Maximum Impedance and Minimum Impedance

We know that,

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

Where, $V_0^+ e^{-\gamma z}$ is forward travelling and
$V_0^- e^{\gamma z}$ is reverse travelling.

Then, at point z = 0, forward travelling voltage, $V_0^+ e^{-\gamma z} = V_0^+$ and reverse travelling voltage, $V_0^- e^{\gamma z} = V_0^-$.

Due to load mismatch both waves may be:

i. Added
ii. Subtracted

So, at point z = 0, the maximum voltage : $V_{max} = V_0^+ + V_0^-$
the minimum voltage : $V_{min} = V_0^+ - V_0^-$

From definition, at any point, the voltage standing wave ratio:

$$\text{VSWR} = \frac{Maximum\,Voltage\ at\ that\ Point}{Minimum\,Voltage\ at\ that\ Point} = \frac{V_{max}}{V_{min}} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}$$

$$\therefore \text{VSWR} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}$$

$$= \frac{V_0^+\left(1 + \frac{V_0^-}{V_0^+}\right)}{V_0^+\left(1 - \frac{V_0^-}{V_0^+}\right)}$$

$$= \frac{1 + \frac{V_0^-}{V_0^+}}{1 - \frac{V_0^-}{V_0^+}}$$

$$= \frac{1+\rho}{1-\rho} \qquad \because \rho = \frac{V_0^-}{V_0^+}$$

Again, we know that, $Z_0 = \frac{V_{max}}{I_{max}} = \frac{V_{min}}{I_{min}}$

Now,

$$Z_{max} = \frac{V_{max}}{I_{min}} = \frac{V_{max}}{I_{min}} \times \frac{V_{min}}{V_{min}} = \frac{V_{max}}{V_{min}} \times \frac{V_{min}}{I_{min}} = VSWR \times Z_0 \quad \text{and,}$$

$$Z_{min} = \frac{V_{min}}{I_{max}} = \frac{V_{min}}{I_{max}} \times \frac{V_{max}}{V_{max}} = \frac{V_{min}}{V_{max}} \times \frac{V_{max}}{I_{max}} = \frac{1}{\frac{V_{max}}{V_{min}}} \times Z_0 = \frac{1}{VSWR} \times Z_0 = \frac{Z_0}{VSWR}$$

Problem 1: A lossless transmission line has $Z_0 = 100\,\Omega$, having maximum voltage of 3 V and minimum voltage of 1 V. It is terminated by unknown load.

Find: i) VSWR and

ii) Z_L

Solution:

Given:

Characteristic impedance: $Z_0 = 100\ \Omega$

Maximum Voltage: $V_{max} = 3$ V

Minimum Voltage: $V_{min} = 1\ V$

Let us assume:

Reflection Coefficient = ρ

We know:

$$\begin{aligned} \text{VSWR} &= \frac{V_{max}}{V_{min}} \\ &= \frac{3}{1} \\ &= 3 \end{aligned}$$

Also, we know:

$$\begin{aligned} \text{VSWR} &= \frac{1+\rho}{1-\rho} \\ \Rightarrow \quad \rho &= \frac{VSWR - 1}{VSWR + 1} \\ &= \frac{3-1}{3+1} \\ &= \frac{2}{4} \\ &= 0.5 \end{aligned}$$

Finally, we know:

$$\begin{aligned} \rho &= \frac{Z_L - Z_0}{Z_L + Z_0} \\ \Rightarrow \quad \rho(Z_L + Z_0) &= Z_L - Z_0 \\ \Rightarrow \quad 0.5(Z_L + 100) &= Z_L - 100 \\ \Rightarrow \quad Z_L - 100 &= 0.5 \times Z_L + 50 \\ \Rightarrow \quad Z_L - 0.5 \times Z_L &= 50 + 100 \\ \Rightarrow \quad 0.5 \times Z_L &= 150 \\ \therefore \quad Z_L &= 150 \end{aligned}$$

Answer: VSWR is 3 and Z_L is 150 Ω.

Problem 2: Find VSWR if characteristic impedance is of 100 Ω and receiving impedance is of $800 + j0\ \Omega$.

