Prediction of Yielding in an Euler-Bernoulli Beam

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Abstract—*This report investigates the prediction of yielding in Euler-Bernoulli beams subjected to bending moments and axial forces. The study utilizes Euler-Bernoulli beam theory to model the structural behavior under various loading conditions. By analyzing the critical load levels that lead to yielding, the study focuses on understanding the influence of beam dimensions, material properties, and loading scenarios. The results provide valuable insights into the conditions under which yielding occurs, offering practical guidance for the design and assessment of beam structures. This research contributes to enhancing the understanding of beam mechanics and informs engineering practices related to Euler-Bernoulli beam applications.*

Keywords—*Euler-Bernoulli beam, yielding, von Mises, Tresca, stress, loading*

1. Objectives

The objectives that we hoped to achieve after the completion of all these experiments are:

1. Plot the Shear Force and Bending Moment Diagrams due to the loading on the bar accurately.
2. Successfully apply the yielding criteria and find the location at which the beam yields first.
3. Plot the location of the point on a 2D space, and plot the curves corresponding to the von Mises and Tresca yield criteria.

2. Problem Statement

Consider an Euler-Bernoulli beam with a certain set of support conditions, and a given set of loads whose magnitudes and positions on the beam can be prescribed. Assuming a fixed value Y for the yield stress, write a code that computes the location of the point on the beam that yields first, and show its location on a 2D space of principle stresses. The plot should also depict the curves corresponding to Von Mises and Tresca yield criteria.

3. Methodology

All the programming for this project was done in WOLFRAM Mathematica. This was because we found that Mathematica was better suited to the needs of this project and the plots and calculations we needed to make.

1. First, we will define the configuration of the beam (refer section 4.2.1).
2. We need to define the loading as a function of length along the beam.
3. Next, we apply equilibrium conditions to find out all the reaction forces acting on the beam.
4. Shear Force and Bending Moment as a function of the x-coordinate have to be derived.
5. Using the above derived quantities, we find out the stress at each point on the beam.
6. To find out which point on the beam yields first, we find the point with maximum stress acting on it. This point would be the first to yield.
7. We plot the curves corresponding to von Mises and Tresca yield criteria, and also plot the location of the yield point on the 2-D plane of principal stresses, and check whether it satisfies the yield criteria.

4. Numerical Implementation

The Euler-Bernoulli beam theory works on the assumption that plane sections remain plane after deformation.

4.1. Formulae used:

Equilibrium conditions:

$$\sum \vec{F}_y = 0 \tag{1}$$

$$\sum M = 0 \tag{2}$$

The loading function input by the user is represented by q(x).

$$\frac{dV}{dx} + q = 0 \tag{3}$$

Here V(x) represents the shear force in the beam. Now to get the bending moment:

$$\frac{dM_b}{dx} + V = 0 \tag{4}$$

To calculate the tensile stress at each point, the following function was used:

$$\sigma_x = \frac{M_b y}{I_{zz}} \tag{5}$$

where I_{zz} represents the second moment of area, and I_{zz}=$\int_{-h/2}^{h/2} y^2 dA$ and evaluating the integral, we get $I_{zz} = \frac{bh^3}{12}$
To calculate the shear stress at each point, the following function was used:

$$\tau_{xy} = \frac{VQ}{bI_{zz}} \tag{6}$$

where $Q = \int_{y_1}^{h/2} y dA$. $y_1 = h/2$, and we can express dA as bdy. Therefore on solving the integral, we obtain $Q = 0$.
Hence, we get the value of $\tau_{xy} = 0$. Also, $\tau_{yz} = 0$ and $\tau_{zx} = 0$.
To calculate the von Mises failure criterion of the beam, the following formula was used:[1]

$$Y = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \tag{7}$$

Where Y is the yield stress.
Since $\sigma_3 = 0$, the above equation simplifies to:

$$Y = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \tag{8}$$

To calculate the Tresca criterion, we use the following formula:

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{Y}{2} \tag{9}$$

If the value of σ_x exceeds the maximum yield stress at any point, the beam yields. If we put in the values obtained from equation.
Also, we use equations 8 and 9 to plot the von Mises and Tresca criteria.

4.2. Code Description

4.2.1. BEAM CONFIGURATION. This section of the code deals with the configuration of the Euler-Bernoulli beam. We have assumed the beam to be supported simply by two supports with variable locations in the x-direction. The code takes the length, width and height of the beam as input, along with the locations (x-coordinates) of both the supports. The code also takes the loading function on the beam as input.

