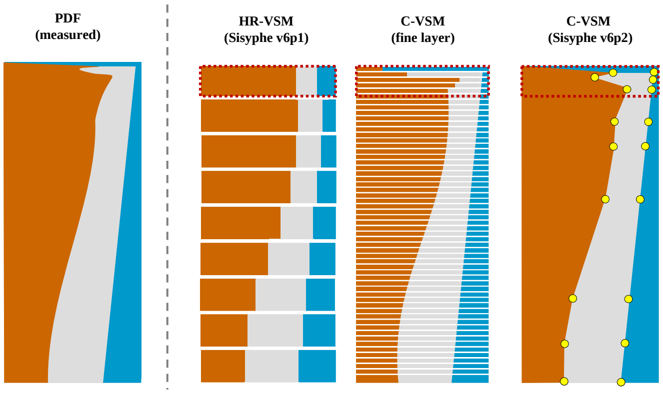

Figure 4. Comparison of stratigraphy abstraction models with active layer (red dotted): Measured Profil (far left); Hirano/Ribberink (left), Continuous model based on very fine layers (middle) , Continuous model based on polylines (right).

Additionally a fraction gradient over depth is modelled as a single trapezoid instead of an approximation with many rectangular layers. This saves resources. The main benefit can be found in simulations where the real layering is finer than the ALT (Fig 4).

This is especially helpful for fine grain lentils or armouring layers within the sediment body which might significantly influence the calculation in a subsequent time step. Figure 4 shows the main advantage of the polyline C-VSM version against the fine layer C-VSM versions. To describe declining fractions of a single material only few points (= double precision variables) are necessary while the fine layer sorting profile needs many datasets more.

1) *Depositing sediments:*

Sedimentation can occur in 2 modes. The first mode is a plain sedimentation of one or all fractions. The already lying sediments are not moved, incoming sediments can only be placed on top. Numerically this is a new polyline section on top of the C-VSM. The bed level is lifted while storing the new material. See Figure 5(c), which is the 5(b) plus sedimentation on top.

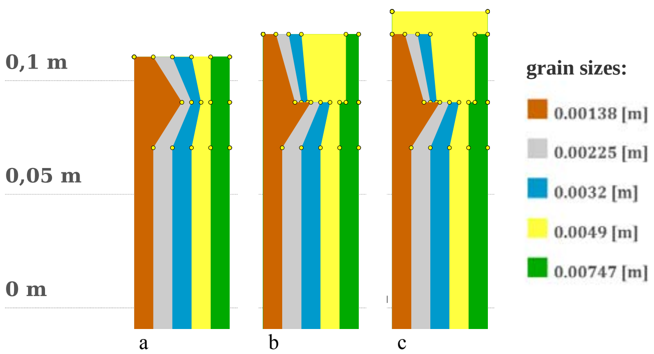

Figure 5. There are 2 modes of adding fractions: a) a random profile in its initial state; b) Sedimentation of fraction #4 mixed in the topmost section of (a); c) Only sedimentation of fraction #4 on top of the profile (b), without mixing in the top section.

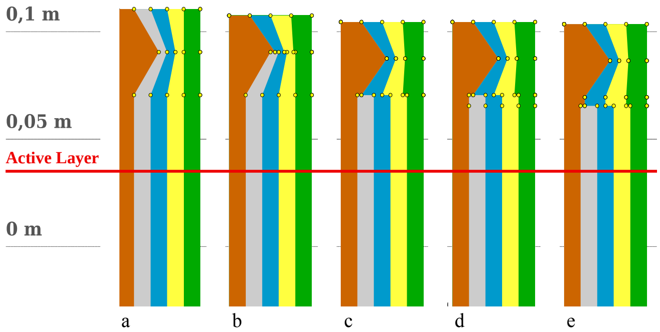

Figure 6. Eroding a certain volume of grain fraction No2 (grey) from the storage model. Example in 4 Steps: a) Initial state; b) & c). Erosion of sorting profile section 1 and 2 leads to a lowering of the surface, while the sum of all fractions is normalized to 100%. d) & e) Section 3 is not eroded completely, but split into two sections where only the upper part is eroded.

The second mode is sedimentation of some coarse grain fractions while finer fractions are eroded from the active part of the bed. Here the sediments within the active layer are also in motion. Therefore depositions are placed as an addition to the active part of the bed, inside the existing top section. See Figure 5(b) which adds material in the top section of (a).