Solution:
Given:
Characteristic impedance: $Z_0 = 100\ \Omega$
Receiving impedance: $Z_L = 800 + j0\ \Omega = 800\ \Omega$
Let us assume:
Reflection Coefficient = ρ

We know:

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{800 - 100}{800 + 100} = \frac{700}{900} = 0.78$$

Again, we know:

$$\text{VSWR} = \frac{1 + \rho}{1 - \rho} = \frac{1 + 0.78}{1 - 0.78} = \frac{1 + 0.78}{1 - 0.78} = 8.09$$

Answer: VSWR is 8.09.

Problem 3: A 50 Ω coaxial cable feeds $100 + j20\ \Omega$ dipole antenna. Find:

i. Reflection Coefficient
ii. Standing Wave Ratio
iii. Maximum Impedance
iv. Minimum Impedance

Solution:
Given:
Characteristic impedance: $Z_0 = 50\ \Omega$
Load impedance: $Z_L = 100 + j20\ \Omega$

i. Reflection coefficient,

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j20\ - 50}{100 + j20\ + 50} = \frac{50 + j20}{150 + j20} = \frac{\sqrt{50^2 + 20^2}\angle tan^{-1}\frac{20}{50}}{\sqrt{150^2 + 20^2}\angle tan^{-1}\frac{20}{150}} = \frac{53.85\angle 21.8°}{151.33\angle 7.59°} = 0.35\angle 14.21°$$

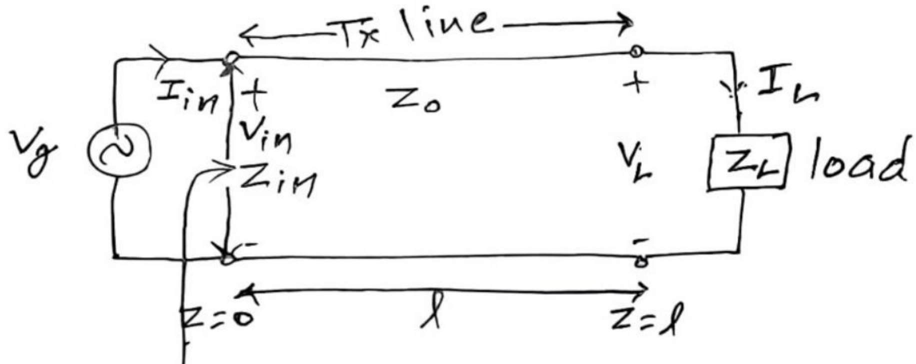
ii. Standing wave ratio, $S = \frac{1+\rho}{1-\rho} = \frac{1+0.35}{1-0.35} = \frac{1.35}{0.65} = 2.04$

iii. Maximum impedance, $Z_{max} = S \times Z_0 = 2.04 \times 50\ \Omega = 102\ \Omega$

iv. Minimum impedance, $Z_{min} = \frac{Z_0}{VSWR} = \frac{50}{2.04}\Omega = 24.51\Omega$

Answer: Reflection Coefficient is $0.35\angle 14.21°$, standing wave ratio is 2.04, maximum impedance is 102 Ω and minimum impedance is 24.51 Ω.

Input Impedance of Transmission Line



Input Impedance (Z_{in})

From the solution of transmission line equation, we know:

$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$

$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$

From the definition we can write:

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$$

So that, $I_0^+ = \frac{V_0^+}{Z_0}$ and $I_0^- = -\frac{V_0^-}{Z_0}$

At, distance $z = l$:

$V_L = V_s(z = l) = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l}$

$\therefore V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l} = V_L \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$

Again At, distance $z = l$:

$$I_L = I_s(z = l) = I_0^+ e^{-\gamma l} + I_0^- e^{\gamma l} = \frac{V_0^+}{Z_0} e^{-\gamma l} - \frac{V_0^-}{Z_0} e^{\gamma l}$$

$\therefore V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l} = I_L Z_0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$

$(1) + (2) \Rightarrow$

$2V_0^+ e^{-\gamma l} = V_L + I_L Z_0$

$\Rightarrow 2V_0^+ e^{-\gamma l} = I_L Z_L + I_L Z_0$

$$\therefore V_0^+ = \frac{(I_L Z_L + I_L Z_0)}{2} e^{\gamma l}$$

$(1) - (2) \Rightarrow$

$2V_0^- e^{\gamma l} = V_L - I_L Z_0$

$\Rightarrow 2V_0^- e^{\gamma l} = I_L Z_L - I_L Z_0$

$$\therefore V_0^- = \frac{(I_L Z_L + I_L Z_0)}{2} e^{-\gamma l}$$

At, distance $z = 0$:

$$V_{in} = V_s(z=0) = V_0^+ e^{-\gamma.0} + V_0^- e^{\gamma.0} = V_0^+ + V_0^-$$

$$I_{in} = I_s(z=0) = I_0^+ e^{-\gamma.0} + I_0^- e^{\gamma.0} = I_0^+ + I_0^- = \frac{V_0^+}{Z_0} + \left(-\frac{V_0^-}{Z_0}\right) = \frac{V_0^+ - V_0^-}{Z_0}$$

$$\therefore Z_{in} = \frac{V_{in}}{I_{in}} = \frac{V_0^+ + V_0^-}{\frac{V_0^+ - V_0^-}{Z_0}} = \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right).Z_0$$

Now, $Z_{in} = Z_0 \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right)$

$$= Z_0 \left[\frac{\left(\frac{I_L Z_L + I_L Z_0}{2}\right) e^{\gamma l} + \left(\frac{I_L Z_L - I_L Z_0}{2}\right) e^{-\gamma l}}{\left(\frac{I_L Z_L + I_L Z_0}{2}\right) e^{\gamma l} - \left(\frac{I_L Z_L - I_L Z_0}{2}\right) e^{-\gamma l}}\right]$$

$$= Z_0 \left[\frac{\left(\frac{Z_L + Z_0}{2}\right) e^{\gamma l} + \left(\frac{Z_L - Z_0}{2}\right) e^{-\gamma l}}{\left(\frac{Z_L + Z_0}{2}\right) e^{\gamma l} - \left(\frac{Z_L - Z_0}{2}\right) e^{-\gamma l}}\right]$$

$$= Z_0 \left[\frac{\frac{Z_L}{2} e^{\gamma l} + \frac{Z_0}{2} e^{\gamma l} + \frac{Z_L}{2} e^{-\gamma l} - \frac{Z_0}{2} e^{-\gamma l}}{\frac{Z_L}{2} e^{\gamma l} + \frac{Z_0}{2} e^{\gamma l} - \frac{Z_L}{2} e^{-\gamma l} + \frac{Z_0}{2} e^{-\gamma l}}\right]$$

$$= Z_0 \left[\frac{Z_L \left(\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right) + Z_0 \left(\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right)}{Z_0 \left(\frac{e^{\gamma l} + e^{-\gamma l}}{2}\right) + Z_L \left(\frac{e^{\gamma l} - e^{-\gamma l}}{2}\right)}\right]$$

$$= Z_0 \left[\frac{Z_L \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_L \sinh \gamma l}\right]$$

$$= Z_0 \left[\frac{\cosh \gamma l \left(Z_L + Z_0 \frac{\sinh \gamma l}{\cosh \gamma l}\right)}{\cosh \gamma l \left(Z_0 + Z_L \frac{\sinh \gamma l}{\cosh \gamma l}\right)}\right]$$

$$= Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}\right]$$

From Euler's formula
$e^{i\theta} = \cos\theta + i\sin\theta$ and
$e^{-i\theta} = \cos\theta - i\sin\theta$ we find:
$\frac{e^{i\theta}+e^{-i\theta}}{2} = \cos\theta$, $\frac{e^{i\theta}-e^{-i\theta}}{2i} = \sin\theta$
and $\sin i\theta = i \sinh\theta$

This equation is for any kind of transmission line. For lossless transmission line:

$\alpha = 0$ and $\gamma = \alpha + j\beta = 0 + j\beta = j\beta$

$$\therefore \tanh \gamma l = \frac{\frac{e^{\gamma l} - e^{-\gamma l}}{2}}{\frac{e^{\gamma l} + e^{-\gamma l}}{2}} = \frac{j\left(\frac{e^{j\beta l} - e^{-j\beta l}}{2j}\right)}{\frac{e^{j\beta l} + e^{-j\beta l}}{2}} = \frac{j \sin\beta l}{\cos\beta l} = j\tan\beta l$$

Then, for lossless transmission line the input impedance will be:

$$\therefore Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tan \text{ɦ}\gamma l}{Z_0 + Z_L \tan \text{ɦ}\gamma l}\right] = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right]$$

For different lengths ($\boldsymbol{l}$) of transmission line:

For, $l = \frac{\lambda}{8}$:

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$$

Then, $\tan \beta l = \tan \frac{\pi}{4} = 1$

$$\therefore Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 . 1}{Z_0 + jZ_L . 1}\right]$$

$$= Z_0 \left[\frac{Z_L + jZ_0}{Z_0 + jZ_L}\right]$$

$$|Z_{in}| = Z_0 \frac{\sqrt{Z_L^2 + Z_0^2}}{\sqrt{Z_0^2 + Z_L^2}}$$

$$= Z_0$$

For, $l = \frac{\lambda}{4}$ (It's called quarter wave transmission line):

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

Then, $\tan \beta l = \tan \frac{\pi}{2} = \infty$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right]$$

$$= Z_0 \left[\frac{\tan \beta l \left(\frac{Z_L}{\tan \beta l} + jZ_0\right)}{\tan \beta l \left(\frac{Z_0}{\tan \beta l} + jZ_L\right)}\right]$$

$$= Z_0 \left[\frac{0 + jZ_0}{0 + jZ_L}\right]$$

$$= Z_0 \frac{Z_0}{Z_L}$$

$$= \frac{{Z_0}^2}{Z_L}$$

Analyzing this equation $Z_{in} = \frac{{Z_0}^2}{Z_L}$ we find:

i. If the load is open circuit, then $Z_L = \infty$ and $Z_{in} = \frac{{Z_0}^2}{\infty} = 0$, means the source acts like short circuit.

ii. If the load is short circuit, then $Z_L = 0$ and $Z_{in} = \frac{{Z_0}^2}{0} = \infty$, means the source acts like open circuit.

iii. If the load is inductive then $Z_L = jx$ and $Z_{in} = \frac{{Z_0}^2}{jx} = \frac{j{Z_0}^2}{j.jx} = \frac{j{Z_0}^2}{j^2x} = -\frac{j{Z_0}^2}{x}$, means the source acts as capacitive.

iv. If the load is capacitive then $Z_L = -jx$ and $Z_{in} = \frac{{Z_0}^2}{-jx} = \frac{j{Z_0}^2}{j.-jx} = \frac{j{Z_0}^2}{-j^2x} = \frac{j{Z_0}^2}{x}$, means the source acts as inductive.

That's why the quarter wave transmission line is also called impedance inversion device.

For, $l = \frac{\lambda}{2}$: $\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$

$\tan\beta l = \tan\pi = 0$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] = Z_0 \left[\frac{Z_L + jZ_0.0}{Z_0 + jZ_L.0}\right] = Z_0 \times \frac{Z_L}{Z_0} = Z_L$$

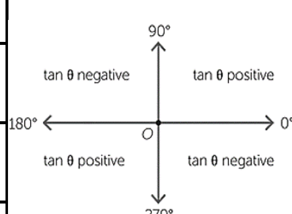
$\therefore Z_{in} = Z_L$ (Same result will be found for $l = \lambda$.

Analyzing the input impedance Z_{in} For different values of load impedance Z_L:

If, $Z_L = 0$ ($Load\ is\ short\ circuit$):

$$\begin{aligned} Z_{in} &= Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] \\ &= Z_0 \left[\frac{0 + jZ_0 \tan\beta l}{Z_0 + 0}\right] \\ &= jZ_0 \tan\beta l \end{aligned}$$

For different ranges of length (l) of transmission line when the load is short circuit ($Z_L = 0$)**:**

<table>
<tr><td>$0 < l < \frac{\lambda}{4}$</td><td>$0 < \beta l < \frac{\pi}{2}$</td><td rowspan="4">
</td><td>$\tan\beta l = +k$</td><td>$Z_{in} = jZ_0k$</td><td>Inductive</td></tr>
<tr><td>$\frac{\lambda}{4} < l < \frac{\lambda}{2}$</td><td>$\frac{\pi}{2} < \beta l < \pi$</td><td>$\tan\beta l = -k$</td><td>$Z_{in} = -jZ_0k$</td><td>Capacitive</td></tr>
<tr><td>$\frac{\lambda}{2} < l < \frac{3\lambda}{4}$</td><td>$\pi < \beta l < \frac{3\pi}{2}$</td><td>$\tan\beta l = +k$</td><td>$Z_{in} = jZ_0k$</td><td>Inductive</td></tr>
<tr><td>$\frac{3\lambda}{4} < l < \lambda$</td><td>$\frac{3\pi}{2} < \beta l < 2\pi$</td><td>$\tan\beta l = -k$</td><td>$Z_{in} = -jZ_0k$</td><td>Capacitive</td></tr>
</table>

If, $Z_L = \infty$ $(Load\ is\ open\ circuit)$:

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$$= Z_0\left[\frac{Z_L\left(1 + \frac{jZ_0}{Z_L}\tan\beta l\right)}{Z_L\left(\frac{Z_0}{Z_L} + j\tan\beta l\right)}\right]$$

$$= Z_0\left[\frac{1 + \frac{jZ_0}{\infty}\tan\beta l}{\frac{Z_0}{\infty} + j\tan\beta l}\right]$$

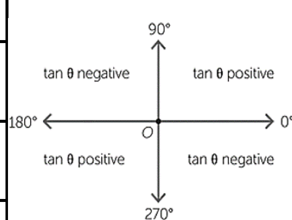
$$= Z_0\left[\frac{1+0}{0 + j\tan\beta l}\right]$$

$$= \frac{Z_0}{j\tan\beta l}$$

$$= \frac{jZ_0}{j^2\tan\beta l}$$

$$= -jZ_0\left(\frac{1}{\tan\beta l}\right)$$

For different ranges of length (l) of transmission line when the load is open circuit ($Z_L = \infty$)**:**

$0 < l < \frac{\lambda}{4}$	$0 < \beta l < \frac{\pi}{2}$		$\tan\beta l = +k$	$Z_{in} = -jZ_0\frac{1}{k}$	Capacitive
$\frac{\lambda}{4} < l < \frac{\lambda}{2}$	$\frac{\pi}{2} < \beta l < \pi$		$\tan\beta l = -k$	$Z_{in} = jZ_0\frac{1}{k}$	Inductive
$\frac{\lambda}{2} < l < \frac{3\lambda}{4}$	$\pi < \beta l < \frac{3\pi}{2}$		$\tan\beta l = +k$	$Z_{in} = -jZ_0\frac{1}{k}$	Capacitive
$\frac{3\lambda}{4} < l < \lambda$	$\frac{3\pi}{2} < \beta l < 2\pi$		$\tan\beta l = -k$	$Z_{in} = jZ_0\frac{1}{k}$	Inductive

If, $Z_L = Z_0$ $(Impedance\ matching)$:

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$$= Z_0\left[\frac{Z_0 + jZ_0 \tan\beta l}{Z_0 + jZ_0 \tan\beta l}\right]$$

$$= Z_0$$

Here, the reflection coefficient $\rho = 0$ and voltage standing wave ratio VSWR will be absent.

Problem 1: A telephone line has the parameters $R = 60\frac{\Omega}{\mathrm{m}}$, $\mathrm{G} = 600\frac{\mu\mathrm{S}}{\mathrm{m}}$, $\mathrm{L} = 0.3\frac{\mu\mathrm{H}}{\mathrm{m}}$ and $C = 0.75\frac{nF}{m}$. If the line operates at 10 MHz, then find Z_0, V_p and the length where the voltage drops by 30 dB in the line.

Answer: $Z_0 = 29.55 - j21.41, V_p = 45.54 \times 10^6\ ms^{-1} and\ 3.36\ m$.

Problem 2: A distortionless line operating at 120 MHz has the parameters $R = 18\frac{\Omega}{\mathrm{m}}$, $\mathrm{L} = 0.9\frac{\mu\mathrm{H}}{\mathrm{m}}$ and $C = 21\frac{pF}{m}$. Find the length of line where 20% of voltage drop happens, propagation constant and length at 45° phase shift.

Answer: $18.5\ \mathrm{m}, 0.0864 + \mathrm{j}3.218\ and\ 0.234\ m$.

Problem 3: A lossless transmission line operating at 4.5 GHz has $\mathrm{L} = 2.6\frac{\mu\mathrm{H}}{\mathrm{m}}$ and $Z_0 = 80\ \Omega$. Find the phase constant and phase velocity.

Answer: 918.167 rad/m and $30.778\times10^6\ \mathrm{ms}^{-1}$.

