



Energy Science and Engineering {unit 2}

Computer Science (Dr. A.P.J. Abdul Kalam Technical University)

Unit-2

Fundamental Forces in the Universe:

The four fundamental forces of Nature are Gravitational force, Weak Nuclear force, Electromagnetic force, Strong Nuclear force. The weak & strong forces are effective only over a very short range and dominate only at the level of subatomic particles. Gravity & Electromagnetic force have infinite range.

- 1) Gravitational force - Weakest force, but infinite range
- 2) Weak Nuclear force - Next weakest force, but short range
- 3) Electromagnetic force - Stronger, with infinite range
- 4) Strong Nuclear force - Strongest, but short range

1) Gravitational force: The gravitational force is weak, but very long ranged. Furthermore, it is always attractive. It acts between any two pieces of matter in the Universe since mass is its source.

2) Weak Nuclear force: The weak force is responsible for radioactive decay. It has a very short range. As its name indicates, it is very weak. The weak force causes beta decay i.e. conversion of a neutron into a proton, an electron and an antineutrino.

3) Electromagnetic force: The electromagnetic force causes electric & magnetic effects such as the repulsion between like electrical charge or the interaction of bar magnets. It is long-ranged, but much weaker than the strong force. It can be attractive or repulsive, and acts

Only between pieces of matter carrying electrical charge, Electric Magnetism & light are all produced by this force.

4) Strong Nuclear Force:

The strong interaction is very strong but very short-ranged. It is responsible for holding the nuclei of atoms together. It is basically attractive, but can be effectively repulsive in some circumstances. The strong force is carried by particles called gluons, that is, when two particles interact through the strong force, they do so by exchanging gluons. Thus, the quarks inside of the protons & neutrons are bound together by exchange of the strong nuclear force.

Structure of the atom

All matter is composed of unit particles.

An atom consists of a relatively heavy, positively charged nucleus and a number of much lighter negatively charged electrons orbiting around the nucleus. The nucleus consists of protons & neutrons, which together are called nucleons. Protons are positively charged, while the neutrons are electrically neutral.

The electric charge on one proton is equal in magnitude but opposite in sign to that on an electron. The atom as a whole is electrically neutral, since the number of protons is equal to number of electrons in orbit.

The No. of protons in the nucleus is called the atomic Number

(Z). The total Number of Nucleons in the nucleus is called the mass Number, A. Nuclear symbols

$${}_{Z}^{A}X \text{ or } {}_{Z}^{A}X$$

where X is the usual chemical symbol.

$$\text{Radius of Nucleus} = 10^{-16} \text{ m}$$

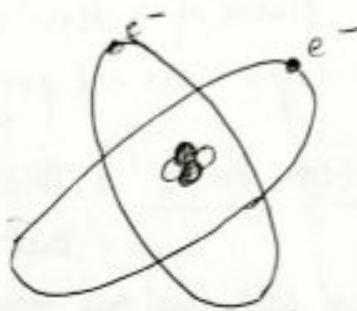
$$\text{Radius of atom} = 10^{-10} \text{ m}$$

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J} = 4.44 \times 10^{-20} \text{ kJ/mJ}$$

(s)



Hydrogen ${}_1^1\text{H}$



Helium ${}_2^4\text{He}$

● → proton
○ → neutron

$$\text{Neutron mass, } m_n = 1.008655 \text{ amu} = 1.674 \times 10^{-27} \text{ kg.}$$

$$\text{Proton mass, } m_p = 1.007277 \text{ amu} = 1.673 \times 10^{-27} \text{ kg}$$

$$\text{Electron mass, } m_e = 0.0005486 \text{ amu} = 9.109 \times 10^{-31} \text{ kg.}$$

amu is unit of mass · amu = atomic mass unit
 $1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg.}$ [amu; defined as one $\frac{1}{12}$ th of the mass of one common carbon atom]

↳ Hydrogen is only atom that has no neutron

↳ ${}_1^2\text{H}$ Deuterium has one proton & one neutron in its nucleus

↳ ${}_2^4\text{He}$ Helium $Z=2 \quad A=4 \quad p=2 \quad n=2-2=2$

Isotopes: Atoms with nuclei having the same number of protons have similar chemical & physical properties and differ mainly in their masses. They are called

isotope are the atoms of the same element having same atomic no. but different mass no.

