M4

a)

$$\frac{d^3 y(t)}{dt^3} + 5\frac{d^2 y(t)}{dt^2} + 4\frac{dy(t)}{dt} = 2\delta(t)$$

$y(0) = 1$

$y^{(1)}(0) = -1$

$y^{(2)}(0) = 1$

$$\mathcal{L}\left\{\frac{d^3 y(t)}{dt^3}\right\} = s^3 Y(s) - s^2 y(0) - s y(0)^{(1)} - y(0)^{(2)} = s^3 Y(s) - s^2 + s - 1$$

$$5\mathcal{L}\left\{\frac{d^2 y(t)}{dt^2}\right\} = 5\left(s^2 Y(s) - s y(0) - y(0)^{(1)}\right) =$$

$$= 5s^2 Y(s) - 5s + 1$$

$$4\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = 4\left(sY(s) - y(0)\right) = 4sY(s) - 4$$

$$2\mathcal{L}\{\delta(t)\} = 2$$

$$s^3 Y(s) - s^2 + s - 1 + 5s^2 Y(s) - 5s + 1 + 4sY(s) - 4 = 2$$

$$s^3 Y(s) + 5s^2 Y(s) + 4sY(s) = s^2 + 4s + 2$$

$$Y(s)\left[s^3 + 5s^2 + 4s\right] = s^2 + 4s + 2$$

$$\varphi(s) = \frac{s^2+4s+2}{s^3+5s^2+4s}$$

$$\frac{B(s)}{A(s)} = \frac{num}{den} = \frac{r(1)}{s-p(1)} + \frac{r(2)}{s-p(2)} + \dots + \frac{r(n)}{s-p(n)} + k$$

matlab

num = [1 4 2]

den = [1 5 4 0]

[r, p, k] = residue(num, den)

$$\cancel{G(s) = \frac{0{,}1667}{r(1) \in M_1}} \quad r(2) \in M_2 \quad r(3) \in M_3$$

$$G(s) = \frac{M_1 e^{j\varphi_1}}{s-p(1)} + \frac{M_2 e^{j\varphi_2}}{s-p(2)} + \dots$$

$$G(s) = \frac{0{,}1667}{s+4} + \frac{0{,}3333}{s+1} + \frac{0{,}5}{s}$$

matlab

$M_n = abs(r(n))$

$\varphi = angle(r(n)) \cdot 180/pi$

$$G(s) = \frac{0{,}1667 e^{j0}}{s+4} + \frac{0{,}333 e^{j0}}{s+1} + \frac{0{,}5 e^{j0}}{s}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s-\alpha}\right\} = e^{\alpha t} \cdot 1(t)$$

$$\mathcal{L}^{-1}\{G(s)\} = 0{,}1667 e^{-4t} + 0{,}333 e^{-1t} + 0{,}5 e^{0} \cdot 1(t)$$

wykres w matlab

t=[0:0.1:10]

y=0,1667*exp(-4*t)+0,0333*exp(-1*t)+0.5

plot(t,y)

przerysować wykres.

b) $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 2e^{-t}$

$y(0) = 0$

$y^{(1)}(0) = -2$

$\mathcal{L}\left\{\frac{d^2y(t)}{dt}\right\} = s^2Y(s) - sy(0) - y^{(1)}(0) = s^2Y(s) + 2$

$5\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = 5(sY(s) - y(0)) = 5sY(s)$

$6y(t) = 6Y(s)$

$2\mathcal{L}\{e^{-t}\} = 2\frac{1}{s+1}$

$s^2Y(s) + \frac{2}{\cancel{sy(0)}} + 5sY(s) + 6Y(s) = \frac{2}{\cancel{2}s+1}$

$s^2Y(s) + 5sY(s) + 6Y(s) = \frac{2}{s+1} - 2$

$Y(s)(s^2 + 5s + 6) = \frac{2}{s+1} - \frac{2(s+1)}{(s+1)}$

$Y(s)(s^2 + 5s + 6) = \frac{2 - 2(s+1)}{s+1}$

$Y(s)(s^2 + 5s + 6) = \frac{2 - 2s - 2}{s+1}$

$Y(s) = \frac{-2s}{(s^2+5s+6)(s+1)}$

$$y(s) = \frac{3}{s+3} + \frac{4}{s+2} + \frac{1}{s+1}$$

$-4e^{j0}$

P =

-3
-2
-1

$$y(s) = \frac{3e^{j0}}{s+3} + \frac{4e^{j180}}{s+2} + \frac{1e^{j0}}{s+1}$$

$$\mathcal{L}^{-1}\{G(s)\} = 3e^{-3t} - 4e^{-2t} + e^{-t}$$

c)

$$\frac{dy(t)}{dt} + 2y(t) = 6\cos 2t$$

$$y(0) = 2$$

$$\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = sY(s) - y(0) = sY(s) - 2$$

$$2\mathcal{L}\{y(t)\} = 2Y(s)$$

$$6\mathcal{L}\{\cos 2t\} = \frac{6s}{s^2+4}$$

$$sY(s) - 2 + 2Y(s) = \frac{6s}{s^2+4}$$

$$sY(s) + 2Y(s) = \frac{6s + 2(s^2+4)}{s^2+4}$$

$$Y(s)(s+2) = \frac{6s + 2s^2 + 8}{s^2+4}$$

$$Y(s) = \frac{2s^2 + 6s + 8}{(s^2+4)(s+2)}$$

$$G(s)=\frac{0,5}{s+2}+\frac{0,75-0,75i}{s-(0+2i)}+\frac{0,75+0,75i}{s-(0-2i)}$$

$$\cancel{\mathcal{L}^{-1}\{G(s)\}=0,5e^{-2t}+}$$

$$G(s)=\frac{0,5e^{j0^\circ}}{s+2}+\frac{1.0607e^{j45^\circ}}{s-(0+2i)}+\frac{1.0607e^{j-45^\circ}}{s-(0-2i)}$$

$$G(s)=\frac{0,5e^{j0^\circ}}{s+2}+\frac{2,1213e^{j45^\circ}}{s-(0+2i)}+\frac{2,1213e^{j-45^\circ}}{s-(0-2i)}$$

$$G(s)=0,5e^{-2t}+2,1213e^{0}\cos(2t+45^\circ)$$

d) $$\frac{d^3y(t)}{dt^3}+4\frac{d^2y(t)}{dt}+3\frac{dy(t)}{dt}=3\delta(t)$$

$y(0)=1$

$y^{(1)}(0)=-1$

$y^{(2)}(0)=1$

$$\mathcal{L}\left\{\frac{d^3y(t)}{dt^3}\right\}=s^3Y(s)-y(0)s^2-y(0)^{(1)}s-y(0)^{(2)}=s^3Y(s)-s^2+s-1$$