2) *Eroding sediments:*

Erosion always starts from bed level and ends where the transport capacity is satisfied. This leads to complete or partial erosion of grains within C-VSM polyline sections. If a grain class within a section (which equals a volume!) is eroded completely, like grain class 2 in Figure 6b & 6c, only the bed level elevation has to be updated and the fraction variables of the remaining sediment have to be normalized to 100%. If the section contains more material than can be eroded, only the necessary volume is extracted. This forces a splitting into two parts (6d). It remains one section without grain class 2 and below another section with grain class 2 (6e).

3) *Avoiding excessive fragmentation:*

After a longer series of sedimentation and erosion cycles the C-VSM is fragmented in many very small sections, which is sometimes only 10-15 m strong. To avoid too excessive memory consumption a compression algorithm reduces the number of sections based on user defined quality threshold values. This algorithm is a modified version of the Douglas Peuker line generalization algorithm. It works iterative until a maximum fraction error or a minimum point number is reached. See figure 7.

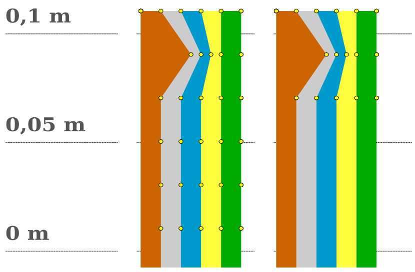

Figure 7. The yellow points mark the polyline PDF sections of the C-VSM. The intial profile (left) is simplified with the line generalization algorithm without significant change in volume.

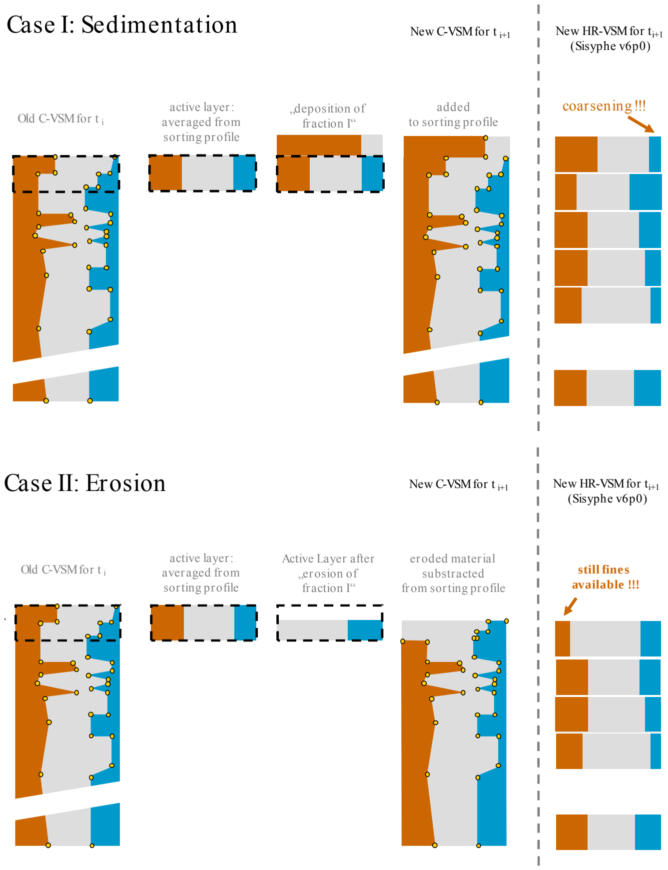

Figure 8. The changed and finer book keeping of Continuous Vertical Sorting Profiles (C-VSM) avoids smearing problems due to averaging in the classic Hirano/Ribberink layer method (HR-VSM). This sketch shows the behavior for sedimentation and erosion of both algorithms within one time step.

4) *Updating the transport model:*

When the bed load formula is calculated, the supplied active layer hasn't got the content of the active layer of the last time step. It is an updated content from the C-VSM by integration of the grain class volumes over the newly estimated active layer thickness.

C. *Reduced smearing effects*

Figure 8 shows clearly how the high resolution vertical sorting is preserved even though the transport model itself works with the averaged and therefore coarse Hirano active layer. It shows 3 sub steps of one time step for both deposition and erosion.

- The active layer is averaged from the bookkeeping.
- Evolution is calculated with any well known transport formula and the Exner equation based on the active layer and without impact on the bookkeeping.
- Evolution is added or subtracted from the top of the PDF.