Isotopes

For ex. ① Deuterium called heavy hydrogen is an isotope of hydrogen

② C-12, C-13, C-14 are isotopes of carbon

③ 92 Uranium $\begin{array}{c} 234\text{U} \\ 92 \\ 235\text{U} \\ 92 \\ 238\text{U} \\ 92 \end{array}$

0.006% 0.712% 99.282% } Natural
Uranium is composed of

mass defect is the mass difference we get between actual mass of the nucleus and the mass we get on adding the no. of protons and neutrons

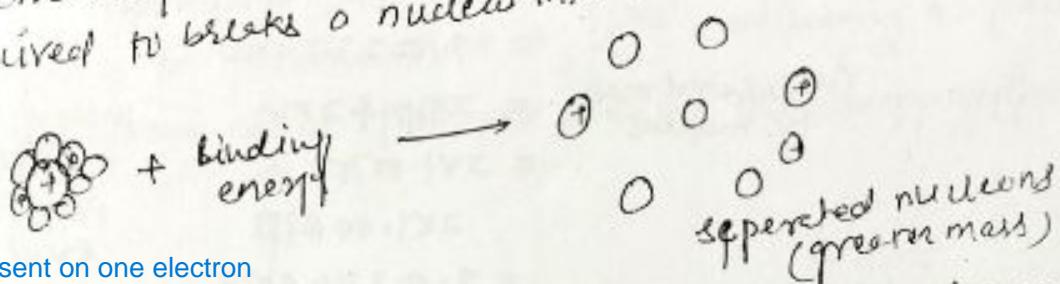
Mass defect and binding energy

Defect/defect: Nucleus comprises of several positively charged protons (equal to atomic number) clumped together in a very small volume. Because of the identical nature of their electrical charge, strong electrostatic repulsive force is exerted by all of them. The magnitude of such force increases with increase in the number of protons in the nucleus. Existence of the nucleus despite such opposing forces indicates towards the presence of a short-range force, which is able to overcome the electrostatic repulsion and bind the nucleus together. The energy associated with this particular force is called the Binding energy.

Alternatively it can be viewed as the amount of energy required to break a nucleus into its constituents

binding energy is defined as the amount of energy needed to break the nucleus into its constituents
 $BE = (m)c^2$

where Δm is the mass defect



$$1 \text{ eV} = 1.6 \times 10^{19} \text{ J}$$

1 eV is the charge present on one electron

Mass Defect: The sum of the masses of the protons & neutrons that comprises the nucleus exceeds the mass of the atomic nucleus. This difference in mass is called the mass defect. The mass defect (Δm) is found by adding up all the individual particle weights & subtracting the actual mass (m) of the atom

$$\Delta m = n m_p + (n - Z) m_n - M_A$$

where n refers to the number of neutrons or protons

The mass defect is converted to energy in a nuclear reaction as given by Einstein's law

$$AE = \Delta m \cdot c^2$$

where $E = \text{energy in J}$

$c = \text{velocity of light} = 3 \times 10^8 \text{ m/s}$

$\Delta m = \text{mass defect}$

$$[4m = Z(1.007228) + (A-Z)(1.008665) - M_A] \text{ amc}^2$$

The energy associated with the mass defect is known as the binding energy of the nucleus. It acts as a "glue" which binds the protons & neutrons together in the nucleus.

- The energy equivalent of 1amu of mass is

$$m = [Z(mp + me) + (A - Z)mn] - m_{atom}$$

$$4E = 1.66 \times 10^{-27} \text{ kg} \times (3 \times 10^8 \text{ m/s})^2$$

$$= 14.94 \times 10^{-11} \text{ J} = 9.31 \times 10^8 \text{ eV}$$

$$= 931 \text{ meV}$$

1 amu of mass = 931 meV

~~By~~ mass of ${}_2^4\text{He}$ atom = 1.00277 amu

mass of ${}_2^4$ He nucleus = mass of Helium atom - mass of two orbital electrons



$$= 1.00277 - 2 \times 0.00055$$

$$= 1.00277 \text{ amu}$$

Calculated mass
He nucleus

$$= 2mp + 2mn$$

$$= 2 \times 1.00759 +$$

$$2 \times 1.00898$$

$$= 1.003314 \text{ amu}$$

mass of e^- is
neglected because
it is very very
small as compared
to proton

$$\text{mass defect} = \Delta m = 1.003314 - 1.00277$$

$$= 0.03037 \text{ amu}$$

$$\Delta E = 0.03037 \times 931 = 28.2 \text{ meV}$$

This is the energy released when two protons & two neutrons are bound together. If we were to change one helium nucleus back to its constituents, we would have to give back this 28.2 MeV to the nucleus. The binding energy per nucleon is then

