

$\forall \epsilon, \delta > 0 \exists m(\epsilon, \delta) \text{ s.t. } \forall m \geq m(\epsilon, \delta)$

$$P_{S \sim D^m} [L_D(A(S)) \leq \epsilon] \geq 1 - \delta$$

$\Leftrightarrow$

$$\lim_{m \rightarrow \infty} E_{S \sim D^m} [L_D(A(S))] = 0$$

$m(\epsilon, \delta) = \frac{1}{\delta + \epsilon}$   $\forall m \geq m(\epsilon, \delta), \forall \epsilon, \delta > 0$  :  $\Downarrow$  (הוכחה)

$$P_{S \sim D^m} (L_D(A(S)) \leq \epsilon) \geq 1 - \delta \rightarrow P_{S \sim D^m} (L_D(A(S)) > \epsilon) \leq \delta$$

$$E_{S \sim D^m} (L_D(A(S))) = \int_0^\infty P(L_D(A(S)) > t) dt =$$

$$= \int_0^\infty P(L_D(A(S)) > t) dt \leq \delta + \int_0^\infty P(L_D(A(S)) > t) dt \leq \delta + \epsilon$$

$$0 \leftarrow 0 \leq E_{S \sim D^m} (L_D(A(S))) \leq \delta + \epsilon \leq \frac{1}{m} \xrightarrow{m \rightarrow \infty} 0 \quad \text{וכן}$$

$$E_{S \sim D^m} (L_D(A(S))) \xrightarrow{m \rightarrow \infty} 0 \quad \text{אנחנו רוצים להוכיח}$$

$$\lim_{m \rightarrow \infty} E_{S \sim D^m} [L_D(A(S))] = 0$$

$$\frac{E_{S \sim D^m} [L_D(A(S))]}{\epsilon} \geq P(L_D(A(S)) \geq \epsilon)$$

$\epsilon, \delta > 0$  יהיו  $\epsilon, \delta$

$$P(L_D(A(S)) \geq \epsilon) \leq \frac{E_{S \sim D^m} [L_D(A(S))]}{\epsilon} \xrightarrow{m \rightarrow \infty} 0$$

$[0, \delta]$  נמצא בקטע  $[0, \delta]$  קיים  $m$  מסוים כך ש

$m(\epsilon, \delta)$  ק"מ  $\delta = \epsilon$  והפרט  $\delta = \epsilon$   $\delta = \epsilon$   $\delta = \epsilon$

$$P(L_D(A(S)) \geq \epsilon) \leq \delta \Rightarrow P(L_D(A(S)) \leq \epsilon) \geq 1 - \delta$$



$$X = \mathbb{R}^2 \quad Y = \{0, 1\} \quad H = \{h_r; r \in \mathbb{R}_+\}^2$$

$$h_r(\bar{x}) = \mathbb{1}_{[\|\bar{x}\|_2 \leq r]} \quad \text{כל } r > 0$$

$$m_H(\epsilon, \delta) \leq \frac{\log(\frac{1}{\delta})}{\epsilon} \quad \underline{m_3}$$

יש להוכיח כי PAC

הוכחה: ~~הוכחה~~ ~~הוכחה~~ ~~הוכחה~~  
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$$A: D^m \rightarrow (X \rightarrow Y)$$

$$A(S) = h_{r_S}(\bar{x}), \quad r_S = \max\{\|\bar{x}\| \mid x \in S\} \quad \text{כל } S$$

ה'  $\epsilon, \delta > 0$  ידוע,  $D$  ~~הוכחה~~ ~~הוכחה~~ ~~הוכחה~~

~~$$P_{S \sim D^m} [L_0(A(S)) > \epsilon] = P_{S \sim D^m} [\exists \bar{x} \in S, \|\bar{x}\| > r_\epsilon]$$~~

~~$$= P_{S \sim D^m} [\exists \bar{x} \in S, \|\bar{x}\| > r_\epsilon]$$~~

$$r_\epsilon = \max\{r \in \mathbb{R}_+ \mid P[\|\bar{x}\|_{\text{real}} \geq r] \geq \epsilon\}$$

כל  $r_\epsilon \leq \|\bar{x}\| \leq r_{\text{real}}$  כן  $\bar{x}$  נכנס  $S$  אל

וכן ~~הוכחה~~ ~~הוכחה~~ ~~הוכחה~~  $L_0(A(S)) \leq \epsilon$  - כל

$$P_{S \sim D^m} [L_0(A(S)) > \epsilon] = P_{S \sim D^m} [\bigcap_{i=1}^m \{\bar{x}_i \notin [r_\epsilon, r_{\text{real}}]\}] =$$

$$= [1 - P(\bar{x}_i \in [r_\epsilon, r_{\text{real}}])]^m \leq \exp(-m P(\bar{x}_i \in [r_\epsilon, r_{\text{real}}]))$$

$$P(\bar{x}_i \in [r_\epsilon, r_{\text{real}}]) \geq \epsilon$$

ההסתברות



2.  $P_{S \sim D^m} [L_D(A(S)) > \epsilon] \leq e^{-\epsilon m}$  ;  $\delta = \epsilon m$