$$4\mathcal{L}\left\{\frac{d^2y(t)}{dt}\right\}=4\left(s^2Y(s)-y(0)s-y(0)^{(1)}\right)=4s^2Y(s)-4s+4$$

$$3\mathcal{L}\left\{\frac{dy(t)}{dt}\right\}=3\left(sY(s)-y(0)\right)=3sY(s)-3$$

$$3\mathcal{L}\{\delta\}=3$$

$$s^3Y(s) - s^2 + s - 1 + 4s^2Y(s) - 4s + 4 + 3sY(s) - 3 = 3$$

$$s^3Y(s) + 4s^2Y(s) + 3sY(s) = 3 + s^2 - s + 1 + 4s - 4 + 3$$

$$Y(s)\left(s^3 + 4s^2 + 3s\right) = s^2 + 3s + 3$$

$$Y(s) = \frac{s^2 + 3s + 3}{s^3 + 4s^2 + 3s}$$

$$G(s) = \frac{0{,}5}{s+3} + \frac{0{,}5}{s+1} + \frac{1}{s}$$

$$G(s) = \frac{0{,}5e^{j0}}{s+3} + \frac{0{,}5e^{j180}}{s+1} + \frac{1e^{j0}}{s}$$

$$\mathcal{L}\{G(s)\} = 0{,}5e^{-3t} - 0{,}5e^{-1t} + 1\,1(t)$$

e) $\frac{dy(t)}{dt} + 2y(t) = \frac{1}{2}t^2$

$y(0) = -1$

$\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = Y(s)s - y(0) = Y(s)s + 1$

$2\mathcal{L}\{y(t)\} = 2Y(s)$

$\mathcal{L}\left\{\frac{1}{2}t^2\right\} = \frac{1}{s^3}$

$Y(s)s + 1 + 2Y(s) = \frac{1}{s^3}$

$Y(s)(s+2) = \frac{1}{s^3} - 1$

$Y(s)(s+2) = \frac{1-s^3}{s^3}$

$Y(s) = \frac{1-s^3}{s^3(s+2)} = \frac{1-s^3}{s^4+2s^3}$

$y(s) = \frac{1{,}125}{s+2} + \frac{0{,}125}{s} + \frac{0{,}25}{s^2} + \frac{0{,}5}{s^3}$

$G(s) = \cancel{1{,}25e^{j\ldots}}\ \frac{1{,}125e^{j180}}{s+2} + \frac{0{,}125e^{j0}}{s} + \frac{0{,}25e^{j180}}{s^2} + \frac{0{,}5e^{j0}}{s^3}$

$G(s) = \frac{-1{,}125e^{j0}}{s+2} + \frac{0{,}125e^{j0}}{s} + \frac{-0{,}25e^{0}}{s^2} + \frac{0{,}5e^{j0}}{s^3}$

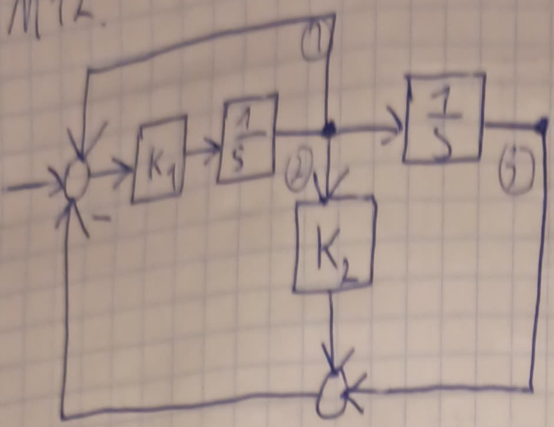
$$\mathcal{L}^{-1}\{G(s)\} = -1{,}125e^{-2t} + 0{,}125e \cdot 1(t) - 0{,}$$

$$+ \cancel{0{,}125}\ \frac{1}{2} \cdot 0{,}5\, t^2 = y(t)$$

$\omega_r = 3$ rad/s
$BW = 6$ rad/s

Z 5

M12.



$P_1 = k_1 \frac{1}{s^2}$ $L_1 = 0$ $L_2 = 0$ $L_3 = 0$ $\Delta = 1$

$L_1 = k_1 \frac{1}{s}$

$L_2 = -k_1 k_2 \frac{1}{s}$

$L_3 = -k_1 \frac{1}{s^2}$

T(s)

$$W(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$W = \frac{k_1 \frac{1}{s^2}}{1 - k_1 \frac{1}{s} + k_1 k_2 \frac{1}{s} + k_1 \frac{1}{s^2}} \Big/ \cdot \frac{s^2}{s^2}$$

$$W = \frac{k_1}{s^2 - k_1 s + k_1 k_2 s + k_1} = \frac{k_1}{s^2 - s(k_1 + k_1 k_2) + k_1}$$

$2\zeta\omega_n = k_1 k_2 - k_1$

$\omega_n^2 = k_1$

$$\begin{cases} \omega_r = \omega_n \sqrt{1 - 2\zeta^2} \text{ dla } 0 < \zeta < 0{,}707 \\ BW = \omega_n \sqrt{(1 - 2\zeta^2) + \sqrt{(4\zeta^4 - 4\zeta^2 + 2}} \end{cases}$$

$$\begin{cases} \omega_n = \frac{\omega_r}{\sqrt{1 - 2\zeta^2}} = \frac{3}{\sqrt{1 - 2\zeta^2}} \\ 6 = \frac{3}{\sqrt{1 - 2\zeta^2}} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} \ / : 3 \end{cases}$$

$$2 = \frac{1}{\sqrt{1 - 2\zeta^2}} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$$4 = \frac{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}{1-2\zeta^2} \quad / \cdot 1-2\zeta^2$$

$$4 - 8\zeta^2 = 1 - 2\zeta^2 + \sqrt{4\zeta^4 - 4\zeta^2 + 2}$$

$$3 - 6\zeta^2 = \sqrt{4\zeta^4 - 4\zeta^2 + 2}$$

$$9 - 36\zeta^2 + 36\zeta^4 = 4\zeta^4 - 4\zeta^2 + 2$$

$$32\zeta^4 - 32\zeta^2 + 7 = 0$$

$$\zeta = -0{,}8227,\ -0{,}5685,\ 0{,}8227,\ 0{,}5685$$

$$\zeta = 0{,}5685$$

$$\omega_n = \frac{3}{\sqrt{1-2(0{,}5685)^2}}$$

$$\omega_n = 5.0449$$

$$k_1 = 25{,}4514$$

$$2\zeta\omega_n = k_1 k_2 - k_1$$

$$2\zeta\omega_n + k_1 = k_1 k_2$$

$$k_2 = \frac{2\zeta\omega_n + k_1}{k_1} = 1{,}2254$$

22 №4

7) $\frac{dy(t)}{dt} + 3y(t) = 4\sin 4t$

$y(0) = 1$

$\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = Y(s)s - y(0) = Y(s)s - 1$