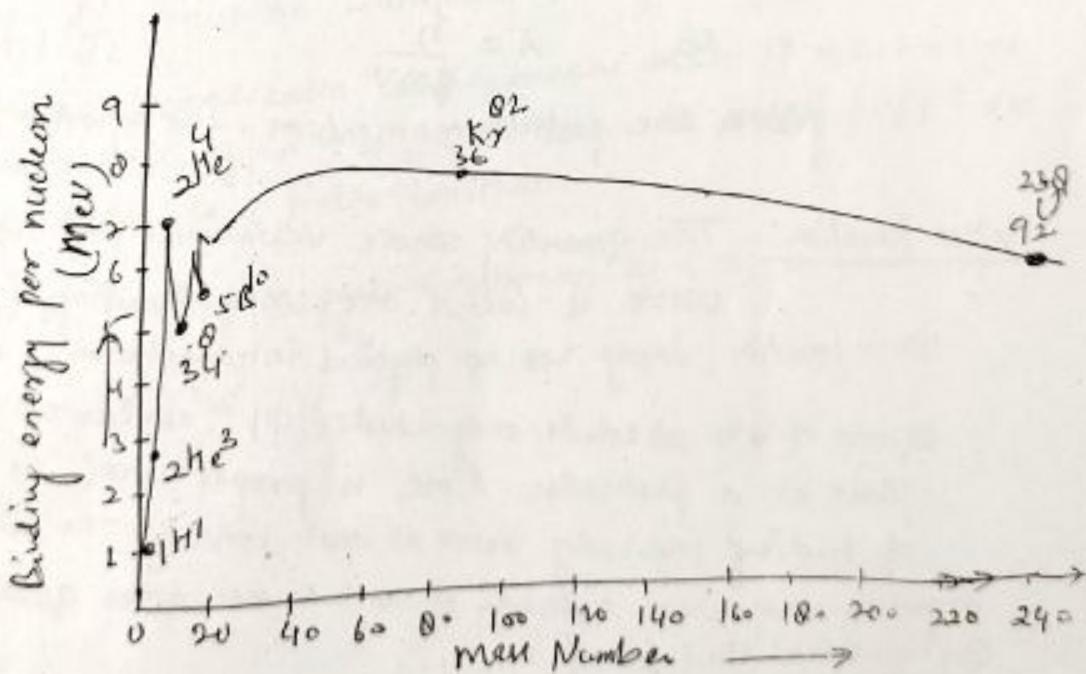
$$BE/\text{nucleon} = 28.2/4 = 7.05 \text{ MeV}$$

for Deuterium it is 1.115 MeV/nucleon.

In this way, the binding energy per nucleon can be calculated for all isotopes. Higher the binding energy per nucleon, higher is the stability of the nucleus.

The binding energy curve shows that one most stable elements (like iron, cobalt, nickel etc) are in intermediate mass number range. If elements of low mass number are fused together, it would lead to more stable elements. The elements of higher mass number are less stable & if they are fissioned, they would form elements of less mass number, which are more stable. Thus light isotopes like hydrogen, deuterium ³⁰~~on~~ are good for fusion reactions, while one ~~most~~ heavier isotopes like uranium are suitable for fission reaction.

It was found that all nuclei of the even-even type i.e. having an even number of protons & even number of neutrons, are very stable. Therefore, a ⁹²₉₂ U²³⁸ atom having 92 protons & 146 neutrons is quite stable & requires very high energy neutrons for fission, whereas a ⁹²₉₂ U²³⁵ atom having 92 protons & 143 neutrons can be fissioned even by low energy neutrons.



binding energy per nucleon varying with mass Number

Quantum Mechanics!

Quantum mechanics is a detailed description of the behaviour of matter and light and all the phenomena occurring at the sub atomic scale.

