

ECE220: Stack Data Structure

Lecture 4

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Outline

- Review of Stack
 - Interface: Push, Pop, Isfull, IsEmpty
 - Implementation choices
- Applications of Stack
 - Example 1
 - Example 2

Stack Data Structure

- Basic operations
 - Push a: puts “a” at the top of the stack
 - Pop: removes the element at the top of the stack and returns it
 - Any other useful operations?
- What if memory is full?

Refresh: Design choices for implementing Stack

- **Definition.** Top of stack: Memory location which is the **first location available** above (less than) the last element pushed onto the stack
- The top of stack is **stored at a memory location** labeled as `STACK_TOP` and loaded to a register (R4 in this case) when needed
- **PUSH a:** First place `a` in memory at location stored in `STACK_TOP` and then **decrement** the value stored in `STACK_TOP`
- **POP:** **First increment** the value stored in `STACK_TOP` and **then read** from the address stored at the incremented `STACK_TOP`
- `STACK_START`: first memory location available for our stack
 - `IsEmpty`: `val in STACK_TOP = val in STACK_START`
- `STACK_END`: last memory location available for our stack
 - `IsFull`: `val in STACK_TOP = val in STACK_END - 1`

Label/address		
x44FF		
x4500		
...		
x45FD		
x45FE	C	
x45FF	B	
x4600	A	
STACK_TOP	x45FD	
STACK_START	x4600	
STACK_END	x4500	

Problems

- What are some potential problems with implementing a stack in memory?
- What could happen if we didn't check underflow when trying to POP?
- What could happen if we didn't check overflow when trying to PUSH?

PUSH subroutine implementation

- Argument

- Value to be pushed onto the stack
- Passed to the subroutine in R0

- Result

- To indicate if push was successful
- Will be returned in R5 (0 – success, 1 – fail)

- Internally uses R3 and R4

```

; IN: R0, OUT: R5
; R3: STACK_END
; R4: STACK_TOP
;
PUSH
;
; prepare registers
;
    ST R3, PUSH_SaveR3 ; save R3
    ST R4, PUSH_SaveR4 ; save R4
    AND R5, R5, #0 ; clear R5, indicates success
    LD R3, STACK_END
    LD R4, STACK_TOP
;
; check for overflow (when stack is full)
;
    ADD R3, R3, #-1
    NOT R3, R3
    ADD R3, R3, #1
    ADD R3, R3, R4; R3 = STACK_TOP - (STACK_END-1)
    BRz OVERFLOW ; stack is full
;

```

```

; store value in the stack
;
    STR R0, R4, #0 ; push onto the stack
    ADD R4, R4, #-1 ; move top of the stack
    ST R4, STACK_TOP ; store top of stack pointer
    BRnzp DONE_PUSH
;
; indicate the overflow condition on return
;
OVERFLOW ADD R5, R5, #1
;
; restore modified registers and return
;
DONE_PUSH
    LD R3, PUSH_SaveR3
    LD R4, PUSH_SaveR4
    RET
;
PUSH_SaveR3 .BLKW #1
PUSH_SaveR4 .BLKW #1
STACK_TOP .FILL x4000
STACK_START .FILL x4000
STACK_END .FILL x3FF0
.END

```

Label/address		
x3FEF		
x3FF0		
...		
x3FFD		
x3FFE		
x3FFF		
x4000		
STACK_TOP	x4000	
STACK_START	x4000	
STACK_END	x3FF0	

POP subroutine implementation

- Argument - none
- Result
 - R0: Value popped of the stack
 - R5: Indicator if pop was successful (0 – success, 1 – fail)
- Internally uses R3 and R4


```

; OUT: R0 OUT: R5
; R3: STACK_START
; R4: STACK_TOP
;
POP
;
; prepare registers
;
    ST R3, POP_SaveR3 ; save R3
    ST R4, POP_SaveR4 ; save R4
    AND R5, R5, #0      ; clear R5 (assume success)
    LD R3, STACK_START
    LD R4, STACK_TOP
;
; check for underflow (when stack is empty)
; STACK_START = STACK_TOP, R4 - R3 = 0
    NOT R3, R3
    ADD R3, R3, #1
    ADD R3, R3, R4
    BRz UNDERFLOW ; stack is empty, no pop
;

