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Executive Summary

The present document is part of the openTELEMAC documentation (http://www.opentelemac.org).

Table des matières

Conventions

In this document, the following conventions are used :
- The names of the different modules are written in CAPS AND SMALL CAPS
- File names are written in sans serif font
- Keywords, variables, subroutines, etc. are written in `monospaced ‘‘typewriter’’ font`

1 Introduction

Sisyphe is a sediment transport and morphodynamic simulation module which is part of the hydroinformatic finite elements and finite volume system Telemac-Mascaret. In this module, sediment transport rates, split into bedload and suspended load, are calculated at each grid point as a function of various flow (velocity, water depth, wave height, etc.) and sediment (grain diameter, relative density, settling velocity, etc.) parameters. The bedload is calculated by using classical sediment transport formulae from the literature. The suspended load is determined by solving an additional transport equation for the depth-averaged suspended sediment concentration. The bed evolution equation (Exner equation) can be solved by using either a finite element or a finite volume formulation.

Sisyphe is applicable to non-cohesive sediments (uniform or graded), cohesive sediments as well as to sand-mud mixtures. The sediment composition is represented by a finite number of classes, each characterized by its mean diameter, grain density and settling velocity. Sediment transport processes can also include the effect of bottom slope, rigid beds, secondary currents, slope failure, etc. For cohesive sediments, the effect of bed consolidation can be accounted for.

Sisyphe can be applied to a large variety of hydrodynamic conditions including rivers, estuaries and coastal applications, where the effects of waves superimposed to a tidal current can be included. The bed shear stress, decomposed into skin friction and form drag, can be calculated either by imposing a friction coefficient (Strickler, Nikuradse, Manning, Chézy or user defined) or by a bed-roughness predictor.

1.1 Coupling with hydrodynamics

In Sisyphe the relevant hydrodynamic variables can be either imposed in the model (chaining method) or calculated by a hydrodynamic computation (internal coupling). It is convenient to use one of the hydrodynamic modules of the Telemac system (Telemac-2d , Telemac-3d or Tomawac) for compatibility reasons (same mesh, same pre- and post-processor, etc.), but the user can also choose a different hydrodynamic model. The different methods which can be used to prescribe the hydrodynamics are described in §2.3.

1.2 Numerical kernel

Sisyphe can be run on Unix, Linux or Windows. The latest release of Sisyphe (version 6.2) uses the version 6.2 of the Bief finite element library (Telemac system library).

1.3 Preprocessing and Postprocessing

We refer the reader to the specific documentation for pre- and post-processing tools, see e.g. [].

1.4 Outline of the manual

The main steps to run a sedimentological computation are given in Section 2. In Section 3, a description of the main sediment and hydrodynamics parameters such that total bed shear stress, skin friction, etc. is presented. In Section 4 the bedload is presented. In Section 5, the suspended load transport is introduced. Section 6 presents the influence of the effect of waves on the sediment transport and bed evolution. In Section 7, the sand grading is introduced.

2 Running a sedimentological computation

2.1 Input files

The minimum set of input files to run a SISYPHE simulation includes the steering file (text file cas), the geometry file (format selafin slf), and the boundary conditions file (text file cli). Additional or optional input files include the fortran file, the reference file, the result file, etc.

2.2 Steering file

The steering file contains the necessary information for running a computation, plus the values of parameters that are different from the defaults. Like in other modules of the TELEMAC system, the model parameters can be specified in the (obligatory) SISYPHE steering file. The following essential information (input/output) should be specified in the steering file :
- Input and output files
- Physical parameters (sand diameter, cohesive or not, settling velocity, etc.)
- Main sediment transport processes (transport mechanism and formulae, etc.)
- Additional sediment transport processes (secondary currents, slope effect, etc.)
- Numerical options and parameters (numerical scheme, options for solvers, etc.)

2.3 Coupling hydrodynamics and morphodynamics

We describe here two methods for linking the hydrodynamic and the morphodynamic models : by *chaining* (the flow is obtained from a previous hydrodynamic simulation assuming a fixed bed) or by *internal coupling* (both the flow and bed evolution are updated at each time step).

2.3.1 Chaining method

- **Principle**
  Both models (hydrodynamic and morphodynamic) are run independently : during the first hydrodynamic simulation the bed is assumed to be fixed. Then, in the subsequent morphodynamic run, the flow rate and free surface are read from the previous hydrodynamic results file. This 'chaining method' is only justified for relatively simple flows, due to the difference in time-scales between the hydrodynamics and the bed evolution. For unsteady tidal flow, SISYPHE can be used in an unsteady mode : the flow field is linearly interpolated between two time steps of the hydrodynamic file. For steady flow, the last time step of the hydrodynamic file is used and the flow rate and free surface assumed to stay constant while the bed evolves.

- **Flow updating**
  At each time step, the flow velocity is updated by assuming simply that both the flow rate and the free surface elevation are conserved, such that, in the case of deposition, the flow velocity is locally increased, whereas in the case of erosion, the flow velocity decreases.
  This rather schematic updating does not take into account any deviation of the flow. It is only suitable for simple flows (2D processes) and assuming relatively small bed evolutions. However it can be responsible for numerical instabilities [].The morphodynamic computation is stopped when the bed evolution reaches a certain percent of the initial water depth. This simple updating of the flow field is no longer valid when the bed evolution becomes greater than a significant percentage of the water depth, specified by the user. At this point, it is recommended to stop the morphodynamic calculation and to recalculate the hydrodynamic variables.

- **Mass continuity**
  It should be noted that with this simple method, the sediment mass continuity may not be satisfied because of potential losses due to changes in the flow depth as the bed evolves.

When the flow is steady (`STEADY CASE = YES`), only the last record of the previous result file will be used. Otherwise (`STEADY CASE = NO`), the `TIDE PERIOD` and `NUMBER OF TIDES OR FLOODS` will be used to specify the sequence to be read on the hydrodynamic files. Hydrodynamic records are interpolated at each time step of the sedimentological computation.
Note : an error may occur when the `TIDE PERIOD` is not a multiple of the graphical time steps of the hydrodynamic file (`hydrodynamic file is not long enough`). In an unsteady case, the keyword `STARTING TIME OF THE HYDROGRAM` gives the first time step to be read. If the starting time is not specified, the last period of the hydrogram will be used for sedimentological computation.

– **Steering/fortran files**
For uncoupled mode, the Sisyphe steering file should specify :
– The time steps, graphical or listing output, duration
– The hydrodynamic file as yielded by Telemac-2d or Telemac-3d (`HYDRODYNAMIC FILE`) or by the subroutine `condim_sisyphe.f`.
– For waves only : the wave parameters can be either calculated by a wave propagation code (Tomawac), or defined directly in Sisyphe (`condim_sisyphe.f`). The effect of waves on bed forms and associated bed roughness coefficient can be accounted with keyword : `EFFECT OF WAVES = YES`.
– A restart from a previous Sisyphe model run, by setting `COMPUTATION CONTINUED = YES` and specification of sedimentological results in `PREVIOUS SEDIMENTOLOGICAL COMPUTATION`
– Flow options : steady or unsteady options, flow period

Keywords
For time step, duration and output :
– `TIME STEP, NUMBER OF TIME STEPS`
– `GRAPHIC PRINTOUT PERIOD`
– `LISTING VARIABLES FOR GRAPHIC PRINTOUTS`
For hydrodynamics (imposed flow and updated) :
– `HYDRODYNAMIC FILE`
– `STEADY CASE =NO`, default option
– `TIDE PERIOD = 44640`, default option
– `STARTING TIME OF THE HYDROGRAM = 0.`, default option
– `NUMBER OF TIDES OR FLOODS = 1`, default option
– `CRITICAL EVOLUTION RATIO = 0.1`, default value
For waves :
– `WAVE FILE, WAVE EFFECTS`

2.3.2 Internal coupling

– **Principle**
Sisyphe can be automatically coupled (internally) with the hydrodynamic model, Telemac-2d or Telemac-3d . Sisyphe is called inside the hydrodynamic model without any exchange of data files. The data to be exchanged between the two programs is now directly shared in the memory, instead of being written and read in a file.
At each time step, the hydrodynamics variables (velocity field, water depth, bed shear stress,...) are transferred to the morphodynamic model, which sends back the updated bed elevation to the hydrodynamics model.

– **Time step and coupling period** The internal coupling method is more CPU time consuming than the chaining method. Various techniques can be set up to reduce the CPU time (e.g. parallel processors). In certain cases, the use of a coupling period > 1 allows the bed load transport rates and resulting bed evolution not to be re-calculated at every time step. For suspended load, a diffusion/transport equation needs to be solved. This transport equation obeys the

same Courant number criteria on the time step than the hydrodynamics, and therefore needs to be solved at each time-step (`COUPLING PERIOD = 1`).
The time step of SISYPHE is equal to the time step of TELEMAC-2D or TELEMAC-3D multiplied by the 'coupling period'. The graphic/listing printout periods are the same as in the TELEMAC computation.
The TELEMAC-2D /3D steering file must specify the type of coupling, the name of the SISYPHE steering file, and the coupling period. In addition, the Fortran file of SISYPHE must sometimes specified in the TELEMAC steering file (if there is no Fortran file for TELEMAC). Some of the keywords of the SISYPHE steering file become obsolete.

Keywords
For internal coupling, the following keywords need to be specified in the TELEMAC-2D or TELEMAC-3D steering files :
– `COUPLING WITH = SISYPHE`
– `COUPLING PERIOD =1`, default value
– `NAME OF SISYPHE STEERING FILE`

All computational parameters (time step, duration, printout, option for friction) need to be specified in the TELEMAC steering file, but are no longer used by SISYPHE . The values of time step, bottom shear stress, etc. are transferred directly from TELEMAC to SISYPHE .

Important
The following keywords are no longer in use is (SISYPHE) steering file :
– `TIME STEP`
– `GRAPHIC PRINTOUT PERIOD`
– `LISTING PRINTOUT PERIOD`
– `LAW OF BOTTOM FRICTION`
– `FRICTION COEFFICIENT`

2.4 Boundary conditions file

The format of the boundary condition file is the same as for TELEMAC-2D or TELEMAC-3D . This file can be created by a mesh generator (for example, BlueKenue [?]) and modified using a text editor. Each line is related to a point along the edge of the mesh. Boundary points are listed in the file in the following way : First the domain outline points, proceeding counterclockwise from the lower left corner, then the islands proceeding clockwise, also from the lower left corner.

The edge points are numbered like the file lines ; the numbering first describes the domain outline in the counterclockwise direction from lower left point (X+Y minimum), then the islands in the clockwise direction. The boundary condition about the bottom depth is imposed at the specific place of the tracer boundary condition in TELEMAC-2D . The following thirteen variables for each edge point are first read out of the boundary conditions file `X1, X2, X3, X4, X5, X6, X7, LIEBOR, EBOR, X10, X11, N, K`. The first seven variables (`X1, X2, X3, X4, X5, X6, X7`), as well as `X10` and `X11`, are specific to the TELEMAC-2D model. They are read in Sisyphe, but remain unused.

3 Flow-sediment interactions

3.1 Sediment and fluid properties

Fine sediment particles of grain size $d_{50} < 60\mu m$* present complex cohesive properties which affect the sediment transport processes. For non-cohesive sediments (median diameter $d_{50} > 60\mu m$), the grain diameter and grain density ρ_s are the key parameters which determine its resistance to erosion and sediment transport rate.

For cohesive sediments ($d_{50} < 60\mu m$), the grain diameter is no longer the key sediment parameter : the settling velocity now depends on the concentration of the sediment, whereas the critical bed shear strength depends on the consolidation state of the sediment bed, see [].

In this section, we consider uniform, non-cohesive sediments characterized by one single value for the grain size d_{50} and grain density ρ_s, with ρ_s =2650 kg m^{-3} which can be transported both as bed-load and suspended load.

Keywords

The physical properties of the sediment are always defined in the Sisyphe steering file using the following keywords :

- `COHESIVE SEDIMENTS` (`= NO` , default option)
- `NUMBER OF SIZE-CLASSES OF BED MATERIAL` (`NSICLA= 1`, default option)
- `SEDIMENT DIAMETERS` ($d_{50} > 60\mu$m for non-cohesive sediments)
- `SEDIMENT DENSITY` ($\rho_s = 2650$ kgm^{-3}, default value)
- `SETTLING VELOCITIES` (The default value is not given. If the user does not give a value, the subroutine `vitchu.f` is used : Stokes, Zanke or Van Rijn formulae depending on the grain size)

The physical properties of the fluid are defined by :

- `WATER VISCOSITY` ($\nu = 1.0 \times 10^{-6}$ m^{2}s^{-1}, by default)
- `WATER DENSITY` ($\rho = 1000$ kgm^{-3}, by default)
- `GRAVITY ACCELERATION` ($g = 9.81$ ms^{-2})

3.2 Bed shear stress

3.2.1 Hydrodynamic model

The current-generated bed shear stress is used in both the shallow water momentum equation as well as the bottom boundary condition for the velocity profile. When Sisyphe is coupled with Telemac-2D , the bed shear stress term $\tau_0 = (\tau_x, \tau_y)^T$ is calculated at each time step from the classical quadratic dependency on the depth–averaged velocity :

$$\tau_{x,y} = \frac{1}{2}\rho C_d (u, v)^T |\mathbf{u}|, \tag{1}$$

where $(u, v)^T$ are the depth-averaged velocity components along the $x-$ and $y-$ Cartesian directions, respectively with transpose $(\cdot)^T$; $|\mathbf{u}| = \sqrt{u^2 + v^2}$ the velocity module ; and friction coefficient C_d.

When Sisyphe is coupled with Telemac-3D , the bed shear stress is aligned with the near bed velocity in order to account for possible vertical flow deviations. The magnitude of the bed shear stress is still related to the depth-averaged velocity, except if the Nikuradse friction law is applied. In that case, the friction velocity u$_*$, defined by

$$\tau_0 = \rho u_*^2 \tag{2}$$

*. d_{50} is the grain size with 50% of the material finer by weight

is related to the near bed flow velocity $u(z_1)$ by a logarithmic velocity profile :

$$u(z_1) = \frac{u_*}{\kappa} \ln\left(\frac{z_1}{z_0}\right) \tag{3}$$

where z_0 is the vertical distance from a rough boundary, expressed as a function of the Nikuradse bed roughness ($z_0 = k_s/30$), with k_s the grain roughness height, z_1 is the near bed distance measured along the vertical coordinate z, and aligned against the direction of the acceleration of gravity of magnitude g. In TELEMAC-3D , z_1 is taken equal to the velocity in the bottom computational layer ; and $\kappa = 0.4$ is the von Kármán constant. For flat beds, the roughness height has been shown to be approximately $k_s \approx 3d_{50}$ [?].

The direction of the bed shear stress and resulting bedload transport rate is assumed to be in the direction of the depth-averaged velocity in SISYPHE (alone or internally coupled with TELEMAC-2D). When SISYPHE is internally coupled to TELEMAC-3D , the bed shear stress and resulting transport rate are assumed to be in the direction of the near bed velocity. The 3D model gives a more accurate estimate of the bottom friction, since it accounts for a possible vertical deviation of the current.

Sisyphe coupled with Telemac-2d

When the model is coupled with TELEMAC-2D , the values of the friction coeffients (and therefore, the bed shear stress) are provided by TELEMAC-2D . The depth-averaged bed shear stress and resulting bedload transport rates are assumed to be in the direction of the mean flow velocity, except when the sediment transport formulation accounts for :
- deviation correction (bed slope effect)
- secondary currents

Both bed shear stress and bed load transport rate are aligned with the near bed velocity. This 3D approach is more physical, and takes into account possible recirculation and veering of the flow, such that corrections for secondary currents for example are no longer necessary.

3.2.2 Uncoupled model

The quadratic friction coefficient C_d which is used to calculate the total bed shear stress can be calculated based on the selected friction law. Different options, which are consistent with the TELEMAC-2D options, are available in SISYPHE and depend on the choice of the keywords `LAW FOR BOTTOM FRICTION` and on the value of the `FRICTION COEFFICIENT` :

- Chézy coefficient C_h (`KFROT = 2`)
$$C_d = \frac{2g}{C_h^2} \tag{4}$$
- Strickler coefficient S_t (`KFROT = 3`)
$$C_d = \frac{2g}{S_t^2} \frac{1}{h^{1/3}} \tag{5}$$
- Manning friction M_a (`KFROT = 4`)
$$C_d = \frac{2g}{h^{1/3}} M_a^2 \tag{6}$$
- Nikuradse bed roughness k_s (`KFROT = 5`)
$$C_d = 2\left[\frac{\kappa}{\log(\frac{12h}{k_s})}\right]^2, \tag{7}$$

Keywords
In the SISYPHE steering file, the total bed shear stress is calculated based on :
– `LAW OF BOTTOM FRICTION` (`KFROT =3`, default option)
– `BOTTOM FRICTION COEFFICIENT` (`St=50`, default value)

Similar keywords are available in both TELEMAC-2D and TELEMAC-3D in the case of internal coupling.

3.2.3 Role of bed forms

A natural sediment bed is generally covered with bed forms (length λ_d and height η_d). The presence of bed forms greatly modifies the boundary layer flow structure, with the formation of recirculation cells and depressions in the lee of bed forms. Depending on the flow and sediment transport rates, the size of bed forms ranges from a few centimeters for ripples to a few tens of meter for mega-ripples. The dimension of dunes scales with the water depth h, such that $\eta_d \approx 0.4h$ and $\lambda_d \approx 6 - 10h$. In most of cases, large scale models do not resolve the small to medium scale bed forms (ripples, mega-ripples) which need therefore to be parameterized by an increased friction coefficient. The total bed shear stress is expressed as the sum of two components :

$$\tau_0 = \tau_0' + \tau_0'' \tag{8}$$

where τ_0 is the total bed shear stress, τ_0' is the grain (or skin) shear stress, and τ_0'' is the form shear stress. The local skin friction component determines the bedload transport rate and the equilibrium concentration for the suspension. The total friction velocity determines the (spatially averaged along bedforms) turbulence eddy viscosity/diffusivity vertical distribution in 3D models, and therefore determines both the velocity vertical profile and the mean concentration profile.

3.3 Skin friction correction

3.3.1 Total bed shear stress decomposition

Bedload transport rates are calculated as a function of the local skin friction component τ'. The total bed shear stress issued from the hydrodynamics model needs to be corrected in the morphodynamics model as follows :

$$\tau' = \mu \tau_0, \tag{9}$$

where μ is a correction factor for skin friction. Physically, the skin bed roughness should be smaller than the total bed roughness (i.e. $\mu \leq 1$). However, in most cases the hydrodynamic friction does not represent the physical bottom friction : the coefficient is generally used as a calibration coefficient in hydrodynamics models. It is adjusted by comparing simulation results with observations of the time-varying free surface and velocity field. Therefore, its model value integrates various neglected processes (side wall friction, possible errors in the bathymetry and input data). Under those conditions, a correction factor $\mu > 1$ can be admitted.