(or) Quantum mechanics is a mathematical formulation that enables us to calculate the wave behavior of material particles.

de Broglie Wave!

A moving body behaves in certain ways as though it has a wave nature.

If a particle moving with momentum p is a wave of wavelength

$$\lambda = h/p$$

where h = Planck's constant

The wavelength defined in this way is generally called the de Broglie wavelength.

The momentum of a particle of mass m & velocity v is

$$p = mv$$

$$\text{so } \lambda = \frac{h}{mv}$$

greater are particle momentum, the shorter is wavelength

Wave function: The quantity whose variations make up matter wave is called the wave function (ψ). While

Wave function itself has no physical interpretation, the

square of its absolute magnitude (ψ^2) evaluated at a particular place at a particular time is proportional to the probability of finding the body there at that time. The linear momentum, angular momentum & energy of the body are other quantities that can be established from ψ .

Wave functions ψ are generally complex with real & imaginary part.

$$\psi = A + Bi$$

But probability density $|\psi|^2$ must be positive real quantity.
The probability density $|\psi|^2$ for a complex ψ is therefore taken as the product $\psi * \psi^*$.

where $\psi^* = A - Bi$

$$|\psi|^2 = \psi * \psi^* = (A+Bi)(A-Bi) \\ = A^2 + B^2$$

Normalization:

Probability density $= |\psi|^2$

$$\int_{-\infty}^{\infty} |\psi|^2 dV = 1$$

A wave fn. that obeys this eqn is said to be normalized.

if $\int_{-\infty}^{\infty} |\psi|^2 dV = 0$ the particle does not exist.

Well Behaved Wave function:

1. ψ must be continuous & single valued everywhere
2. $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{d\psi}{dz}$ must be continuous & single valued everywhere
3. ψ must be normalizable which means that ψ must go to 0 as $x \rightarrow \pm\infty, y \rightarrow \pm\infty, z \rightarrow \pm\infty$ in order $\int |\psi|^2 dV$ over all space be a finite constant.

L Probability of finding a particle between x_1 & x_2 is given by

$$P_{x_1 x_2} = \int_{x_1}^{x_2} |\psi|^2 dx$$

(1)

The Wave Equation:

Schrodinger's Equation: Which is the fundamental equation of quantum mechanics.

Time dependent form :- We assume one ψ for a particle moving freely in the $+x$ direction is specified by

$$\psi = A e^{i\omega(t-x/v)} \quad \text{--- (1)}$$

$$\text{put } \omega = 2\pi\nu + k = \lambda v \quad \therefore \omega = 2\pi\nu$$

$$\psi = A e^{-2\pi\nu i(t - x/\lambda v)} \quad \therefore \nu = 1/v$$

$$\psi = A e^{-2\pi i [vt - x/\lambda]} \quad \begin{matrix} v = \text{velocity} \\ \nu = \text{frequency} \end{matrix}$$

We know that $v + \nu$ in terms of total energy & momentum p of the particle being described by ψ .

$$E = \hbar\nu = 2\pi\hbar\nu \quad \text{and } \lambda = \frac{\hbar}{p} = \frac{2\pi\hbar}{P} \quad \left[\frac{\hbar}{2\pi} = \hbar \text{ (h-bar)} \right]$$

for free particle.

$$\psi = A e^{-i(Et - px/\hbar)}$$

$$\psi = A e^{-i[\frac{E}{\hbar}t - \frac{px}{\hbar}]}$$

$$\psi = A e^{-i\frac{Et}{\hbar}[Et - px]} \quad \left[\frac{1}{\hbar} = 1.054 \times 10^{-34} \text{ J-s} \right] \quad \text{--- (2)}$$

Above ψ describes the wave equivalent of an unrestricted particle of total energy E & momentum p in $+x$ direction. Above ψ is valid only for freely moving particles. However we are most interested in situations where the motion of a particle is subject to various restrictions. For eg. an electron is bound to an atom by the electric field of its nucleus.

What we must now do is obtain the fundamental differential equation for ψ , which we can then solve for ψ in a specific situation.