4.2.2. CALCULATION OF SHEAR FORCE AND BENDING MOMENT. This section deals with the calculation of the shear force and bending moment as functions of the x-coordinate ($V(x)$ and $M(x)$). It performs the calculations to obtain the reaction forces at the supports, and then calculate $V(x)$ and $M(x)$. This part also uses the calculated functions to plot the shear force and bending moment diagrams to better visualise their distribution.

4.2.3. YIELD POINT CALCULATION. This section deals with the calculation of the yield point. Using the bending moment function, the tensile stress (σ_x) as a function of the x-coordinate was calculated using equation 5 and its corresponding x-coordinate is found which gives us the yield point of the beam.

4.2.4. YIELD CRITERIA. After obtaining the yield point, the graphs for the von Mises and Tresca yield criteria are plotted using their respective equations in the plane of principal stresses utilising the value of the maximum yield stress Y and passing it through the equation and plotting a contour plot.

5. Results and Discussions

Let us consider the following example for our analysis:
The loading function was input into the program and the visualised loading scenario was obtained as shown: Here, W=4.5 kN/m and P=4.5 kN.
P acts at a distance of 1.5 m from the left end, and W acts across a length of 1.5 m at the far end of the beam.

> **Note**
> *We can define and perform the beam analysis for any user-defined loading function.*

Figure 1. *Visualised Loading Scenario Example*

After this, the shear force and bending moment diagrams were obtained as shown below:

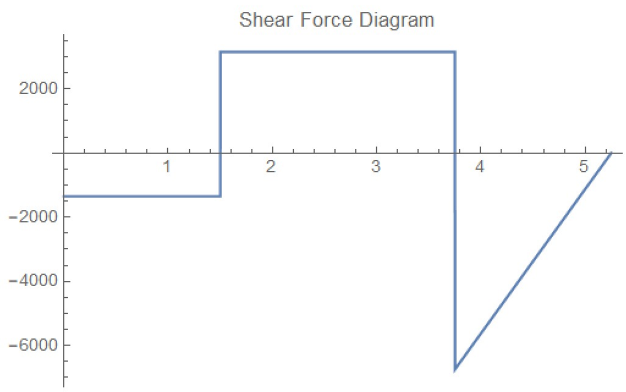

Figure 2. *Shear Force Diagram*

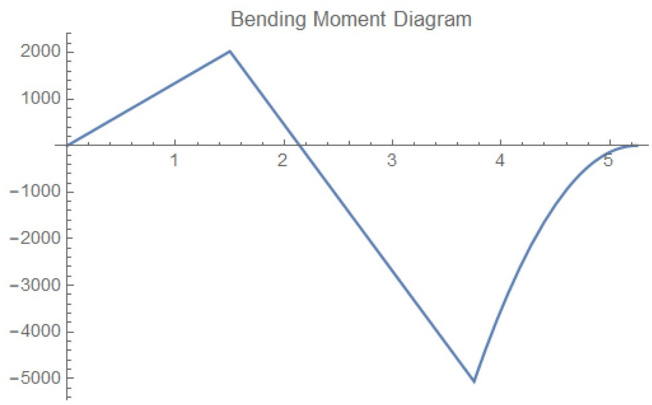

Figure 3. *Bending Moment Diagram*

We calculated the absolute values of the bending moment in order to determine the location of the point where the bending moment is maximum, which is also the point where the tensile stress is maximum (refer section 4.2.3).

$$\{2.43 \times 10^8, \{x \rightarrow 3.75\}\}$$

Figure 5. *Maximum Tensile Stress and its Location*

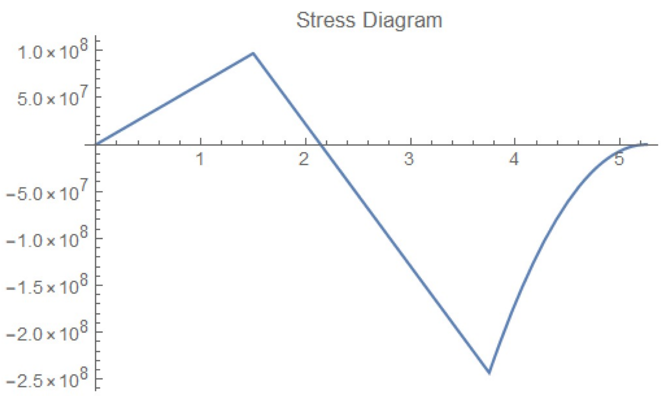

Figure 4. *Diagram of Tensile Stress Distribution*

We then obtain the stress distribution, and following that we obtain the value of the magnitude of the maximum tensile stress and its position as shown below: We then plot the graphs for the von Mises and Tresca criteria, and also plot the point of maximum tensile stress in the 2-D plane of principal stresses as shown in the below graphs.