Different methods are programmed in SISYPHE in order to calculate the bedform correction factor μ, according to the keyword `SKIN FRICTION CORRECTION` :

– `ICR = 0` : no correction, the total friction issued from TELEMAC is directly used for sand transport calculations ($\mu = 1$).
– `ICR = 1` : the skin roughness is assumed to be proportional to the sand grain diameter like in the case of flat beds ($k_s' \sim d_{50}$). The proportionality coefficient is specified by the keyword `RATIO BETWEEN SKIN FRICTION AND MEAN DIAMETER`, defined as :

$$\mu = \frac{C_d'}{C_d}, \tag{10}$$

where C_d et C'_d are both quadratic friction coefficients related to total friction and skin friction,respectively. C_d is obtained from TELEMAC-2D or TELEMAC-3D and C'_d is calculated from k'_s, as follows :

$$C'_d = 2\left[\frac{\kappa}{\log(\frac{12h}{k'_s})}\right]^2 \tag{11}$$

– `ICR = 2` the bedform predictor is used to calculate the bedform roughness k_r in order to account for the effect of ripples. Both k_r and k'_s should influence the transport rates. It is assumed that :

$$\mu = \frac{C_d'^{0.75} C_r^{0.25}}{C_d}, \tag{12}$$

where the quadratic friction C_r due to bedforms is calculated as a function of k_r. For currents only, the ripple bed roughness is function of the mobility number, see [?] :

$$k_r = \begin{cases} d_{50}(85 - 65\tanh(0.015(\Psi - 150))) & \text{for } \Psi < 250 \\ 20 d_{50} & \text{otherwise} \end{cases}$$

with $\Psi = U^2/(s-1)gd_{50}$.

Keywords

The option selected for the skin friction correction is based on keywords :
– `SKIN FRICTION CORRECTION` (`ICR=0`, default option)
– `RATIO BETWEEN SKIN FRICTION AND MEAN DIAMETER` (`KSPRATIO=3`, default value)

3.4 Bed roughness predictor

3.4.1 Total bed roughness (uncoupled)

Different options are programmed in SISYPHE to predict the total bed roughness through the associated keywords `BED ROUGHNESS PREDICTION` and `BED ROUGHNESS PREDICTOR OPTION`. It is recalled that the bed friction option of SISYPHE is not used in the case of internal coupling with TELEMAC-2D or TELEMAC-3D .

– For `IKS = 1` : the bed is assumed to be flat $k_s = k'_s = \alpha d_{50}$, with α a constant (assumed to be equal to 3.
– `IKS = 2` : for waves and combined waves and currents, bedform dimensions are calculated as a function of wave parameters following the method of Wiberg and Harris [?]. The wave-induced bedform bed roughness k_r is calculated as a function of the wave-induced bedform height η_r :

$$k_r = \max(k'_s, \eta_r). \tag{13}$$

– `IKS = 3` : for currents only, the van Rijn's total bed roughness predictor [? ?] has been implemented. The total bed roughness can be decomposed into a grain roughness k'_s, a small-scale ripple roughness k_r, a mega-ripple component k_{mr}, and a dune roughness k_d :

$$k_s = k'_s + \sqrt{k_r^2 + k_{mr}^2 + k_d^2}. \tag{14}$$

Both small scale ripples and grain roughness have an influence on the sediment transport laws, while the mega-ripples and dune roughness only contribute to the hydrodynamic model (total friction).

3.5 Sediment transport

3.5.1 Bedload and suspended load

When the current-induced bed shear stress increases above a critical threshold value, sediment particles start to move as bedload, while the finer particles are transported in suspension. Bedload occurs in a very thin high concentrated near-bed layer, where inter-particle interactions develop. The suspended load is defined as the depth-integrated flux of sediment concentration, from the top of the bedload layer up to the free surface. For dilute suspension concentration values, clear flow concepts (turbulence diffusion, eddy viscosity, logarithmic velocity profile) are considered to be valid. The total sediment load Q_t includes both a bedload Q_b and suspended load Q_s :

$$Q_t = Q_b + Q_s. \tag{15}$$

3.5.2 Shields parameter

The critical Shields number or dimensionless critical shear stress Θ_c is defined by :

$$\Theta_c = \frac{\tau_c}{g(\rho_s - \rho)d_{50}} \tag{16}$$

where τ_c is the critical shear stress for sediment incipient motion. Values of Θ_c can be either specified in the SISYPHE steering file by use of keyword `SHIELDS PARAMETERS` or calculated by the model as a function of non-dimensional grain diameter $D_* = d[(\rho_s/\rho - 1)g/\nu^2]^{1/3}$. It is implemented in subroutine `init_sediment.f`, see also [?]

$$\frac{\tau_c}{g(\rho_s - \rho)d_{50}} = \begin{cases} 0.24D_*^{-1}, & D_* \leq 4 \\ 0.14D_*^{-0.64}, & 4 < D_* \leq 10 \\ 0.04D_*^{-0.10}, & 10 < D_* \leq 20 \\ 0.013D_*^{0.29}, & 20 < D_* \leq 150 \\ 0.045, & 150 \leq D_* \end{cases}$$

where τ_c and d are in $\mathrm{N\,m^{-2}}$ and m, respectively. The default option (no Shields number given in steering file) is to calculate the shields parameter as a function of sand grain diameter (see logical `CALAC`).

4 Bed-load transport

4.1 Exner equation

4.1.1 Equilibrium conditions

Bed-load occurs in a thin near-bed high-concentrated region. The bed-load layer therefore adapts very rapidly to any changes in the flow conditions, such that equilibrium conditions can be considered to be valid. The bed-load transport rate can then be calculated by use of some equilibrium sediment transport formula, as a function of various flow and sediment parameters, assuming that the transport rate corresponds to saturation conditions.

4.1.2 Bed evolution

To calculate the bed evolution, SISYPHE solves the Exner equation :

$$(1-n)\frac{\partial Z_f}{\partial t} + \nabla \cdot Q_b = 0, \tag{17}$$

where n is the non-cohesive bed porosity ($n \approx 0.4$ for non cohesive sediment), Z_f the bottom elevation, and Q_b ($\mathrm{m^2s^{-1}}$) the solid volume transport (bedload) per unit width.

Equation (17) states that the variation of sediment bed thickness can be derived from a simple mass balance and it is valid for equilibrium conditions.

Keywords
- `BEDLOAD = YES` (default option)
- `SUSPENDED LOAD = NO` (default option)
- `NON COHESIVE BED POROSITY` (default option = `0.4`).

4.1.3 Bedload transport formulae

For currents only (no wave effects), a large number of semi-empirical formulae can be found in the literature to calculate the bedload transport rate. SISYPHE offers the choice among different bedload formulae including the Meyer-Peter and Müller, Engelund-Hansen and Einstein-Brown formulae.

Most sediment transport formulae assume threshold conditions for the onset of erosion (e.g. Meyer-Peter and Müller, van Rijn and Hunziker). Other formulae are based on similar energy concept (e.g. Engelund-Hansen) or can be derived from a statistical approach (e.g. Einstein-Brown, Bijker, etc.). The non-dimensional current-induced sand transport rate Φ_s, is expressed as :

$$\Phi_s = \frac{Q_s}{\sqrt{g(s-1)d_{50}^3}} \tag{18}$$

with ρ_s the sediment density ; $s = \rho_s/\rho$ the relative density ; d the sand grain diameter ($= d_{50}$ for uniform grains) ; and g the gravity. As presented next, the non-dimensional sand transport rate Φ_s is, in general, expressed as a function of the non-dimensional skin friction or Shields parameter θ', defined by :

$$\theta' = \frac{\mu \tau_0}{(\rho_s - \rho) g d_{50}}, \tag{19}$$

with μ the correction factor for skin friction.

4.2 Sediment transport formulae

4.2.1 Choice of formulae

The choice of a transport formula depends on the selected value of the model parameter `ICF`, as defined in the steering file by the keyword `BED-LOAD TRANSPORT FORMULA`. By setting `ICF = 0`, the user can program a specific transport formula through the subroutine `qsform.f`. Other sediment transport formulae, described in Chapters 6 and **??**, have been programmed in SISYPHE to account for the effects of waves (cf. Bijker, Bailard, Dibajnia and Watanabe, etc.) or sand grading (cf. Hunziker).

Keywords
– `BED-LOAD TRANSPORT FORMULA` (default option : Meyer-Peter-Muller formula `ICF=1`)

– `ICF = 1` (Meyer-Peter-Mueller formula) : this classical bed-load formula has been validated for coarse sediments in the range (0.4 mm $< d_{50} <$ 29 mm). It is based on the concept of initial entrainment :

$$\Phi_b = \begin{cases} 0 & \text{if } \theta' < \theta_c \\ \alpha_{mpm} & (\theta' - \theta_c)^{3/2} \quad \text{otherwise} \end{cases}$$

with α_{mpm} a coefficient (= 8 by default), θ_c the critical Shields parameter (= 0.047 by default). The coefficient α_{mpm} can be modified in the steering file by the keyword `MPM COEFFICIENT`.

– `ICF = 2` (Einstein-Brown formula) : this bed-load formula is recommended for gravel ($d_{50} > 2$ mm) and large bed shear stress $\theta > \theta_c$. The solid transport rate (see [12]) is expressed as :

$$\Phi_b = F(D_*)f(\theta'), \tag{20}$$

with

$$F(D_*) = \left(\frac{2}{3} + \frac{36}{D_*}\right)^{0.5} - \left(\frac{36}{D_*}\right)^{0.5}, \tag{21}$$

and

$$f(\theta') = \begin{cases} 2.15\exp(-0.391/\theta') & \text{if } \theta' \leq 0.2 \\ 40\,\theta'^3 & \text{otherwise} \end{cases}$$

where the non-dimensional diameter D_* is defined according to Equation (**??**).

– `ICF = 3 or 30` (Engelund-Hansen formula) : this formula predicts the total load (bedload plus suspended load). It is recommended for fine sediments, in the range (0.2 mm $< d_{50} <$ 1 mm but beware that the use of a total load formula is only suitable under equilibrium conditions (quasi-steady and uniform flow). The two different forms of the same equation are : programmed in SISYPHE :

  – `ICF = 30` corresponds to the original formula, where the transport rate is related to the skin friction without threshold :

$$\Phi_s = 0.1\,\theta'^{5/2} \tag{22}$$

  – `ICF = 3` corresponds to the version modified by Chollet and Cunge [8] to account for the effects of sand dunes. The transport rate is related to the total bed shear stress as :

$$\Phi_s = \frac{0.1}{C_d}\hat{\theta}^{5/2}, \tag{23}$$

  where the dimensionless bed shear stress $\hat{\theta}$ is calculated as a function of the dimensionless skin friction θ' :

$$\hat{\theta} = \begin{cases} 0 & \text{if } \theta' < 0.06 \text{ (flat bed regime - no transport)} \\ \sqrt{2.5(\theta' - 0.06)} & \text{if } 0.06 < \theta' < 0.384 \text{ (dune regime)} \\ 1.065\theta'^{0.176} & \text{if } 0.384 < \theta' < 1.08 \text{ (transition regime)} \\ \theta' & \text{if } \theta' > 1.08 \text{ (flat bed regime - upper regime)} \end{cases}$$

Attention
To avoid the suspended load to be calculated twice, in the case of coupling between bedload and suspended load (`SUSPENSION = YES`), the total load formula (`ICF = 3 or 30`) should not be used.

– `ICF = 7` (van Rijn's formula) : this formula was proposed by van Rijn [29] to calculate the bedload transport rate for particles of size 0.2 mm $< d_{50} <$ 2 mm :

$$\Phi_b = 0.053 D_*^{-0.3} \left(\frac{\theta_p - \theta_{cr}}{\theta_{cr}} \right)^{2.1}. \tag{24}$$

4.2.2 Validity range of sediment transport formulae

Most sediment transport formulae are based on experiments performed under fluvial, unidirectional flows. These formulae shown a rapid variation of the bedload transport prediction, as a function of the mean flow intensity. Therefore, an increasing of the current velocity by 10% will result, depending on the formula being used, in an increasing of the transport rate of over 30% (Meyer-Peter), 60% (Engelund-Hansen) or almost 80% (Einstein-Brown). Therefore, any error made when calculating the hydrodynamics will be significantly amplified by the sediment transport rates estimates. On the other hand, under variable flow conditions (e.g. tidal regime), the average transport will be highly influenced by the stronger currents and will not be directly related to the mean flow.

The validity range of the different formulae can be summarized in Table 1.

Formula	Meyer-Peter	Einstein-Brown	Engelund-Hansen
`IFC`	`1`	`2`	`3 or 30`
Mode of transport	bedload	bedload	total load
Validity range (d_{50})	$0.4 - 3$ mm	$0.3 - 29$ mm	$0.2 - 1$ mm
Rippled bottoms	Yes	Yes	Yes
Low bedload flows	Yes	Yes	No
High bedload flows	Yes	Yes	Yes

TABLE 1 – Validity range of some of the sand transport formulae programmed in SISYPHE (for currents only).

4.3 Bed slope effect

The effect of a sloping bottom is to increase the bedload transport rate in the downslope direction, and to reduce it in the upslope bedload direction. In SISYPHE , a correction factor can be applied to both the magnitude and direction of the solid transport rate, before solving the bed evolution equation. The bed slope effect is activated if the keyword `SLOPE EFFECT` is present in the SISYPHE steering file.

Two different formulations for both effects are available depending on the choice of the keywords `FORMULA FOR SLOPE EFFECT`, which chooses the magnitude correction and `FORMULA FOR DEVIATION`, which chooses the direction correction.

4.3.1 Correction of the magnitude of bedload transport rate (formula for slope effect)

– `SLOPEFF = 1` : this correction method is based on the Koch and Flokstra's formula [22]. The solid transport rate intensity Q_{b0} is multiplied by a factor $1 - \beta \partial Z_f / \partial s$, thus :

$$Q_b = Q_{b0} \left(1 - \beta \frac{\partial Z_f}{\partial s} \right), \tag{25}$$

with s the coordinate in the current direction and β an empirical factor, which can be specified with the associated keyword `BETA` (default value = `1.3`). This effect of bed slope is then similar to adding a diffusion term in the bed evolution equation. It tends to smooth the results and is often used to reduce unstabilities.

– `SLOPEFF=2` : this correction is based on the method of Soulsby (1997), in which the threshold bed shear stress θ_{co} is modified as a function of the bed slope χ, the angle of repose of the sediment ϕ_s, modified in the SISYPHE steering file with the keyword `FRICTION ANGLE OF THE SEDIMENT` (= `40`° by default), and the angle of the current to the upslope direction ψ :

$$\frac{\theta_c}{\theta_{co}} = \frac{\cos\psi\sin\chi + (\cos^2\chi\tan^2\phi_s - \sin^2\psi\sin^2\chi)^{0.5}}{\tan\phi_s}, \tag{26}$$

with θ_c the modified threshold bed shear stress.

4.3.2 Correction of the direction of bedload transport rate (formula for deviation)

The change in the direction of solid transport is taken into account by the formula :

$$\tan\alpha = \tan\delta - T\frac{\partial Z_f}{\partial n}, \tag{27}$$

where α is the direction of solid transport in relation to the flow direction, δ is the direction of bottom stress in relation to the flow direction, and n is the coordinate along the axis perpendicular to the flow.

– `DEVIA = 1` : according to Koch and Flochstra, the coefficient T depends exclusively on the Shields parameter

$$T = \frac{4}{6\theta} \tag{28}$$

– `DEVIA = 2` : the deviation is calculated based on Talmon *et al.* [**?**] and depends on the Shields parameter and an empirical coefficient β_2

$$T = \frac{1}{\beta_2\sqrt{\theta}} \tag{29}$$

The empirical coefficient β_2 can be modified by the keyword `PARAMETER FOR DEVIATION` (= `0.85` by default).

Keywords
– `SLOPE EFFECT` (= `NO`, default option)
– `FORMULA FOR SLOPE EFFECT` (`SLOPEFF = 1`, default option)
– `FORMULA FOR DEVIATION` (`DEVIA = 1`, default option)
– `FRICTION ANGLE OF THE SEDIMENT` ($\phi_s = 40°$, default value)
– `BETA` ($\beta = 1.3$, default value)
– `PARAMETER FOR DEVIATION` ($\beta_2 = 0.85$, default value)

4.4 Bedload transport in curved channels

The bedload movement direction deviates from the main flow direction due to helical flow effect [39]. Different authors proposed empirical formulas for evaluating this deviation. The Engelund formula [**?**], based on the assumption that the bottom shear stress, the bed roughness and the mean water depth are constant in the cross-section, is

$$\tan\delta = 7\frac{h}{r} \tag{30}$$

where δ is the angle between the bedload movement and main flow direction, h the mean water depth and r the local radius of curvature. Yalin and Ferrera da Silva [38] have showed that δ is proportional to h/r. The local radius r can be computed based on the the cross sectional variation of the free surface [?] :

$$r = -\rho\alpha' \frac{U^2}{g\frac{\partial Z_s}{\partial y}},$$

with α' a coefficient($\alpha' \approx 1.0$).

Keywords
- SECONDARY CURRENTS (= NO, default option)
- SECONDARY CURRENTS ALPHA COEFFICIENT (= 1.0, default option)

Attention
This option should only be activated when the flow is calculated by a depth-averaged model.

4.5 Treatment of rigid beds

4.5.1 Different methods

Non-erodible beds are treated numerically by limiting bedload erosion and letting incoming sediment pass over. The problem of rigid beds is conceptually trivial but numerically complex. In finite elements the minimum water depth algorithm allows a natural treatment of rigid beds, see [17]. The sediment is managed as a layer with a depth that must remain positive, and the Exner equation is solved similarly to the shallow-water continuity equation. Different algorithms can be chosen with the keyword `OPTION FOR THE TREATMENT OF NON ERODABLE BEDS` and the space location and position of the rigid bed can be changed in subroutine `noerod.f`.

Keywords
- OPTION FOR THE TREATMENT OF NON ERODABLE BEDS (= NO, default option)
  - = 0 : erodable bottoms everywhere (default option)
  - = 1 : minimisation of the solid discharge
  - = 2 : nul solid discharge
  - = 3 : minimisation of the solid discharge in FINITE ELEMENTS / MASS-LUMPING
  - = 4 : minimisation of the solid discharge in FINITE VOLUMES

Attention
When the rigid bed can be reached during the computation, it is advised to use the method 3 or the method 4.

4.6 Tidal flats

Tidal flats are the areas of the computational domain where the water depth can become zero during the simulation. In Sisyphe , tidal flats can be treated with the keyword `TIDAL FLATS`. The companion keyword `OPTION FOR THE TREATMENT OF TIDAL FLATS` allows to choose between two different approaches :

- `OPTBAN = 1` : is the default option, and the equations are solved everywhere. The water depth is set to a minimum value, controlled by the keyword `MINIMAL VALUE OF THE WATER HEIGHT`, with a default value = `1.D-3`m.

– `OPTBAN = 2` : this option removes from the computational domain the elements with points that present a water depth less than a minimum value. This option should be avoided because it is not exactly mass-conservative.

Keywords
– `TIDAL FLATS` (= `YES`, default option)
– `MINIMAL VALUE OF THE WATER HEIGHT` (= `1.D-3m`, default value)
– `OPTION FOR THE TREATMENT OF TIDAL FLATS` (`OPTBAN = 1`, default option)

4.7 Boundary conditions for imposed sediment transport rates

Attention
From v6.2, as a general case the user has to specify **two different boundary conditions files** : a conlim file for the hydrodynamics module (e.g., for TELEMAC-2D) and a different conlim file for SISYPHE . This allows the user to apply different boundary conditions for a tracer and for bed evolution or concentration, for TELEMAC-2D and SISYPHE , respectively. However, in most simple applications, the same boundary condition can be used.