```

```

; remove value from the stack
;
    ADD R4, R4, #1 ; move top of the stack
    LDR R0, R4, #0 ; read value from the stack
    ST R4, STACK_TOP ; store top of stack pointer
    BRnzp DONE_POP
;
; indicate the underflow condition on return
;
UNDERFLOW ADD R5, R5, #1
;
; restore modified registers and return
;
DONE_POP
    LD R3, POP_SaveR3
    LD R4, POP_SaveR4
    RET
;
POP_SaveR3 .BLKW #1
POP_SaveR4 .BLKW #1

```

Exercises

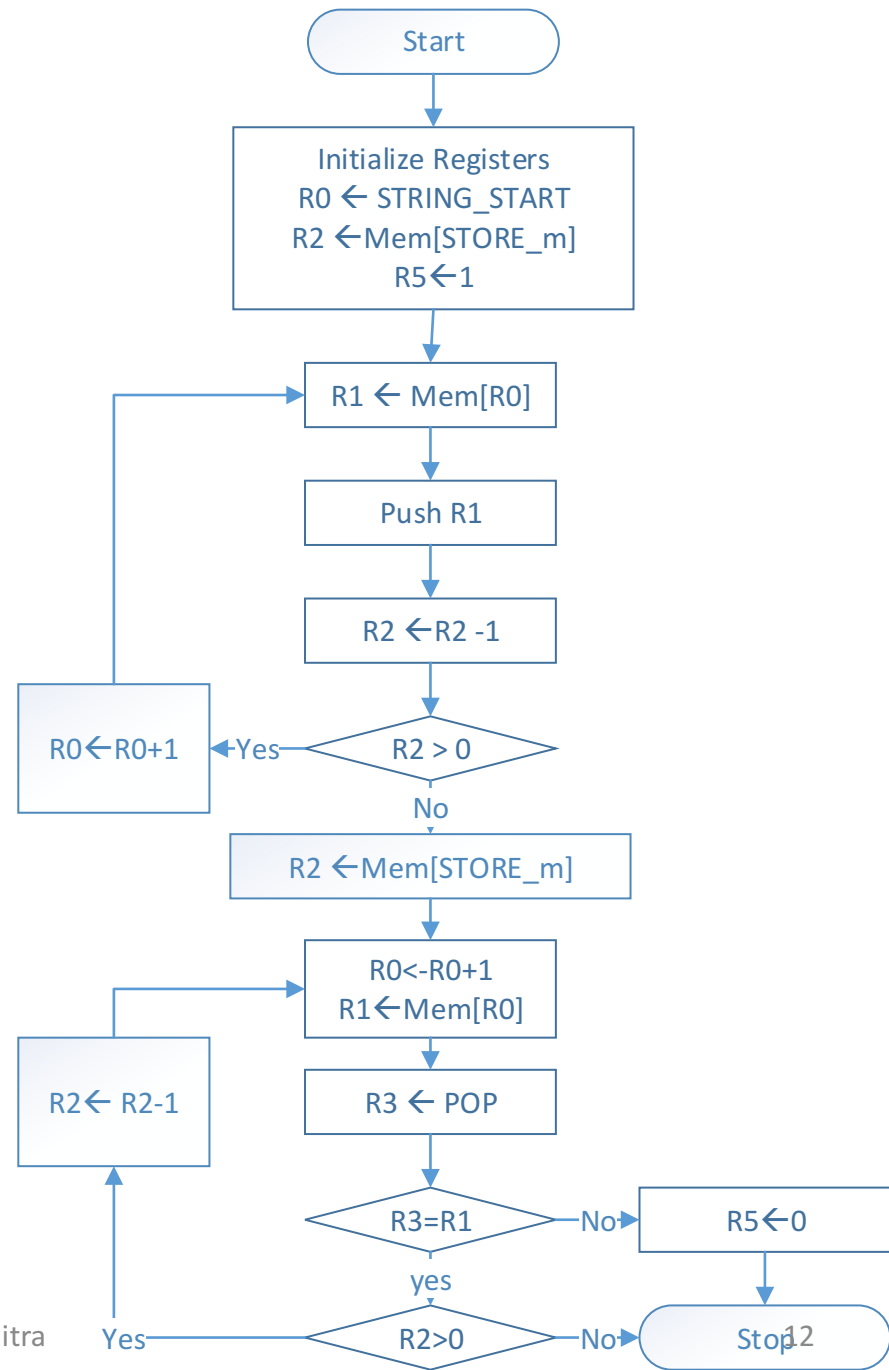
- Implementation of stack growing downward in memory
- Implement a stack of characters
- How to make it work for any data type?