Now differentiate eqⁿ ③ w.r.t x , which gives

$$\frac{d\psi}{dx} = A e^{-i/\hbar(Et - px)} \cdot \left(+ \frac{p_i}{\hbar} \right)$$

diff again

$$\frac{d^2\psi}{dx^2} = A e^{-i/\hbar(Et - px)} \cdot \left(+ \frac{p^2}{\hbar^2} \right)$$

$$\frac{d^2\psi}{dx^2} = \psi \left(- \frac{p^2}{\hbar^2} \right)$$

$$p^2 \psi = - \frac{\hbar^2}{m} \frac{d^2\psi}{dx^2} \quad \text{--- } ④$$

Now Differentiate eqⁿ ③ w.r.t t , which gives

$$\frac{d\psi}{dt} = - \frac{Ei}{\hbar} \cdot A e^{-i/\hbar(Et - px)}$$

$$\frac{d\psi}{dt} = - \frac{Ei}{\hbar} \psi \quad \text{--- } ⑤$$

$$E\psi = - \frac{\hbar}{i} \frac{d\psi}{dt} \quad \text{--- } ⑥$$

At speeds small compared with that of light, the total energy E of a particle is the sum of its kinetic energy $p^2/2m$ & its potential energy U where U is in general a fn of position x & time t .

$$E = \frac{p^2}{2m} + U(x, t) \quad \text{--- } ⑦ \quad R_E = \frac{1}{2} m v^2$$

The fn U represents the influence of the rest of universe on the particle.

Multiply both side by the wave function ψ in eqⁿ ③

$$\psi E = \frac{p^2}{2m} \psi + U \psi$$

Now we substitute $E\psi$ & $p^2\psi$ from eqⁿ ⑤ & ⑥ to obtain the time dependent form of Schrodinger's equation.

$$-\frac{\hbar}{i} \frac{d\psi}{dt} = \frac{1}{2m} -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + U\psi \quad (1)$$

$$+ i \frac{\hbar}{\tau^2} \frac{d\psi}{dt} = -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + U\psi$$

$$\boxed{i \frac{\hbar}{\tau} \frac{d\psi}{dt} = -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + U\psi}$$

Time dependent
Schrodinger equation in
one dimension

In three dimensions the time dependent form of Schrodinger's eqn

$$\boxed{i\hbar \frac{d\psi}{dt} = -\frac{\hbar^2}{2m} \left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \right) + U\psi}$$

where the particle's potential energy U is some function of x, y, z & t .

Expectation Values: Let us calculate the expectation value ($\langle \rangle$) of the position of a particle confined to x -axis that is described by the wave function $\psi(x, t)$.

$$\langle \hat{x} \rangle = \frac{\int_{-\infty}^{\infty} x |\psi|^2 dx}{\int_{-\infty}^{\infty} |\psi|^2 dx}$$

$$\bar{x} = \frac{N_1 x_1 + N_2 x_2}{N_1 + N_2}$$

If ψ is normalized wave fn, denominator of above eqn equals the probability that the particle exist somewhere between $x = -\infty$ and $x = \infty$ & therefore has value 1. In this case,

Expected value for position

$$\langle \hat{x} \rangle = \int_{-\infty}^{\infty} x |\psi|^2 dx$$

Ex: A particle limited to x axis has the wave fn $\psi = \pi x$ between $x=0$ and $x=1$; $\psi=0$ elsewhere

- (a) find the probability that the particle can be found b/w $x=0.45$ and $x=0.55$
- (b) find the expectation value (\bar{x}) of the particle position

Solⁿ (a) The probability is

$$\begin{aligned} \int_{x_1}^{x_2} |\psi|^2 dx &= \int_{0.45}^{0.55} |ax|^2 dx \\ &= a^2 \int_{0.45}^{0.55} x^2 dx = a^2 \frac{x^3}{3} \Big|_{0.45}^{0.55} \\ &= \frac{a^2}{3} [(0.55)^3 - (0.45)^3] = 0.257 a^2 \quad \text{Ans} \end{aligned}$$

(b) The expectation value is

$$\begin{aligned} \langle x \rangle &= \int_0^L x |\psi|^2 dx \\ &= \int_0^1 x |ax|^2 dx \Rightarrow a^2 \int_0^1 x^3 dx \\ &= a^2 \frac{x^4}{4} \Big|_0^1 = \frac{a^2}{4} \quad \text{Ans} \end{aligned}$$