For example, for an imposed flow rate `LIUBOR = 5` and `LIVBOR = 5` and imposed tracer `TBOR = 5`, the information contained in the hydrodynamics boundary condition file includes :

```
LIHBOR     LIUBOR     LIVBOR     ...   LITBOR     TBOR
4          5          5          ...   5          0.
```

In the sediment transport boundary condition file, the following boundary condition types can be imposed :

4.7.1 Imposed zero bed evolution

For this case, `LIEBOR = 5`

```
(not used)    LIQBOR     (not used)     ...    LIEBOR     EBOR
(4)         4          (5)              ...    5          0.
```

equivalent to :

```
(not used)    LIQBOR     (not used)     ...    LIEBOR     EBOR
(4)         5          (5)              ...    5          0.
```

That implies that the same boundary condition file can be used.

4.7.2 Imposed sediment transport rates

For this case, `LIEBOR = 4` and `LIQBOR = 5`

```
(not used)    LIQBOR     (not used)    Q2BOR    ...    LIEBOR     EBOR
(4)           5          (5)           1.E-5    ...    4          0.
```

For this case, a boundary condition file for SISYPHE is needed. `LIHBOR` is actually not used but is now available for users. `LIQBOR` is the boundary condition on solid discharge (that was before given by user in subroutine `conlis.f`, after initialization to `KSORT` in SISYPHE). On liquid boundaries you must now have `LIQBOR=5 (KENT)` and `LIEBOR=4 (KSORT)` for boundaries with prescribed solid discharge or `LIEBOR=5 (KENT)` and `LIQBOR=4 (KSORT)` for boundaries with prescribed variation of elevation.

`Q2BOR` is the prescribed solid discharge given at boundary points, and expressed in m^2s^{-1}, excluding voids. If the keyword `PRESCRIBED SOLID DISCHARGES` is given in the parameter file, these values will be taken as such to give `QBOR` per point, which is in m^3s^{-1}, excluding voids. When the keyword `PRESCRIBED SOLID DISCHARGES` is given it supersedes `Q2BOR`, which is then taken as a profile.

All these values are meant for the total if there are sediment classes, and in this case they will be distributed to every class with the help of the fractions (array `AVAIL`), this is done for `EBOR` and `QBOR` in `conlis.f` and may be changed.

5 Suspended load

5.1 Suspended load transport equation

5.1.1 Passive scalar hypothesis

For non-equilibrium flow conditions, the bedload and the suspended load are dealt separately. The interface between the bedload and suspended load is located at $z = Z_{ref}$:

– In the thin high-concentrated bedload layer ($z < Z_{ref}$) inter-particle interactions and flow-turbulent interactions strongly modify the flow structure. Equilibrium conditions are however a reliable assumption to relate the bed-load to the current induced bed shear stress.
– In the upper part of the flow ($z > Z_{ref}$), for dilute suspension clear flow concepts still apply, and the sediment grains can be regarded as a passive scalar which follows the mean and turbulent flow velocity, with an additional settling velocity term.

5.1.2 Settling velocity

The settling velocity w_s is an important parameter for the suspension. It can be either specified or calculated by the model as a function of grain diameter. The van Rijn formula [30, 31] which is valid for non-cohesive spherical particles and dilute suspensions, has been implemented in Sisyphe as follows :

$$w_s = \begin{cases} \dfrac{(s-1)gd_{50}^2}{18\nu}, & \text{if } d_{50} \leq 10^{-4} \\ \dfrac{10\nu}{d_{50}}\left(\sqrt{1+0.01\dfrac{(s-1)gd_{50}^3}{18\nu^2}}-1\right), & \text{if } 10^{-4} \leq d_{50} \leq 10^{-3} \\ 1.1\sqrt{(s-1)gd_{50}}, & \text{otherwise} \end{cases}$$

with $s = \rho_s/\rho_0$ is the relative density and g is gravity.

Outside the bedload layer sediment particles are assumed to follow the mean flow velocity, $\mathbf{u} = (u, v, w)^T$, with an additional vertical settling velocity w_s.

5.1.3 Three-dimensional sediment transport equation

The suspended sediment load can be calculated by solving the full three-dimensional (3D) advection-diffusion equation for the suspended sediment concentration distribution, expressed as :

$$\frac{\partial c}{\partial t} + \frac{\partial (uc)}{\partial x} + \frac{\partial (vc)}{\partial y} + \frac{\partial (wc)}{\partial z} - \frac{\partial (w_s c)}{\partial z} = \frac{\partial}{\partial x}\left(\varepsilon_s \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varepsilon_s \frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(\varepsilon_s \frac{\partial c}{\partial z}\right) \tag{31}$$

with $c = c(\mathbf{x}, t)$ the volumetric suspended sediment concentration, $w_s > 0$ the vertical-settling sediment velocity, ε_s the turbulent diffusivity of sediment, often related to the eddy viscosity ν_t by $\varepsilon_s = \nu_t/\sigma_c$, with σ_c being the turbulent Schmidt number (between 0.5 and 1.0). The advection-diffusion equation (31) is completed with initial conditions specifying $C = C(\mathbf{x}, 0)$ and boundary conditions as follows.

At the inlet boundary, a local equilibrium concentration profile can be specified. This profile can be derived from equation (31) by assuming uniform and steady flow conditions. If a parabolic distribution of turbulent eddy diffusivity coefficient is adopted, then the vertical distribution of suspended sediment concentration is the classical Rouse profile. At the outlet, the normal gradients of the concentration are set equal to zero. A similar boundary condition can be specified at the sidewalls of the model.

At the free surface $z = Z_s$, the net vertical sediment flux is set to zero, thus :

$$\left(\varepsilon_s \frac{\partial c}{\partial z} + w_s c\right)_{z=Z_s} = 0.$$

At the interface between the bed-load and the suspended load $z = Z_{ref}$, a Neumann type boundary condition is specified, in which the total vertical flux equals the net sediment transport :

$$\left(\varepsilon_s \frac{\partial c}{\partial z} + w_s c\right)_{z=Z_{ref}} = D - E.$$

The deposition D and entrainment E fluxes can be expressed as $D = w_s c|_{z=z_b}$ and $E = w_s c_{ref}$, respectively. Further details are given next. The 3D advection-diffusion equation (31), with suitable initial and boundary conditions are solved with SEDI-3D, the set of subroutines of TELEMAC-3D for three-dimensional sediment transport modelling. We refer the reader to the corresponding documentation [**?**].

5.1.4 Two-dimensional sediment transport equation

The two-dimensional (2D) sediment transport equation for the depth-averaged suspended-load concentration $C = C(x, y, t)$ is obtained by integrating the 3D sediment transport equation (31) over the suspended-load zone. By applying the Leibniz integral rule, adopting suitable boundary conditions and assuming that the bedload zone is very thin, the depth-integrated suspended-load transport equation is obtained :

$$\frac{\partial hC}{\partial t} + \frac{\partial (hUC)}{\partial x} + \frac{\partial (hVC)}{\partial y} = \frac{\partial}{\partial x}\left(h\epsilon_s \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(h\epsilon_s \frac{\partial C}{\partial y}\right) + E - D, \tag{32}$$

with $h = Z_s - Z_f \approx Z_s - Z_{zref}$ the water depth, assuming that the bedload layer thickness is very thin, and $(U, V)^T$ are the depth-averaged velocity components in the $x-$ and $y-$directions, respectively.

If the effects of the heterogeneous vertical distribution of suspended sediment are taken into account, the following 2D depth-averaged horizontal suspended sediment transport model in its non-conservative form is obtained :

$$\frac{\partial C}{\partial t} + \underbrace{U\frac{\partial C}{\partial x} + V\frac{\partial C}{\partial y}}_{\text{advection}} = \underbrace{\frac{1}{h}\left(\frac{\partial}{\partial x}\left(h\epsilon_s \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(h\epsilon_s \frac{\partial C}{\partial y}\right)\right)}_{\text{diffusion}} + \frac{E - D}{h}, \tag{33}$$

where U and V are the depth-averaged convective flow velocities in the $x-$ and $y-$ directions, respectively.

Attention

Different schemes are available for solving the non-linear advection terms depending on the choice of the parameter the classical characteristics schemes. To the diffusive schemes SUPG and PSI, it is recommended to use conservative schemes.

5.1.5 Treatment of the diffusion terms

According to the choice of the parameter `OPDTRA` of the keyword `OPTION FOR THE DIFFUSION OF TRACER`, the diffusion terms in (33) can be simplified as follows :

– `OPDTRA = 1` : the diffusion term is solved in the form $\nabla \cdot (\varepsilon_s \nabla T)$

– `OPDTRA = 2` : the diffusion term is solved in the form $\frac{1}{h}\nabla \cdot (h\varepsilon_s \nabla T)$

The value of the dispersion coefficient ϵ_s depends on the choice of the parameter `OPTDIF` of the keyword `OPTION FOR THE DISPERSION` :

– `OPTDIF = 1` : the values of the constant longitudinal and transversal dispersivity coefficients ($T1$ and $T2$ respectively) are provided with the keywords `DISPERSION ALONG THE FLOW` and `DISPERSION ACROSS THE FLOW`
– `OPTDIF = 2` : the values of the longitudinal and transversal dispersivity coefficients are computed with the Elder model $T1 = \alpha_l u^* h$ and $T2 = \_alpha_t u^* h$, where the coefficients α_l and α_t can be provided with the keywords `DISPERSION ALONG THE FLOW` and `DISPERSION ACROSS THE FLOW`.
– `OPTDIF = 3` : the values of the dispersion coefficients are provide by TELEMAC-2D

The diffusion term can also be set to zero with keyword `DIFFUSION = NO` (`YES` is the option by default).

5.1.6 Treatment of the advection terms

The convective velocity can be corrected from the depth-averaged mean velocity in order to account for the fact that most sediment is transported near the bed.

Tidal flats

In presence of tidal flats, it is recommended to use conservative scheme based on the calculation of flux per segments. The scheme `14` should be used only if the convection velocity differs from the depth-averaged velocity and no longer satisfies the shallow-water continuity equation, that is when the keyword `CORRECTION ON CONVECTION VELOCITY = YES`.

5.2 Bed evolution

5.2.1 Bed evolution due to the suspension

By considering only suspended load sediment transport, the bed evolution is function of the net vertical sediment flux at the interface between the bedload and the suspended load given by :

$$(1-n)\frac{\partial Z_f}{\partial t} + (E-D)_{z=Z_{ref}} = 0 \tag{34}$$

with Z_f the bed elevation, Z_{ref} the interface between the bedload and suspended load, and n the bed porosity. Once the flow variables are determined by solving the hydrodynamics and suspended sediment transport equations, changes of bed level are computed from Equation (34) for the cell coincident to the bed, calculating at each time step the net sediment flux $E - D$ at the bedload-suspended load interface ($z = Z_{ref}$), as explained later. Further details on the derivation of the mass balance equation (34) and coupling strategies can be found in [39].

Attention

In Equation (34), the net sediment flux $D - E$ is expressed in ms^{-1}. For consistency with bedload units, expressed in m^2s^{-1}), concentration is dimensionless. However, the user can choose concentration by mass per unit volume of the mixture (kg m^{-3}) by using the keyword `MASS CONCENTRATION`. The relation between concentration by volume and concentration by mass is C_m (kg m^{-3}) = $\rho_s C$.

5.3 Equilibrium concentrations

5.3.1 Erosion and deposition rates

For non-cohesive sediments, the net sediment flux $E - D$ is determined based on the concept of equilibrium concentration, see [7] :

$$(E-D)_{Z_{ref}} = W_s \left(C_{eq} - C_{Z_{ref}} \right) \tag{35}$$

where C_{eq} is the equilibrium near-bed concentration and C_{zref} is the near-bed concentration, calculated at the interface between the bed-load and the suspended load, $z = Z_{ref}$. The keyword `REFERENCE CONCENTRATION FORMULA` access to four differents semi-empirical formulas :

- `ICQ= 1` : Zyserman and Fredsoe formula

$$C_{eq} = \frac{0.331(\theta' - \theta_c)^{1.75}}{1 + 0.72(\theta' - \theta_c)^{1.75}}, \tag{36}$$

  where θ_c is the critical Shields parameter and $\theta' = \mu\theta$,the non-dimensional skin friction which is related to the Shields parameter, by use of the ripple correction factor.
- `ICQ= 2` : Bijker (1992) formula The equilibrium concentration corresponds to the volume concentration at the top of the bed-load layer. It can related to the bed load transport rate :

$$C_{eq} = \frac{Q_s}{b Z_{ref} u_*}, \tag{37}$$

  with an empirical factor $b = 6.34$.

  Attention
  The near bed concentration computed with the Bijker formula is related to the bedload. Therefore, this option cannot be used without bedload transport (`BED-LOAD = YES`). Furthermore, both bedload and suspended load transport must be calculated at each time step, with the keyword `PERIOD OF COUPLING` set equal to 1.
- `ICQ= 3` : van Rijn formula
- `ICQ= 4` : Soulsby-van Rijn formula

5.3.2 Reference level

The reference elevation Z_{ref} corresponds to the interface between bedload and suspended load. For flat beds, the bed-load layer thickness is proportional to the grain size, whereas when the bed is rippled, the bedload layer thickness scales with the equilibrium bed roughness (k_r). The definition of the reference elevation needs also to be consistent with the choice of the equilibrium near-bed concentration formula.

- `ICQ = 1` : According to Zyserman and Fredsoe, the reference elevation should be set to $Z_{ref} = 2d_{50}$. In SISYPHE we take $Z_{ref} = k'_s$, where k'_s is the skin bed roughness for flat bed ($k'_s \approx d_{50}$) , the default value of proportionality factor is `KSPRATIO =3`).
- `ICQ = 2` : According to Bijker, $Z_{ref} = k_r$.

5.3.3 Vertical concentration profile

We assume here a Rouse profile for the vertical concentration distribution, which is theoretically valid in uniform steady flow conditions :

$$C(z) = C_{Z_{ref}} \left(\frac{z-h}{z} \frac{a}{a-h} \right)^R, \tag{38}$$

where R is the Rouse number defined by

$$R = \frac{w_s}{\kappa u_*}, \tag{39}$$

with κ the von Karman constant ($\kappa = 0.4$), u_* the friction velocity corresponding to the total bed shear stress (see § **??**), and a the reference elevation above the bed elevation.

5.3.4 Ratio between the reference and depth-averaged concentration

By depth-integration of the Rouse profile (40), the following relation can be established between the mean (depth-averaged) concentration and the reference concentration :

$$C_{Zref} = FC,$$

where :

$$F^{-1} = \left(\frac{Z_{ref}}{h}\right)^R \int_{Zref/h}^{1} \left(\frac{1-u}{u}\right)^R du. \tag{40}$$

In SISYPHE , we use the following approximate expression for F :

$$F^{-1} = \frac{1}{(1-Z)} B^R \left(1 - B^{(1-R)}\right), \quad \text{for } R \neq 1$$
$$F^{-1} = -B \log B, \quad \text{for } R = 1,$$

with $B = Z_{ref}/h$.

5.4 Convection velocity

A straightforward treatment of the advection terms would imply the definition of an advection velocity and replacement of the depth-averaged velocity U in Eq. (33) by :

$$U_{conv} = \overline{UC}/C.$$

A correction factor is introduced in SISYPHE , defined by :

$$F_{conv} = \frac{U_{conv}}{U}$$

The convection velocity should be smaller than the mean flow velocity ($F_{conv} \leq 1$) since sediment concentrations are mostly transported in the lower part of the water column where velocities are smaller. We further assume an exponential profile concentration profile which is a reasonable approximation of the Rouse profile, and a logarithmic velocity profile, in order to establish the following analytical expression for F_{conv} :

$$F_{conv} = -\frac{I_2 - \ln\left(\frac{B}{30}\right) I_1}{I_1 \ln\left(\frac{eB}{30}\right)},$$

with $B = k_s/h = Z_{ref}/h$ and

$$I_1 = \int_B^1 \left(\frac{(1-u)}{u}\right)^R du, \quad I_2 = \int_B^1 \ln u \left(\frac{(1-u)}{u}\right)^R du.$$

For further details, see [19].

Keywords

- CORRECTION ON CONVECTION VELOCITY (= NO, default option)

5.5 Initial and boundary conditions for sediment concentrations

5.5.1 Initial conditions

The initial values of volume concentration for each class can be either imposed in the subroutine `condim_susp.f` or specified in the steering file through the keyword `INITIAL SUSPENSION CONCENTRATIONS`. It will not be used if `EQUILIBRIUM INFLOW CONCENTRATION = YES`.

5.5.2 Boundary conditions

For boundary conditions, the concentration of each class can be specified in the steering file through keyword `CONCENTRATION PER CLASS AT BOUNDARIES`. To avoid unwanted erosion or sedimentation at the entrance of the domain, it may be also convenient to use keyword `EQUILIBRIUM INFLOW CONCENTRATION = YES` to specify the value of the concentration at inflow, according to the choice of the `REFERENCE CONCENTRATION FORMULA`. The depth-averaged equilibrium concentration is then calculated assuming equilibrium concentration at the bed and a Rouse profile correction for the F factor. Input concentrations can be also directly specified in the subroutine `conlit.f`.

6 Wave effects

6.1 Introduction

In coastal zones, the effect of waves superimposed to a mean current (wave-induced or tidal), modifies the bed structure. Due to the reduced thickness of the bed boundary layer, the bottom shear stress is largely increases and the resulting sand transport rate is, in many cases, of an order of magnitude greater than in the case of currents alone. Furthermore, the wave-generated ripples may also have a strong effect on the bed roughness and sand transport mechanisms. In Sisyphe , these effects can be incorporated into the numerical simulation when the keyword `EFFECT OF WAVES` is activated.

To compute sediment transport rates due to the action of waves, the wave height, period and direction need to be specified. This information can be provided from a fortran file, by the subroutine `condim_sisyphe.f`, from a file containing the variables computed previously by the wave module, e.g. Tomawac [?], or by internal coupling.

The wave orbital velocity U_0 is calculated by Sisyphe assuming linear theory :

$$U_0 = \frac{H_s \omega}{2\sinh(kh)},$$

where H_s is the significant wave height, $\omega = 2\pi/T_p$ is the intrinsic angular frequency, $k = 2\pi/L$ is the wave number, with L the wave length. The wave number is calculated from the dispersion relation (neglecting surface tension and amplitude effects) :

$$\omega^2 = gk\tanh(kh).$$

6.1.1 Wave information

The wave information, required as input parameters for the morphodynamic computation, are the significant wave height H_s, the peak wave period T_p and the wave direction θ_w. If θ_w is not provided, Sisyphe assumes the default value $\theta_w = 90°$, which means that the direction of propagation of the wave is parallel to the x-axis. When using the Bailard's transport formula, this variable must be always provided.

If the wave field is provided from a file containing the variables computed previously on the same mesh by the wave module, e.g. Tomawac , the keyword `WAVE FILE` must be specified in the Sisyphe steering file. This file must contain the wave variables `HM0`, significant wave height H_s (`HM0`), wave peak period T_p (`TPR5`) and mean wave direction θ_w, relative to y-axis (`DMOY`).

Attention

For the keyword `WAVE FILE`, variables are given by the last record of the file.

If the currents and wave field is provided from a file containing the variables computed previously on the same mesh by the currents and wave modules, e.g. Telemac-2d and Tomawac , respectively, the keyword `HYDRODYNAMIC FILE` must be specified in the Sisyphe steering file. This file must contain the hydrodynamic variables (water height and velocity file) as well as wave variables `HM0`, significant wave height H_s (`HM0`), wave peak period T_p (`TPR5`) and mean wave direction θ_w, relative to y-axis (`DMOY`). The users has also the choice of provide the wave variables from the subroutine `condim_sisyphe.f`.