$3\mathcal{L}\{y(t)\} = 3Y(s)$

$4\mathcal{L}\{\sin 4t\} = 4\frac{4}{s^2+16} = \frac{16}{s^2+16}$

$Y(s)s - 1 + 3Y(s) = \frac{16}{s^2+16}$

$Y(s)s + 3Y(s) = \frac{16+s^2+16}{s^2+16}$

$Y(s)(s+3) = \frac{s^2+32}{s^2+16}$

$Y(s) = \frac{s^2+32}{(s^2+16)(s+3)}$

$G(s) = \frac{-32-24i}{s-(0+4i)} + \frac{-32+24i}{s-(0-4i)} + \frac{1{,}64}{s+3}$

$G(s) = \frac{0{,}4e^{j-143{,}13}}{s-(0+4i)} + \frac{0{,}4e^{j143{,}13}}{s-(0-4i)} + \frac{1{,}64}{s+3} =$

$= \frac{\frac{1}{2}\cdot 0{,}8e^{j-143{,}13}}{s-(0+4i)} + \frac{\frac{1}{2}\cdot 0{,}8e^{j143{,}13}}{s-(0-4i)} + \frac{1{,}64}{s+3}$

$\mathcal{L}^{-1}\{G(s)\} = 0{,}8e^{0}\cos(4t - 143{,}13) + 1{,}64e^{-3t}$

g) $$\frac{d^2 y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 5y(t) = 1(t)$$

$y(0) = -3$

$y^{(1)}(0) = 2$

$$\mathcal{L}\left\{\frac{d^2 y(t)}{dt^2}\right\} = Y(s)s^2 - sy(0) - y^{(1)}(0) =$$

$$= Y(s)s^2 + 3s - 2$$

$$\mathcal{L}\left\{4\frac{dy(t)}{dt}\right\} = 4(Y(s)s - y(0)) = 4Y(s)s + 12$$

$$\mathcal{L}\{5y(t)\} = 5Y(s)$$

$$\mathcal{L}\{1(t)\} = \frac{1}{s}$$

$$Y(s)s^2 + 3s - 2 + 4Y(s)s + 12 + 5Y(s) = \frac{1}{s}$$

$$Y(s)s^2 + 4Y(s)s + 5Y(s) = \frac{1}{s} - 3s + 2 - 12$$

$$Y(s)(s^2 + 4s + 5) = \frac{1}{s} - 3s - 10$$

$$Y(s)(s^2 + 4s + 5) = \frac{1}{s} - \frac{3s^2}{s} - \frac{10s}{s}$$

$$Y(s) = \frac{1 - 3s^2 - 10s}{s(s^2 + 4s + 5)} = \frac{-3s^2 - 10s + 1}{s^3 + 4s^2 + 5s}$$

$$G(s) = \frac{-1{,}6 + 2{,}2i}{s-(-2+1i)} + \frac{-1{,}6 - 2{,}2i}{s-(-2-1i)} + \frac{0{,}2}{s}$$

$$G(s) = \frac{2{,}7203e^{j-126{,}02}}{s-(-2+1i)} + \frac{2{,}7203e^{j-126{,}02}}{s-(-2+1i)} + \frac{0{,}2e^{j0}}{s}$$

$$G(s) = \frac{\frac{1}{2}\cdot 5{,}4406e^{j126{,}02}}{s-(-2+1i)} \quad e^{j-126{,}02} \ \text{-||-} \quad + \frac{0{,}2e^{j0}}{s}$$

$$\mathcal{L}^{-1}\{G(s)\} = 5{,}4406e^{-2t}\cos(1t + 126{,}02) + 0{,}2\cdot 1(t)$$

h) $\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 3e^{-3t}$

$y(0) = 0$

$y(0)^{(1)} = 1$

$\mathcal{L}\left\{\frac{d^2y(t)}{dt^2}\right\} = Y(s)s^2 - y(0)s - y0^{(1)} = Y(s)s^2 - 1$

$\mathcal{L}\left\{3\left\{\frac{dy(t)}{dt}\right\}\right\} = 3(Y(s)s - y(0)) = 3Y(s)s$

$\mathcal{L}\{2\{y(t)\}\} = 2Y(s)$

$\mathcal{L}\{3\{e^{-3t}\}\} = \frac{3}{s+3}$

$Y(s)s^2 - 1 + 3Y(s)s + 2Y(s) = \frac{3}{s+3}$

$Y(s)s^2 + 3Y(s)s + 2Y(s) = \frac{3}{s+3} + 1$

$Y(s)(s^2 + 3s + 2) = \frac{s+6}{s+3}$

$Y(s) = \frac{s+6}{(s+3)(s^2+3s+2)}$

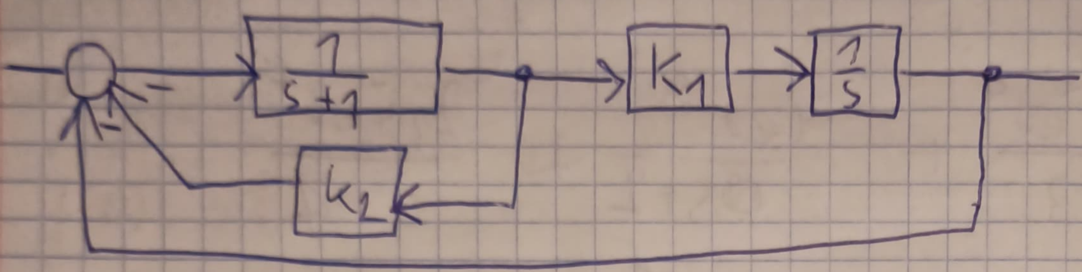
$$G(s) = \frac{1{,}5}{s+3} + \frac{-4}{s+2} + \frac{2{,}5}{s+1}$$

$$G(s) = \frac{1{,}5e^{j0}}{s+3} + \frac{4e^{j180}}{s+2} + \frac{2{,}5e^{j0}}{s+4}$$

$$G(s) = -\|- + \frac{-4e^{j0}}{s+2} + -\|-$$

$$\mathcal{L}^{-1}\{G(s)\} = 1{,}5e^{-3t} - 4e^{-2t} + 2{,}5e^{-4t}$$

N11.