$P_{S \sim D^m} [L_D(A(S)) \leq \epsilon] \geq 1 - e^{-\epsilon m}$   $\epsilon'' \leq$

$m_{\epsilon}(\epsilon, \delta) = \frac{\log(\frac{1}{\delta})}{\epsilon}$   $\delta$   $\epsilon$   $m$

~~$$\begin{aligned} e^{-\epsilon m} &\leq \delta \\ -\epsilon m &\leq \ln \delta \\ -m &\leq \frac{\ln \delta}{\epsilon} \\ m &\leq \frac{\ln(\frac{1}{\delta})}{\epsilon} \end{aligned}$$~~

$P_{S \sim D^m} [L_D(A(S)) \leq \epsilon] \geq 1 - \delta$  :  $\delta$   $\epsilon$   $m$

$m_{\epsilon}(\epsilon, \delta)$   $\epsilon$   $\delta$   $m$   $\epsilon$   $\delta$   $m$

PAC  $m_{\epsilon}(\epsilon, \delta) = \frac{\log(\frac{1}{\delta})}{\epsilon}$   $\epsilon$   $\delta$   $m$

$\epsilon - m$



$$X = \{0,1\}^n \quad Y = \{0,1\} \quad .3$$

$$h_I(\bar{x}) = \left( \sum_{i \in I} x_i \right) \bmod 2 \quad I \subseteq [n] \quad \text{כך}$$

$$\bar{e}_i = \begin{pmatrix} 0 \\ \vdots \\ 1 \text{ at } i \text{ position} \\ \vdots \\ 0 \end{pmatrix} \quad C = \{ \bar{e}_i \}_{i=1}^n \quad \text{כך} \quad \text{מבנה}$$

$$\text{VC-dim}(H_n) = n \quad \text{על ידי}$$

$$\bar{y} \in \{0,1\} \quad \text{כך} \quad \text{מבנה}$$

$$I = \{ i \mid \bar{y}_i = 1 \} \quad \text{כך} \quad \text{כך}$$

$$h_I(\bar{x}_i) = \bar{y}_i \quad : i \in [n] \quad \text{כך}$$

$$H_n \text{ shatters } C \quad \text{כך}$$

$$H_n \text{ shatters } C \quad \text{כך} \quad |C| > n \quad \text{כך} \quad \text{כך}$$

$$H_n = \{ h_I \mid I \subseteq [n] \} \Rightarrow |H_n| = 2^n \quad \text{כך} \quad \text{כך}$$

$$H_n \text{ shatters } C \quad \text{כך} \quad \text{כך} \quad \text{כך}$$

$$2^{|C|} \quad \text{כך} \quad \text{כך}$$

$$|H_n| = 2^n < 2^{|C|} \quad \text{כך} \quad \text{כך}$$

$$\text{כך} \quad \text{כך}$$

$$\text{כך} \quad \text{כך}$$



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$$\left. \begin{array}{l} e \in R \\ a_i \leq b_i \end{array} \right\}$$