Attention

For the keyword `HYDRODYNAMIC FILE`, variables are given by the last record of the file if the case is steady or by the time steps describing the tide or flood for the unsteady case.

6.2 Wave-induced bottom friction

6.2.1 Wave-induced bottom friction

The maximum stress due to waves is calculated at each time step as a function of the wave-orbital velocity U_w by use of a quadratic friction coefficient f_w due to waves :

$$\tau_w = \frac{1}{2}\rho f_w U_w^2.$$

The wave friction factor f_w is calculated as a function of relative density :

$$f_w = f_w\left(A_0/k_s\right),$$

where $A_0 = U_w/\omega$ is the orbital amplitude on the bed and k_s the bed roughness. In Sisyphe , the formule proposed by Swart *et al.* [27] is provided

$$f_w = \begin{cases} \exp\left(-6.0 + 5.2\left(\frac{A_0}{k_s}\right)^{-0.19}\right), & \text{if } \frac{A_0}{k_s} > 1.57 \\ 0.30, & \text{otherwise} \end{cases}$$

6.2.2 Wave-current interaction

For combined waves and currents, the wave-induced (mean and maximum) bottom stresses are, in many cases, of an order of magnitude larger than in the case of currents alone. Different models can be found in the literature to calculate the wave and current bottom stresses τ_{cw}, as a function of the bottom shear stress due to currents only τ_c and the maximum shear stress due to waves only τ_w. Following Bijker [5] :

$$\tau_{cw} = \tau_c + \frac{1}{2}\tau_w. \tag{41}$$

The mean τ_{mean} and maximum $\tau_{\max}$ shear stresses can also be calculated following the Soulsby's method [26]. Non-dimensional variables are defined :

$$X = \frac{\tau_0}{\tau_0 + \tau_w}; \quad Y = \frac{\tau_{mean}}{\tau_0 + \tau_w}; \quad Z = \frac{\tau_{\max}}{\tau_0 + \tau_w}$$

They can be parameterized as :

$$\begin{aligned} Y &= X\left(1 + bX^p(1-X)^q\right), \\ Z &= 1 + aX^m(1-X)^n \end{aligned}$$

The various model coefficients (a, b, m, n, p, q) are empirical functions of friction coefficients f_w and C_D, and the wave-current angle ϕ :

$$\begin{aligned} a &= \left(a_1 + a_2\left|\cos\phi\right|^I\right) + \left(a_3 + a_4\left|\cos\phi\right|^I\right)\log_{10}\left(\frac{2f_w}{C_D}\right), \\ b &= \left(b_1 + b_2\left|\cos\phi\right|^J\right) + \left(b_3 + b_4\left|\cos\phi\right|^J\right)\log_{10}\left(\frac{2f_w}{C_D}\right), \\ &\ldots, \end{aligned}$$

with similar expressions for m, n, p, and q. The fitted constants (a_i, b_i, m_i, n_i, p_i, q_i, I and J) depend on the turbulence model selected. Table 2 shows the coefficients used in Sisyphe corresponding to the model of Huynh-Thanh and Temperville [20], for $I = 0.82$, $J = 2.7$.

i	a_i	b_i	m_i	n_i	p_i	q_i
1	-0.07	0.27	0.72	0.78	-0.75	0.89
2	1.87	0.51	-0.33	-0.23	0.13	0.40
3	-0.34	-0.10	0.08	0.12	0.12	0.50
4	-0.12	-0.24	0.34	-0.12	0.02	-0.28

TABLE 2 – Constants issued from the turbulence model of Huynh-Thanh and Temperville [20], see also Soulsby [26], page 91.

6.2.3 Friction coefficient under combined waves and current

The characteristic shear stress to represent wave current interactions $\langle|\overrightarrow{\tau_{cw}}(t)|\rangle$, is related to the time-averaged mean velocity :

$$\langle|\overrightarrow{\tau_{cw}}(t)|\rangle = \rho f_{cw} \left\langle \left|\overrightarrow{U(t)}\right|^2 \right\rangle,$$

where

$$\left\langle \left|\overrightarrow{U(t)}\right|^2 \right\rangle = U_c^2 + \frac{1}{2}U_w^2$$

According to Camenen [6], the characteristic shear stress is taken as a weighted average between τ_{mean} and $\tau_{\max}$:

$$\langle|\overrightarrow{\tau_{cw}}(t)|\rangle = X\tau_{mean} + (1-X)\tau_{\max},$$

which is equivalent to :

$$\langle|\overrightarrow{\tau_{cw}}(t)|\rangle = Y\tau_c + Z\tau_w.$$

The final expression for the wave-current interaction factor is :

$$f_{cw} = \frac{Y\tau_c + Z\tau_w}{U_c^2 + \frac{1}{2}U_w^2}. \tag{42}$$

This expression of the wave and current friction factor is used by Bailard [?] and Dibajnia and Watanabe [10] to compute sand transport rates.

6.3 Wave-induced ripples

Equilibrium ripples are generally observed outside the surf zone. Their dimensions can be predicted as a function of waves (orbital velocity U_0 and wave period $T = 2\pi/\omega$), for a given uniform sediment diameter d_{50}. In SISYPHE , the procedure of Wiberg and Harris has been implemented [35]. This formulation is only applicable to oscillatory flow conditions and does not account for the effect of a superimposed mean current.

Ripples can be classified into three types depending on the value of the ratio of wave orbital diameter to mean grain diameter, D_0/d_{50}, with $D_0 = 2U_0/\omega$. The ripples wave length λ and height η, can be estimated as follows :

- Under moderate wave conditions (orbital regime), the ripple dimensions are proportional to D_0 :

$$\lambda = 0.62 D_0, \quad \eta = 0.17\lambda.$$

- For larger waves (anorbital regime), ripple dimensions scale with the sand grain diameter :

$$\begin{aligned} \lambda &= 535\, d_{50}, \\ \eta &= \lambda \exp\left(-0.095\left(\log\frac{D_0}{\eta}\right)^2 + 0.042\log\frac{D_0}{\eta} - 2.28\right). \end{aligned}$$

The effect of ripples is to increase the bed roughness. Based on the Bijker's formula, in SISYPHE it is assumed that :

$$k_s = \max(\eta, 3d_{50}).$$

The effect of the mean current superimposed to the waves is accounted by introducing a correction factor α to the orbital velocity, following Tanaka and Dang [28] :

$$\alpha = 1 + 0.81\left(\tanh(0.3S_*^{2/3})\right)^{2.5}\left(\frac{U}{U_w}\right)^{1.9},$$

with $S_* = d_{50}\sqrt{(s-1)gd_{50}}/4\nu$.

6.4 Wave-induced sand transport

The effect of waves modifies the sand transport rates when it is superimposed to currents. This effect can be accounted by using one of the following wave sand transport formula programmed in SISYPHE (keyword `BED-LOAD TRANSPORT FORMULA`), namely the Bijker's formula [5], the Soulsby-van Rijn's formula [26], the Bailard's formula [?] and the Dibajnia and Watanabe's formula [10].

– `ICF = 4` : the Bijker's formula (1968) can be used for determining the **total transport rate** Q_s (bed-load + suspension), with its two components the bedload Q_{sc} and the suspended load Q_{ss} determined separately.
For bedload transport, Bijker extended the steady bed-load formula proposed by Frijlink (1952), by adding the effect of the wave. In non-dimensional variables, the bedload transport rate is :

$$\Phi_b = b\theta_c^{0.5}\exp\left(-0.27\frac{1}{\mu\theta_{cw}}\right),$$

where θ_c is the non-dimensional shear stress due to currents alone, θ_{cw} the non-dimensional shear stress due to wave-current interaction, and μ is a correction factor which accounts for the effect of ripples. The shear stress under combined wave and current is calculated by Equation (41).
In the original formulation, the coefficient b is $b = 5$. Recent studies [?] showed that $b = 2$ provides better results when comparing with field and experimental data. Be default, in SISYPHE $b = 2$ but this value can be modified with the keyword `B VALUE FOR THE BIJKER FORMULA`. The ripple factor correction μ is calculated in the same way as in the Meyer-Peter-Mueller formula for currents only.
The suspended load transport is solved in a simplified manner, the concentration profile is assumed to be in equilibrium. The inertia effects related to the water column loading and unloading will then in no way be modelled. Furthermore, no exchange takes place with the bed load layer, therefore only the continuity of concentration is ensured. After depth-integration and by assuming a Rouse profile for the concentration and a logarithmic velocity profile for the mean velocity profile, the suspended load can be written as :

$$Q_{ss} = Q_{sc}I,$$

where

$$I = 1.83 \times 0.216\frac{B^{A-1}}{(1-B)^A}\int_B^1\left(\frac{1-y}{y}\right)^A \ln\left(\frac{33y}{B}\right)dy,$$

with

$$A = \frac{W_s}{\kappa u_*}, \quad u_* = \sqrt{\frac{\tau_{cw}}{\rho}}, \quad B = k_s/h.$$

– `ICF = 5` : Soulsby-van Rijn formula, the transport rate due to the combined action of waves and current is provided by the following equation :

$$Q_{b,s} = A_{b,s}U\left[\left(U^2 + 2\frac{0.018}{C_D}U_0^2\right)^{0.5} - U_{cr}\right]^{2.4}.$$

This formula can be applied to estimate both components of the total sand transport rate (bedload Q_b and suspension Q_s), and it is suitable for rippled bed regimes. The bedload and suspended load coefficients, $A_{b,s}$ are computed :

$$A_b = \frac{0.005h\,(d_{50}/h)^{1.2}}{((s-1)gd_{50})^{1.2}}$$
$$A_s = \frac{0.012d_{50}D_*^{-0.6}}{((s-1)gd_{50})^{1.2}},$$

where U is the depth-averaged current velocity, U_0 is the RMS orbital velocity of waves, and C_D is the quadratic drag coefficient due to current alone. This formula has been validated assuming a rippled bed roughness with $k_s = 0.18$ m. The critical entrainment velocity U_{cr} is given by :

$$U_{cr} = \begin{cases} 0.19 d_{50}^{0.1} \log_{10}\left(\dfrac{4h}{D_{90}}\right), & \text{if } 0.1mm \leq d_{50} \leq 0.5mm \\ 8.5 d_{50}^{0.6} \log_{10}\left(\dfrac{4h}{D_{90}}\right), & \text{if } 0.5mm \leq d_{50} \leq 2.0mm. \end{cases}$$

The diameter D_{90}, characteristic of the coarser grains, is specified with the keyword `D90`. The validity range for the Soulsby-van Rijn formula is $h = 1 - 20$ m, $U = 0.5 - 5\text{ms}^{-1}$, and $d_{50} = 0.1 - 2\text{mm}$.

– `ICF = 8` : the Bailard's formula is based on an energetic approach. The bedload and the suspended load components of the sand transport rate are expressed respectively as the third- and fourth-order momentum of the near-bed time-varying velocity field $\overrightarrow{U(t)}$, as follows

$$Q_b = \frac{f_{cw}}{g(s-1)} \frac{\epsilon_c}{\tan\varphi} \left\langle \left|\overrightarrow{U}\right|^2 \overrightarrow{U} \right\rangle$$
$$Q_s = \frac{f_{cw}}{g(s-1)} \frac{\epsilon_s}{W_s} \left\langle \left|\overrightarrow{U}\right|^3 \overrightarrow{U} \right\rangle$$

with f_{cw} the friction coefficient which accounts for wave-current interactions, $\epsilon_s = 0.02$, $\epsilon_c = 0.1$ empirical factors, φ sediment friction angle ($\tan\varphi = 0.63$) and $< \cdot >$ time-averaged over a wave-period.

Under combined wave and currents, the time-varying velocity vector $\overrightarrow{U(t)}$ is composed of a mean component U_c and an oscillatory component of amplitude U_0, assuming linear waves ; ϕ is the angle between the current and the wave direction, ϕ_c is the angle between the mean current direction and the $x-$axis, and ϕ_w is the angle between the wave direction of propagation and the x-axis. The time-varying velocity field verifies :

$$\overrightarrow{U(t)} = U_c \exp^{i\phi_c} + U_0 \cos\omega t \exp^{i\phi_w}.$$

For the third-order term, one can derive an analytical expression in the general case, whereas for the fourth-order momentum, we have to assume the waves and currents to be co-linear ($\phi = 0$), in order to find an analytical expression. For the general case of a non-linear non-colinear wave, we would have to integrate numerically the fourth-order velocity term [6]. This method is however not efficient.

Third-order term :

$$\left\langle \left|\overrightarrow{U}\right|^2 \overrightarrow{U} \right\rangle = \overrightarrow{U_c}(U_c^2 + \frac{1}{2}U_0^2) + \overrightarrow{U_0}(\overrightarrow{U_c} \cdot \overrightarrow{U_0})$$
$$\left\langle \left|\overrightarrow{U}\right|^2 \overrightarrow{U} \right\rangle_x = \left[U_c^3 + U_c U_0^2(1 + \frac{1}{2}\cos 2\phi)\right] \cos\phi_c - \frac{1}{2}U_c U_0^2 \sin 2\phi \sin\phi_c$$
$$\left\langle \left|\overrightarrow{U}\right|^2 \overrightarrow{U} \right\rangle_y = \left[U_c^3 + U_c U_0^2(1 + \frac{1}{2}\cos 2\phi)\right] \sin\phi_c + \frac{1}{2}U_c U_0^2 \sin 2\phi \cos\phi_c$$

Fourth-order term :

$$\left\langle \left|\overrightarrow{U}\right|^3 \overrightarrow{U} \right\rangle_x = \frac{1}{8}(24U_0^2U_c^2 + 8U_c^4 + 3U_0^4)\cos\phi_c$$
$$\left\langle \left|\overrightarrow{U}\right|^3 \overrightarrow{U} \right\rangle_y = \frac{1}{8}(24U_0^2U_c^2 + 8U_c^4 + 3U_0^4)\sin\phi_c$$

– `ICF = 9` : the Dibajnia and Watanabe (1992) formula is an unsteady formula, which accounts for inertia effects. The sand transport rate is calculated by :

$$\frac{\overrightarrow{Q_s}}{W_s d_{50}} = \alpha \frac{\overrightarrow{\Gamma}}{|\Gamma|^{1-\beta}}, \tag{43}$$

with $\alpha = 0.0001$ and $\beta = 0.55$ empirical model coefficients and $\overrightarrow{\Gamma}$ is the difference between the amounts of sediments transported in the onshore and offshore directions. This formula is used to estimate the intensity of the solid transport rate, the direction is assumed to follow the mean current direction.
In the wave direction, the time-varying velocity is given by :

$$U(t) = U_c \cos\phi + U_0 \sin\omega t,$$

r is defined by the asymmetry parameter :

$$r = \frac{U_0}{U_c \cos\phi}.$$

The wave cycle is decomposed into two parts :

1. During the first part of the wave-cycle ($0 < t < T_1$), the velocities are directed onshore ($U(t) > 0$).
2. During the second part ($T_2 = T - T_1$), the velocities are negatives ($U(t) < 0$).

The sand transport rate in the wave direction is :

$$\Gamma_{X'} = \frac{u_1T_1(\Omega_1^3 + \Omega_2'^3) - u_2T_2(\Omega_2^3 + \Omega_1'^3)}{(u_1 + u_2)T}$$

where Ω_1 and Ω_2 represent the amount of sand transported in the onshore and offshore directions which will be deposited before flow reversal, respectively, Ω_1' and Ω_2' represent the remaining part which stay in suspension after flow reversal. The inertia of sediment depends on the ratio d_{50}/w_s, and represented by parameter ω_i :

$$\omega_i = \frac{u_i^2}{2(s-1)gW_sT_i},$$

with

$$u_i^2 = \frac{2}{T_i}\int_{u>\text{or}<0} u^2 dt,$$

and two different cases :

1. All sediment is deposited before flow reversal $\omega_i \leq \omega_{crit}$

$$\Omega_i = \omega_i \frac{2W_sT_i}{d_{50}}, \quad \Omega_i' = 0$$

2. Part of the sediment stays in suspension after flow reversal $\omega_i \geq \omega_{crit}$

$$\Omega_i = \omega_{crit}\frac{2W_sT_i}{d_{50}}, \quad \Omega_i' = (\omega_i - \omega_{crit})\frac{2W_sT_i}{d_{50}}.$$

The critical value of ω_{crit} is calculated as function of the wave-current non-dimensional friction :

$$\Theta_{cw} = \frac{< \tau_{cw} >}{\rho} \frac{1}{(s-1)gd_{50}}.$$

According to Temperville et Guiza [**?**] :

$$\omega_{crit} = \begin{cases} 0.03, & \text{if } \Theta_{cw} \leq 0.2 \\ 1 - \sqrt{1 - ((\Theta_{cw} - 0.2)/0.58)^2}, & \text{if } 0.2 \leq \Theta_{cw} \leq 0.4 \\ 0.8104\sqrt{\Theta_{cw}} - 0.4225, & \text{if } 0.4 \leq \Theta_{cw} \leq 1.5 \\ 0.7236\sqrt{\Theta_{cw}} - 0.3162, & \text{otherwise} \end{cases}$$

Attention

In SISYPHE , the solid discharge is assumed to be oriented in the direction of the mean current. The possible deviation of the transport in the direction of waves, which can be important in the near shore zone due to non-linear effects, is not accounted for.

7 Sand grading effects

7.1 Sediment bed composition

7.1.1 Granulometry distribution

The number of classes of bed material is specified in the steering file and limited to $\texttt{NSICLA} \leq 20$. For uniform sediment, the granulometry distribution is represented by one single value $\texttt{NSICLA} = 1$ for the whole domain. The mean grain diameter and corresponding settling velocity are defined in the steering file.

For sediment mixtures, the granulometry distribution is discretized in a number of classes. Each class of sediment `1 < j < NSICLA` is defined by its mean diameter $d_{50}(\texttt{j})$ and volume fraction in the mixture at every node `AVAI(j)`. The characteristics of each class of sediment, for example the Shields number $\theta_c(\texttt{i})$, settling velocity $w_s(\texttt{i})$ of each class can be specified in the steering file or calculated by the model, as for a single sediment class.

The percent of each class of material need to verify, such that :

$$\sum_{\texttt{j}=1,\texttt{NSICLA}} \texttt{AVAI}(\texttt{j}) = 1 \quad \text{and} \quad 0 \leq \texttt{AVAI}(\texttt{j}) \leq 1. \tag{44}$$

The initial bed composition can be defined in the steering file for uniform bed. For complex bed configuration, e.g. spatial variation, vertical structure, etc., the user subroutine `init_compo.f` is used to define the initial state.

The mean diameter D_m, `ACLADM`, is calculated as :

$$D_m = \sum_{\texttt{j}=1,\texttt{NSICLA}} \texttt{AVAI}(\texttt{j})\,\texttt{D}(\texttt{j}). \tag{45}$$

7.1.2 Bed structure

The bed is stratified into a number of layers `NOMBLAY` that enables vertical variations of the sediment bed composition. The percentage of each class `j` of material, `AVAI(i,j,k)` as well as the thickness of each layer E_s are defined at each point `i` and for each layer `k`.

The composition of transported material is computed using the composition of the upper layer, see below the definition of *active layer*. The initial composition of the bed (number of layers, thickness of each layer E_s, composition of each layer `AVAI` are specified in user's subroutine `init_compo.f`.

The subroutine `init_avail.f` verifies if the structure of the initial bed composition is compatible with the position of the rigid bed, as defined in user's subroutine `noerod.f`) :

$$Z_f(\texttt{i}) - Z_r(\texttt{i}) = \sum_{\texttt{k}=1,\texttt{NOMBLAY}} E_s(\texttt{k}). \tag{46}$$

In general, the thickness of the bed is taken to be large (by default, 100 m), so the bottom layer thickness need to be increased.