The resulting C-VSM surface is free from eroded and buried materials, while the HR-VSM mixes in other materials from the active stratum as well.

D. *Dynamic active layer thickness*

The original Hirano idea assumes the active layer bottom as the limit of the moving part of the bed. It is clear that a fix active layer thickness can not account for changing hydraulics, morphology and grain sorting.

This empirical mean value is hard to measure and

- has growing uncertainties the coarser the spatial steps get (mesh width),
- is sensitive to the length of the observed morphological activity (time step),
- and is dependent on the shear stress magnitude.

Replacing these influence factors with mean values increases the morphological uncertainties. A collection of formulas for dynamic ALT approximations during a simulation are available in Malcherek (2007), using the bottom shear stress τ_B, the critical shear stress τ_C, the characteristic diameters d_{50}, d_{90}, d_{MAX} and transport stage parameter D^*.

- Hunziker & Günther

$$ALT = 5 * d_{MAX} \tag{1}$$

- Fredsoe & Deigaard 1992

$$ALT = \frac{2 \cdot \tau_B}{(1-n) \cdot g \cdot (\rho_S - \rho) \tan\phi} \tag{2}$$

- van RIJN 1993

$$ALT = 0.3 \cdot D_*^{0,7} \cdot \left(\frac{\tau_B - \tau_C}{\tau_B}\right)^{0,5} \cdot d_{50} \tag{3}$$

- Wong 2006

$$ALT = 5 \cdot \left(\frac{\tau_B}{(\rho_S - \rho) \cdot g \cdot d} - 0,0549\right)^{0,56} \cdot d_{50} \tag{4}$$

- Malcherek 2003

$$ALT = \frac{d_{90}}{1-n} \max\left(1, \frac{\tau_B}{\tau_C}\right) \tag{5}$$

Other parameters: ρ_S … density solid; ρ … density water; n … porosity; $\tan\phi$ … friction angle

The implementation of these formulas is possible for HR-VSP with very long morphological time steps. But it is limited, due to the smearing problem shown in Figure 2. Especially in pulsating eddy zones the ALT changes by several 100% within few time steps instead of the above shown 0.0001%. This increases the smearing problem.

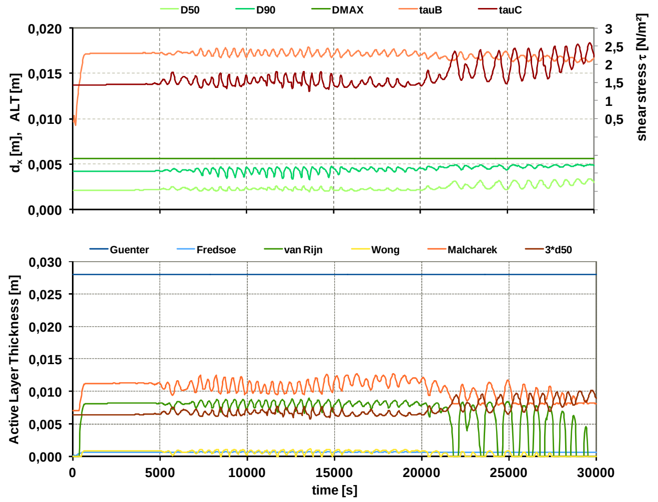

Figure 9. Development of the shear stress, the characteristic diameters and the active layer thicknesses for a random point in the middle of the Guenter flume, calculated with the five above mentioned formulas. Strong variations between the formulas force a careful selection.

With the new C-VSP this problem is obsolete and the formulas for a dynamic ALT can be used over longer simulation periods in coupled morphodynamic and hydrodynamic models.

No further recommendation is given on these formulas, as their usability is strongly dependent to the project. Figure 9 shows the strong variance of these formulas for the later described Guenter flume.

E. *Possible future extensions*

Another advantage of the separate bookkeeping method is the possibility to add other geotechnical algorithms like time dependent compacting, shrinking and changes of the porosity, as well. Furthermore we recommend the development of a vertical mixing algorithm which accounts for moving sediments without sedimentation or erosion.

III. Validation of the New C-VSM

The capabilities of the C-VSM were tested against 3 laboratory flume experiments with a total of 25 different parameter combinations. Stability, usability and the bandwidth of the C-VSM is demonstrated in this chapter by picking demonstrative aspects of one suitable flume. Even though we did not calibrate all 25 setups, it is technically possible. The following examples show some new possibilities and the model behavior.