Problem 1: Palindromes

- Examples of palindromes
 - Madam
 - Kayak
 - Tennis set won now Tess in net
 - Was it a car or a cat I saw?
 - Aibophobia
- Problem:
 - INPUTS: String starting from memory location `STRING_START`
 - Length of the string is $2m$, m is stored in memory location `STORE_m`.
 - OUTPUT: $R5 = 1$ if palindrome and $R5 = 0$ indicates not a palindrome
 - Assume that the string is NUL terminated; no spaces and punctuations.
 - An implementation of stack (Push/Pop) is given

Design choices & decomposition

- R0: address of character being read
- R1: current character being read
- R3: 'mirror' character
- R2: (m - #characters read)



Simple usage

```
.ORIG x3000
; R2 <-- some value
; R3 <-- another value
; store R3 and R2 in the stack
; push one value onto the stack

LD R2, ITEM1 ; prepare argument
LD R3, ITEM2 ; prepare argument
ADD R0, R2, #0 ; prepare argument for Push
JSR PUSH
; push another value onto the stack
ADD R0, R3, #0 ; prepare argument for Push
JSR PUSH
; pop from the stack
JSR POP
ST R0, RITEM1
JSR POP
ST R0, RITEM2
HALT

ITEM1 .FILL x10FF
ITEM2 .FILL x20FF
RITEM1 .BLKW 1
RITEM2 .BLKW 1
```

- Pushes two values into the stack then pops them and stores the results in 2 memory locations
- Does not check for success of Push or Pop but how would we check? Then, how would we respond once we knew?

Exercises

- How to handle strings of odd length ($2m+1$)?
- What if the length of string is not known a priori?
- How to handle punctuations and space?

Problem 2. Postfix evaluation

- **Definition.** A postfix expression is defined as
 - $\langle \text{operand1} \rangle \langle \text{operand2} \rangle \langle \text{operator} \rangle$
 - Example: $3\ 4\ +$ evaluates to $(3 + 4) = 7$
- The operands may be postfix subexpressions
 - Example: $3\ 4\ 5\ +\ -$ evaluates to $3 - (4 + 5)$
- The advantage of using postfix: no need for parentheses, i.e., unambiguous
 - Infix $3 + 4 \times 5$ can have two values $(3 + 4) \times 5 = 35$ OR
 - $3 + (4 \times 5) = 23$
 - The above two expressions in postfix will be $3\ 4\ +\ 5\ \times$ and $3\ 4\ 5\ \times\ +$, respectively.

Problem 2

- Problem statement. Given a valid postfix expression with numerals and '+', '-', 'x' in the form of a string, evaluate it and store the answer in R5.
- Algorithm.
 - Read the string (postfix expression) left to right;
 - push the numbers in the expression on the stack;
 - for an operator, pop the top two elements, compute the answer and push it on stack.
- Example: $3\ 4\ +\ 5\ *$ and $3\ 4\ 5\ +\ *$
- How would you write $7-4*(6-2)$?