Assuming the porosity n of each class to be identical and constant, the total mass of sediments per unit area is :

$$M_s(\texttt{i}) = \sum_{\texttt{k}=1,\texttt{NOMBLAY}} \rho_s E_s(\texttt{k})(n-1). \tag{47}$$

In each layer, the percent of each class `AVAI`, which is defined as the volume of each class of material per the total volume of material, is considered to be a constant. For each layer and at each point of the domain, the following constraints need to be satisfied :

$$\sum_{\mathtt{j=1,NSICLA}} \mathtt{AVAI(i,k,j)} = 1 \quad \text{and} \quad 0 \leq \mathtt{AVAI(i,k,j)} \leq 1.$$

Keywords

The initial sediment bed composition is defined by :
- `NUMBER OF SIZE-CLASSES OF BED MATERIAL` : `NSICLA = 1`, by default
- `SEDIMENT DIAMETERS` : `FDM = .01`, by default
- `SETTLING VELOCITIES` : if `XWC` is not given, the subroutine `vitchu_sisyphe.f` is used to compute the settling velocity based on the Stokes, Zanke or Van Rijn formulae as a function of the grain size
- `SHIELDS PARAMETERS` : for multi grain size, the Shields parameter needs to be specified for each class. If only one value is specified, the shields parameter will be considered constant for all classes. The default option, no Shields parameter given in steering file, is to calculate the Shields parameter as a function of the sand grain diameter, see logical `CALAC`.
- `INITIAL FRACTION FOR PARTICULAR SIZE CLASS` : `AVA0 = 1.;0.;0.;...,` `by default`

For more than one-size classes, the user should specify `NSICLA` values for the mean diameter, initial fraction, etc. For example, for a mixture of two classes :

Example
- `NUMBER OF SIZE-CLASSES OF BED MATERIAL = 2`
- `SEDIMENT DIAMETERS = 0.5; 0.5`
- `SETTLING VELOCITIES = 0.10; 0.05`
- `SHIELDS PARAMETERS = 0.045; 0.01`
- `INITIAL FRACTION FOR PARTICULAR SIZE CLASS = 0.5; 0.5`

7.2 Sediment transport of sediment mixtures

The method programmed in SISYPHE for the treatment of mixtures of sediment with variable grain sizes is classical and based on previous models from the literature (as for example the 1D model Sedicoup developed at Sogreah). There are two layer concepts implemented in SISYPHE . The active layer model based on Hirano's concept [15] (default) and a newly developed continues vertical sorting model (C-VSM) from Merkel [**? ? ? ?**].

7.2.1 Active layer and stratum

Since only the thin upper layer will be transported, the upper layer is therefore subdivided into a thin *active layer* and an *active stratum*. The composition of the active layer is used to calculate the composition of transported material and the intensity of transport rates for each class of sediment :

$$Q_s = \sum_{\mathtt{j=1,NSICLA}} \mathtt{AVA0(j)} Q_s(\mathtt{j}), \tag{48}$$

where `AVA0(j)` is the percentage of the class `j` in the active layer.

The active stratum is the layer that will exchange sediment with the active layer in order to keep the active layer to a fixed size. When there is a lot of erosion and when this active stratum becomes too thin, the stratum lying underneath will be merged with it.

The active layer thickness concept is not well defined and depends on the flow and sediment transport characteristics [30]. According to Yalin [37], it is proportional to the sand diameter of the upper layer. The active later thickness generally scales with the bed roughness, which is typically of the order of a few grain diameters for flat beds up to few centimeters in the case of rippled beds. In the presence of dunes, the active layer thickness should be half the dune height [24].

In SISYPHE , the active layer thickness is an additional parameter of the model, noted ELAY0 , which can be estimated by calibration and specified by the user in the steering file by keyword ACTIVE LAYER THICKNESS. With the option CONSTANT ACTIVE LAYER THICKNESS = NON, it is possible to use a space and time varying active layer thickness during the simulation. In SISYPHE it is assumed that ELAY0 $= 3D_m$, with D_m the mean diameter of the upper layer.

The erosion rate is restricted by the total amount of sediments in the active layer, which therefore acts as a rigid bed. The same methods applied for rigid beds are applicable for active layer formulation. When dealing with graded sediment and thin thickness active layers, it is advised to use finite elements combined with the positive depth algorithm, as used for the treatment of rigid bed, in order to avoid numerical problems such as negative sediment fractions. The error message `time step should be reduced` can also appear in the listing file when the erosion is greater than half of the active layer thickness. It is strongly recommended to follow this message.

7.2.2 Continuous vertical sorting model

The C-VSM overcomes many limitations of the classic layer implementation of Hirano (HR-VSM). Even though it is a different way to manage the grain sorting, it is just another interpretation of Hiranos original idea with fewer simplifications. So still an active layer is defined. But the grain distribution in this layer is computed each time step from a depth dependent bookkeeping model for each grain size fraction with unlimited numerical resolution for each horizontal node. The new model doesn't overcome the need to carefully calibrate the same input parameters as all other models, but the new interpretation has the following advantages :

– It is possible to keep minor but prominent grain mixture variations even after a high number of time steps. Smearing effects and bookkeeping accuracy is defined by user defined thresholds or the computational resources, rather than through a fix value.
– Various functions for the impact depth of the shear stress can be chosen to the projects demands. The result is a much more accurate vertical grain sorting, which results in better prognoses for bed roughness, bed forms and erosion stability.

A couple of new keywords must be set in your sis.cas file. The full C-VSM output can be found in the new Selafin files VSPRES & VSPHYD in the tmp-folders. As the higher resolution of the C-VSM needs resources, you can reduce the printout period, or suppress the output at all. The common SISYPHE result files only show the averaged values for the active layer. Even more disk space can be saved, if only few points are printed out as .VSP.CSV files in the subfolder /VSP/. We recommend using between 200 and 1000 vertical sections. More will not improve the accuracy much, and less will lead to increasing data management, as the profile compression algorithms are called more often.

Keywords

The initial sediment bed composition is defined by :

- `VERTICAL GRAIN SORTING MODEL = 1`
  - `0 = Layer = HR-VSM` (Hirano + Ribberink as until Sisyphe v6p1)
  - `1 = C-VSM`
- `C-VSM MAXIMUM SECTIONS = 100`
  - Should be at least $4 + 4\times$ `Number of fractions`,
  - better > 100, tested up to 10000
- `C-VSM FULL PRINTOUT PERIOD = 1000`
  - `0 => GRAPHIC PRINTOUT PERIOD`
  - Anything greater `0 =>` Sets an own printout period for the `CVSP`
  - Useful to save disk space
- `C-VSM PRINTOUT SELECTION = 0|251|3514|...|...`
  - Add any 2D Mesh Point numbers for `.CSV-Ascii` output of the CVSP
  - Add `0` for full `CVSP` output as Selafin3D files (called `VSPRES + VSPHYD`)
  - All files are saved to your working folder and in `/VSP` & `/LAY` folders below
- `C-VSM DYNAMIC ALT MODEL = 5`
  - Model for dynamic active layer thickness approximation
  - `0` = constant (Use Keyword : `ACTIVE LAYER THICKNESS`)
  - `1` = Hunziker & Guenther
$$ALT = 5d_{MAX}$$
  - `2` = Fredsoe & Deigaard (1992)
$$ALT = \frac{2\tau_B}{(1-n)g(\rho_S-\rho)\tan\Phi}$$
  - `3` = van RIJN (1993)
$$ALT = 0.3D_*^{0.7}\frac{\tau_B-\tau_C}{\tau_B}^{0.5} d_{50}$$
  - `4` = Wong (2006)
$$ALT = 5\frac{\tau_B}{(\rho_S-\rho)gd} - 0.0549)^{0.56} d_{50}$$
  - `5` = Malcharek (2003)
$$ALT = \frac{d_{90}}{1-n}\max(1, \frac{\tau_B}{\tau_C})$$
  - `6 = 3*d50` within last time steps ALT'
$$ALT = 3d_{50}$$

7.2.3 General formulation

Bedload transport rates are computed separately for each class using classical sediment transport formulae, corrected for sand grading effects. The Exner equation is then solved for each class of sediment. The individual bed evolution due to each class of bed material is then summed to give the global evolution due to bedload. Similarly, the suspended transport equation is solved for each class of sediment and the resulting bed evolution for each class is then summed to give the global evolution due to the suspended load.

At each time step, the bed evolution due to bedload and to suspension transport is used to compute the new bed structure. The algorithm which determines the new bed composition considers two possibilities, namely deposit or erosion, and ensures mass conservation for each individual class of material. The algorithm is explained in Gonzales de Linares [14].

7.2.4 Hiding exposure

In order to calculate bedload sediment transport rates for each class of sediment, it is necessary to consider the effect of hiding exposure. That means that in a sediment mixture, bigger particles will be more exposed to the flow than the smaller ones and therefore, prediction of sediment transport rates with classical sediment transport formulas, need to be corrected by use of a *hiding-exposure* correction factor. In SISYPHE , the keyword `HIDING FACTOR FOR PARTICULAR SIZE CLASS` sets the value of hiding factor for a particular size class and the keyword `HIDING FACTOR FORMULA` allows the user to choose among different formulations. Two formulas, Egiazaroff [11] and Ashida & Michiue [1], have been written based on the Meyer-Peter and Müller formula. Both formulas modify the critical Shields parameter that will be used in the Meyer-Peter formula. The formula of Karim and Kennedy [21] can be used in combination with any bedload transport predictor. This formula modifies directly the bedload transport rate.

– `HIDFAC = 1` : formulation of Egiazaroff

$$\theta_{cr} = 0.047\zeta_i, \quad \text{with } \zeta_i = \left[\frac{\log(19)}{\log(19D_i/D_m)}\right]^2$$

– `HIDFAC = 2` : formulation of Ashida & Michiue
– `HIDFAC = 4` : formulation of Karim *et al.*

Keywords
Sand grading effects are defined by :
– `HIDING FACTOR FORMULA` : if `HIDFAC = 0` (by default), the user needs to give `HIDING FACTOR FOR PARTICULAR SIZE CLASS`
– `ACTIVE LAYER THICKNESS` : `= 10000`, by default
– `CONSTANT ACTIVE LAYER THICKNESS` : `= YES`, by default
– `NUMBER OF BED LOAD MODEL LAYERS` : `NOMBLAY = 2`, by default

7.2.5 Sediment transport formula

The formula of Hunziker [18] is an adaptation of the Meyer-Peter Müller formula for fractional transport. The volumetric sediment transport per sediment class is given by :

$$\Phi_b = 5p_i \left[\zeta_i \left(\mu\theta_i - \theta_{cm}\right)\right]^{3/2} \quad \text{if} \quad \mu\theta_i > \theta_{cr}, \tag{49}$$

with p_i the fraction of class i in the active layer, θ_i the Shields parameter, $\theta_{cm} = \theta_{cr}\left(D_{mo}/D_m\right)^{0.33}$ the corrected critical Shields parameter, θ_{cr} the critical Shields parameter ($\theta_{cr} = 0.047$), $\xi_i = (D_i/D_m)^{-\alpha}$ the hiding factor, D_i the grain size of class i, D_m the mean grain size of the surface layer (m), D_{mo} the mean grain size of the under layer (m), and α a constant. The critical Shields parameter is calculated as a function of the dimensionless mean grain size D_*. It should be noted that according to Hunziker, stability problems may occur outside the parameter range $\alpha \leq 2.3$ and $D_i/D_m \geq 0.25$.

8 Cohesive sediment

†

8.1 Introduction

8.1.1 Sediment bed composition

The non-cohesive sediment composition is represented by a finite number of classes, each characterized by its mean diameter, grain density and constant settling velocity. The fine cohesive particles made of silt and clay present specific properties (flocculation, consolidation) for a grain diameter less than a limiting value of about 60 μm.

For cohesive sediments, the bed is generally non-uniform : as a result of underweight consolidation, the bed becomes stratified, with density increasing with distance from the surface. The top layer is therefore made of soft mud which can be viewed as the active layer. It is indeed eroded under the action of regular currents, while the sediment in suspension will be deposited in this first layer. The bottom consolidated layers present higher resistance and can only be eroded under extreme events. This vertical stratification of the bed is therefore a key issue, which controls the amount of material to be put in suspension.

Cohesive sediments are transported only in suspension (no bedload), such that the Exner equation for the bed evolution is no-longer solved. The bed evolution is obtained as a mass balance between the erosion and deposition fluxes, which are calculated in SISYPHE using the Krone and Partheniades laws : the erosion flux need specific treatment in order to correctly account for the vertical increase in the bed shear strength as the bed gets eroded.

The effect of flocculation can be represented in the model by specifying a higher settling velocity parameter which can be an order of magnitude greater than the individual or primary particle settling velocity. More physically based relation have been programmed in the 3D library of TELEMAC-3D following the method of Van Leussen (1994).

The model presents different options to represent the effect of consolidation : different schemes can be used. In a so-called iso-pycnal scheme, the bed is represented by a number of layers of increasing concentration. The concentration of each layer is fixed, while their thickness varies in time as the bed undergoes erosion/deposition/consolidation. This scheme is used in a semi-empirical model, originally developed by Villaret and Walther (2008). A new scheme developed by Lan Anh Van (2012) is based on the Gibson's theory (Gibson et al, 1967, 1981). An other third scheme has been implemented which is similar to the Gibson consolidation model developed in TELEMAC-3D by LeNormand (1993). In this third model, the bed is discretised in layers of given thicknesses and time-varying concentrations. This third model has been shown to be unstable and CPU time-consuming (cf. Van, 2012).

This chapter is organized as follows. Section 2 presents the initialization of cohesive sediment properties. Section 3 concentrates on the erosion/deposition and Section 4 on the consolidation. Appendix 1 gives a list of keywords for cohesive applications and user-subroutine. In Appendix 2, the Gibson equation is derived for consolidation modelling.

†. This chapter has been written by D. Phan van Bang, Lan and Villaret

8.2 Initialization of cohesive sediment

8.2.1 Sediment properties

Non cohesive/cohesive sediment

Fine sediment particles of diameters less than 60 μm present complex cohesive properties which affect the sediment transport processes. For non-cohesive sediments ($D_{50} > 60\mu$m), the grain diameter and grain density ρ_s are the key parameters which determine its resistance to erosion, settling velocity and sediment transport rate. Non cohesive material are generally made of quartz, such that the density of sand particles is constant and equal to 2650 kg/m^3.

For cohesive sediments ($D_{50} < 60\mu$m), the primary grain diameter is no longer the key sediment parameter. Due to electro-static attractive forces (van der Waals), individual particles tend to form floc. Their size can be an order of magnitude greater than the initial particle diameter, while the floc density decreases : the floc parameters (size and density) are more relevant to compute the terminal velocity.

The settling velocity of cohesive particles therefore depends on the suspended sediment concentration as well as to other physico-chemical properties (pH, salinity, cations for instance) of the suspension which influence the flocculation process. The effect of turbulence level determines also the flocculation state. The critical bed shear strength depends on the consolidation state or age of the sediment bed (Migniot, 1968).

Attention

The separation value at 60μm to discriminate non-cohesive from cohesive sediment is conventional (UK classification). The value is different depending on the country (63μm in Netherlands, 75μm in USA as pointed by Winterwerp and Van Kesteren, 2004). Moreover, aggregation of flocs can lead to the formation of macro-flocs larger than 100μm.

We consider in Sections 3-5, the simpler case of uniform, cohesive sediments, characterized by one single value for the primary grain size D_{50} and density $\rho_s = 2650$ kg/m^3 for quartz particles, which is transported in suspension (no bedload). In SISYPHE , the effect of flocculation is represented by the choice of settling velocity considered to be constant (for simplicity). More physically based relations will be implemented in the future. The effect of consolidation is modeled through physical as well as empirically-based models.

Keywords

In the SISYPHE steering file, the physical properties of the sediment are defined :
- `COHESIVE SEDIMENT = YES` (= `NO`, default option)
- `SEDIMENT DIAMETERS` ($D_{50} > 0.00006$ m, for non-cohesive sediment)
- `SEDIMENT DENSITY` ($\rho_s = 2650$ Kg/m^3, default value)
- `SETTLING VELOCITIES`
- `SUSPENDED LOAD = YES` (= `NO`, default option)
- `BED LOAD = NO` (= `YES`, default option)

Attention

1. The settling velocity needs to be specified, if not, it will be calculated by the Stokes law as a function of the individual particle diameter in subroutine `vitchu_sisyphe.f`. As a result of flocculation, the settling velocity can be approximately an order of magnitude greater.
2. Both key words `SUSPENDED LOAD = YES`, `BED LOAD = NO` are set automatically, in order to be consistent with the selected type of sediment, when `COHESIVE SEDIMENT = YES`

8.2.2 Sediment bed structure

The cohesive sediment bed properties are generally not uniform : as a result of self-weight consolidation, the top layer is made of freshly deposited mud. This soft layer represents the active layer, which is regularly eroded under the action of regular currents in mean flow conditions. The concentration increases with distance from the surface, whereas its resistance to erosion increases. The vertical bed structure determines both the erosion flux and the resulting bed evolution.

In SISYPHE , cohesive sediment bed can be represented by a fixed number `NOMBLAY` of layers of iso-pycnal (constant concentrations). The number of layers is fixed with maximum number up to 20. Each layer is represented by a constant concentration. The concentration of each layer is generally fixed and specified by keyword `MUD CONCENTRATION PER LAYER` expressed in kg/m^3. It may also vary in space and time (see consolidation model 3).

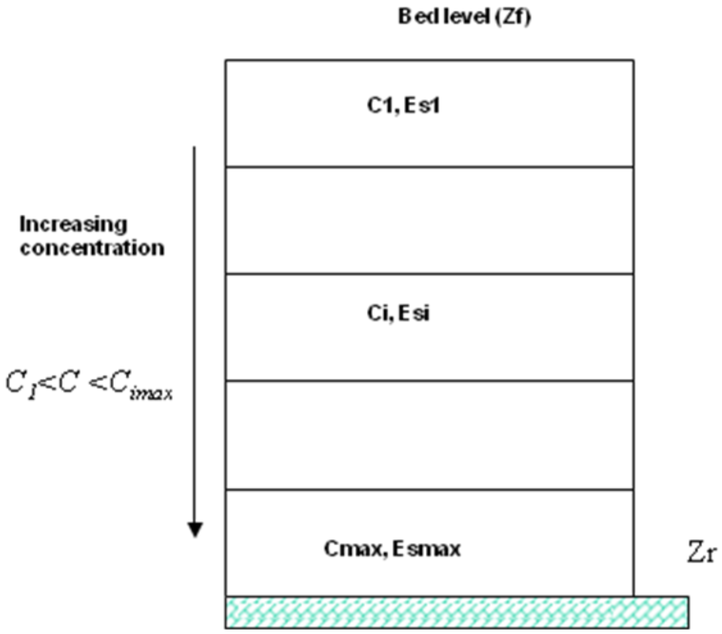

FIGURE 1 – Schema of vertical bed structure. `NOMBLAY` is the number of layers.