A. *Validation case: BLOM FLUME*

1) *Setup:*

Astrid Blom (2003a, 2003b, 2008a, 2008b) conducted flume experiments at Delft Hydraulic Laboratories in 1998 to investigate vertical sorting processes. She used the measured data to develop her own numerical vertical sorting model. The authors decided to use these experiments as validation cases as well.

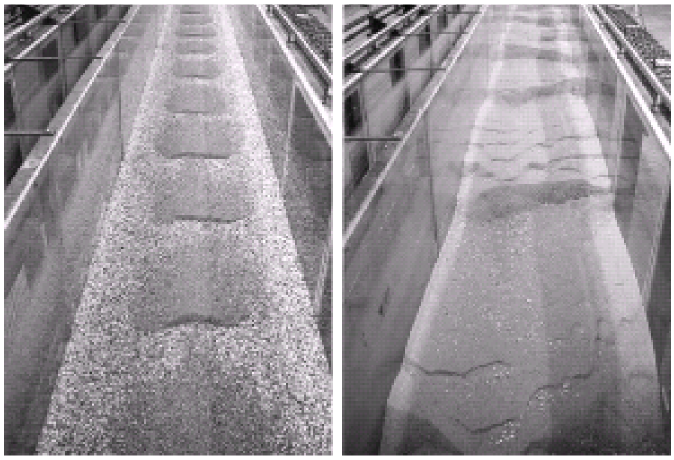

Figure 10. Pictures by Astrid Blom (2003) of flume experiment before (left) and after (right) experiment B2.

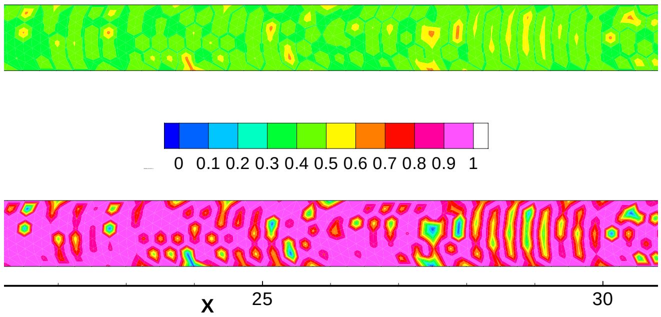

Figure 11. Top view a.k.a. flume surface. Development of the fine grain fraction after 4h. Initial values (t=0) for all grain fractions f and both models have been fi = 0.3333.

This rectangular profile laboratory flume was 50m long, 1m wide and filled with an artificial 3 modal grain mixture (d50 = 0.00068 m, 0.0021 m and 0.0057 m, 33.3% each). Several flume configurations can be found in her publications, we calculated all with Telemac and the C-VSM, but want to focus on the "Series B" for this publication (see Figure 10 for a picture).

For a discharge of 0.267 m3/s a slope of 0.0018 produced a normal flow depth of 0.386 m for case "B2". The sediments were recirculated. Both, the physical and numerical experiments require the first half of the flume length to gain undisturbed hydro- and morphodynamic conditions, thus only the second half is used for comparison.

2) *Results:*

Results of the C-VSM are shown in Figure 12 (averaged) and Figure 11 (2D top view). Bed forms occurred and fine material moved over coarse material.

The C-VSM clearly shows that the surface material has no coarse fraction any more as the fines move in form of dunes on top of them. This effect has been observed by Astrid Blom (see Figure 12), but is clearly missing in the HR-VSM.

These results subsequently show a better grain and form roughness approximation for the C-VSM.

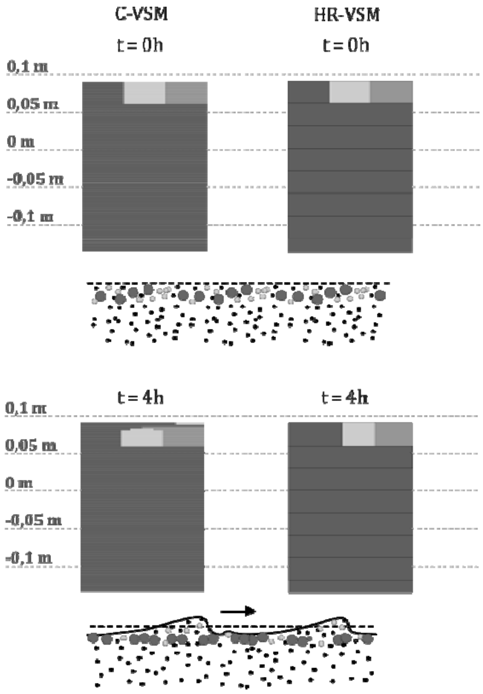

Figure 12. Development of bed form and grain sorting within 4h for the Blom series B. Calculated with Telemac / Sisyphe v6p1. C-VSM and HR-VSM show averaged values between flume stage 25 m to 47 m.