$$P_1 = \frac{1}{s+1} k_1 \frac{1}{s} = k_1 \frac{1}{s^2+s} \qquad \Delta = 1$$

$$L_1 = -\frac{1}{s+1} k_2 = -k_2 \frac{1}{s+1}$$

$$L_2 = -\frac{1}{s+1} k_1 \frac{1}{s} = -k_1 \frac{1}{s^2+s}$$

$t_p = 1\ [s]$

$t_r = 2\ [s]$

$$W_{0} = \frac{k_1 \frac{1}{s^2+s}}{1 + \frac{1}{s+1} k_2 + k_1 \frac{1}{s^2+s}} = \frac{k_1 \frac{s^2}{s^2+s}}{s^2 + \frac{s^2}{s+1} k_2 + k_1 \frac{s^2}{s^2+s}}$$

$$\begin{cases} \omega_n^2 = k_1 \cancel{\frac{1}{s^2+s}}\ k_1 \cancel{\frac{s^2}{s^2+s}} \\ 2\zeta\omega_n = k_2 \cancel{\frac{s^2}{s+1}} \end{cases}$$

$$\begin{cases} t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \\ t_r = \frac{4{,}6}{\zeta\omega_n} \end{cases}$$

$$\begin{cases} 1 = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \\ 2 = \frac{4{,}6}{\zeta\omega_n} \end{cases}$$

$$2\omega_n = \frac{4{,}6}{\zeta}$$

$$\omega_n = \frac{4{,}6}{2\zeta}$$

$$\omega_n = \frac{2,3}{\zeta}$$

$$K_1 = 20,2113$$

$$K_2 = 4,6$$

$$1 = \frac{\pi}{\frac{2,3}{\zeta}\sqrt{1-\zeta^2}}$$

$$\frac{2,3}{\zeta}\sqrt{1-\zeta^2} = \pi$$

$$\frac{5,29}{\zeta^2}\cdot\left(1-\zeta^2\right) = \pi^2$$

$$\cancel{\frac{5,29}{\zeta^2} - \frac{\zeta^2 5,29}{\zeta^2} = \pi^2}$$

$$\frac{5,29-5,29\zeta^2}{\zeta^2} = \pi^2$$

$$5,29-5,29\zeta^2 = \zeta^2\pi^2$$

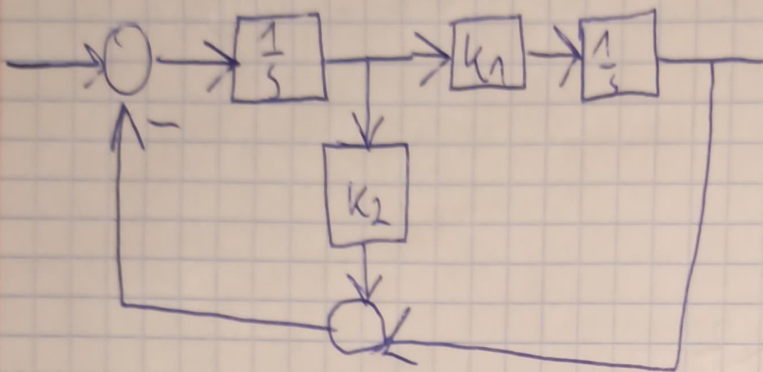
$$0 = \zeta^2\pi^2 + 5,29\zeta^2 - 5,29$$

$$\zeta = -1.0476 \quad \vee \quad 0,5116$$

$$\omega_n = \frac{2,3}{\zeta} = 4,4957$$

M10. $M_r = 3$

$BW = 5$



$P = k_1 \frac{1}{s^2}$ $\quad \Delta = 1$

$L_1 = -k_2 \frac{1}{s}$

$L_2 = -k_1 \frac{1}{s^2}$

$$W = \frac{k_1 \frac{1}{s^2}}{1 + k_2 \frac{1}{s} + k_1 \frac{1}{s^2}} = \frac{k_1}{s^2 + k_2 s + k_1}$$

$\omega_n^2 = k_1$

$2\zeta\omega_n = k_2$

$$\begin{cases} 3 = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}} \\ 5 = \omega_n \sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} \end{cases}$$

$$3(2y\sqrt{1-y^2}) = 1$$

$$\cancel{2y\sqrt{1-y^2} = \frac{1}{3}}$$

$$\cancel{6y(1-y^2) = 1}$$

$$4y^2(1-y^2) = \frac{1}{9}$$

$$\cancel{-6y^3 + 6y - 1 = 0}$$

$$4y^2 - 4y^4 = \frac{1}{9}$$

$$-4y^4 + 4y^2 - \frac{1}{9} = 0$$

$$2y(1-y^2) = \frac{1}{9}$$

$$2y - 2y^3 = \frac{1}{9}$$

$$-2y^3 + 2y - \frac{1}{9} = 0$$

$$3 = \frac{1}{2y\sqrt{1-y^2}}$$

$$3 \cdot (2y\sqrt{1-y^2}) = 1$$

$$3(-2y^3 + 2y) = 1$$

$$-6y^3 + 6y - 1 = 0$$

$M_r =$

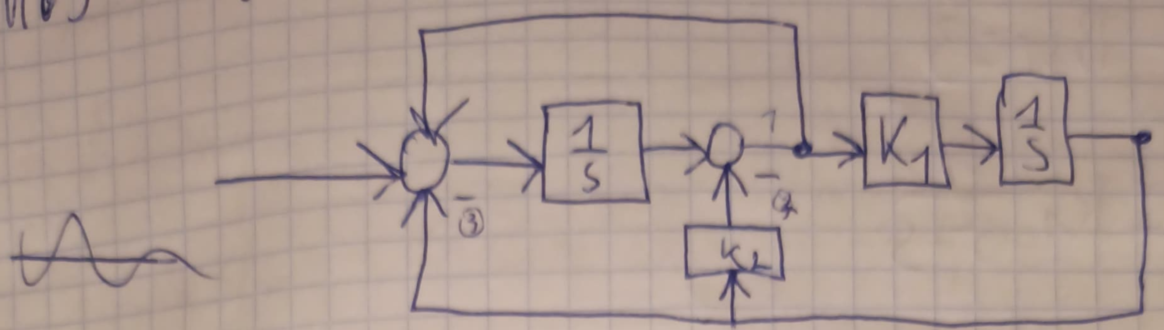
$$1{,}4125\left(2\zeta\sqrt{1-\zeta^2}\right)=1$$