8.2.3 Initialization of the sediment bed structure

The initial concentrations (CONC_VASE (j)) are generally specified by keywords.
The initial layer thicknesses are specified in user-subroutine init-compo-coh.f. It is called by subroutine init_mixte.f which just checks if the sum of all layers is equal to the total bed thickness as specified by the initial bathymetry Zf and rigid bed Zr :

$$Z_f(i) - Z_r(i) = \sum_{j=1,NOMBLAY} Es(i,j)$$

(unknown char) i is the number of point,
(unknown char) j the number of layer.
(unknown char) Zf : the bed level
(unknown char) Zr : the non-erodible bed level

Each layer j ($1 \le j \le NOMBLAY$) is defined by its concentration C_s, thickness E_s and mass per surface area M_s, which depends on both layer concentration and thickness, such that

M_s(i, j)= C_s (j) E_s(i,,j),
(unknown char) Ms is the mass per unit surface area (Kg/m^2),
(unknown char) Cs is the concentration (kg/m3),
(unknown char) Es, the layer thickness.

For particular application, the bed concentrations C_S (i, j) can be also allowed to vary from point to point. The initial distribution can be specified in subroutine init_compo-coh.f.
Key words
¬ 'NUMBER OF LAYERS OF THE CONSOLIDATION MODEL'
o NCOUCH_TASS = 1 (default value) should be less than the maximum number of layers (NLAYMAX = 20)
¬ 'MUD CONCENTRATION PER LAYER' in (Kg/m^3) : CONC_VASE

Remarks
¬ The keywords 'NUMBER OF BED LOAD MODEL LAYERS' and 'NUMBER OF LAYERS OF THE CONSOLIDATION MODEL' are essentially the same except that the default values are different. For cohesive sediments, it is possible to have only one uniform layer, whereas for sand grading algorithm we need at least 2 layers (the active layer and the substratum).

The subroutine lecdon.f specifies at the end NOMBLAY = NCOUCH_TASS

8.3 Erosion/deposition properties

8.3.1 Transport in suspension

Transport equation

Fine cohesive sediments are transported in suspension. The two-dimensional (2D) model solves a two-dimensional (2D) transport equation for the depth-averaged suspended sediment transport concentration C, which is derived by depth-integration of the 3D classical transport/diffusion equation.

$$X = \overline{x} = \frac{1}{h}\int_{Z_f}^{Z_s} x(z)dz$$

(1)

where h =Z_s-Z_f is the water depth, assuming the bed-load layer thickness to be small.
After simplification of the advection terms and using the continuity equation, the following approximate depth-averaged transport equation can be solved in its non-conservative form :

$$\frac{\partial C}{\partial t} + \underbrace{U\frac{\partial C}{\partial x} + V\frac{\partial C}{\partial y}}\,Adv = \underbrace{\frac{1}{h}\left[\frac{\partial}{\partial x}\left(h\epsilon_s\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(h\epsilon_s\frac{\partial C}{\partial y}\right)\right]}\,Diff + \frac{(E-D)}{h}$$

(2)
(unknown char) U and V are the depth-averaged convective flow velocities in the x and y directions,
(unknown char) E and D are respectively the erosion and deposition fluxes at the bed and represent the exchange terms between the suspension and the sediment bed. They are defined at the interface between the bed and the suspension.

Units
In SISYPHE, the volume concentration is the main variable : such that E and D are expressed in m/s However, the user can choose mass concentration for graphic printouts, by use of keyword 'MASS CONCENTRATION'.
The relation between volume concentration C and mass concentration C_s (Kg/m^3) is : C_s = ρ_s C,

where ρ_s is the solid density (ρ_s = 2650 Kg/m^3)
Convection
For non cohesive sediments, the convective velocity generally differs from the depth-averaged velocity (issued from Telemac-2d). A correction term is applied on the depth-averaged mean velocity to account for the fact that most sediments is transported near the bed. This correction term is expressed as a function of the Rouse parameter as explained in the user-manual (cf. Report H-P74-2012-02004-EN).
For fine cohesive sediments, the Rouse parameter

$$R = \frac{W_s}{\kappa u_*}$$

(where W_s is the settling velocity, κ , the von Karman constant (κ=0.4), and u_* the friction velocity) is generally less than one and the sediment can be regarded as fairly uniformly distributed in the vertical. The convection velocity can be taken as the depth-averaged flow velocity.
Diffusion
The horizontal diffusion terms can be set to zero by use of keyword 'DIFFUSION' = NO.
According to the choice of the parameter 'OPDTRA' (keyword), the diffusion terms in (2) can be simplified and equation (2) becomes :

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} + V\frac{\partial C}{\partial y} = \left[\frac{\partial}{\partial x}\left(\epsilon_s\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(\epsilon_s\frac{\partial C}{\partial y}\right)\right] + \frac{(E-D)}{h}$$

(3)

Keywords
(unknown char) 'correction on convection velocity' (NO = Default value).
(unknown char) 'MASS CONCENTRATION' = YES (NO, default value)

Treatment of advection terms

The choice of the numerical scheme is based on keyword ('TYPE OF ADVECTION'). The following finite elements methods available are :
- 1. Method of characteristics
o Main advantages : unconditionally stable and monotonous
o Diffusive for small time steps
o Not mass conservative
- 2. Method Streamline Upwind Petrov Galerkin (SUPG)
o Courant number criteria
o Not mass conservative
o Less diffusive for small time steps
- 3 or 4. Conservative N-scheme (similar to finite volumes)
o solves the continuity equation under its conservative form
o (recommended when the correction on convection velocity is accounted for)
o Courant number limitation (sub iterations are included to reduce the time step
- 13, 14 are the same as 3 and 4 but here adapted to the presence of tidal flats based on positive water depth algorithm
- 5. Distributive schemes (PSI) : like N scheme 4, but non-linear ie the fluxes are corrected according to the tracer value itself. This relaxes the courant number criteria and it is also less diffusive than scheme 4 and 14. The CPU time is however increased. This method does not apply for tidal flats.

It is therefore recommended to use scheme 4 or 14 (if tidal flats) as a good compromise (accuracy/computational time)
Keywords
The following keywords determine the numerical scheme options, choice of advection scheme, accuracy of solver, coefficient of implicitation . . ..
(unknown char) 'TETA SUSPENSION'(TETA_SUSP = 1, by default)

(unknown char) 'TYPE OF ADVECTION'(=1, default option : characteristics)
o The method of characteristics is non-conservative
o Use method 4 (or 14 in the presence of tidal flats) to check mass continuity

(unknown char) 'SOLVER FOR SUSPENSION'(=3, conjugate gradient, by default)
(unknown char) 'PRECONDITIONING FOR SUSPENSION'
(unknown char) 'SOLVER ACCURACY FOR SUSPENSION' (= 10^{-8}, by default)

It is important to have a small value here, when working with volume concentration <<<1
(unknown char) 'MAXIMUM NUMBER OF ITERATIONS FOR SOLVER FOR SUSPENSION'(=50, per default)

It is important to have a small value here, when working with volume concentration <<<1
(unknown char) OPTION FOR THE DISPERSION(=1, constant dispersion coefficient per default)
(unknown char) DISPERSION ALONG THE FLOW =$10^{-2}(unknownchar) \quad DISPERSIONACROSSTHEFLOW$
10^{-2}

See also :
(unknown char) 'MATRIX-VECTOR PRODUCT', 'MATRIX STORAGE...'

The user should refer to the "Sisyphe.dico" file, if he would like to modify some of the numerical options and parameters. See also Hervouet (2007, chapter 6).

8.3.2 Erosion flux for cohesive sediment

Uniform bed
The classical Partheniades formula is applied for cohesive sediments. Assuming the bed to be uniform :

$$\left.\begin{array}{l} E = M\left[\left(\frac{u_*}{u_{*_e}}\right)^2 - 1\right] \quad for \quad u_* > u_{*_e} \\ E = 0 \;\; for \;\; u_* < u_{*_e} \end{array}\right\} \quad (4)$$

where u_* is the friction velocity related to skin friction, u_{*e} is the critical shear stress velocities for erosion.
(unknown char) u_{*ce} is the critical erosion velocity,
(unknown char) u* the friction velocity, defined as a

$$u_* = \sqrt{\frac{\tau'}{\rho}}$$

(unknown char) τ' the shear stress corrected for skin friction,
(unknown char) ρ the fluid density.

The empirical coefficient M is a dimensional coefficient. The Partheniades coefficient M is specified in the steering file (in Kg/m^2/s). The erosion rate is expressed in m/s, by converting the M coefficient to m/s (subroutine lecdon.f).

Non uniform bed

$$\tau_{ce} = \rho u_{*ce}^2 \approx C^\beta$$

In the multi-layer model, the critical erosion shear stress increases as the mud concentration increases. The following semi-empirical formulae have been established (e.g. Migniot, 1968) :
(unknown char) τ_{ce} is the critical erosion bed shear stress,
(unknown char) C the mud concentration,
(unknown char) β an empirical coefficient.

In the multi-layer model, each layer is characterized by its density and critical bed shear stress. The erosion rate of each layer E(j) decreases with distance from the surface.

For each layer j and at each time step, the erosion rate E(j) is calculated as a function of the difference between the applied bed shear stress and the critical bed shear strength, using (4). For each E(j)>0, the layer is erodible.
The mean erosion flux E is determined as the mean erosion rate, averaged over the eroded depth :

$$E = \frac{1}{\Delta zer}\int_0^{\Delta zer} E(z)dz = \frac{1}{\rho s Dt}\int_0^{Dt} dMs$$

(unknown char) dM_s is the eroded mass per unit surface area (Kg/m^2),
(unknown char) Δzer is the maximum depth to be eroded,
(unknown char) E(z) is the erosion flux (m/s) at distance z from the bed interface,
(unknown char) Dt, the time step.

Iterative procedure
The potential depth to be eroded is estimated using the iterative procedure described below and programmed in subroutine suspension_erosion_coh.f.
At each time step, the top layer is first eroded if E(1)>0. Once it is empty, the next layer is eroded etc... For each erodible layer j with erosion rate (E(j) >0), the time interval Dt(j) to erode it completely is estimated by a simple mass balance from the mass of the layer Ms(j) :

$$Dt(j) = \frac{M_s(j)}{\rho_s E(j)} = \frac{E_s(j) * C_s(j)}{\rho_s E(j)}$$

For the first layer (j=1), there are two possibilities :
(unknown char) Dt(1)>Dt : layer 1 is emptied partially :

$$-C_s(1)dE_s(1) = \rho_s Dt E\,(j)$$

(unknown char) Dt(1)<Dt : layer 1 is emptied completly, and the next (second) layer can be eroded :

dEs(1)=-Es(1)

The last (non-empty) layer (noted j_{max}) to be eroded is obtained by emptying all successive top layers until :

$$\sum_{j=1}^{j\max -1} Dt(j) < Dt < \sum_{j=1}^{j\max} Dt(j)$$

For j_{max}, the time left to erode the last layer is

$$Dt_{\max} = Dt - \sum_{j=1}^{j\max -1} Dt(j)$$

The mass potentially eroded during this process is

$$\overline{M_s} = \sum_{j=1}^{j\max -1} M_s(j) + \rho_s E(j_{\max}) Dt_{\max}$$

The erosion flux (m/s) is therefore estimated :

$$E = \frac{\overline{M_s}}{\rho_s Dt}$$

Keywords
(unknown char) 'COHESIVE SEDIMENTS' = YES (NO ; default option),
(unknown char) 'PARTHENIADES CONSTANT' (M = 10^{-3} Kg/m^2/s, by default),
(unknown char) 'CRITICAL EROSION SHEAR STRESS OF THE MUD' (TOCE_VASE : τ_{ce} = 0.01 N/m^2 by default), in N/m^2

8.3.3 Deposition flux for cohesive sediment

The deposition flux is calculated as a function of the near bed concentration, which in the case of cohesive sediments is considered equal to the depth-averaged concentration. In the transport equation (3), the deposition flux is therefore an implicit term, whereas the erosion flux is explicit.

$$\left.\begin{array}{ll} D = 0 & for \quad u_* > u_{*_d} \\ D = W_s C\left[1-\left(\frac{u_*}{u_{*_d}}\right)^2\right] & for \;\; u_* < u_{*_d} \end{array}\right\} \quad (5)$$

(unknown char) C is the depth-averaged concentration
(unknown char) W_s is the settling velocity
(unknown char) U_{*d}, the critical deposition velocity which represents the limiting shear velocity, above which the sediment flocs are broken and resuspended.

In the case of cohesive sediments, the settling velocity is generally an order of magnitude greater the settling velocity based on the individual particle size, as a result of flocculation.
The flocculation process is not yet programmed in Sisyphe and the settling velocity should be specified by the user as a calibration parameter.
Remark
In Eq(5), the sediment concentration is assumed to be uniform over depth, which is a reasonable assumption for fine cohesive sediments (W_s<<u*).

Keywords
(unknown char) 'CRITICAL SHEAR VELOCITY FOR MUD DEPOSITION' (VITCD : = 1000 m/s , by default : for no deposition),
(unknown char) 'SETTLING VELOCITIES' (By default, W_s is calculated by the model based on the Stokes law and individual particle diameter)

8.3.4 Bed evolution

In the case of cohesive sediments, there is no bed-load. The bed evolution is only due to the suspension. This is the mass conservation equation to be solved at each time step :

$$C_s \frac{\partial Z_f}{\partial t} + \rho_s (E - D) = 0$$

(6)
(unknown char) Z_f is the bottom elevation (m),
(unknown char) C_s is the mass concentration of the cohesive bed which increases from the top layer to the bottom in the multi-layer discretization of the bed (Kg/m^3).
(unknown char) E-D is the net sediment flux (m/s),
(unknown char) ρ_s the solid particles density (Kg/m^3).

The subroutine evol_susp_coh.f updates the bed level dZ_f and layer thicknesses E_S, as well as the mass of mud per layer M_s.
For uniform bed (NOMBLAY = 1)
The bed is characterized by a single density C_s= C_s(1). The bed evolution at each time step is therefore :

$$dZ_f = \frac{\rho_s (D-E)Dt}{C_s}$$

(dZ_f < 0 for net erosion and dZ_f < 0 for net deposition).

For non-uniform beds
Two different cases occur. In the case of net deposition (D-E>0), sediment is deposited in the first top layer $C_s(1)$:

$$dZ_f = dE_s(1) = \frac{\rho_s (D-E)Dt}{C_s(1)} > 0$$

In the case of net erosion (E-D>0), sediment is eroded layer by layer dEs(j) <0, from the surface to the bottom dense layer, until the eroded mass balances the eroded flux :

$$\sum_{j=1}^{j\max} M_s(j) = \rho_s (E-D)Dt$$

(unknown char) j_{max} is the last layer to be eroded,
(unknown char) M_s(j) is the total mass per surface area of layer j :

$$M_s(j) = C_s(j)E_s(j)$$

The last layer to be eroded j_{max} is determined in order to satisfy :

$$\sum_{j=1}^{j\max -1} M_s^n(j) < \rho_s(E-D)Dt < \sum_{j=1}^{j\max} M_s^n(j)$$

Top layers are emptied (dEs (j)= - Es(j) for j=1 up to j_{max}-1) ; the last layer is eroded up to a the depth dE_s(jmax)<0 in order to satisfy mass conservation :

$$\underbrace{\sum_{j=1}^{j\max -1} M_s^n(j) - dE_s(j\max)C_s(j\max)}\, erosed\ mass = \rho_s(E-D)Dt$$

The variation of bed elevation is therefore :

$$dZ_f = -\left(\sum_{j=1}^{j\max -1} E_s^n(j)\right) + dE_s(j\max) < 0$$

The thickness and mass of each layer are updated at the end of each time step (n+1), from their initial values (n) :

$$for\, j = 1, j_{\max} - 1 \begin{cases} E_s^{n+1}s(j) = 0 \\ M_s^{n+1}(j) = 0 \end{cases}$$
$$for\, j = j_{\max} \begin{cases} E_s^{n+1}(j_{\max}) = E_s^n(j_{\max}) + dE_s(j_{\max}) \\ M_s^{n+1}(j_{\max}) = E_s^{n+1}(j_{\max}) * C_s(j_{\max}) \end{cases}$$

Underneath layers (from j_{max}+1 to NOMBLAY) are not eroded and keep their thickness and mass.

Keywords
¬ 'MUD CONCENTRATION PER LAYER' in (Kg/m^3)

¬ SEDIMENT DENSITY (ρ_s =2650 Kg/m^3)
¬ Due to the default value of NOMBLAY for sand grading effects (NOMBLAY =2),For uniform beds both keywords need to be specified

'NUMBER OF BED LOAD MODEL LAYERS' = 1
'NUMBER OF LAYERS OF THE CONSOLIDATION MODEL' = 1

8.3.5 Initial concentrations, boundary conditions

Initial conditions

The initial concentration for the suspended load can be either imposed within condim_susp.f or specified in the steering file through the keyword 'Initial suspension concentrations ' initializes the value of the volume concentration for each class.
Boundary conditions
For the boundary conditions, the concentration of each class can be specified in the steering file through keyword : 'concentration per class at boundaries'. It may be also convenient to use keyword 'equilibrium inflow concentration =Yes' : the concentration at the entrance of the domain and at t=0 is set to its equilibrium value, according to the choice of the 'reference concentration formula'. Input concentrations can be also directly specified (user subroutines : conlit.f).
Equilibrium conditions

The concentrations at the entrance of the domain can be calculated by SISYPHE assuming equilibrium conditions in order to avoid unwanted bed-evolution at the entrance of the domain, and also at the first time step, it is possible to impose the concentration to its equilibrium value, by activating the keyword 'EQUILIBRIUM INFLOW CONCENTRATION'.
The equilibrium (depth-averaged) concentration is then calculated assuming equilibrium concentration at the bed and a Rouse profile correction for the F factor.
Keywords
(unknown char) 'INITIAL SUSPENSION CONCENTRATIONS' (C_{S0}=0, default value)
(unknown char) 'EQUILIBRIUM INFLOW CONCENTRATION (=NO, default option)
(unknown char) 'CONCENTRATION PER CLASS AT BOUNDARIES' (=0, default value)

User subroutines
The subroutine condim_susp.f can be used to specify the initial conditions for the sediment concentration (see § V.4.2).
The subroutine conlit.f can be used to specify the concentration at the entrance of the domain.

8.3.6 Mass conservation algorithm

The subroutine suspension_bilan_coh calculates at each time step the mass of sediments in the computational domain and ensures that the sum of the different components is compensated by the fluxes at the liquid boundaries :
(unknown char) Mass of sediment in suspension : M_1

$$M_1 = \rho_s \iiint c(x,y,z)\delta v = \rho_s \iint_S C(x,y)h\delta s$$

$(unknownchar)$ C is the (depth-averaged) volume concentration of sediment in suspension
(unknown char) h the water depth,
(unknown char) S is the surface of the computational domain.

In finite elements, each 2D variable is decomposed on basis function ϕ_i :

$$M_1 = \rho_s \sum_{i=1,Npoin} C_i h_i \iint \phi_i \delta s = \rho_s \sum_{i=1,Npoin} C_i h_i S_i$$

(unknown char) S_i are the integral of basis function

(unknown char)   Mass of sediment in the bed : M_2

$$M_2 = \iiint C_s \delta v = \iint \left(\int_{Zr}^{Zf} C_s \delta z \right) \delta s$$

(unknown char) C_s is the sediment bed mass concentration.
(unknown char) Z_f : the bed level
(unknown char) Z_r : the non erodible bed level

After discretization of the bed layers into layers of variable thickness E_s and concentration C_s w

$$M_2 = \iint \left(\sum_{j=1,Nomblay} M_s(j) \right) \delta s = \sum_{i=1,Npoin} \left[\sum_{j=1,Nomblay} M_s(j) \right]_i S_i$$

Where M_{si} is the total mass per surface area at node i.