B. Validation case: VIPARELLI FLUME

1) Setup:

Enrica Viparelli et al. setup a rectangular profile flume of 17 m length and 0.61 m width with a slope between 0.0046 and 0.0079. A sediment mixture between 1 and 10mm (d50=7.8 mm) was filled in 18.5 cm strong. 9 Experiments were run with various discharge and duration configurations resulting in water depths H between 6.9 and 8.7 cm.

Transport rates, water depth, resulting slope and the shift of surface grading lines are documented well in Viparelli (2010). A Telemac2D / Sisyphe v6p1 model with 3800 points was used to calculate the flume experiments. The initial grain mixture was set to 5 grain sizes with a fraction of 0.2 each.

2) Results:

Viparelli describes the results of the flume as free from bed forms and lateral effects. This is the same for the C-VSM results. There is no clear trend of the grain sorting in neither direction, maybe due to the equilibrium conditions. But we have a strong variance on single points especially close to the inflow and outflow boundaries.

Figure 13 shows 5 C-VSM profiles from probing points in the middle of the flume after 36000 hydraulic time steps of 0.1 s, what equals 1h. Due to an extended morphological time step this equals 720 calculations of the morphology. These 5 profiles with totally different character are selected because they show the capability of the bookkeeping system. Each of the shown profiles has its own development history out of erosion and sedimentation cycles and consists of 12 to 48 sections.

The black line marks the active part of the bed at the last time step, calculated with van Rijn formula. The active layer thickness varies between 2.5 and 4.5 cm as the shear stress varies and the different grain mixtures of the active zone have a different d50 value. An interesting fact is that the last 2 profiles are not mixed in as deep as the average active layer thickness reaches. This means that so far no erosion took the maximum of the available material. Maybe the dynamic ALT estimation according to van Rijn overestimates this case and one of the other formulas shown in Figure 9 would perform better. This problem also inspires to develop a new formula for the ALT, dependent on the maximum impact thickness of the last time steps morphological processes.

3) Development of the C-VSM sections: Another important result proofs the robustness of the dynamic C-VSM data management. Initial fears that an uncontrolled fragmentation of the C-VSM might increase the memory demands significantly can be disarmed.

Starting with only 12 sections for the C-VSM in Figure 14, sediment movements fragmented the profiles and the number of data points rises until a threshold of 80 is reached. Now the profile simplification algorithm merges neighbouring line sections in the profile while respecting a maximum fraction error which is user defined 10-12 here. The number of sections drops below 30. As the Viparelli flume is in equilibrium state the cycle of growing and shrinking is in equilibrium as well.

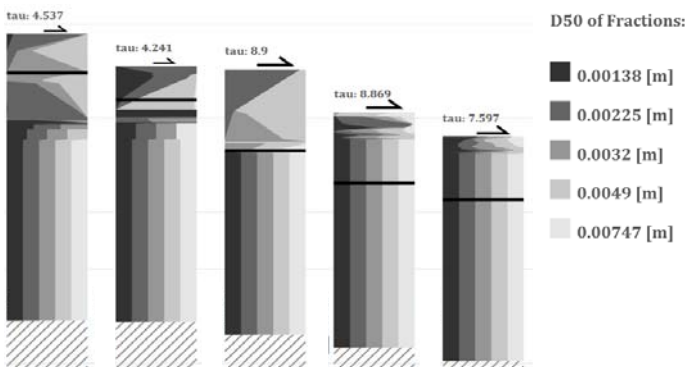

Figure 13. C-VSM profiles from 5 stations of the flume show a different morphological history, even though the surface mixture d50 is almost 3mm for all of them. Pulsating shear stress values τ_B(tau) of the last time steps are shown in N/m² and the resulting active layer thickness according to the van Rijn formula is marked as a thick black line.