Microwave Components:

Microwave components do the following functions:

- Sample fixed amounts of power
- Split the wave into paths
- Control the direction of the wave
- Transmit or absorb fixed frequencies
- Transmit power in one direction
- Terminate the wave
- Shift the phase of the wave
- Switch power
- Reduce power
- Detect and mix waves

The waveguide components generally encountered are:

- Directional couplers
- Impedance changing devices
- Waveguide terminating devices
- Slotted sections
- Ferrite devices
- Isolator switches
- Mixers
- Tee junctions
- Attenuators
- Circulators
- Cavities
- Wavemeters
- Filters
- Detectors

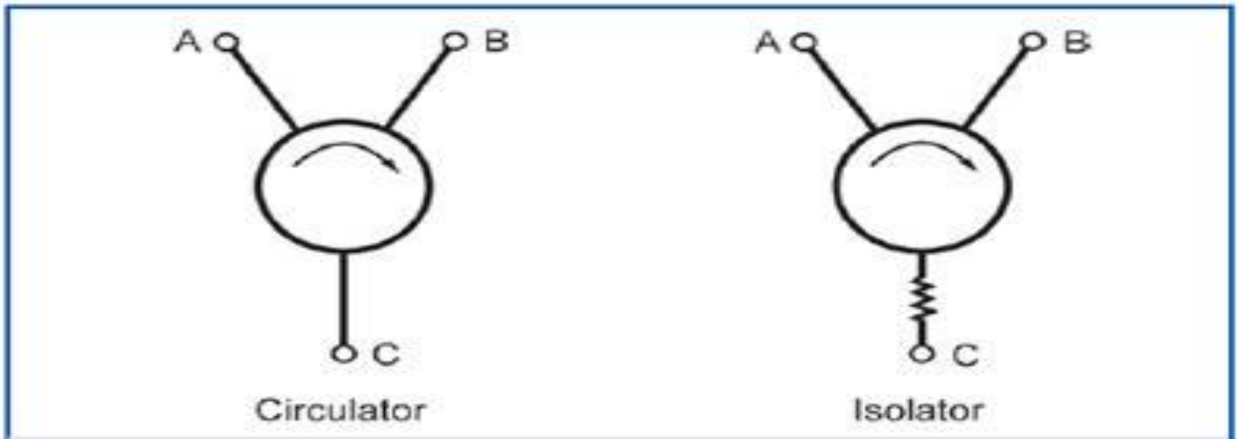


Figure 1

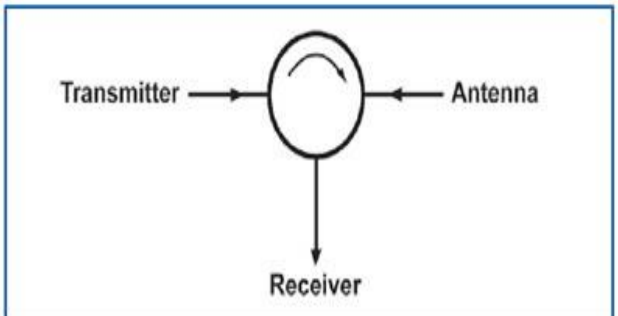


Figure 2: Duplexor

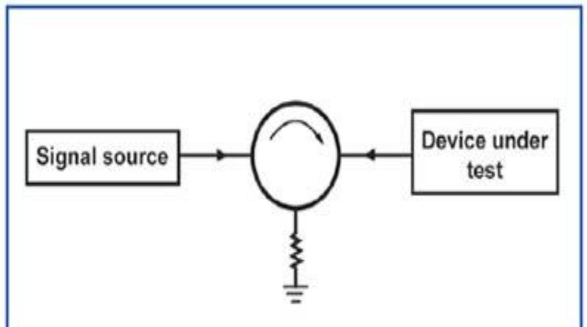


Figure 3

Circulator

An RF circulator is a three-port ferromagnetic passive device used to control the direction of signal flow in a circuit and is a very effective, low-cost alternative to expensive cavity duplexers.

The interaction of the magnetic field to the ferrite material inside circulators creates magnetic fields similar to the water flow in the cup. The rotary field is very strong and will cause any RF/microwave signals in the frequency band of interest at one port to follow the magnetic flow to the adjacent port and not in the opposite direction. If there is a mismatch at port B, the reflected signal from port B will be directed to port C.

A common application for a circulator is as an inexpensive duplexer (a transmitter and receiver sharing one antenna). When the transmitter sends a signal, the output goes directly to the antenna port and is isolated from the receiver. Good isolation is key to ensure that a high-power transmitter output signal does not get back the receiver front end.