$$= 24$$

~~(2) 183 3/4 1/2 1/4 1/8 1/16 1/32 1/64 1/128 1/256 1/512 1/1024 1/2048 1/4096 1/8192 1/16384 1/32768 1/65536 1/131072 1/262144 1/524288 1/1048576 1/2097152 1/4194304 1/8388608 1/16777216 1/33554432 1/67108864 1/134217728 1/268435456 1/536870912 1/1073741824 1/2147483648 1/4294967296 1/8589934592 1/17179869184 1/34359738368 1/68719476736 1/137438953472 1/274877906944 1/549755813888 1/1099511627776 1/2199023255552 1/4398046511104 1/8796093022208 1/17592186044416 1/35184372088832 1/70368744177664 1/140737488355328 1/281474976710656 1/562949953421312 1/1125899906842624 1/2251799813685248 1/4503599627370496 1/9007199254740992 1/18014398509481984 1/36028797018963968 1/72057594037927936 1/144115188075855872 1/288230376151711744 1/576460752303423488 1/1152921504606846976 1/2305843009213693952 1/4611686018427387904 1/9223372036854775808 1/18446744073709551616 1/36893488147419103232 1/73786976294838206464 1/147573952589676412928 1/295147905179352825856 1/590295810358705651712 1/1180591620717411303424 1/2361183241434822606848 1/4722366482869645213696 1/9444732965739290427392 1/18889465931478580854784 1/37778931862957161709568 1/75557863725914323419136 1/151115727451828646838272 1/302231454903657293676544 1/604462909807314587353088 1/1208925819614629174706176 1/2417851639229258349412352 1/4835703278458516698824704 1/9671406556917033397649408 1/19342813113834066795298816 1/38685626227668133590597632 1/77371252455336267181195264 1/154742504910672534362390528 1/309485009821345068724781056 1/618970019642690137449562112 1/1237940039285380274899124224 1/2475880078570760549798248448 1/4951760157141521099596496896 1/9903520314283042199192993792 1/19807040628566084398385987584 1/39614081257132168796771975168 1/79228162514264337593543950336 1/158456325028528675187087900672 1/316912650057057350374175801344 1/633825300114114700748351602688 1/1267650600228229401496703205376 1/2535301200456458802993406410752 1/5070602400912917605986812821504 1/10141204801825835211973625643008 1/20282409603651670423947251286016 1/40564819207303340847894502572032 1/81129638414606681695789005144064 1/162259276829213363391578010288128 1/324518553658426726783156020576256 1/649037107316853453566312041152512 1/1298074214633706907132624082305024 1/2596148429267413814265248164610048 1/5192296858534827628530496329220096 1/10384593717069655257060992658440192 1/20769187434139310514121985316880384 1/41538374868278621028243970633760768 1/83076749736557242056487941267521536 1/166153499473114484112975882535043072 1/332306998946228968225951765070086144 1/664613997892457936451903530140172288 1/1329227995784915872903807060280344576 1/2658455991569831745807614120560689152 1/5316911983139663491615228241121378304 1/10633823966279326983230456482242756608 1/21267647932558653966460912964485513216 1/42535295865117307932921825928971026432 1/85070591730234615865843651857942052864 1/170141183460469231731687303715884105728 1/340282366920938463463374607431768211456 1/680564733841876926926749214863536422912 1/1361129467683753853853498429727072845824 1/2722258935367507707706996859454145691648 1/5444517870735015415413993718908291383296 1/10889035741470030830827987437816582766592 1/21778071482940061661655974875633165533184 1/43556142965880123323311949751266331066368 1/87112285931760246646623899502532662132736 1/174224571863520493293247799005065324265472 1/348449143727040986586495598010130648530944 1/696898287454081973172991196020261297061888 1/1393796574908163946345982392040522594123776 1/2787593149816327892691964784081045188247552 1/5575186299632655785383929568162090376495104 1/11150372599265311570767859136324180752990208 1/22300745198530623141535718272648361505980416 1/44601490397061246283071436545296723011960832 1/89202980794122492566142873090593446023921664 1/178405961588244985132285746181186892047843328 1/356811923176489970264571492362373784095686656 1/713623846352979940529142984724747568191373312 1/1427247692705959881058285969449495136382746624 1/2854495385411919762116571938898990272765493248 1/5708990770823839524233143877797980545530986496 1/1141798154164767904846628775559596109106197299~~

$$C = (1, 2, \dots, 20) \rightarrow \text{[scribbles]} \rightarrow \text{[scribbles]}$$

$$y \in \{0, 1\}^{2K} \quad (f) \quad y'$$

$$[a_i, b_i] = \begin{cases} [i - \frac{1}{2}, i + \frac{1}{2}] & y_i = 1 \\ [-1, 0] & \text{and} \end{cases}$$

•  $x_i \in C$  and  $h_A(x_i) = y_i$  and  $\|x_i - x_j\|$

$|C| \geq 2k$  וכן  $C \subseteq R$  וכן  $C \subseteq R$  וכן  $C \subseteq R$

2k+1 1/2 C 2 0' 25/10

$$: + p \delta n \rangle \quad G \delta \sim \quad |1\rangle \sim |1\rangle \quad X_{\downarrow} \dots X_{2k+1} \quad |N \sim 0\rangle$$

$$y_i = \begin{cases} 1 & \text{46151/6 i} \\ 0 & \text{4615 i} \end{cases}$$

1)  $\exists$  ק"מ"ם  $k$  וקטע  $AB$  כך ש

$$h_A(x_i) = y_i$$

$x_i \in [a_j, b_j]$

[illegible]



5.  $H$  מחלקת היכולת,  $H$  חזקה PAC

1. יהי  $\delta \in (0,1)$ , ויהי  $0 < \epsilon_1 \leq \epsilon_2 < 1$

2.  $m_H(\epsilon_1, \delta) \geq m_H(\epsilon_2, \delta)$  : "3

מחלקת PAC-יכולת  $H$  פ'ק'  $m \geq m_H(\epsilon_2, \delta)$  נדע

$$P_{S \sim D^m} [L_D(h_S) \leq \epsilon_2] \geq 1 - \delta$$

נניח בהשעיה ש  $m_H(\epsilon_1, \delta) < m_H(\epsilon_2, \delta)$  (\*)

$$P_{S \sim D^{m_H(\epsilon_1, \delta)}} [L_D(h_S) \leq \epsilon_2] \geq 1 - \delta$$

$$P_{S \sim D^{m_H(\epsilon_1, \delta)}} [L_D(h_S) \leq \epsilon_1] \geq 1 - \delta$$