Schrodinger's Eqⁿ: steady state form

(13)

one dimensional wave function ψ of a free particle is given by

$$\psi = A e^{-\frac{i}{\hbar} (E t - p x)}$$

$$\psi = A e^{-\frac{i}{\hbar} Et} e^{\frac{i}{\hbar} px}$$

$$\psi = \psi e^{-i\hbar Et} \quad \psi = A e^{i\hbar px}$$

We know Schrodinger's time dependent eqⁿ

$$i\hbar \frac{d\psi}{dt} = -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + U(x,t) \psi$$

$$\cancel{i\hbar} \psi e^{-i\hbar Et} \left[\cancel{i\hbar} \frac{d\psi}{dt} \right] = -\frac{\hbar^2}{2m} e^{-\frac{i}{\hbar} Et} \frac{d^2\psi}{dx^2} + U(x,t) \psi e^{-\frac{i}{\hbar} Et}$$

$$E \psi = -\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + U(x) \psi$$

$$\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + (E - U) \psi = 0$$

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - U) \psi = 0}$$

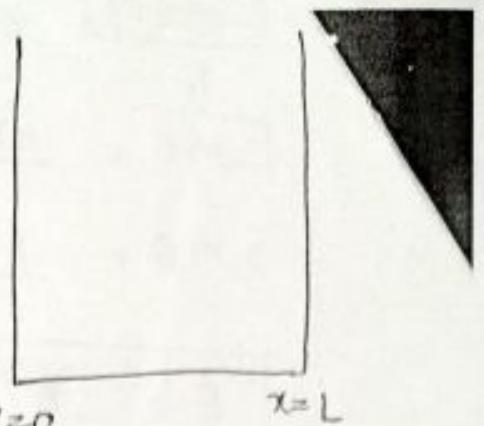
↳ One dimension steady state
Schrodinger eqⁿ

$$\boxed{\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} + \frac{2m}{\hbar^2} (E - U) \psi = 0}$$

↳ steady state Schrodinger in
three dimension

Particle in Box :

Particle is trapped in a box with infinitely hard wall. It is restricted to traveling along the x-axis by $x \geq 0$ & $x = L$ by infinitely hard walls. A particle does not lose energy when it collides with such walls. So total energy stays const.



$$\psi(x) = 0 \quad x \geq 0$$

$$\psi(x) = 0 \quad x = L$$

Potential energy is zero inside box.

With in box we know steady state Schrodinger eqn

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} (E - U) \psi = 0$$

$$\therefore U = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E \psi = 0$$

Solⁿ of above eqn is

$$\psi(x) = C_1 \sin \frac{\sqrt{2mE}}{\hbar} x + C_2 \cos \frac{\sqrt{2mE}}{\hbar} x$$

This solⁿ is subjected to boundary condition

$$\psi(x) = 0 \quad \text{at } x = 0$$

$$0 = C_1 \sin \frac{\sqrt{2mE}}{\hbar} (0) + C_2 \cos \frac{\sqrt{2mE}}{\hbar} (0)$$

$$\boxed{C_2 = 0}$$

$$\text{Now } \psi(x) = 0 \quad \text{at } x = L$$

$$0 = C_1 \sin \frac{\sqrt{2mE}}{\hbar} L$$

$$C_1 \neq 0 \quad \frac{\sqrt{2mE}}{\hbar} L = n\pi \quad \text{where } n = 1, 2, 3$$

$$\frac{\sqrt{2mE}}{\hbar} L = n\pi$$

$$\frac{\sqrt{2mE}}{\hbar} = \frac{n\pi}{L}$$

$$2mE = \frac{n^2\pi^2\hbar^2}{L^2}$$

$$E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

$$n = 1, 2, 3$$

The wave-function of a particle in a box whose energies are E_n are

$$\psi_n = A \sin \frac{\sqrt{2mE_n}}{\hbar} x$$

$$\boxed{\psi_n = A \sin \frac{n\pi x}{L}} \quad \therefore \frac{\sqrt{2mE_n}}{\hbar} = \frac{n\pi}{L}$$

The values for which energy for which Schrodinger steady state equation can be solved are called eigenvalues & the corresponding wave function ψ_n are called eigenfunctions.

