

Q1:

- a) $O(n \log n)$
- b) $O(n^3)$
- c) $O(n^2)$
- d) $O((\log n)^2)$

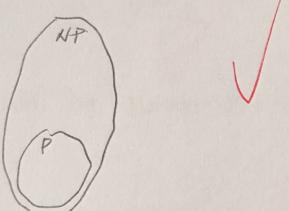
Q2:

Use external merge sort.

Chop the input file into smaller pieces, then call merge sort on each piece, then merge all pieces into one.

What is the question of $P = NP$? Describe under three sentences. (worth 5 points)

Class P is any question that can be solved by not exponential time and Class NP is that the question can only solved by exponential time. $P=NP$ is trying to figure out if all questions could be solved by non-exponential time.



b) Is Towers of Hanoi in NP-Complete? Explain in under three sentences. (worth 5 points)

Yes, Towers of Hanoi is in NP-Complete (Hard).

We can not turn the solution method of Towers of Hanoi into class P. It has to be exponential. (At least we're not finding any method to solve it in P).

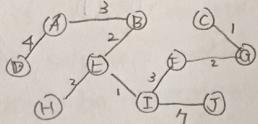
W~~DOWN~~ IN THE FIRST LIST OF EDGES
considered by the algorithm in the order they were considered. Assume that the algorithm terminates as soon as the MST has been found.

In the lists, use (x,y) to indicate an edge connecting vertices x and y .

a) Edges that form part of the MST, in order considered:

$$\Rightarrow (E, I), (C, G), (E, H), (E, B),$$
$$(F, G), (A, B), (I, F), (A, D),$$

(I, J)



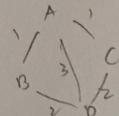
- ① ~~(E, I), (C, G)~~
- ② ~~(E, H), (E, B), (F, G)~~
- ③ ~~(A, B), (I, F), (B, F)~~
- ④ ~~(A, D), (A, E), (H, I)~~
- ⑤ ~~(D, E)~~
- ⑥ ~~(D, H), (C, F)~~
- ⑦ ~~(I, J)~~
- ⑧ ~~(J, G)~~
- ⑨
- ⑩ ~~(B, C)~~
- ⑪ ~~(E, F)~~

b) Other edges considered, but not included in the MST, in order considered:

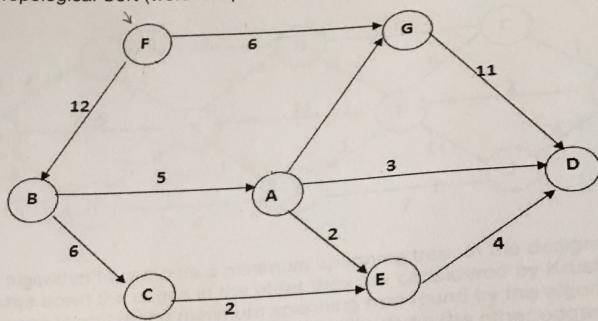
~~$(B, F), (A, E), (H, I), (D, E), (D, H), (C, F)$~~

c) Is the MST you found unique? Why?

No, Kruskal's algorithm will not guarantee uniqueness if the weight of the edge is not unique.



Question 4: Topological Sort (worth 10 points)



a) Starting from node F, what is a topological sort? (worth 6 points)

A	B	C	D	E	F	G
1	1	1	3	2	0	2
1	0	1	3	2	x	1
0	x	0	3	2	x	1
x	x	0	2	1	x	0
x	x	x	2	0	x	0
2	v	x	0	v	x	x

b) Is the sort you found unique? Why? (worth 4 points)

No, it is not unique because topological sort will not produce unique answer due to only consider the in-degree of vertex in graph.

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vertices. Assume no cycles. Please use pseudocode. You can use Java

```
Void topoSrt (Vertex root) {  
    if (root.inDegZero()) {  
        remFromList (root);  
        print (root);  
    }  
    else {  
        topoSrt (root.next);  
    }  
}
```

```
Void remFromList (Vertex v) {
```

```
    check = v.next  
    while (check != v) {  
        vertex traverse = check.adjlist.  
        while (traverse != null) {  
            if (traverse == v)  
                delete (traverse);  
            break;  
            traverse = traverse.next;  
        }  
    }  
}
```

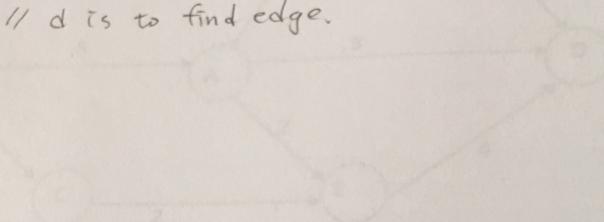
```
bool inDegZero (Vertex v) {
```

Except if statement

```
    if (traverse == v)  
        return false;
```

- d) Provide an O (Big-Oh) bound on the time to check whether two vertices are adjacent using the adjacency list representation. (worth 2 points)

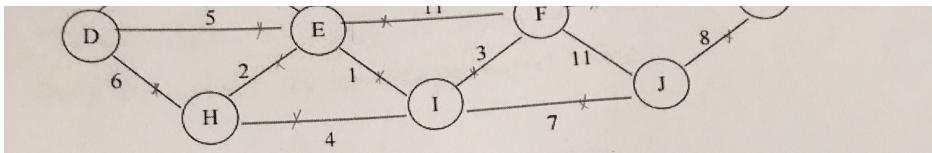
$O(V \cdot d)$ // d is out degree of the source.
// $|E|$ is to get to the beginning vertex
// d is to find edge.