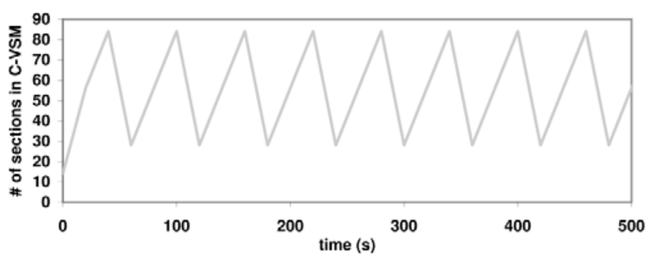

Figure 14. The number of sections in the C-VSM, averaged over all points in the middle of the flume, increases and decreases during changing erosion and sedimentation cycles. If the number of sections gets too high, recompression is performed, but an accuracy threshold is kept.

C. Validation Case: GUENTER FLUME

1) Setup:

Arthur Guenter published his laboratory flume experiments at the ETH Zürich in 1971. The rectangular flume dimensions were X=40m / Y=1m / Ie~0.002 / H<0.1m. Different to the other 2 test cases he didn't use recirculation. Therefore this flume is especially useful to observe the development of armoring layers and the influence of turbulence on the critical shear stress.

We use a Telemac2D + Sisyphe model based on a 5000 point irregular triangle mesh. All simulations are based on a 6 size fraction sieve line.

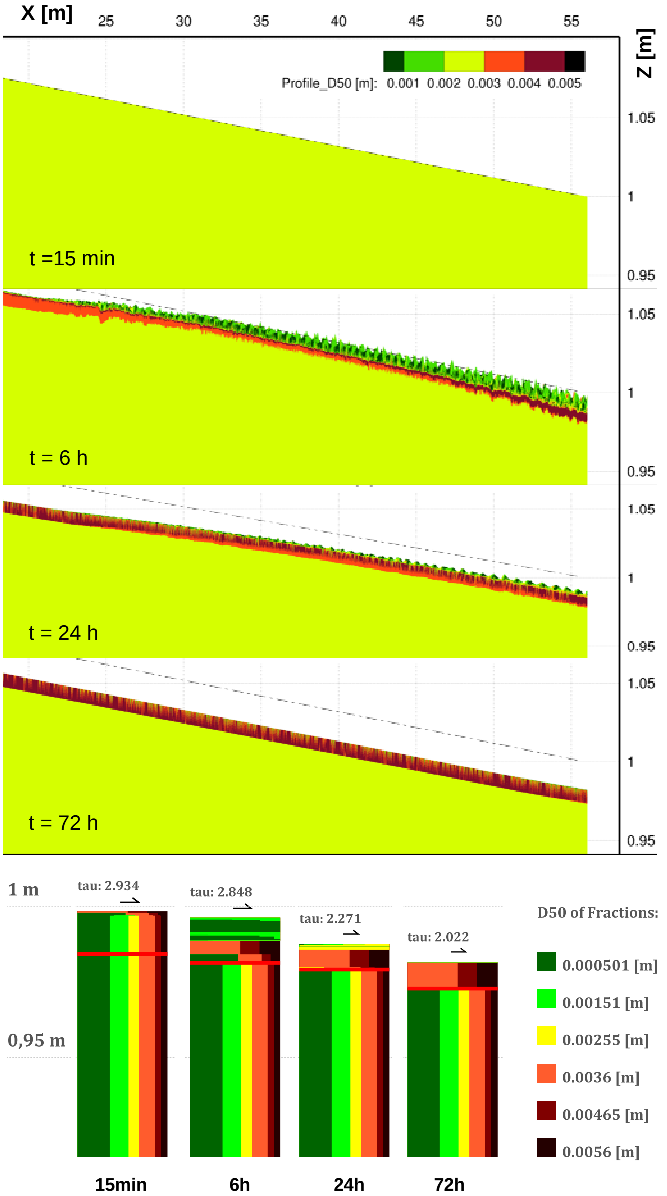

Figure 15. Top: d50 [m] - longitudinal cut through the Günther flume for T=15min; 6h; 24h; 72h. Bottom: According vertical sorting profiles of flume middle axis for x =55m.

2) Results:

Figure 15 shows a longitudinal cut for d50 through the middle of the Guenter flume along the X axis and the corresponding C-VSM charts for X=55m; Y=0.5m.