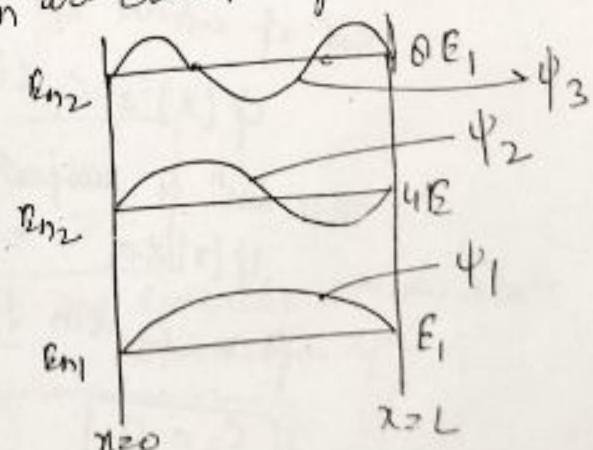
$$\psi_n(x) = A \sin \frac{n\pi x}{L} \quad 0 < n \leq 2$$

$$= 0$$

$$n < 0$$

$$= 0$$

$$n > L$$



if wave fn is normalized

(16)

$$\int_{-\infty}^{\infty} |\psi_n|^2 dx = 1$$

$$\int_{-\infty}^{\infty} |A \sin \frac{n\pi x}{L}|^2 dx = 1$$

$$A^2 \int_{-\infty}^{\infty} \sin^2 \frac{n\pi x}{L} dx = 1 \Rightarrow A^2 \int_{-\infty}^{\infty} \left[1 - \cos \frac{2n\pi x}{L} \right] dx = 1$$

$$\frac{A^2}{2} \left[x - \frac{L}{2n\pi} \sin \frac{2n\pi x}{L} \right]_{-\infty}^{\infty} = 1$$

$$\frac{A^2}{2} [L - 0 - 0 - 0] = 1 \Rightarrow A^2 = \frac{2}{L}$$

$$A = \sqrt{\frac{2}{L}}$$

Now wave function is

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}}$$

on $x \in L$

Ex: Find the probability that a particle trapped in a box L wide can be found between $0.45L$ & $0.55L$ for one ground & first excited states.

Sol: for ground state $n=1$ $\psi_1 = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$

$$\psi_1 = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

for first state $n=2$ $\psi_2 = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}$

for ground state

$$P_{\text{ground}} = \int_{n_1}^{n_2} |\psi_1|^2 dx = \int_{0.45L}^{0.55L} \left| \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \right|^2 dx$$

$$= \frac{2}{L} \left[1 - \left(\cos \frac{2n\pi x}{L} \right) \frac{L}{2\pi} \right]_{0.45L}^{0.55L}$$

$$\boxed{P_{\text{ground}} = 0.190 = 19.0\%}$$

for first state

$$\boxed{P_{\text{first}} = 0.65\%}$$

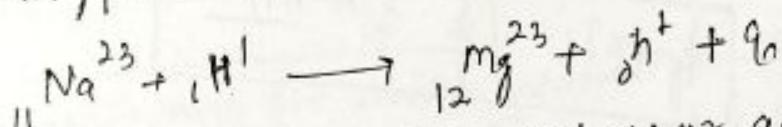
Nuclear Reactions

During a nuclear reaction, any change in the mass of the particle represents an release or an absorption of energy. If the total mass of the particle after the reaction is reduced, the process releases an energy. Consequently an increase in the mass of the resultant particle, will cause an absorption of energy.

The eq's are much similar to chemical reactions. The energy variation is also of the order of meV. In simple terms the equation shows the balance of one neutron and proton.

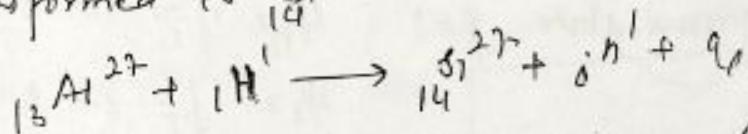
Some of the type of reactions are given below

- ① When $^{23}\text{Na}^{23}$ is bombarded with protons possessing high energy, it is converted to $^{23}\text{Mg}^{23}$



where q_0 = release or absorption of energy in the reaction

- ② When $^{27}\text{Al}^{27}$ is bombarded with high energy protons it is transformed to $^{27}\text{Si}^{27}$



