where $\phi > 1$. Typically, this case is ignored and $\phi = 1$ is used to characterise the non-stationarity because $\phi > 1$ does not describe many data series in economics and finance, but $\phi = 1$ has been found to describe accurately many financial and economic time series. Moreover, $\phi > 1$ has an intuitively unappealing property: shocks to the system are not only persistent through time, they are propagated so that a given shock will have an increasingly large influence. In other words, the effect of a shock during time t will have a larger effect in time $t+1$, a larger effect still in time $t+2$, and so on. To see this, consider the general case of an AR(1) with no drift

$$y_t = \phi y_{t-1} + u_t \tag{8.4}$$

Let ϕ take any value for now. Lagging (8.4) one and then two periods

$$y_{t-1} = \phi y_{t-2} + u_{t-1} \tag{8.5}$$

$$y_{t-2} = \phi y_{t-3} + u_{t-2} \tag{8.6}$$

Substituting into (8.4) from (8.5) for y_{t-1} yields

$$y_t = \phi(\phi y_{t-2} + u_{t-1}) + u_t \tag{8.7}$$

$$y_t = \phi^2 y_{t-2} + \phi u_{t-1} + u_t \tag{8.8}$$

Substituting again for y_{t-2} from (8.6)

$$y_t = \phi^2(\phi y_{t-3} + u_{t-2}) + \phi u_{t-1} + u_t \tag{8.9}$$

$$y_t = \phi^3 y_{t-3} + \phi^2 u_{t-2} + \phi u_{t-1} + u_t \tag{8.10}$$

T successive substitutions of this type lead to

$$y_t = \phi^{T+1} y_{t-(T+1)} + \phi u_{t-1} + \phi^2 u_{t-2} + \phi^3 u_{t-3} + \cdots + \phi^T u_{t-T} + u_t \tag{8.11}$$

There are three possible cases:

(1) $\phi < 1 \Rightarrow \phi^T \rightarrow 0$ as $T \rightarrow \infty$
So the shocks to the system gradually die away – this is the *stationary case.*

(2) $\phi = 1 \Rightarrow \phi^T = 1 \;\forall\, T$
So shocks persist in the system and never die away. The following is obtained

$$y_t = y_0 + \sum_{t=0}^{\infty} u_t \text{ as } T \rightarrow \infty \tag{8.12}$$

So the current value of y is just an infinite sum of past shocks plus some starting value of y_0. This is known as the *unit root case*, for the root of the characteristic equation would be unity.

(3) $\phi > 1$. Now given shocks become more influential as time goes on, since if $\phi > 1$, $\phi^3 > \phi^2 > \phi$, etc. This is the *explosive case* which, for the reasons listed above, will not be considered as a plausible description of the data.

Going back to the two characterisations of non-stationarity, the random walk with drift

$$y_t = \mu + y_{t-1} + u_t \tag{8.13}$$