$$2{,}8251\,\zeta\sqrt{1-\zeta^2}=1$$

$$7{,}981\,\zeta^2(1-\zeta^2)=1$$

$$7{,}981\,\zeta^2-7{,}981\,\zeta^4-1=0$$

$M_4 \quad t_0 = 1 \quad M_{rav} = 2 \approx 1{,}2589$



$\omega_n = 1 + 0{,}7\zeta$

$P = \frac{1}{s^2} k_1 = k_1 \frac{1}{s^2}$

$L_1 = \frac{1}{s}$

$L_2 = -k_1 \frac{1}{s} k_2 = -k_1 k_2 \frac{1}{s}$

$L_3 = -\frac{1}{s} k_1 \frac{1}{s} = -k_1 \frac{1}{s^2}$

$2\zeta + 1 = k_1 k_2$

$k_2 = \frac{2\zeta + 1}{k_1}$

$$W(s) = \frac{k_1 \frac{1}{s^2}}{1 - \frac{1}{s} + k_1 k_2 \frac{1}{s} + k_1 \frac{1}{s^2}} = \frac{k_1}{s^2 - s + k_1 k_2 s + k_1} = \frac{k_1}{s^2 + s(-1 + k_1 k_2) + k_1}$$

$\omega_n^2 = k_1$

$2\zeta\omega_n = -1 + k_1 k_2$

$$\begin{cases} 1 = \frac{1 + 0{,}7\zeta}{\omega_n} \\ 1{,}2589 = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \end{cases}$$

$\cancel{1{,}2589}$

$2{,}5179\,\zeta\sqrt{1-\zeta^2} = 1$

$6{,}3396\,\zeta^2(1-\zeta^2) = 1$

$6{,}3396\,\zeta^2 - 6{,}3396\,\zeta^4 - 1 = 0$

$\zeta = 0{,}4430$

№8. $M_r = 1,2 \quad = 1,1482$

$BW = 3$

$\cancel{\#}\ P = k_1 \frac{1}{s^2+s}$

$L_1 = -\frac{1}{s+1} k_2$

$L_2 = -k_1 \frac{1}{s^2+s}$

$$W(s) = \frac{k_1 \frac{1}{s^2+s \Rightarrow s(s+1)}}{1+\frac{1}{s+1}k_2 + k_1 \frac{1}{s^2+s}}$$

$$W(s) \frac{k_1}{s(s+1)+sk_2+k_1} = \frac{k_1}{s^2+s+k_2 s+k_1}$$

$w_n^2 = k_1$

$2\zeta w_n = 1+k_2$

$$1,1482 = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$5,273\,\zeta^2(1-\zeta^2) - 1 = 0$

$5,273\,\zeta^2 - 5,273\,\zeta^4 - 1 = 0$

$\zeta = 0{,}5043$

Rw1

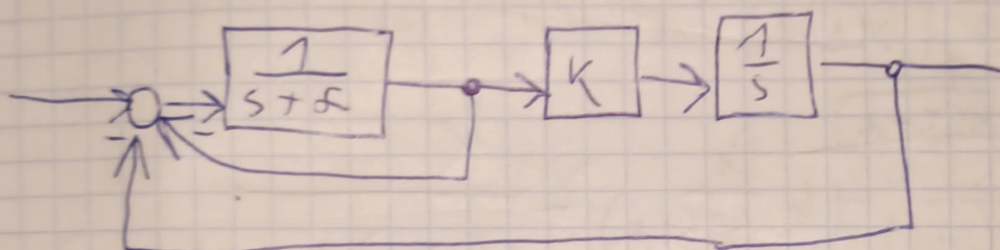
$3 = \omega_n \sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$

$\frac{3}{\omega_n} = \sqrt{\; - \; 11 \; -}$

$\sqrt{\quad}$ $\quad \omega_n \cdot 1{,}3$

M1.7 $\quad M_p = 14\%$

$t_0 = 0{,}6$



$P = \cancel{\;\;} \; k_1 \frac{1}{(s+\alpha)s}$

$L_1 = -\frac{1}{s+\alpha}$

$L_2 = -k_1 \frac{1}{(s+\alpha)s}$

$$W(s) = \frac{k_1 \frac{1}{(s+\alpha)s}}{1 + \frac{1}{s+\alpha} + k_1 \frac{1}{(s+\alpha)s}} = \frac{k_1}{(s+\alpha)s + s + k_1}$$

$s^2 + \alpha s$

$\omega_n^2 = k_1$

$2\zeta\omega_n = \alpha + 1$

$M_p = 14$

$t_0 = 0,6$

$0,6\,\omega_n = 1 + 0,2\,\zeta$

$\omega_n = \dfrac{1 + 0,2\,\zeta}{\omega_n}$

16.

$P = k\,\dfrac{1}{s(s+\alpha)}$

$L_1 = -k\,\dfrac{1}{s(s+\alpha)}$

$L_{11} = -\dfrac{s}{s(s+\alpha)}$

$$W(s) = \dfrac{k\,\dfrac{1}{s(s+\alpha)}}{1 + \dfrac{s}{s(s+\alpha)} + k\,\dfrac{1}{s(s+\alpha)}} = \dfrac{k}{s(s+\alpha) + s + k} =$$

$$= \dfrac{k}{s^2 + s\alpha + s + k}$$

f

$BW = 4$

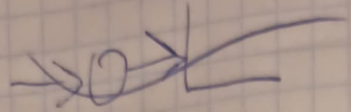
$\omega_n^2 = k$

$2\zeta\omega_n = \alpha + 1$

N5. $t_p = 1$

$t_r = 2$

$\zeta = 0,5907$



$$P = k\frac{1}{s(s+\alpha)}$$

$$L_1 = -\frac{\cancel{4}0,5s}{s(s+\alpha)}$$

$$L_2 = -k\frac{1}{s(s+\alpha)}$$

$$W(s) = \frac{k\frac{1}{s(s+\alpha)}}{1+\frac{0,5s}{s(s+\alpha)}+k\frac{1}{s(s+\alpha)}} = \frac{k}{s(s+\alpha)+0,5s+k} =$$

$$= \frac{k}{s^2+\alpha s+0,5s+k}$$

$$0 = 5,29 - 5,29\zeta^2 - \pi^2\zeta^2$$

$$0 = \zeta^2(-5,29-\pi^2) + 5,29$$

$\omega_n^2 = k$

$2\zeta\omega_n = \alpha + 0,5$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$1 = \frac{\pi}{\frac{2,3}{\zeta}\sqrt{1-\zeta^2}}$$

$$t_r = \frac{4,6}{\zeta\omega_n}$$

$$\pi = \frac{2,3}{\zeta}\sqrt{1-\zeta^2}$$

$$\pi^2 = \frac{5,29 - 5,29\zeta^2}{\zeta^2}$$

$$2 = \frac{4,6}{\zeta\omega_n}$$

$$\pi^2\zeta^2 = 5,29 - 5,29\zeta^2$$

$$\omega_n = \frac{4,6}{2\zeta} = \frac{2,3}{\zeta}$$

$$\cancel{0 = 5,29 - 52,2702\,\zeta^2}$$

M4

$R+r=4 \quad 2\% \quad \frac{4}{\sigma} \quad \sigma=\zeta w_n$

$w_r=1$

$P=k\frac{1}{s(s+\alpha)} \qquad W=\frac{k\frac{1}{s(s+\alpha)}}{1+\frac{1}{s+\alpha}+k\frac{1}{s(s+\alpha)}}=\frac{k}{s(s+\alpha)+s+k}$