8.4 CONSOLIDATION MODEL

8.4.1 Theoretical background

Once the sedimentation process is achieved, a sediment bed is formed. For non-cohesive bed, no evolution with time will be observed if no erosion or further sedimentation occurs. For cohesive bed, the concentration will increase with time as the result of self-weight consolidation or compaction. Two stages, namely the primary and secondary consolidation (Been and Sills, 1981), are classically considered to explain the progressive compaction of the cohesive sediment bed with time. As the formed bed resulting from the sedimentation of mud is very soft, it tends to compact under its self-weight. During this process, the void ratio tends to diminish so that pore water is expulsed upward as a result of water incompressibility. The dynamic of this primary stage of consolidation is therefore controlled by the permeability of the bed. At the end of the primary consolidation, the excess pore pressure is fully dissipated (Figure 2) but the cohesive bed records further compaction. This secondary stage of consolidation is no longer related to pore pressure but rather to the solid skeleton property. The secondary consolidation is considered as a result of time effect (viscous or ageing) in the stress-strain relationship of the soil. Two effects of time are currently considered in modern soil mechanic : the creep of the soil (viscous behaviour of cohesive geomaterial) and ageing (thixotropy of mud). Even through the dynamic of the secondary consolidation is much slower than the primary consolidation, this stage is important for long term prediction.

FIGURE 2 : Vertical variation of density and effective stress within the cohesive sediment bed

Remarks :
1. Most consolidation theories describe the primary consolidation since they are based on the theory by Terzaghi (1923).
2. The theory by Terzaghi (1923) relates the deformation of the bed to the permeability and the effective (or solid) stress which is defined as the difference between the total stress and the pore pressure (principle of effective stress, Terzaghi, 1923). It corresponds to the stress that is transmitted directly through the contacts between solid particles. It is also called osmotic pressure by some authors. Been and Sills (1981) evidenced the progressive increase of effective stress during the consolidation process.
3. The theory by Terzaghi (1923) was originally formulated for infinitesimal strains so that permeability and compressibility could be assumed constant. The theory was extended for large deformations by Gibson et al (1967, 1981) as pointed out by Been and Sills (1981).

As an illustration of the process in the water column, Figure 3 proposes a schematic representation of both, the sedimentation and the consolidation. In the right side of Figure 3, we present the validity range of the Kynch theory of sedimentation and of the Gibson theory of large strain consolidation.
Both theories are unified by Toorman (1996, 1999) which pointed out the fact that Gibson model can also describe the sedimentation of particle depending on the choice of closure equations.

FIGURE 3 : Diagram of different processes involved in the settling transport (left : non-cohesive, right : cohesive)

The general model (Eq.7a, 7b, 7c) proposed by Gibson et al. (1967, 1981) represents the different stages of consolidation. This equation is based on a two-phase approach by considering continuity and motion equations for both fluid and solid phase to obtain the general equation that reads in material co-ordinate ζwhich represents the volume of solids :

$$\frac{\partial e}{\partial t} + \left(\frac{\rho_s - \rho_f}{\rho_f}\right) \frac{d}{de}\left(\frac{k}{1+e}\right)\frac{\partial e}{\partial \zeta} + \frac{\partial}{\partial \zeta}\left(\frac{k}{g\rho_f(1+e)}\frac{d\sigma'}{de}\frac{\partial e}{\partial \zeta}\right) = 0$$

(7a)
(unknown char) e is the void ratio,
(unknown char) k the hydraulic permeability (in m/s),
(unknown char) σ' the effective (or solid) stress.

The Gibson equation can be also written in Eulerian framework as :

$$\frac{\partial e}{\partial t} + (1+e)^2 \left(\frac{\rho_s - \rho_f}{\rho_f}\right) \frac{\partial}{\partial z}\left(\frac{k}{(1+e)?}\right) + \frac{(1+e)?}{g\rho_f}\frac{\partial}{\partial z}\left(\frac{k}{1+e}\frac{\partial \sigma'}{\partial z}\right) = 0$$

(7b)

This equation is equivalent to :

$$\frac{\partial \phi}{\partial t} - \frac{\partial}{\partial z}\left[\left(k(s-1)\phi + \frac{k}{\gamma_f}\frac{\partial \sigma'}{\partial z}\right)\phi\right] = 0$$

(7c)
(unknown char) Φ stands for the sediment volume concentration,
(unknown char) k the hydraulic permeability,
(unknown char) s the density ratio between sediment and fluid (=ρ_s/ρ_f),
(unknown char) γ_f the unit weight of fluid (=gρ_f, g being the acceleration of gravity),
(unknown char) z the vertical coordinate (positive upward),
(unknown char) t the time.

The equation of Gibson has been widely used in various numerical consolidation models (Been and Sills (1981), Toorman (1996, 1999), Bartholomeeusen et al. (2002) for instance) as well as compared with the experimental results. It has been implemented in Telemac-3d by Lenormand (1993) or coupled with Telemac-2D by Thiebot (2008). Their method of resolution has been adapted to SISYPHE by Lan Anh Van (2012) and will be described in §4.3 and 4.4.

The main difficulty in using the Gibson model is related to the choice for closing the problem. Two closure equations, for the permeability k and for the effective stress σ', is indeed required to obtain the time evolution of vertical concentration profiles. However, the formulation of these closure equations remains an open problem and a shared protocol to determine their parameter values are still lacking as reported by Toorman (1996, 1999), Bartholomeeusen et al. (2002) or Lan Anh Van (2012). In annexe 9.3, we present the derivation of the Gibson equation in Eulerian coordinate from a two-phase approach and give the way for obtaining the original Gibson equation in material coordinate.

8.4.2 Multi-layer empirical algorithm

Time evolution
The consolidation effect is reproduced by assuming that the vertical flux of sediment from layer j to underneath layer j+1 is proportional to the mass of sediments, Ms (kg/m2), contained in the layer j.

$$\frac{dM_s(j)}{dt} = a_i M_s(j)$$

(8)

The transfer mass coefficients a$_j$ (in s^{-1}) are specified in the steering file ('MASS TRANSFER PER LAYER'). They correspond to a characteristic timescale to transfer mass from one layer to another.
This multilayer consolidation model is not related to Gibson equation so that it should be considered as empirical. Despite its apparent simplicity, it can qualitatively reproduce an increase of mud bed deposit with time, while ensuring mass conservation (transfer coefficient of the last bottom layer is zero). The model results are highly sensitive to the specified values of the mass transfer coefficients

a_i which are however difficult to calibrate. The main advantage of this iso-pycnal formulation relies in the fact that no vertical grid is required to compute the concentration evolution of the different layers.

Discretization

The vertical resolution of semi-empirical equation (8) is based on the multi-layer iso-pycnal model with fixed concentrations. The consolidation is then reproduced by mass transfer between layers of the model. The transfer coefficients are fixed for each layer.

The set of mass-transfer coefficients a(j) in (s^{-1}) are selected by calibration, in order to find best agreement of time-varying concentration profiles between model and experiment. Physically, they represent the inverse of the residence time per layer. The values a(j) are found to decrease from top to bottom, as the time scale of consolidation (or residence time) increases. The mass transfer of the last layer is set to zero, in order to insure no mass loss at the rigid bed level (impermeability condition).

Keywords

¬ 'COHESIVE SEDIMENTS' = YES (default , NO ; NO ; . . .)
¬ 'MUD CONSOLIDATION' =YES (=NO, default)
¬ ' CONSOLIDATION MODEL' = 1

¬ 'NUMBER OF LAYERS OF THE CONSOLIDATION MODEL' (NOMBLAY=10, default, maximum value is 20)'
¬ 'MUD CONCENTRATION PER LAYER' in Kg/m^3 (E= 50. ; 100. ;..,by default)
¬ 'MASS TRANSFER PER LAYER' (= 5.10-5, ...0., by default)

8.5 Multi-layer iso-pycnal Gibson's model

8.5.1 Theoretical background

The previous equation presents the advantage of not using a vertical mesh. However it is not physically based model since it neglects the Gibson theory. Improvement of this category of model was proposed by Sanchez (1992) and Thiebot (2008). In these formulations, the sediment flux from one layer to the other is given not in term of empirical mass transfer coefficient (a_i) but in term of theoretical flux which is given by the Gibson theory.

8.5.2 Numerical discretization

The model originally developed by Sanchez (1992) and Thiebot (2008) is a 1DV sedimentation-consolidation ≪ multi-layer ≫ model, based on an original technique to solve Gibson equation. The advantage of this representation (9a) if compared with previous one (8) relies on the right hand side term which relates the mass of sediments, Ms_i (kg), to the net flux entering or leaving the layer. This equation enables therefore to consider the flux of sedimentation and consolidation as provided by the Gibson theory.

$$\frac{dMs_i}{dt} = (F_i(t) - F_{i+1}(t))\Delta t \pi r^2$$

(9a)

In equation (9a), a circular shape (of radius r) is assumed for the surface. Equivalent to (9a), equation (9b) is expressed in term of layer thickness Es_i (m), recalling Ms_i= Cs_i πr^2 Es_i.. This last expression becomes independent on the settling column geometry.

$$\frac{dEs_i}{dt} = \frac{F_i(t) - F_{i+1}(t)}{Cs_i}$$

(9b)

The concentration of the different bed layers Cs(i) are fixed. As the sedimentation and consolidation progress, the sediment is transferred to the more concentrated layers, and the thickness of these layers increase as well.

The mass conservation is ensured by requiring at each moment, in each layer, the equality between the mass contained in a layer at time t + $\Delta t$and the mass present in this layer at time t in which the outgoing mass was removed and the incoming mass was added (means the mass that crossed the upper and lower sections respectively during the time Δt). The outgoing and incoming masses are taken into account by sediment flux noted F_i(t).

$$F_i(t) = \frac{(V_{s,i}(t) - V_{s,i-1}(t))C_{i-1}C_i}{C_{i-1} - C_i}$$

(10)

In which $V_{s,i}$ is the falling velocity of the layer i, and can be defined as :

$$SiC_i \leq C_{gel} V_{s,i} = V_{s,i}(C_i) = k(C_i)C_i \left(\frac{1}{\rho_s} - \frac{1}{\rho_f}\right)$$

$$SiC_i > C_{gel} V_{s,i} = k(C_i)C_i \left(\frac{1}{\rho_s} - \frac{1}{\rho_f}\right) + k(C_i)\frac{\sigma'(C_{i-1}) - \sigma'(C_i)}{\frac{1}{2}(Ep_{i-1}(t) + Ep_i(t))}$$

(11)

Where k is the permeability, $\sigma'$is the effective stress, C_{gel} the transition concentration between sedimentation and consolidation schemes (Camenen & Pham Van Bang, 2011). The closure equation are presented in section 4.5.

8.5.3 Vertical grid Gibson's model

"Vertical-grid" models propose a natural resolution of the Gibson equation as a vertical grid is used to compute the concentration at each point. This category of model uses common techniques (finite difference or finite volume or finite element methods) for resolving partial differential equations.
As they use a vertical grid, the connection with SISYPHE which is depth integrated model is not straightforward.

In the new version of SISYPHE, such a category of model is also available. Strictly speaking, the original Gibson model (in material coordinate) used in TELEMAC-3D which was developed by Lenormant (1993) is connected to SISYPHE.

The finite difference method is used. The implicit scheme leads to a tridiagonal matrix that is solved by using a classical double sweep algorithm. More details are given in TELEMAC-3D user guide, or Lenormant (1993) or Lan Anh Van (2012).

8.5.4 Closure equations for permeability and effective stress

All of the Gibson based models, i.e. Multi-layer iso-pycknal Gibson's model (section 4.3) and vertical grid Gibson model (section 4.4), requires two closure equations. The Multi-layer empirical model (section 4.2) only needs closure on ai, the mass transfer coefficients.
In this section, some available empirical formula are detailed. In general, a decreasing (or increasing) function of solid concentration is considered for permeability K (or effective solid stress). These expressions are logarithmic or exponential or power law functions (see Bartholomeeusen et al. 2002).

4.5.1 Closure equations
Bed permeability k

Bartholomeeusen et al. (2002) introduced typical functions, in the form of either power or exponential, to relate the permeability k with the void ratio e :

$$\begin{cases} k = A_1 e^{A_2} \\ k = A_1 \phi_s^{-A_2} \\ k = \exp(-A_1 + A_2 e) \end{cases} \tag{12}$$

The value of these coefficients A_1, A_2 depends on grain size distribution, organic content, activity and pore size distribution.

Effective stress σ'

The similar way can be opted in the determination of $\sigma'(e)$ ($\sigma'(C)$,

$$\sigma'(\phi)$$

) , which gives :

$$\begin{cases} e = -B_1 \sigma'^{B_2} + B_3 \\ \sigma' = B_1 \phi_s^{B_2} \\ e = B_1(\sigma' + B_2)^{-B_3} \\ \sigma' = \exp(B_4 + B_5 e) \end{cases} \tag{13}$$

According to Winterwerp & van Kesteren (2004), the validity range of the power-type functions (eg.

$$k = A_1 e^{A_2}$$

,

$$\sigma' = B_1 \phi_s^{B_2}$$

) is much larger than that of the exponential-type functions. Moreover, there is a physical insight in the formulation of the power-type functions. Therefore, it is recommended to use power-type functions. However, two different power functions for permeability may be necessary to represent the two separate processes : sedimentation and consolidation.

4.5.2 Determination of parameters

No standard is found in the literature to recommend a specific methodology for calibrating the empirical functions for both permeability and effective stress. Most of study reported some fitting exercice on settling curve, i.e. the position of supernatant/suspension interface. They considered mostly the least square technique for the adjustment to experimental results. However as stated by Toorman (1999) a more relevant adjustment is offered when concentration profiles are available from Gamma-ray techniques (Been and Sills, 1981, or Bartholomeeusen et al, 2002 for instance), X-ray technique (Villaret et al. 2010 for instance) or MRI techniques (Pham Van Bang et al, 2008 for instance). In such a situation, the density or concentration profile are adjusted on the measurement for different time. A specific procedure has been recently proposed by Thiebot et al. 2011, also based on least square method.
The user should consider the previous method (adjustment on settling curve or on concentration profile) as conventional even through no shared procedure has been internationality recognized as the best one (as previously discussed in 4.1).

An recent alternative is offered in the next paragraph and was successfully applied to Gironde mud (Lan Anh Van, 2012). Since the test case of Gironde mud is proposed for the SISYPHE 6.2 release, we will detail the method used in the next paragraph.

4.5.3 Space-time based method to determine parameters of closure equations

The new methodology to determine parameters is based on space-time analysis of data. It requires data on time evolution of concentration (vertical) profile. If such an information is not available, the proposed method becomes useless. If only the supernatant/suspension interface position is detailed by data, the user should consider the classical fitting method.
If the space-time resolved data is available, this new methodology could be applied easily. The method is based on a theoretical framework which is an advantage to help during the calibration procedure. From time evolution of concentration profiles which are provided by non-intrusive techniques (here the X-ray technique), the procedure uses self-similar analytical solutions to determine the closure equation relative to the convection (or sedimentation) part and to the diffusion (or consolidation) part of Gibson equation.

$$\frac{\partial \phi}{\partial t} - \frac{\partial}{\partial z}\left[V(\phi)\phi\right] - \frac{\partial}{\partial z}\left[D(\phi)\frac{\partial \phi}{\partial z}\right] = 0$$

(14)

Where V(ϕ)=K(s-1) ϕ and D(ϕ)=Kϕ/γ_f dσ'/dϕ in order to match with Gibson equation (7c) in Eulerian coordinate.

Considering a separation regime between sedimentation and consolidation (or between convection and diffusion), self-similar solutions are obtained for each regime. The separation between both problems is justified by the fact that effective stress should be zero for a suspension since there is no direct contact between particles.

Determination of closure equation for the sedimentation

If the concentration of the suspension is lower than a given threshold (the so-called gelling point for cohesive sediment), the inter-particle contacts are negligible, i.e. there is no solid (or effective) stress (Camenen & Pham Van Bang, 2011). In such a situation the Gibson equation (7c or 14) is simply reduced to the equation of Kynch (15) :

$$\frac{\partial \phi}{\partial t} - \frac{\partial}{\partial z}\left[V(\phi)\phi\right] = 0$$

(15)

where $\phi = C/\rho_s$is the volume fraction of solids, V(ϕ) is the settling velocity of the suspension at concentration ϕ that is equal to K(s-1)ϕ.
Considering the self-similar variable ζ=z/t and similarity solution U, i.e. ϕ(z,t)=U(ζ), equation (15) leads to :

$$\left(\frac{df}{dU} - \zeta\right)\frac{dU}{d\zeta} = 0$$

where f is the solid (or sedimentation) flux, which is equal to –V(ϕ)ϕ.

The method is equivalent to the so-called method of characteristics (Leveque, 2002). The iso-concentration pattern presents different straight lines in the space-time (z-t) plot. The slopes of iso-concentration straight lines in the z-t plane are measured to obtain the first derivative of the solid flux. The sedimentation flux proposed for the Gironde mud (Villaret et al, 2010 ; L.A. Van, 2012) is given by equation (16a or 16b). The first derivative of this closure equation is straightforward (Villaret et al. 2010) : the determination and validation of its parameters (V_{st}, ϕ_{gel}, n) has been presented in details in the test case of settling column of Gironde mud :

$$f(\phi) = V_{st}(1-\phi)\left(1-\frac{\phi}{\phi_{gel}}\right)^n \phi \quad \text{for} \quad \phi < \phi_{gel}$$

(16a)

$$f(C) = V_{st}(1-\frac{C}{\rho_s})\left(1-\frac{C}{C_{gel}}\right)^n \frac{C}{\rho_s} \quad \text{for} \quad C < C_{gel}$$

(16b)

where V_{st} is the Stokes velocity of an equivalent sphere, ϕ_{gel} (C_{gel}) is the gelling concentration, n is an exponent. Both parameters, ϕ_{gel} (C_{gel}) and n, have physical meanings in terms of rheology (Pham Van Bang et al., 2007). Indeed, ϕ_{gel} (C_{gel}) is the concentration value from which the effective viscosity of the suspension diverges. And n is a parameter describing the transition from suspension to a structured bed. It is worth noting that the so-called Richardson & Zaki (1954) empirical model is recovered by setting ϕ_{gel}=1.

