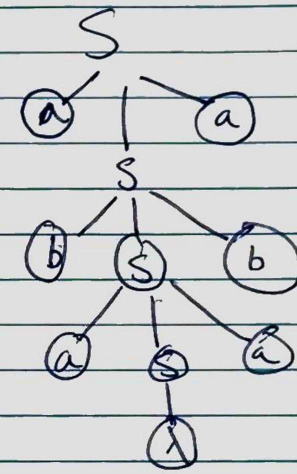
Talal Jawaid - Homework 2 - Professor Meiliu Lu

Thursday, March 14, 2019 2:36 PM

Ch 5 HW

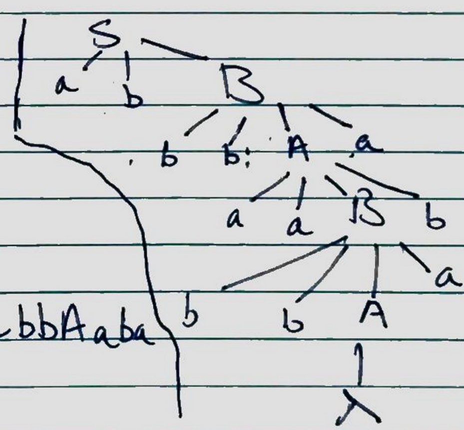
5.1 #2 Draw the derivation tree corresponding to the derivation in Ex. 5.1

$S \Rightarrow aSa$
$S \Rightarrow bSb$
$S \Rightarrow \lambda$



#3 Give a derivation tree for
$w = abbbaabbaba$
Use derivation tree to find a left most derivation.

$S \Rightarrow abB$
$A \rightarrow aaBb$
$B \rightarrow bbAa$
$A \rightarrow \lambda$



$S \Rightarrow abB \Rightarrow abbbAa$
$\Rightarrow abbbaaBba \Rightarrow abbbaabbAaba$
$\Rightarrow abbbaabbaba$

#7 Find CFGs for following languages (with $n \geq 0, m \geq 0$)

(a) $L = \{a^n b^m : n \leq m+3\}$

solve $n = m+3$ → add more b's

$$S \to aaaA$$
$$A \to aAb \mid B$$
$$B \to Bb \mid \lambda$$

$$S \to \lambda \mid aA \mid aaA$$

(d) $L = \{a^n b^m : 2n \leq m \leq 3n\}$

$$S \to aSbb \mid aSbbb \mid \lambda$$

(f) $L = \{w \in \{a, b\}^* : n_a(v) \geq n_b(v)$, where v is any prefix of $w\}$

$G = (\{S, A\}, \{a, b\}, S, P)$ w/production

abaabb $\quad S \xrightarrow{1} aSb \mid \overset{2}{S}S \mid \overset{3}{A}$

$\quad A \to \overset{4}{a}A \mid \overset{5}{\lambda}$

1

aaabb

$S \overset{1}{\Rightarrow} aSb \overset{1}{\Rightarrow} aaSbb \overset{3}{\Rightarrow} aaAbb \overset{4}{\Rightarrow} aaaAbb \overset{5}{\Rightarrow} aaabb$

Find CFG for following languages w/ $(n \geq 0, m \geq 0, K \geq 0)$

8. (a) $L = \{a^n b^m c^K : n = m \text{ or } m \leq K\}$

$n = m$ K is arbitrary

$S_1 \to AC$
$A \to aAb \mid \lambda$
$C \to Cc \mid \lambda$

or $S_2 \to BD,$
$B \to aB \mid \lambda,$
$D \to bDc \mid E,$
$E \to Ec \mid \lambda$

Then $S \to S_1 \mid S_2$

(b) $L = \{a^n b^m : n = m \text{ or } m \neq K\}$

Case $n = m$

$A \to aAb \mid \lambda$ $S_1 \to AC$
$C \to Cc \mid \lambda$

Case $m > K$

$S_2 \to BD$
$B \to aB \mid \lambda$
$D \to bDc \mid D_b \mid D_c$
$D_b \to bD_b \mid b$

Case $m < K$

$D_c \to cD_c \mid c$

(8) (d) $L = \{a^n b^m c^k : n + 2m = k\}$

$S \rightarrow aSc \mid A$

$A \rightarrow bbAc \mid A$

$S \Rightarrow aSc \Rightarrow aAc \Rightarrow ac$

$S \Rightarrow aSc \Rightarrow aAc \Rightarrow abbAcc \Rightarrow abbcc$

(h)

$L = \{a^n b^n c^k : k \geq 3\}$

$S \rightarrow ACccc$

$A \rightarrow aAb \mid \lambda$

$C \rightarrow cC \mid \lambda$

(1) $S \rightarrow ACccc$

$A \xrightarrow{(2)} aAb \mid \lambda$ (3)

$C \xrightarrow{(4)} cC \mid \lambda$ (5)

