Project, IE 420 – Financial Engineering, Spring 2020

- You must upload your report (with codes in appendix, in a single pdf file) on http://compass2g.illinois.edu before 11am on Thursday 4/23/2020. **Half or all points will be taken off of a late project**.
  - Combine your report and your codes (with codes in Appendix) into a single pdf file and upload it online for originality screening.
  - Submit a hardcopy of your report (without codes) in class.
- This project is about pricing European and American vanilla options using the binomial model.
- **Programming and analysis**: any programming language can be used. C/C++ is preferred. Excel can be used for the analysis and presentation of data.
- **Grading criteria**: (1) Are the results complete and correct? (2) Is the report well organized? (3) Are the results well explained graphically? (4) What is done to reduce computational time? (5) Is the report generally impressive?
- **Option prices you compute should have an accuracy of 10^{-3}.**

1. Implement the CRR binomial model to price European and American puts and calls on a stock paying continuous dividend yield:

   $$Binomial(Option, K, T, S_0, \sigma, r, q, N, Exercise)$$

   where $Option = C$ for calls and P for puts, K is the strike, T is the time to maturity, S_0 is the initial stock price, σ is the volatility, r is the continuous compounding risk free interest rate, q is the continuous dividend yield, N is the number of time steps, and $Exercise = A$ for American options and E for European options. Your program should output both option price and computational time.
2. Consider a 1-year European call option with strike $K = 100$. The current stock price is 100. Other parameters are $r = 0.05$, $q = 0.04$, $\sigma = 0.2$. Use the Black-Scholes formula to compute the price of the call option. In the binomial model, take a sequence of increasing numbers of time steps N. Verify that the binomial option prices converge to the Black-Scholes option price as N increases. Construct a table/plot to visualize the convergence.
3. Consider American puts with $K = 100$, $\sigma = 0.2$, $r = 0.05$, $q = 0$. For time to maturity varying from 1 month to 12 months, investigate the number of time steps needed to achieve the required 10^{-3} accuracy. Calculate and plot the price of a 12-month put as a function of S_0. For time to maturity $i/12$, $i = 0, 1, \cdots, 12$, find the critical stock price $S^*(i)$ on the early exercise boundary (see notes below about how to determine $S^*(i)$). Report and plot the early exercise boundary $\{S^*(i), 0 \leq i \leq 12\}$ as a function of the time to maturity. Repeat the above for $q = 0.04$. How do put prices and the early exercise boundary change when the continuous dividend yield goes from 0 to 0.04? What is the intuition behind this dependence on the dividend yield?
4. Consider American calls with $K = 100$, $\sigma = 0.2$, $r = 0.05$, $q = 0.04$. Investigate the number of time steps needed to achieve 10^{-3} accuracy for 1-month to 12-month calls. Calculate and plot the price of a 12-month call as a function of S_0. For time to maturity $i/12$, $i = 0, 1, \cdots, 12$, find the critical stock price $S^*(i)$ on the early exercise boundary. Report and plot the early exercise boundary $\{S^*(i), 0 \leq i \leq 12\}$ as a function of the time to maturity. Repeat the above for $q = 0.08$. How do call prices and the early exercise boundary change when the continuous dividend yield goes from 0.04 to 0.08? What is the intuition behind this dependence on the dividend yield?

- **Notes**: To plot the option price as a function of S_0, calculate an array of option prices with a reasonably small interval in S_0. **For $S^*(i)$'s, keep two digits after the decimal point.** To determine $S^*(i)$, compare the option price with the option intrinsic value. For a put, $S^*(i)$ is the largest stock price at which the difference between the option price and the intrinsic value is less than 0.005. For a call, $S^*(i)$ is the smallest stock price at which the difference between the option price and the intrinsic value is less than 0.005.