$L_1=-\frac{1}{s+\alpha}$

$L_2=-k\frac{1}{s(s+\alpha)} \qquad =\frac{k}{s^2+s\alpha+s+k}$

$w_n^2=k$

$2\zeta w_n=\alpha+1$

$w_r=w_n\sqrt{1-2\zeta^2}$

$4=\frac{4}{\zeta w_n} \qquad 1=\frac{1}{\zeta}\sqrt{1-2\zeta^2}$

$1=\frac{1-2\zeta^2}{\zeta^2}$

$w_n=\frac{\cancel{4}\,1}{\zeta\,\cancel{4}}=\frac{1}{\zeta} \qquad \zeta^2=1-2\zeta^2$

$0=-3\zeta^2+1$

$M_3 Q$ $\quad M_r = 1{,}5 \Rightarrow 1{,}1885$

$w_r = 2$

$P = k\frac{1}{s^2}$ $\quad W(s) = \dfrac{k}{1s^2 + ks + k}$

$L_1 = -k\frac{1}{2s}$ $\quad w_n^2 = k_1$

$L_2 = -k\frac{1}{s^2}$ $\quad 2\zeta w_n = k_2$

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$1{,}1885 = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$\cancel{2{,}1}$ $\cancel{3{,}13}$ $2{,}377\,\zeta\sqrt{1-\zeta^2} = 1$

$$5{,}6502\zeta^2 - 5{,}6502\zeta^4 - 1 = 0$$

$M_2 \quad t_r = 1$

$w_r = 2$

$P = k_1 \frac{1}{s^2}$

$L_1 = k_2 \frac{1}{s}$

$L_2 = -2k_1 \frac{1}{s}$

$L_3 - k_1 \frac{1}{s^2}$

$$W(s) = \frac{k_1 \frac{1}{s^2}}{1 - k_2 \frac{1}{s} + 2k_1 \frac{1}{s} + k_1 \frac{1}{s^2}} = \frac{k_1}{s^2 - k_2 s + 2k_1 s + k_1}$$

$$= \frac{k_1}{s^2 + s(-k_2 + 2k_1 + k_1}$$

$w_n^2 = k_1$

$2\xi w_n = -k_2 + 2k_1 \Rightarrow k_2 = -2\xi w_n + 2k_1$

$$w_r = w_n\sqrt{1-2\zeta^2}$$

$$t_n = \frac{\pi}{w_n\sqrt{1-\zeta^2}}$$

$$2 = w_n\sqrt{1-2\zeta^2}$$

$$w_n = \frac{2}{\sqrt{1-2\zeta^2}}$$

$$1 = \frac{\pi}{\frac{2}{\sqrt{1-2\zeta^2}}\sqrt{1-\zeta^2}}$$

$$\frac{2}{\sqrt{1-2\zeta^2}}\cdot\sqrt{1-\zeta^2} = \pi$$

$$\frac{4(1-\zeta^2)}{1-2\zeta^2} = \pi^2$$

$$4-4\zeta^2 = \pi^2(1-2\zeta^2)$$

$$4-4\zeta^2 = \pi^2-\pi^2 2\zeta^2$$

$$-4\zeta^2+\pi^2 2\zeta^2-\pi^2+4 = 0$$

$$\zeta^2(-4+2\pi^2$$

$t_r = 1{,}8 \qquad M_p = 2{,}7 \Rightarrow 1{,}3646$

$$P = k_1 \frac{10}{s(s+2)}$$

$$L_1 = -k_2 s \frac{10}{s(s+2)}$$

$$L_2 = -k_1 \frac{10}{s(s+2)}$$

$$W(s) = \frac{k_1 \frac{10}{s(s+2)}}{1 + k_2 s \frac{10}{s(s+2)} + k_1 \frac{10}{s(s+2)}} =$$

$$= \frac{10k_1}{s(s+2) + 10k_2 s + 10k_1} =$$

$$= \frac{10k_1}{s^2 + 2s + 10k_2 s + 10k_1}$$

$$w_n^2 = 10k_1 \Rightarrow k_1 = \frac{w_n^2}{10}$$

$$2\zeta w_n = 2 + 10k_2 \Rightarrow k_2 = \frac{2\zeta w_n - 2}{10}$$

4) $G(s) = \dfrac{K(5s+7)}{s^4+5s^3+13s^2+14s+6}$

$$s^4+5s^3+13s^2+(14+5K)s+(6+7K)=0$$

s^4	1	13	$6+7K$
s^3	5	$14+5K$	0
s^2	$\frac{51-5K}{5}$	$6+7K$	
s^1			
s^0	$6+7K$		

$$-\frac{\begin{vmatrix} 1 & 13 \\ 5 & 14+5K \end{vmatrix}}{5} = \frac{65-14-5K}{5}$$

$$= \frac{51-5K}{5}$$

$$-\frac{\begin{vmatrix} 1 & 6+7K \\ 5 & 0 \end{vmatrix}}{5} =$$

$$= \frac{30+35K}{5} = 6+7K$$

$$-\frac{\begin{vmatrix} 5 & 14+5K \\ \frac{51-5K}{5} & 6+7K \end{vmatrix}}{\frac{51-5K}{5}} =$$