Isolator

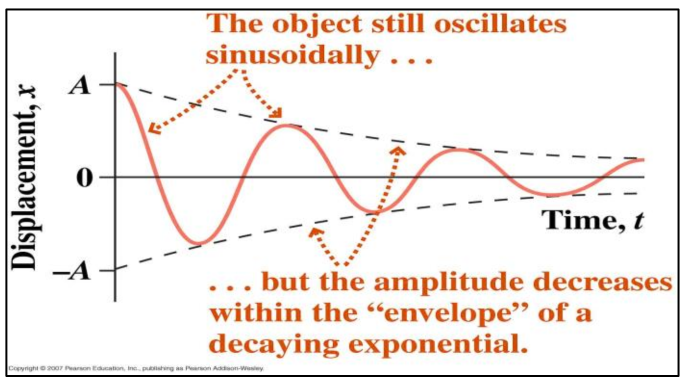
An RF isolator is a two-port ferromagnetic passive device which is used to protect other RF components from excessive signal reflection. Isolators are common place in laboratory applications to separate a device under test (DUT) from sensitive signal sources.

An isolator is a circulator with the third port terminated. The arrows represent the direction of the magnetic fields and the signal when applied to any port of these devices. Example: If a signal is placed at port A, and port B is well matched, the signal will exit at port B with very little loss (typically 0.4dB).

The isolator is placed in the measurement path of a test bench between a signal source and the device under test (DUT) so that any reflections caused by any mismatches will end up at the termination of the isolator and not back into the signal source.

Damping

Damping is an influence within or upon an oscillatory system that has the effect of reducing, restricting or preventing its oscillations. In physical systems, damping is produced by processes that dissipate the energy stored in the oscillation. The damping ratio is a dimensionless measure describing how oscillations in a system decay after a disturbance.

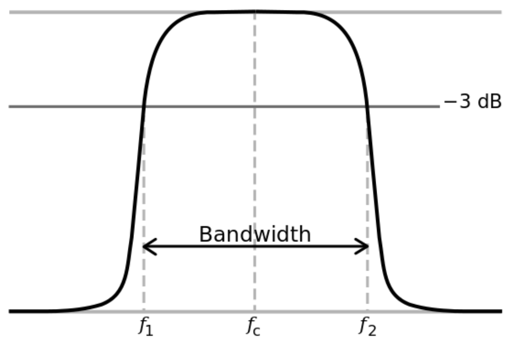


There are some damping cases in oscillation:

- **Undamped:** Where the spring–mass system is completely lossless, the mass would oscillate indefinitely.
- **Overdamped:** If the system contained high losses, for example if the spring–mass experiment was conducted in a viscous fluid, the mass could slowly return to its rest position without ever overshooting.
- **Underdamped:** Commonly, the mass tends to overshoot its starting position, and then return, overshooting again. With each overshoot, some energy in the system is dissipated, and the oscillations die towards zero.

Between the overdamped and underdamped cases, there exists a certain level of damping at which the system will just fail to overshoot and will not make a single oscillation. This case is called *critical damping*. The key difference between critical damping and overdamping is that, in critical damping, the system returns to equilibrium in the minimum amount of time.

The quality factor or Q factor is a dimensionless parameter that describes how underdamped an oscillator or resonator is and characterizes a resonator's bandwidth relative to its center frequency.



The bandwidth $\Delta f = f_2 - f_1$ of a damped oscillator is shown on a graph of energy versus frequency. The Q factor of the damped oscillator, or filter, is $f_c/\Delta f$. The higher the Q, the narrower and 'sharper' the peak is.

- A system with low quality factor ($Q < \frac{1}{2}$) is said to be overdamped.
- A system with **high quality factor** ($Q > \frac{1}{2}$) is said to be underdamped.
- A system with an intermediate quality factor ($Q = \frac{1}{2}$) is said to be critically damped.

Resonator

A resonator is a device or system that exhibits resonance or resonant behavior, that is, it naturally oscillates at some frequencies, called its resonant frequencies, with greater amplitude than at others. The oscillations in a resonator can be either electromagnetic or mechanical (including acoustic).

Cavity Resonator

A **microwave cavity** or *radio frequency (RF) cavity* is a special type of resonator, consisting of a closed (or largely closed) metal structure that confines electromagnetic fields in the microwave region of the spectrum.

- The structure is either hollow or filled with dielectric material.
- The microwaves bounce back and forth between the walls of the cavity. At the cavity's resonant frequencies, they reinforce to form standing waves in the cavity.
- Therefore, the cavity functions similarly to an organ pipe or sound box in a musical instrument, oscillating preferentially at a series of frequencies, its resonant frequencies.
- Thus, it can act as a bandpass filter, allowing microwaves of a particular frequency to pass while blocking microwaves at nearby frequencies.