Problem 2

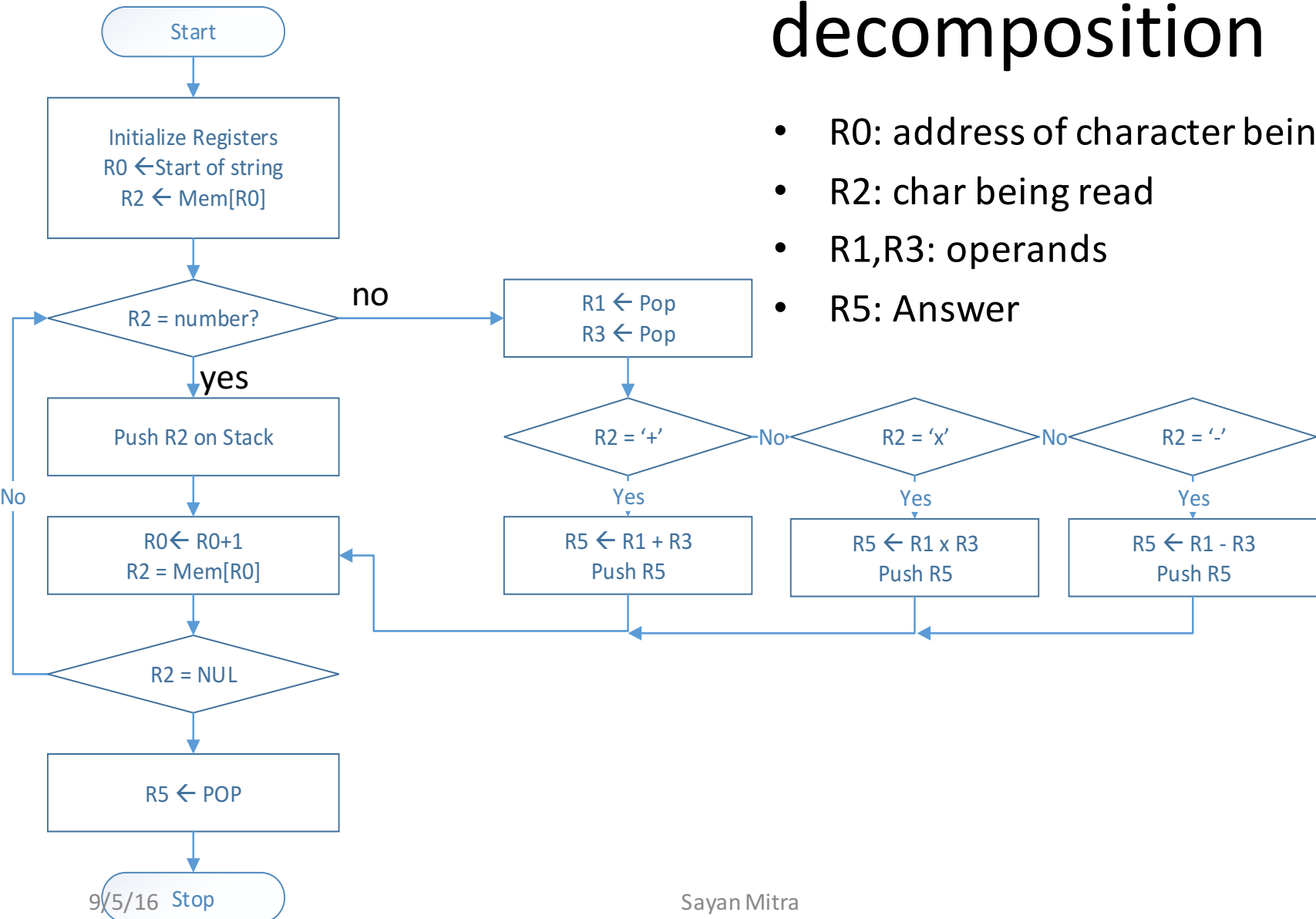
- Example: 3 4 + 5 *
 - input 3: push 3
 - input 4: push 4
 - input +: $R1 \leftarrow \text{pop}(4)$, $R2 \leftarrow \text{pop}(3)$, push ($R1 + R2 = 7$)
 - input 5: push 5
 - input *: $R1 \leftarrow \text{pop}(5)$, $R2 \leftarrow \text{pop}(7)$, push ($R1 * R2 = 35$)Ans
- Example: 3 4 5 + *
 - $3*(4+5)$

How to Evaluate

- Input: String of numbers and operators “3 4 + 5 *”
- Output : 35
- Assume each **numerical** argument is a single char (Exercise: how to relax this?)
- Stack implementation is given
- Idea: Push one argument (char) in string at a time onto a stack
 - If the argument is a number then do nothing
 - Else (**operator**) pop last two elements from stack, perform operation and push the result back in stack
 - Done when input expression is completely read

Design choices and decomposition

- R0: address of character being read
- R2: char being read
- R1,R3: operands
- R5: Answer



Exercises

- When would this implementation fail?
 - What type of expressions are “bad” for this implementation
- Write the code with subroutine calls to Push, Pop, Mult, Sub