$$= \frac{-30-35K+\frac{714-70K+255K-25K^2}{5}}{\frac{51-5K}{5}}$$

z1 wzmocnione k, gdzie jest k

$$\frac{51-5k}{5} > 0 \Rightarrow k < 10,2$$

$$-30-25k + \frac{714-70k+255k-25k^2}{5} > 0$$

$$6+7k > 0 \Rightarrow k > -0,8571$$

~~-150 - 175k + 714 - 70k~~

$$-150-175k+714-20k+255k-25k^2 > 0$$

$$-25k^2+10k+564 > 0 \Rightarrow 4,9539$$

k^1

wielomian nowaczy

$$\frac{51-5k}{5} s^2 + 6 + 7k = 0$$

$\pm 2,78401$

$$T_{os} = \frac{2\pi}{\omega}$$

~~2 * pi / 2,784~~

$T_{os} = 2,2564$

dobrane nastawy
z tabel

$0{,}45 \cdot 4{,}9534$

$\frac{T_{os}}{1{,}2}$

$y(0) = 1$

$$\frac{y(2) - y(1)}{T} = -1$$

$$\frac{y(2) - 2y(1) + y(0)}{T^2} = 1$$

b) $\frac{d^2 y(t)}{\cancel{T^2} \; dt^2} + 5 \frac{dy(t)}{dt} + 6y(t) = 2e^{-t}$

$$\frac{\Delta^2 y(kT)}{T^2} + 5 \frac{\Delta y(kT)}{T} + 6\,y(kT) = 2e^{-kT}$$

$$\frac{y_{k+2} - 2y_{k-1} + y_k}{T^2} + 5 \frac{y_{k+1} - y_k}{T} + 6 y_k = 2e^{-kT}$$

$$y_{k+2} - 2y_{k-1} + y_k + 5T(y_{k+1} - y_k) + 6T^2 y_k =$$
$$= 2T^2 e^{-kT}$$

$$y_{k+2} + y_{k-1}(\overset{a_1}{-2 + 5T}) + y_k(\overset{a_2}{1 - 5T + 6T^2}) =$$
$$= 2T^2 e^{-kT}$$

$$y_{k+2} + -1{,}5\, y_{k-1} + 0{,}56\, y_k = 0{,}02 \cdot (0{,}9048)^k$$

$y(0) = 0$

$y^{(1)}(0) = -1$

$y^{(1)} \cancel{dx}$

$$\frac{dy(0)}{dt} = \frac{\Delta dy(k)}{T} = \frac{y(1) - y(0)}{T} = -1$$

$$\frac{y(1)}{T} = -1$$

$$y(1) = -1 \cdot 0{,}1 = -0{,}1$$

$$\frac{y_k - 2y_{k-1} + y_{k-2}}{T^2} + 5\,\frac{y_k - y_{k-1}}{T} + 6y(k) =$$
$$= 2e^{-kT}$$

$$y_k - 2y_{k-1} + y_{k-2} + 5T(y_k - y_{k-1}) + 6T^2 y_k =$$
$$= 2T^2 e^{-kT}$$

c) $\frac{dy(t)}{dt} + 2y(t) = 6\cos 2t$

$y(0) = -2$

$\frac{\Delta y(kT)}{T} + 2\Delta y\,kT = 6\cos 2kT$

$\frac{y_{k+1} - y_k}{T} + 2y_k = 6\cos 2kT$

$y_{k+1} - y_k + 2Ty_k = 6T\cos 2kT$

$\cancel{y_{k+1}(0-1)}$

$y_{k+1} + y_k(\underbrace{-1+2T}_{a_1}) = \underbrace{6T\cos 2kT}_{b}$

$\frac{y_k - y_{-1}}{T} + 2y_k = 6\cos 2kT$

$y_k - y_{-1} + 2Ty_k = 6T\cos 2kT$

$y_k(1+2T) - y_{-1} = 6T\cos 2kT$

d)

$$\frac{d^3y(t)}{dt^3} + 4\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} = 3\delta(t)$$

$y(0) = -1$

$y^{(1)}(0) = -1 \rightarrow \frac{y(1) - y(0)}{T} = -1 \; ; \; y(1) = -1 \cdot T + y(0)$

$y^{(2)}(0) = 1 \rightarrow \frac{y(2) - 2y(1) + y(0)}{T^2} = 1$

$$\frac{\Delta^3 y_{kT}}{T^3} + 4\frac{\Delta^2 y_{kT}}{T^2} + 3\frac{\Delta y_{kT}}{T} = 3\delta(kT)$$

?? zle odnosz?

$$\frac{y_{k+3} - 3y_{k+2} + 3y_{k+1} - y_k}{T^3} + 4\frac{y_{k+2} - 2y_{k+1} + y_k}{T^2} + 3\frac{y_{k+1} - y_k}{T} =$$

$$= 3\frac{1}{T}\delta(kT)$$

$$y_{k+3} - 3y_{k+2} + 3y_{k+1} - y_k + 4T(y_{k+2} - 2y_{k+1} + y_k) + 3T^2(y_{k+1} - y_k) =$$

$$= 3T^2\delta(kT)$$

$$y_{k+3} + y_{k+2}(-3 + 4T) + y_{k+1}(3 - 8T + 3T^2) + y_k(-1 + 4T - 3T^2)$$

$$= 3T^2\delta(kT)$$

$$\frac{y_k - 3y_{k-1} + 3y_{k-2} - y_{k-3}}{T^3}$$

$$+4\frac{y_k - 2y_{k-1} + y_{k-2}}{T^2} + 3\frac{y_k - y_{k-1}}{T} =$$

$$= 3\frac{1}{T}\delta(k)$$

$$y_k(1 + 4T + 3T^2) + y_{k-1}(-3 - 8T - 3T^2) +$$

$$+ y_{k-2} = 3T^2\delta(k)$$

e) $\frac{dy(t)}{dt} + 2y(t) = \frac{1}{2}t^2$

$y(0) = -1$

$\frac{\Delta y(kT)}{T} + 2y(kT) = \frac{1}{2} \cancel{t} kT^2$

$\frac{y_{k+1} - y_k}{T} + 2y_k = \frac{1}{2}kT^2$

$y_{k+1} - y_k + 2Ty_k = \frac{1}{2}kT^3$

$y_{k+1} + y_k(-1 + 2T) = \frac{1}{2}kT^3$

$\frac{y_k - y_{k-1}}{T} + 2y_k = \frac{1}{2}kT^2$ $\qquad \frac{T}{2} kT^2$

$y_k(1 + 2T) - y_{k-1}\frac{\cancel{2-k}}{2} = \frac{\frac{1}{2}kT^3}{2}$

h) $\frac{d^2y(t)}{dt^2} + \frac{4\,dy(t)}{dt} + 5y(t) = 1(t)$

$y(0) = -3$

$y^{(1)}(0) = 2$

$\mathcal{L}\left\{\frac{d^2y(t)}{dt^2}\right\} = Y(s)s^2 - sy(0) - y^{(1)}(0) = \cancel{Y(s)s^2 + 3s - 2}$

$4\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = 4\left(Y(s)s - y(0)\right) = 4Y(s)s + 12$