Determination of closure equation for the consolidation

Still considering separation regime between sedimentation (convective problem) and consolidation (diffusion problem), for cohesive sediment and concentration larger than the gelling point, the effective solid stress build up. The diffusion term is no longer negligible and becomes the leading term in the Gibson equation. For concentration larger than the gel point, the convective part is annihilated in the proposed formulation so that only the diffusion term remains. As this term takes origin from the competition between the seepage flow through a porous media and the effective stress of the solid skeleton (Camenen & Pham Van Bang, 2011), the diffusion is expressed by an expression with the permability K and the effective stress ($d\sigma'/d\phi$). The problem is now related to the non-linearity of the diffusion :

$$\frac{\partial \phi}{\partial t} - \frac{\partial}{\partial z}\left[D(\phi)\frac{\partial \phi}{\partial z}\right] = 0$$

(17)

where $D(\phi)$ is the non linear diffusion term, equal to $K\phi(d\sigma'/d\phi)/\gamma_f$ in order to match with (7c). The power law is assumed for the diffusion coefficient. Indeed, if we consider a power law for the permeability and a power law for the effective stress, the diffusion coefficient will also be a power function of concentration. Here the possible time dependence of the effective stress is investigated so that the secondary consolidation is also taken into consideration (cf. 4.1). As a result, the non linear diffusion coefficient is assumed to depend on both concentration and time by an empirical power law, i.e. $D(\phi)=D_0\phi^a t^b$. Here the time dependence of the consolidation is introduced to mimic the thixotropic behaviour of mud. Introducing as a self-similar variable, $\chi=z/t^\theta$ with $\theta=(1+b)/(2+a)$ and the similarity solution h, i.e. $\phi(z,t)=h(\chi)/t^\theta$ in equation (17) leads to :

$$\frac{\partial \phi}{\partial t} - \frac{\partial}{\partial z}\left[\left(D_0\phi^a t^b\right)\frac{\partial \phi}{\partial z}\right] = 0 \quad \rightarrow \quad \frac{d}{d\chi}\left[h^a(\chi)\frac{dh}{d\chi} - \theta\chi h(\chi)\right] = 0$$

The similarity solution, h, is obtained after integration of the previous ODE equation (see in L.A. Van, 2012 for the details). The parameter M is a constant whose value corresponds to the total mass of sediment in the system.

$$h(\chi) = \begin{cases} \left(M - \dfrac{a(1+b)\chi^2}{2(2+a)}\right)^{1/a} \; for\ \chi \in \left[0, \left(\dfrac{-2M(2+a)}{a(1+b)}\right)^{\dfrac{1}{2}}\right] \\ 0 \;\; for\ \chi \geq \left(\dfrac{-2M(2+a)}{a(1+b)}\right)^{\dfrac{1}{2}} \end{cases}$$

(18)

In order to find out the closure equation for effective stress, from the above equations, it is needed to determine the two variables : a, b. This procedure will be presented in detail in test case “Tassement_2”, with a given experimental result of settling column.

Keywords

¬ 'COHESIVE SEDIMENT' = YES
¬ 'MUD CONSOLIDATION' = YES
¬ ' CONSOLIDATION MODEL' = 2
¬ ' GEL CONCENTRATION' = CONC_GEL
¬ ' MAXIMUM CONCENTRATION' = CONC_MAX
¬ ' PERMEABILITY COEFFICIENT' = COEF_N
¬ ' NUMBER OF LAYERS OF THE CONSOLIDATION MODEL' = NOMBLAY (< 20)
¬ ' MUD CONCENTRATION PER LAYER' =

8.6 Application Test Cases

8.6.1 Erosion/deposition experiments

Description of experiments

This part can be issued from the simulation of Aachen test cases on Gironde mud. The description of the experiment, test results and simulation results are fully detailed in Lan Anh Van (2012). This study has been published in the 6^{th} International Conference on Scour Erosion (Paris, August 27-31, 2012). This paper is available from the SISYPHE website.
The annular flume at Aachen University (Germany) is used to imposed stepwise increase (erosion test) and stepwise decrease (deposition test) of bottom shear stress on cohesive bed. Figure 4 presents the hydraulic facility and describes the forcing. The Gironde mud is used as cohesive sediment. The bed is initially prepared at 300g/L.

FIGURE 4 : Annular flume (RWTH, Aachen, Germany) on the left, the middle figure shows the step wise increase of bed shear stress during the erosion phase, while the figure on the right hand side shows the decrease during the deposition phase.

Erosion test

In SISYPHE, we consider the hydraulic flume as straight in order to simplify. The bed is modeled by four layers having increasing concentration. The top layer (layer 1) has initial concentration of 150g/L and thickness 0.4cm (see Table 1 below). The erosion parameters (critical shear stress, τ_{*e}, and kinetic parameter, M) of the Partheniades law (Eq. 4) for the considered material (Gironde mud) of each layer is presented in the Table. The value are obtained from best fitting exercice on the test results which are presented in Figure 5 with the simulation results.

TABLE 1 : Example of cohesive sediment bed composition

FIGURE 5 : Comparison between modelled and measured concentration during the erosion phase

Since erosion parameters differ from one layer to the other, both erosion processes, the floc erosion and the mass erosion, can be reproduced. Regarding the test results, a minimal number of four layer is required for the simulation exercice.

Deposition test

After the stepwise increase of the bottom shear stress, the decreasing phase (or deposition test) takes place during the experiment. The sediment bed is fully eroded : all the sediment are transported as suspension having a depth averaged concentration equal to 33.8 g/L. From the test results on deposition tests, the critical deposition velocity, u_{*d}, is measured as equal to 0.016m/s. And the measurement from Owen tube provides the relationship between the settling velocity,W_s, and the concentration, C :

$$W_s(mm/s) = \begin{cases} 0.15C^{2.1} & \text{if C¡4.5g/L} \\ 3.5 & \text{if C¿4.5g/L} \end{cases}$$

Figure 6 illustrates the agreement between experiment and simulation which is obtained from the formula by Krone (Eq. 5) and the parameter values described previously.

FIGURE 6 : Comparison between modelled and measured concentration during the deposition

phase

8.6.2 Consolidation tests

The different consolidation models developed in SISYPHE have been validated against measurements made by Lan Anh Van (2012) in a settling column. In this part we will present the experimental device (X-ray) used to obtain space-time resolved data on the process. This test case is available as a reference test in release 6.2.

Data set

Gironde mud is tested in a settling column which is instrumented by X-ray technique. The prototype was initially developed at the Commissariat à l'Energie Atomique (CEA, Saclay, France) and improved for this study at Chatou. The final version of the prototype is illustrated in Figure 7. More details on the measuring principle (attenuation of signal or transmitometry), the calibration procedure (Beer-Lambert law), the experimental conditions of testing (initial homogeneous sample) are available in Lan Anh Van (2012).

FIGURE 7 - X-ray settling column device (CEA/DRT/LIST, Saclay) : a) supply of the X-Ray generator ; b) X-Ray generator ; c) collimator (5mm slot) ; d) photon detector ; e) computer controlled unit ; f) acquisition data unit ; g) step motor ; h) endless screw.

FIGURE 8 : time evolution of concentration profile during the sedimentation-consolidation of Gironde mud. The settling column is 20.7cm height, the suspension was initially prepared at solid fraction equal to 2.96% (Villaret et al. 2010).

Figure 8 (left, first 3 hours of the test) presents concentration (vertical) profiles having an upward convex shape near the bottom. This shape becomes downward convex in Figure 8 (right, long term). This difference is explained by the different nature of the governing process (See L.A. Van, 2012). At short term, the hindered settling (convection term) is the governing process. For moderate concentration (sufficiently far from the gelling point), the upward convex shape is obtained if we consider the Richardson-Zaki law. At long term and large concentration (larger than the gelling point), the consolidation (diffusion term) is dominant. As a consequence, the near bottom concentration profile becomes downward convex.

Model 1

The multi-layer empirical algorithm initially developed by Walther & Villaret (2008) is used on the data presented by figure 8. Here 20 layers were used to model the vertical profile of concentration. Table 2 presents the model parameters (mass transfer coefficient, ai) which are adjusted on data for best agreement.

Layer	1	2	3	4	5	6	7	8	9	10											
Coef. a	110^{-2}	810^{-3}	610^{-3}	410^{-3}	210^{-3}	110^{-3}	810^{-4}	610^{-4}	410^{-4}	210^{-4}											
Layer	11	12	13	14	15	16	17	18	19	20											
Coef. a	110^{-4}	810^{-5}	610^{-5}	410^{-5}	210^{-5}	110^{-5}	110^{-5}	110^{-5}	110^{-5}	0											

Table 2 - Calibrated parameters of Model 1

Figure 9 presents the simulation results by using the multi-layer empirical approach with the parameters given in Table 2.

FIGURE 9 : simulation results from multi-layer empirical approach : a) sedimentation ; b) consolidation

Model 2

Model 2 (multi layer iso-pycnal Gibson model, see 4.3) and 3 (vertical grid Gibson model, see 4.4) are more rigorously based on Gibson theory than previous Model 1. The space-time methodology for the determination of closure equation and parameters is therefore enabled.
Application of the space-time procedure (see 4.5.3) provides the closure equations with parameters which are presented in the following table. From isoconcentration (straight) lines, slopes are measured which correspond to the first derivative of the solid flux. Integration of this last result provides the solid flux and consequently the hindered settling velocity.

FIGURE 10 : Determination of the hindered settling flux, f (Eq. 16)

FIGURE 11 : Determination of the consolidation (concentration and time dependent non linear diffusion) parameters used in the similarity solution, h (Eq. 18).

The space-time new methodology applied to the test case on Gironde mud leads to the calibration results (Table 3)

Model	Model 2	Model 3
Closure equation for k for C< C_{gel}	$k(C) = \frac{V_{st}}{s-1}(1-\frac{C}{\rho_s})\left(1-\frac{C}{C_{gel}}\right)^n$	$k(e) = \frac{\rho_s V_{st}}{s-1}\left(\frac{e}{1+e}\right)\left(\frac{e-e_{gel}}{1+e}\right)^n$
Closure equation for k for CC_{gel}	$k(C) = \frac{V_{st}}{s-1}(1-\frac{C}{\rho_s})\left(1-\frac{C}{C_{\max}}\right)^n$	$k(e) = \frac{\rho V_{st}}{s-1}\left(\frac{e}{1+e}\right)\left(\frac{e-e_{\max}}{1+e}\right)^n$
Closure equation for ′	$\frac{\partial\sigma'}{\partial C} = -\left[11.55\left(\frac{C}{C_0}\right)^{12.} t^{-3.4}\right]\frac{\gamma_w}{kC}$	$\frac{\partial\sigma'}{\partial e} = -\left[11.55\left(\frac{1+e_0}{1+e}\right)^{12.} t^{-3.4}\right]\frac{\gamma_w}{k/(1+e)}$
V_{stokes} (m/s)	0.0018	0.0018
Gel point	C_{gel}=312 (g/l)	e=7.33 (-)
Maximum concentration	C_{max}=400 (g/l)	e_{max}=5.5(-)
Exponent n	8	8

Table 3 - Parameters of “Model 2” & “Model 3”

We recall here that model 2 (multi layer iso-pycnal Gibson’s model) is solving the problem in Eulerian coordinate and in mass concentration whilst model 3 (vertical grid Gibson’s model) is concerning the same problem in material coordinate (see Annexe 9.3 for the description of the transformation between both system of coordinate) and in term of void ratio, e. Indeed both models run with the same closure equation and parameter values.

FIGURE 12 : simulation results from multi-layer iso-pycnal Gibson model : a) sedimentation ; b) consolidation.

Model 3

Both models 2 and 3 use the same closure equations for the convection and diffusion terms (Table 2.6). However, Model 3 is formulated in material coordinate in term of void fraction, e, whilst the model 2 considers the eulerian coordinate and the mass concentration. The coordinate and parameter transforms are detailed in Appendix 9.3.

Figure 13 presents the simulation results from model 3.

FIGURE 13 : simulation results from model 3 (vertical grid Gibson's model) with parameters presented in Table 2.6

9 Mixed sediments

‡

9.1 Sediment bed composition

Characteristics of each class
Sand-mud mixture can be represented in SISYPHE by using two classes of bed sediments (NSICLA= 2). This recent developments available in release 6.2 require further testing and improvement.
The first class (noted 1) is non-cohesive and represented by its grain diameter D_1 and can be transported as bed-load or suspended load. The settling velocity W_{s1} is a function of the relative sediment density (s=1.65) and grain diameter D_1. So far we assume only suspended load, which implies that the model is devoted to mixtures of fine sand grains and mud.
The second class (noted 2) is cohesive , grain diameter D_2 less than 60 mm. The settling velocity W_{s2} is a function of flocs properties which differs from the individual cohesive particles, and needs to be specified.
Vertical structure of the sediment bed
The vertical structure can be stratified as for pure mud. The bed is discretized into vertical layers up to a maximum number of layers (NOMBLAY<20). Each layer is characterized by a constant value of the mass concentration for the mud, which can be specified in the steering file (C_{si} for i=1, NOMBLAY).
Note : the mass concentration of the mud component is defined for the mud only (i.e. mass of sediments per volume of mud). The initial bed layer thicknesses can be specified in subroutine init_mixte.f, and varies as the bed undergoes erosion/deposition. The consolidation of sand/mud mixture is not yet developed.
The time and spatial variation in the sediment composition is obtained by variation in the composition of each layers (Percent of the mud and mass of sand per layers).
Percent of mud/sand mixture
For layer j

Phase 1 : Sand grains, density ρ_s

Phase 2 : Mud bed mass density C_s

Es

The composition of the sediment bed depends on the percent of each class (P_1,P_2) needs to be specified. In consistency with the algorithm for sand grading effects, P_i represents the volume of class i divided by the total volume, such that $P_2 + P_1 = 1$

$$P_i = \frac{V_i}{V_t}$$

where V_i is the volume occupied by component i, and $V_t = V_2 + V_1$, the total volume.
The total thickness of layer j is decomposed into Es= Es_1 + Es_2 with Es_i = p_i Es.

The total mass of the sand-mud mixture (M_t) is :

$$M_t = C_s V_2 + \rho_s V_1 = (C_s P_2 + \rho_s P_1)\, V_t$$

With ρ_s = 2650 Kg/m^3 (sand density) which is constant, while for the mud, C_s depends on the consolidation state and is constant per sediment bed layer. It is specified by using keyword 'MUD CONCENTRATION PER LAYER'.
Mass balance
The total mass of the sand-mud mixture (M_t) is :

‡. This chapter has been written by D. Phan van Bang, Lan and Villaret

$$M_t = C_s V_2 + \rho_s V_1 = (C_s P_2 + \rho_s P_1)\, V_t$$

We define for each class the mass per surface area and per layer :

$$MS\_VASE = CsP_2Es$$
$$MS\_SAND = \rho_s P_1 Es$$

For the mass balance of the mud phase (phase 2)

$$M_2 = \sum_{i=1,Npoin} \left[\underbrace{\sum_{j=1,Nomblay}}\; P_2 Cs Es\; Ms\_vase \right]_j S_i$$

Initialization

The initial percent of each class (p_1,p_2) needs to be specified. The keyword 'INITIAL FRACTION FOR PARTICULAR SIZE CLASS' can be applied, if the initial distribution is constant (per layer and per node).

In the mass balance for each class, the total mass per unit surface area must account for the fraction of each class :

Keywords :

Type of sediments

¬ 'MIXED SEDIMENT' (default MIXTE = No)

¬ If Mixte =Yes :

o 'COHESIVE SEDIMENTS' (SEDCO) =YES, NO

o 'NUMBER OF SIZE-CLASSES OF BED MATERIAL' (NSICLA) = 2

¬ 'SEDIMENT DIAMETERS' = D1>0.00006 (non-cohesive sediment), D2(D_2<0.00006 m, for cohesive sediment)

¬ 'SEDIMENT DENSITY' (ρ_s= 2650 Kg/m^3, default value)

¬ 'settling velocities' = Ws_1, Ws_2

Bed composition

¬ 'NUMBER OF LAYERS OF THE CONSOLIDATION MODEL' (NOMBLAY=10, default)'

¬ 'MUD CONCENTRATION PER LAYER' in Kg/m^3 (CONC_VASE= 50. ; 100. ;..,by default)

¬ INITIAL FRACTION FOR PARTICULAR SIZE CLASS : p_1 ; p_2

9.2 Erosion/deposition fluxes

The erosion/deposition fluxes now depends on the mass % of each class in the surface layer (Panagiotopoulos et al., 1997). If the mass percent of the mud class is greater than 50% the bed is considered as pure 'cohesive' and the erosion/deposition laws follow the Partheniades classical law. (see below for the calculation of the bed shear strength in a sand/mud mixture).

If the mud percentage is less than 30 %, the bed is considered as non cohesive, and has little effect on the bed shear strength.

Once sediment particles have been put in suspension, they are transported independently, by solving for each class, a transport equation for the volume concentration of the individual sediment particles (deflocculated) defined as :

$$C_i = \frac{C_i}{\rho_s}$$

Mud flocs in suspension

Sand grains in suspension

$$\frac{\partial C_i}{\partial t} + U_{conv}\frac{\partial C_i}{\partial x} + V_{conv}\frac{\partial C_i}{\partial y} = \left[\frac{\partial}{\partial x}\left(\epsilon_s\frac{\partial C_i}{\partial x}\right) + \frac{\partial}{\partial y}\left(\epsilon_s\frac{\partial C_i}{\partial y}\right)\right] + \frac{(E_i - D_i)}{h}$$

The erosion flux is determined for each class of sediments as a function of the bed composition. The deposition flux D= W_{si} C_i is an implicit term.

Erosion law for sand mud mixture

The rate of erosion is based on the Partheniades erosion law, where the critical bed shear strength of the mud class depends on the consolidation state.

The bed shear strength of the sand mud mixtures depends on the % of mud (P_2 at the surface top layer). We follow here the method of Waeles (2005, [36]).
The erosion rate $E_{(1+2)}$ is calculated for the mixture and then for each class Ei.

E_i= P_i $E_{(1+2)}$

¬ C>50% : mud dominant
We apply the Krone erosion law :

$$E_{(1+2)} = M\left[\left(\frac{u_*}{u_{*e}}\right)^2 - 1\right] \quad for \ \ \tau_b = \rho u_*^2 > \tau_{ce} = \rho u_{*e}^2$$
$$E_{(1+2)} = 0 \ \ if \, \tau_b < \tau_{ce}$$

¬ C<30% : sand dominant
We apply the Zyserman and Fredsoe equilibrium concentration :

$$E_{(1+2)} = W_{s1}C_{eq} \ \ for \ \ \tau_b = \rho u_*^2 > \tau_{ce} = \rho u_{*e}^2$$

¬ 30%<C<50% : intermediate range.
We assume a linear interpolation of erosion rates for each class.

9.3 Bed evolution

The bed evolution for both phases is calculated differently following the method for pure mud or pure sand ; This is not entierely correct and should be modified in the near future :

For sand (phase 1).
We assume the first layer to be greater than the active layer thickness, such that the sand percent can be considered to be constant and equal to the percent of the top layer.

$$P_1 dZ_{f1} = (D_1 - E_1)Dt$$

In this mass balance for sand, we do not consider the void ratio, since it is entirely filled by the fine sediments (the volume concentration of the sand phase is 1, instead of (n-1), where n is the bed porosity in case of either pure sand or sand mixtures).
If there is net erosion $(E_1 - D_1) > 0$: the first layer Es_1 is decreased Es_1=Es_1-P_1dZf_1 which is only correct if Es_1-P_1dZf_1>0.
If there is net deposition $(D_1 - E_1) > 0$: the sediment is deposited in the first top layer Es1 is increased Es_1=Es_1-P1dZf_1

For mud (phase 2)
We follow the method described in part. The top layers are successively eroded until we match the erosion flux.
In case of deposition $(D_2 - E_2) > 0$: the sediment is deposited in the top layer

$$P_2 Cs_2 \, DZf_{f_2} = \rho_s (D_2 - E_2) Dt$$

The top layer thickness is increased in order to match : Es_2= Es_2 + dZf_2
In case of erosion the procedure consist in determining the maximum layer to be eroded

$$\sum_{j=1}^{j\max -1} Ms_2(j) < \rho_s (E_2 - D_2) Dt < \sum_{j=1}^{j\max} Ms_2(j)$$

All top layers (up to jmax-1) are emptied :
For the last one, the mass balance is now

$$\sum_{j=1}^{j\max -1} Ms_2(j) + dEs_2(j\max) Cs(j\max) = \rho_s (E_2 - D_2) Dt$$

Reactualisation of the bed composition
The percent of each class of sediment needs to be recalculated (end of suspension main).

$$P_i = \frac{Es_i}{Es}$$

I

With Es= Es_1+Es_2

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