Lecture 2 Hands-On

(1) $$m\frac{dv(t)}{dt} = -\zeta v(t)$$

変数分離法で解く.

$$\frac{1}{v(t)}dv(t) = -\frac{\zeta}{m}dt$$

$$\int \frac{1}{v(t)}dv(t) = -\frac{\zeta}{m}\int dt$$

$$\log|v(t)| = -\frac{\zeta}{m}(t+\tau)$$

$$|v(t)| = \exp\left\{-\frac{\zeta}{m}(t+\tau)\right\}$$

$$v(t) = \pm e^{-\frac{\zeta}{m}t}\cdot e^{-\frac{\zeta}{m}\tau}$$

$$v(t) = V\exp\left(-\frac{\zeta}{m}t\right)$$

$$\left(\because V \equiv e^{-\frac{\zeta}{m}\tau}\right)$$

initial condition $\leftarrow$ by

$$v(0) = \frac{10a\zeta}{m}$$

$$\therefore V = \frac{10a\zeta}{m}$$

$$\boxed{\therefore v(t) = \frac{10a\zeta}{m}\exp\left(-\frac{\zeta}{m}t\right)}$$

$$\left(v(t) = 10\frac{a}{t_0}\exp\left(-\frac{t}{t_0}\right)\right)$$

Euler法により方程式を分離する

$$m\frac{dv(t)}{dt} = -\zeta v(t)$$

$$\frac{dv(t)}{dt} = -\frac{\zeta}{m}v(t)$$

$$\frac{v(t+\Delta t) - v(t)}{\Delta t} + O(\Delta t) = -\frac{\zeta}{m}v(t)$$

$$v(t+\Delta t) \fallingdotseq v(t) - \frac{\zeta}{m}\Delta t\, v(t)$$

$$= \left(1 - \frac{\zeta}{m}\Delta t\right) v(t)$$

無次元化のために

$$t \longrightarrow t_0\tilde{t}$$

$$v \longrightarrow v_0\tilde{v} = \frac{a}{t_0}\tilde{v} \qquad (\because a \equiv v_0 t_0)$$

とおく

$$\frac{a}{t_0}\tilde{v}(\tilde{t}+\Delta\tilde{t}) = \left(1 - \frac{\zeta}{m}\cdot t_0\Delta\tilde{t}\right)\frac{a}{t_0}\tilde{v}(\tilde{t})$$

$$\tilde{v}(\tilde{t}+\Delta\tilde{t}) = \left(1 - \frac{\zeta}{m}t_0\Delta\tilde{t}\right)\tilde{v}(\tilde{t})$$

ここで再び

$$\frac{\zeta}{m}t_0 = 1$$

即ち

$$t_0 = \frac{m}{\zeta}$$

と定義する

よって数値計算する方程式は

$$\tilde{v}(\tilde{t}+\Delta\tilde{t}) = (1 - \Delta\tilde{t})\tilde{v}(\tilde{t})$$

となる。

元の v, t に戻す際には、

$$\tilde{v} = \frac{t_0}{a} v = \frac{m}{a\zeta} v$$

$$\tilde{t} = \frac{1}{t_0} t = \frac{\zeta}{m} t$$

とすればよい。

解析解

$$v(t) = \frac{10a\zeta}{m} \exp\left(-\frac{\zeta}{m} t\right)$$

は、

$$\frac{a}{t_0} \tilde{v}(\tilde{t}) = \frac{10a\zeta}{m} \exp\left(-\frac{\zeta}{m} \frac{m}{\zeta} \tilde{t}\right)$$

$$\frac{\zeta}{m} a \tilde{v}(\tilde{t}) = \frac{10a\zeta}{m} \exp\left(-\tilde{t}\right)$$

$$\boxed{\tilde{v}(\tilde{t}) = 10 \exp\left(-\tilde{t}\right)}$$