Obviously only the 3 finer fractions are moving in higher percentage.

After 6h most of the fine fractions are in motion on top of the coarser ones. A clear stratigraphy can be seen now. The available fine sediments are already getting less in the upper half of the flume. At the end of the flume the coarse intermediate layer is strongest, as all the fine material of this part has meanwhile been eroded and out of the flume. The fine material on top of this flume part is mainly originated from upper flume parts.

After 24h the upper part of the flume almost reached the final state. The lower part still has a small rest of fine sediments coming from the deeper zones below the coarse surface layer now sporadically reached by turbulent pulses.

After 72h the coarse zone nearly reached a constant strength, which equals almost the mean value over all points of the maximum estimated active layer thickness (here: Malcherek formula; option 5). Since even the shear stress peaks now do not reach deeper available fine sediments any more, erosion per time is very small now. An armouring layer has developed and protects the lower sediments.

We tried different formulas, which lead to a different strength of the armouring layer. And of course other parameters like hiding effects or the slope effect, estimation of the critical shear stress and others influence the resulting sieve lines and the time scales.

IV. SUMMARY AND RECOMMENDATION

The continuous vertical sorting model (C-VSM) overcomes many limitations of the classic layer implementation (HR-VSM).

Even though this paper shows a different way to manage the grain sorting, it is just another interpretation of Hiranos original idea with fewer simplifications. The new model doesn't overcome the need to carefully calibrate the same input parameters as all other models, but the new interpretation has the following advantages:

- It is possible to keep minor but prominent grain mixture variations even after a high number of time steps. Smearing effects and book keeping accuracy is defined by user defined thresholds or the computational resources, rather than through a fix value.
- A dynamic active layer thickness is not biased by these effects any more. Various functions for the impact depth of the shear stress can be chosen to the projects demands.
- The result is a much more accurate vertical grain sorting, which results in better prognoses for bed roughness, bed forms and erosion stability.

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/*************************************************************************
/ New keywords for the Continuous VERTICAL SORTING MODEL by Uwe Merkel
/*************************************************************************
/
VERTICAL GRAIN SORTING MODEL  = 1
/         0 = Layer = HR-VSM (HIRANO + RIBBERINK as until SISYPHE v6p1)
/         1 = C-VSM
C-VSM MAXIMUM SECTIONS        = 100
/         Should be at least 4 + 4x Number of fractions,
/         better > 100, tested up to 10000
C-VSM FULL PRINTOUT PERIOD    = 1000
/         0 => GRAPHIC PRINTOUT PERIOD
/         Anything greater 0 => Sets an own printout period for the CVSP
/         useful to save disk space!!!
C-VSM PRINTOUT SELECTION =
0|251|3514|1118|1750|2104|3316|1212|1280|2186|3187|1356|3027|1535|485
/        Add any 2D Mesh Point numbers for .CSV-Ascii output of the CVSP
/        Add 0 for full CVSP output as Selafin3D files
/            (called VSPRES + VSPHYD)
/            All files are saved to your working folder and
/            in /VSP & /LAY folders below
C-VSM DYNAMIC ALT MODEL       = 5
/        'MODEL FOR DYNMIC ACTIVE LAYER THICKNESS APROXIMATION'
/         0 = CONSTANT (Uses Keyword: ACTIVE LAYER THICKNESS)
/         1 = Hunziker & Guenther
/         2 = Fredsoe & Deigaard (1992)
/         3 = van RIJN (1993)
/         4 = Wong (2006)
/         5 = Malcharek (2003)
/         6 = 3*d50 within last time steps ALT'
```

Figure 16. Example configuration for Sisyphe v6p2

The modular implementation provides an interface for important future developments. Further validation in practical projects is in progress.

V. HOW TO USE IT IN SISYPHE V6P2

A couple of new keywords enables the C-VSM. Add the lines of Figure 16 to your sis.cas file.

The full C-VSM output can be found in the Selafin files VSPRES & VSPHYD in the tmp-folders. As the higher resolution of the C-VSM needs resources, you can reduce the print output period, or suppress the output at all. The common Sisyphe result files only show the Hirano output. Even more disk space can be saved, if only few points are printed out as .VSP.CSV files in the subfolder /VSP/. We recommend using between 200 and 1000 vertical sections. More will not improve the accuracy much, and less will lead to increasing data management, as the profile compression algorithms are called more often.

Please feel free to report your experiences with our new development.

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