$5\mathcal{L}\{y(t)\} = 5Y(s)$

$\mathcal{L}\{1(t)\} = \frac{1}{s}$

$Y(s)s^2 + 3s - 2 + 4Y(s)s + 12 + 5Y(s) = \frac{1}{s}$

$Y(s)(s^2 + 4s + 5) = \frac{1}{s} - 3s + 2 - 12$

$\cancel{Y(s) = \frac{-3s - 10}{s(s^2+4s+5)}}$

$Y(s)\cancel{(s^2+4s+5)} = \frac{-3s^2 - 10s + 1}{s(s^2+4s+5)}$

$$G(s) = \frac{-1{,}6 + 2{,}2i}{s - (-2 + 1i)} + \frac{-1{,}6 - 2{,}2i}{s - (-2 - 2i)} + \frac{0{,}2}{s}$$

$$G(s) = \frac{2{,}7203\, e^{i\,126{,}0274}}{s - (-2 + 2i)} + \frac{2{,}7203\, e^{-i\,126{,}0274}}{s - (-2 + 2i)} + \frac{0{,}2\, e^{i0}}{s}$$

$$\mathcal{L}^{-1}\{G(s)\} = 5{,}4406\, e^{-2t} \cos(2t + 126{,}0274) + 0{,}2 \cdot 1(t)$$

h) $\frac{d^2 y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 3e^{-3t}$

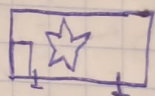
$y(0)=0 \quad y^{(1)}(0)=1$

$\mathcal{L}\{\frac{d^2}{dt^2}y\} = Y(s)s^2 - y(0)s - y^{(1)}(0) = Y(s)s^2 - 1$

$3\mathcal{L}\{\frac{d}{dt}y\} = 3(Y(s)s - y(0)) = 3Y(s)s$

$2\mathcal{L}\{y(t)\} = 2Y(s)$

$3\mathcal{L}\{e^{-3t}\} = \frac{3}{s+3}$



$Y(s)s^2 - 1 + 3Y(s)s + 2Y(s) = \frac{3}{s+3}$

$Y(s)(s^2+3s+2) = \frac{3}{s+3} + 1$

$Y(s) = \frac{3+(s+3)}{(s+3)(s^2+3s+2)}$

$$G(s) = \frac{1{,}5\ M1}{s+3} + \frac{-4\ M2}{s+2} + \frac{2{,}5\ M3}{s+1}$$

$-4e^{j180}$

$$G(s) = \frac{1{,}5e^{j0}}{s+3} + \frac{-4e^{j0}}{s+2} + \frac{2{,}5e^{j0}}{s+1}$$

$$\mathcal{L}^{-1}\{G(s)\} = 1{,}5e^{-3t} - 4e^{-2t} + 2{,}5^{-1t}$$

e) $$\frac{dy(t)}{dt} + 2y(t) = \frac{1}{2}t^2$$

$y(0) = -1$

$$\mathcal{L}\left\{\frac{dy(t)}{dt}\right\} = Y(s)s + 1$$

$$\mathcal{L}\{2y(t)\} = 2Y(s)$$

$$\frac{t^2}{2} = \frac{1}{s^3}$$

$$Y(s)s + 1 + 2Y(s) = \frac{1}{s^3}$$

$$Y(s)(s+2) = \frac{1}{s^3} - 1$$

$$Y(s) = \frac{1-s^3}{s^3(s+2)}$$

$$G(s) = \frac{-1,125}{s+2} + \frac{0,125}{s} + \frac{-0,25}{s^2} + \frac{0,5}{s^3}$$

$$G(s) = \frac{-1,125e^{j0}}{s+2} + \frac{0,125e^{j0}}{s} + \frac{-0,25e^{j0}}{s^2} + \frac{0,5e^{j0}}{s^3}$$

$$\mathcal{L}^{-1}\{G(s)\} = -1,125e^{-2t} + 0,125 \cdot 1(t) - 0,25 \cdot t + 0,025t^2$$

N3 $M_r = 1{,}5$

$\omega_r = 2$

$P = k_1 \frac{1}{s^2}$

$L_1 = -k_2 \frac{1}{s}$

$L_2 = -k_1 \frac{1}{s^2}$

$$W(s) = \frac{k_1 \frac{1}{s^2}}{1 + k_2 \frac{1}{s} + k_1 \frac{1}{s^2}} = \frac{k_1}{s^2 + k_2 s + k_1}$$

$\omega_n^2 = k_1$

$2\zeta\omega_n = k_2$

$\omega_r = \omega_n \sqrt{1 - 2\zeta^2}$

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$2 =$

$$1{,}1885 = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$k_1 = 7{,}4010$

$2{,}3770\,\zeta\sqrt{1-\zeta^2} = 1$

$k_2 = 2{,}6082$

$5{,}6502\,\zeta^2(1-\zeta^2) = 1$

$5{,}6502\,\zeta^2 - 5{,}6502\,\zeta^4 - 1 = 0$

$\zeta = 0{,}4794$

$\omega_n = 2{,}7206$

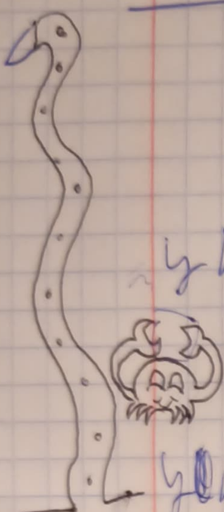
$y(0)=1$

$y^{(1)}(0)=-2$

1) $$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = 5\cdot 1(t)$$

$$\frac{\Delta^2 y(kT)}{T^2} + 2\frac{\Delta y(kT)}{T} + y(kT) = 5\, 1(kT)$$

$$\frac{y_{k+2} - 2y_{k+1} + y_k}{T^2} + 2\frac{y_{k+1} - y_k}{T} + y_k = 5\, k\cdot T$$



$$y_{k+2} - 2y_{k+1} + y_k + 2T(y_{k+1} - y_k) + T^2 y_k = 5k\cdot T^3$$

$$y_{k+2} + y_{k+1}(\overbrace{-2 + 2T}^{a_1}) + y_k(\overbrace{1 - 2T + T^2}^{a_2}) = \overbrace{5k\cdot T^3}^{b_1}$$

$$y_{k+2} - 1{,}8\, y_{k+1} + 0{,}81 y_k = 0{,}005\, k$$

$y(0)=1$

$y^{(1)}(0) \quad \frac{y(1) - y(0)}{T} = -2$

$y^{(1)}(0) = 0{,}8$

$$y_{k+2} = +1{,}8 y_{k+1} - 0{,}81 y_k + 0{,}005k$$