Cavity resonators are used as bandpass filters, tuners etc. They are used for amplification and generation of microwave in devices like Klystron, Reflex Klystrons, Magnetrons, Travelling wave tubes etc.

Types of Cavity Resonator

Depending on the cavity structure:
- Cubic
- Rectangular
- Circular
- Coaxial etc.

Mode of tuning:
- Capacitive
- Inductive

Depending on regulation:
- Regulated
- Un regulated

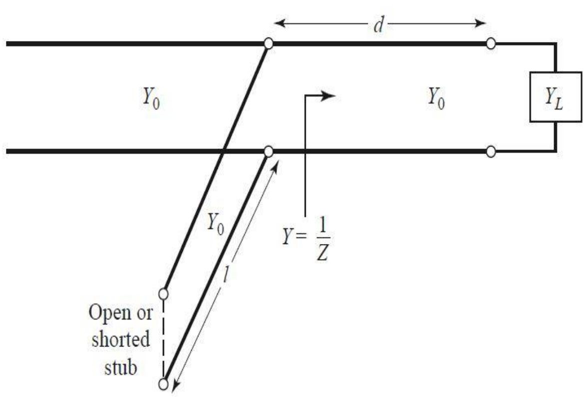
Stub Matching

It is a form of impedance matching in transmission lines where sections of open or short-circuited lines called stubs are connected with the main line in series or parallel at a certain distance from the load. These are of two types:

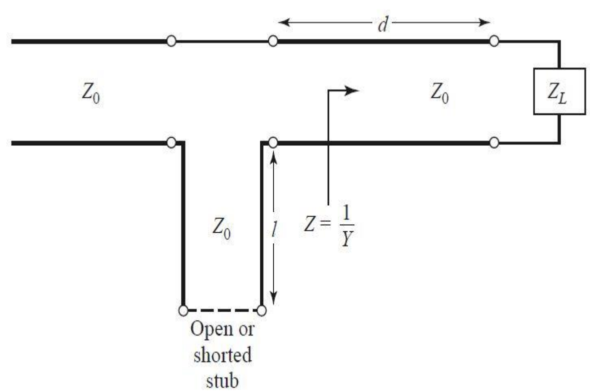
i. Single Stub
ii. Double Stub

Single Stub

This technique uses a single open-circuited or short-circuited length of transmission line (a stub) connected either in parallel or in series with the transmission feed line at a certain distance from the load. Single Stub tuning circuit is often very convenient because the stub can be fabricated as part of the transmission line media of the circuit, and lumped elements are avoided. Shunt stubs are preferred for microstrip line or stripline, while series stubs are preferred for slotline or coplanar waveguide.



Single Shunt Stub

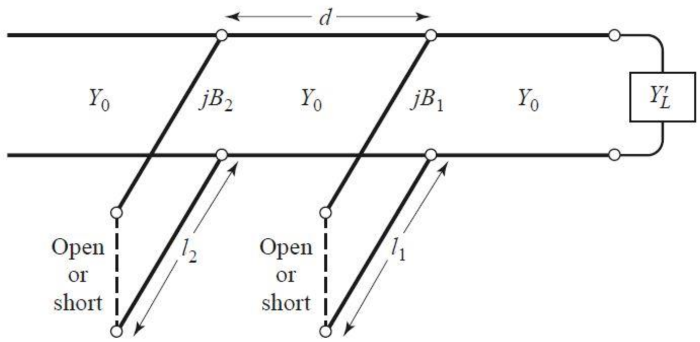


Single Series Stub

N.B. The proper length of an open or shorted transmission line section can provide any desired value of reactance or susceptance.

Double Stub

The double-stub tuner, uses two tuning stubs in fixed positions. Such tuners are often fabricated in coaxial line with adjustable stubs connected in shunt to the main coaxial line.



Advantages and Disadvantage of Single Stub and Double Stub

The single-stub tuner is able to match any load impedance (having a positive real part) to a transmission line, but suffers from the disadvantage of requiring a variable length of line between the load and the stub.

A double-stub tuner cannot match all load impedances.

Open vs. Short Stub

For transmission line media such as microstrip or stripline, open-circuited stubs are easier to fabricate since a via hole through the substrate to the ground plane is not needed.

For lines like coax or waveguide, however, short-circuited stubs are usually preferred because the cross-sectional area of such an open-circuited line may be large enough (electrically) to radiate, in which case the stub is no longer purely reactive.