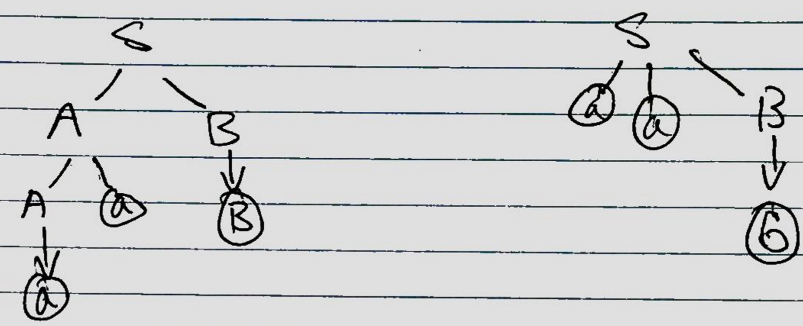
$S \xrightarrow{(1)} ACccc \xrightarrow{(2)} aAbCccc$

$\xrightarrow{(3)} abCccc \xrightarrow{(5)} abccc$

5.2 #6 Show that following grammar is ambiguous.

$$S \to AB \mid aaB,$$
$$A \to a \mid Aa,$$
$$B \to b.$$

$\omega = aab$ $\qquad\qquad$ $\omega = aab$



Same ω w/ 2 diff. deriv. trees.
$\therefore$ grammar must be ambiguous.

10. Give unambiguous grammar that generates the set of all regular expressions on $\Sigma = \{a, b\}$.

$\{\lambda, a, b, ab$
$\downarrow \ \downarrow \ \downarrow \ \downarrow$
SS

$(a+b)^*$

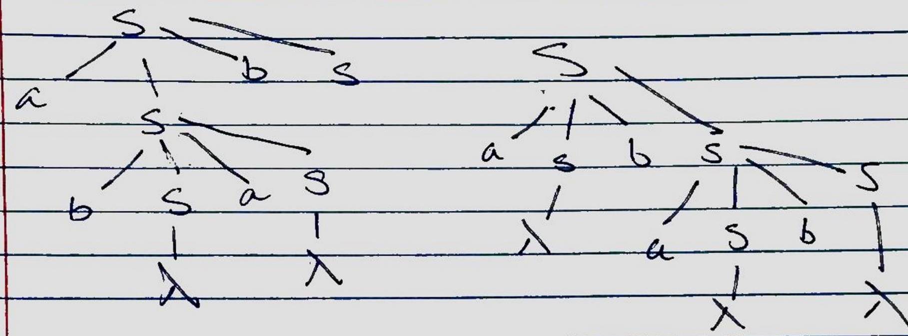
$S \to aS / bS / \lambda$

$S \to aS / bS / a / b$

$S \to$

#B Show that grammar is ambiguous

$S \to aSbS \mid bSaS \mid \lambda$



Ch 4

Section 4.3

#3 Show that the language $L = \{w : n_a(w) = n_b(w)\}$ is not regular. Is L^* regular

$w = a^m b^m \in L$

$|w| \geq m \qquad w = xyz \quad |xy| \leq m \quad |y| \geq 1$

$x = a^{m-r}, \; y = a^r, \; z = b^m, \; r \geq 1$

~~If~~ $i = 0 \qquad xy^0z = \cancel{am} \; a^{m-r} b^m \notin L$

$m - r \neq m$

not in language

$\therefore L$ is not regular

L^* is not regular since $L^* = L$

4 a). Prove that $L = \{a^n b^l a^k : k \geq n+1\}$ is not regular

$w = a^m b^m a^{2m}$

$y = a^k$

$w_i = a^{m + (i-1)k} b^m a^{2m}$

$i \geq 2,$

$m + (i-1)k > m$

$\boxed{w_i \notin L}$

4d. Prove L is not regular

$$L = \{a^n b^t : n \geq 1\}$$

$$w = a^m b^m$$

$$y = a^m$$

$$w_i = a^{m+(i-1)k} b^m$$

$i=2$

$$\underset{i=2}{w_2} = a^{m+k} b^m \notin L$$

<u>not in L, not Regular</u>

15. Consider the languages
Make a conjecture on whether it is regular or not

(a) $L = \{a^n b^l a^k : n+l+k > 5\}$

language is regular. when $l=0$, $k=0$, $n>5$ is still valid strings/reg exprsns.

(b) $L = a^n b^l a^k : n>5, l>3, k \leq l\}$

Language is not regular.
For ex.: ~~when~~ given $w = aaaaaab^m a^m$

if y is all a's, then $i=0$ violates $n>5$
if y is all b's, then $k \leq l$ is violated

(f) $L = \{a^n b^l : n \geq 100, l \leq 100\}$

Language is not regular
for ex: give ~~ea~~ $a^m b^m$, it fails when $i=0$

(g) $L = \{a^n b^l : |n-l| = 2\}$

Language is regular
$i=0$ $a^m b^{m+1}$
$a^k b^{k+1}$