Q3. In what order will a topological sort? (worth 6 points)

Which of these two representations was emphasized as being much more common in representing real world problems, and why? (worth 2 points)

The adjacent list is more commonly used because in real world problems the number of vertices is usually far greater than the number of edges. (sparse). Thus, the adjacent list would be a better solution.



Use Kruskal's algorithm to compute a minimum spanning tree. In the designated boxes below, write down the edges in the order they are considered by Kruskal's algorithm. If the edge is part of the minimum spanning tree found by the algorithm, write it in the first list of edges that form the MST. Write down the other edges considered by the algorithm in the order they were considered. Assume that the algorithm terminates as soon as the MST has been found.

In lists, use (x,y) to indicate an edge connecting vertices x and y .

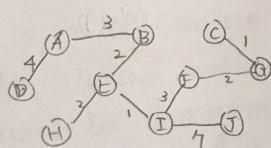
Edges that form part of the MST, in order considered:

$(E, I), (E, G), (E, H), (E, B),$

$(E, G), (A, B), (I, F), (A, D),$

(I, J)

✓



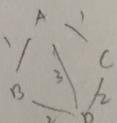
- ① $(E, I), (E, G)$
- ② $(E, H), (E, B), (F, G)$
- ③ $(A, B), (I, F), (B, F)$
- ④ $(A, D), (A, E), (H, I)$
- ⑤ (D, E)
- ⑥ $(D, H), (C, F)$
- ⑦ (I, J)
- ⑧ (J, G)
- ⑨ (B, C)
- ⑩ (E, F)

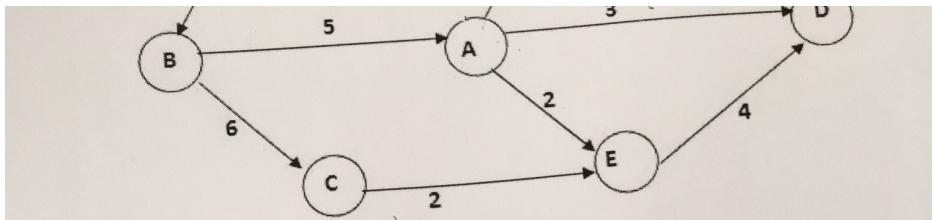
Other edges considered, but not included in the MST, in order considered:

$(B, F), (A, E), (H, I), (D, E), (D, H), (C, F)$

Is the MST you found unique? Why?

No, Kruskal's algorithm will not guarantee uniqueness if the weight of the edge is not distinct.



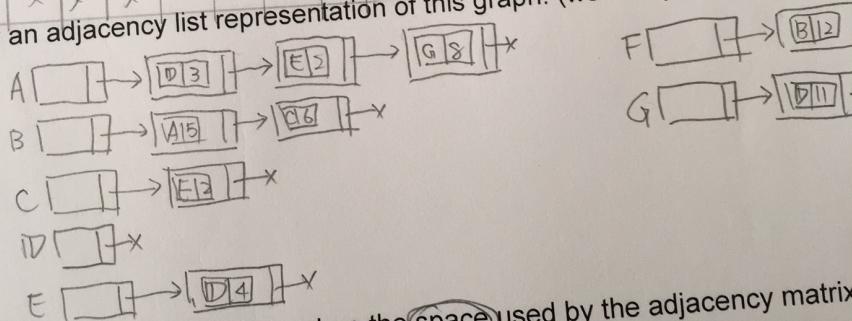


Draw an adjacency matrix representation of this graph. (worth 2 points)

	A	B	C	D	E	F	G
A	X	X	X	3	2	X	8
B	5	X	6	X	X	X	8
C	X	X	X	X	2	X	X
D	X	X	X	X	X	X	X
E	X	X	X	4	X	X	X
F	X	12	Y	X	X	X	6
G	X	Y	X	11	X	X	X

X = null.

Draw an adjacency list representation of this graph. (worth 2 points)



Provide an O (Big-Oh) bound on the space used by the adjacency matrix (worth 2 points)

$$O(|V|^2)$$

Final

show your work for any credit. For full credit give your answer as a number or a simplified fraction (not a formula). (worth 5 points)

$$S = \frac{1}{5}$$

$$\begin{array}{r} 0 \\ 5 \overline{) 0.8 } \\ 4 \overline{) 8 } \\ -8 \\ \hline 0 \\ 5 \overline{) 1.6 } \\ 5 \overline{) 6 } \\ -5 \\ \hline 1 \\ 5 \overline{) 3.0 } \\ 5 \overline{) 0 } \\ -0 \\ \hline 0 \end{array}$$

Question 1:

Question 2:

Question 3:

Question 4:

Question 5:

Question 6:

Question 7:

Question 8:

Question 9:

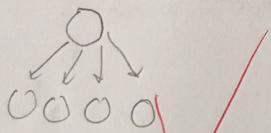
$$\frac{T_p}{T_p} = \frac{1}{S + \frac{(1-S)}{P}} = \frac{1}{0.2 + \frac{0.8}{5}} = \frac{1}{0.36} \approx 2.8 \times$$

$$\begin{array}{r} 2.8 \\ 0.36 \overline{) 10.0 } \\ 72 \\ \hline 28 \\ 28 \\ \hline 0 \end{array}$$

It will be almost 2.8 times faster.

b) What is the difference between parallelism and concurrency? Explain in under three sentences. (worth 5 points)

Parallelism is distributing a task into smaller tasks and handle those tasks with threads. (increase throughput).



Concurrency is about synchronizing the shared memory.
e.g. many chiefs but only fixed number of slaves.