> **Note**
> *All the values for the above graphs are in SI Units.*

Yield Criteria:
A yield criterion is a concept that outlines the conditions under which a material transitions from elastic behavior to plastic deformation under various stress combinations.

There are different yield criteria used to describe how materials behave when stressed. Let's focus on two types particularly relevant to metals.

To better grasp stress combinations, it's helpful to introduce the notion of principal stress space. Here, the principal stress axes are mutually perpendicular, although they may not align with the crystal axes of the material.[2]

The value of the maximum tensile stress is negative since the beam is in compression at that point, and according to convention tensile stresses are positive and compressive stresses are negative.

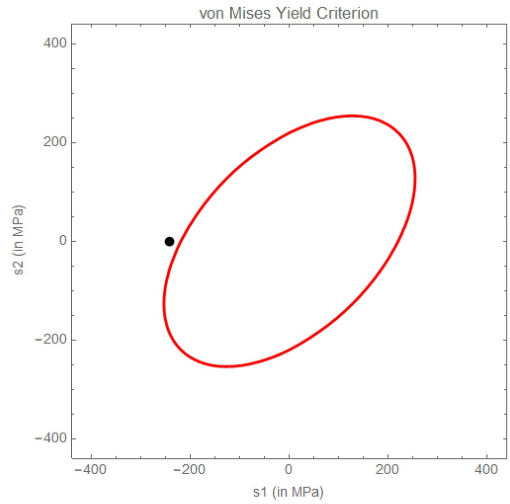

Figure 6. *von Mises Yield Criterion*

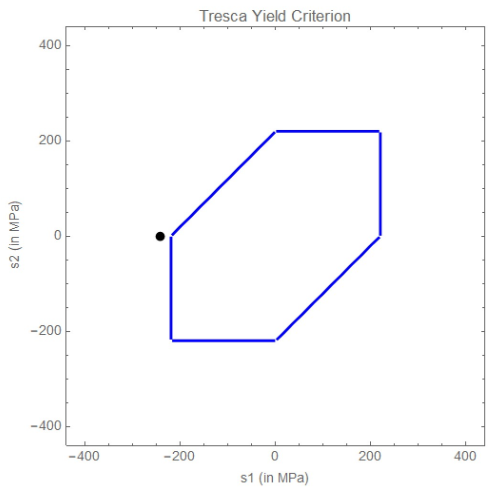

Figure 7. *Tresca Yield Criterion*

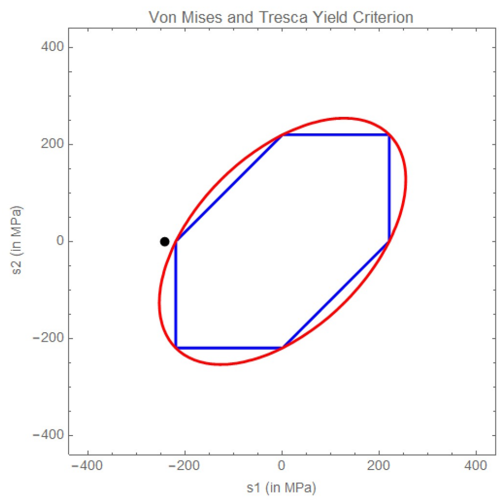

Figure 8. *von Mises and Tresca Criteria*

Since the point lies outside the area bounded by both the von Mises and Tresca curves, we can confirm that the stress experienced by it is higher than the yield stress and the point yields as expected.

6. Learning Outcomes

1. **Loading and Yielding of an Euler-Bernoulli Beam:** By doing this project, we achieved greater insights in how an Euler-Bernoulli beam yields under very different loading conditions. We were able to apply the theory we learnt to a simulated beam.

2. **A New Programming Language:** By opting to write the code for this project in WOLFRAM Mathematica, we learnt its programming language, and we also realised its usefulness and power as a tool in assisting with large and difficult calculations.

3. **Dynamic Problem Solving:** We encountered a lot of challenges during the course of completing this project. A particular part we got stuck on was figuring out how to plot the graphs for the von Mises and Tresca criteria. Learning to figure out solutions to the challenges as we went ahead was key in completing this project.

4. **Working in a Team:** We learnt how to work in a team. We managed each other, distributed work, and worked together to integrate everyone's work to complete the project.

7. Acknowledgements

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References

[1] S. H. Crandall, N. C. Dahl, and T. J. Lardner, *An Introduction to the Mechanics of Solids (in SI Units)*, 3rd. McGraw-Hill Education, 2012, ISBN: 9780071263714.

[2] DoITPoMS, University of Cambridge, *Yield Criteria*, https://www.doitpoms.ac.uk/tlplib/metal-forming-1/yield_criteria.php, [Online; Accessed 11-April-2024], accessed 2024.