נניח  $m_H(\epsilon_1, \delta) < m_H(\epsilon_2, \delta)$  -  $\rightarrow$  sample-complexity פ'ק' :

~~מחלקת~~

$$m_H(\epsilon_1, \delta) < m_H(\epsilon_2, \delta) \wedge m_H(\epsilon_2, \delta) \leq m_H(\epsilon_1, \delta)$$

אכן, נרמ ש"פ, נסמך  $\delta$  (\*)

2. יהי  $\delta \in (0,1)$ , ויהי  $0 < \delta_1 \leq \delta_2 < 1$

$$m_H(\epsilon, \delta_1) \geq m_H(\epsilon, \delta_2)$$

נניח בהשעיה ש  $m_H(\epsilon, \delta_1) < m_H(\epsilon, \delta_2)$

מחלקת PAC-יכולת  $H$  פ'ק'  $m \geq m_H(\epsilon, \delta_2)$

$$P_{S \sim D^m} [L_D(h_S) \leq \epsilon] \geq 1 - \delta_1 \geq 1 - \delta_2$$

$$P_{S \sim D^{m_H(\epsilon, \delta_2)}} [L_D(h_S) \leq \epsilon] \geq 1 - \delta_2$$

פ'ק' sample-comp.  $m \geq m_H(\epsilon, \delta_2)$  נדע

$$m_H(\epsilon, \delta_1) = m_H(\epsilon, \delta_2)$$

נסמך  $\delta$  בהשעיה

ל.ע.נ



•  $h: X \rightarrow \{0,1\}^n$   $h_1 \sim h_2$   $H_1 \subseteq H_2$   $VC\text{-dim}(H_1) \leq VC\text{-dim}(H_2)$   $6$

$$d_{1,2} = VC\text{-dim}(H_{1,2})$$

$H_1$  shatters  $C$   $VC\text{-dim}$   $H_1$   $H_2$   $H_1 \subseteq H_2$   $C$

$2^{|C|}$   $H_1$   $C$   $H_1$   $C$   $H_1$   $C$   $H_1$   $C$

$$|\{h_c | h \in H_1\}| = 2^{|C|}$$

$H$   $C$   $H$   $C$   $H$   $C$   $H$   $C$

$C - \delta$

$H_1 \subseteq H_2$   $C$   $H_1$   $C$   $H_1$   $C$

$$\{h_c | h \in H_1\} \subseteq \{h_c | h \in H_2 \supseteq H_1\} \subseteq 2^C$$

$$|\{h_c | h \in H_2\}| = 2^{|C|}$$

$H_2$  shatters  $C$   $C$   $H_2$   $C$   $H_2$   $C$

$VC\text{-dim}$   $H_2$   $C$   $H_2$   $C$   $H_2$   $C$

$$d_2 \geq d_1$$

7.  $m_H^{UC}(0,1) \rightarrow N$  א"כ  $\forall h \in H$

3"1:  $M_H(\epsilon, \delta) \leq m_H^{UC}(\frac{\epsilon}{2}, \delta)$  אם PAC-הצגה  $H$

א"כ  $m_H^{UC}(\frac{\epsilon}{2}, \delta)$  ו  $0 < \epsilon, \delta < 1$  נכנס

$$D^m(\{S \in (X \times Y)^m \mid \forall h \in H |L_S(h) - L_D(h)| < \epsilon\}) \geq 1 - \delta$$

א"כ  $m \geq m_H^{UC}(\frac{\epsilon}{2}, \delta)$  נכנס

$$D^m(\{S \in (X \times Y)^m \mid \forall h \in H |L_S(h) - L_D(h)| < \frac{\epsilon}{2}\}) \geq 1 - \delta$$

א"כ  $m \geq m_H^{UC}(\frac{\epsilon}{2}, \delta)$  נכנס  
 (כאשר  $\frac{\epsilon}{2}$  הוא המרחק בין  $L_S$  ל  $L_D$  לכל  $S \in (X \times Y)^m$ )

$$D^m(\{S \in (X \times Y)^m \mid \forall h \in H L_S(h) < L_D(h) + \epsilon\}) \geq 1 - \delta$$

א"כ  $M_H(\epsilon, \delta) \leq m_H^{UC}(\frac{\epsilon}{2}, \delta)$  אם PAC-הצגה  $H$