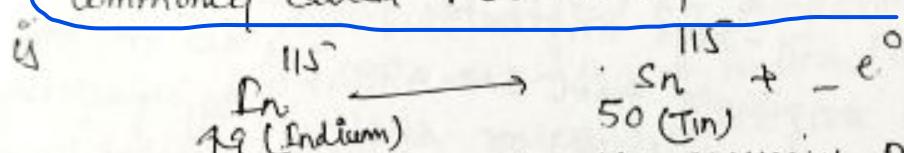
- ③ When $^{27}\text{Al}^{27}$ is bombarded with deuterons $^{27}\text{Al}^{28}$ & proton may be produced



(18)

Radioactivity. Most isotopes that occurs in Nature are stable. Some isotopes of heavy element like Thallium ($Z=81$), lead ($Z=82$) and bismuth ($Z=83$), and all isotopes of heavier elements starting with polonium ($Z=84$) are not stable (binding energy per nucleon being small) & emit radiation till a more stable nucleus is reached. Thus a spontaneous disintegration process, called radioactive decay, occurs. The resulting nucleus is called the daughter and the original nucleus is called the parent. The daughter product may be stable or radioactive.

radioactive ^{natural &} Radioactive isotopes, both natural & man made, are commonly called radioisotopes. An example of radioactivity

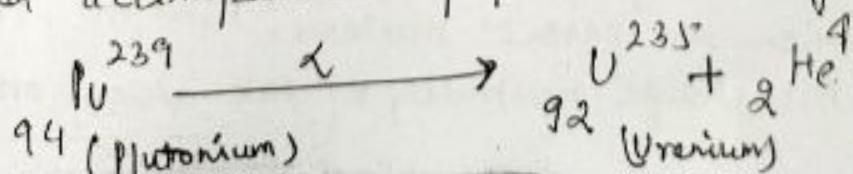


where In^{115} is a naturally occurring radioactive isotope of Indium. In^{115} decays into Sn^{115} which is stable. Radioactivity is always accompanied by a decrease of mass or liberation of energy. The energy thus liberated shows up in the form of kinetic energy of the emitted particle and an electromagnetic radiation (γ -rays). Such activities are called

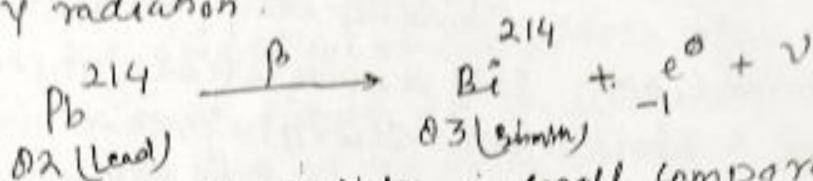
emitted particle and an electromagnetic wave. Those elements which exhibits this activity are called radioactive element. e.g - Uranium, Polonium, radium, radon, thorium, thorium, etc.

Naturally occurring radioisotopes emit
① α - particles ② β - particles ③ γ - radiation

① Alpha Decay: Alpha particles are helium nuclei, commonly emitted by heavier radioactive nuclei and accompanied by γ -radiation. Eg.



- (2) Beta Decay: it is equivalent to the emission of an electron & raises the atomic number by one, while the mass number remains the same. It is usually accompanied by the emission of neutrino (ν) & γ radiation.



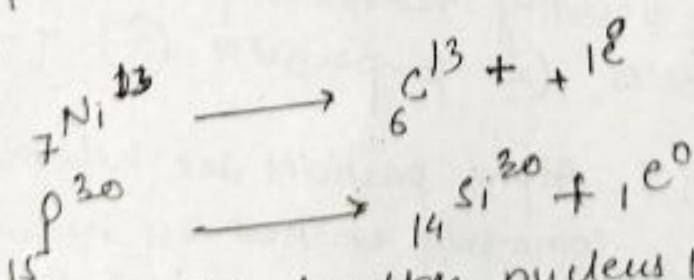
The penetrating power of β -particles is small compared to γ -rays but is larger than that of α -particles.

- (3) Gamma Radiation or photon: This is an electromagnetic radiation of extremely short wavelength & very high frequency & hence, high energy ($E=h\nu$). γ -rays originates from the nucleus, while X-rays originate from the atom (as the orbital electrons change energy levels or shells). By emitting γ -radiation, an excited daughter nucleus falls back into its stable ground (lowest) energy state.



γ -radiation is highly penetrating & does not affect either the atomic or mass number.

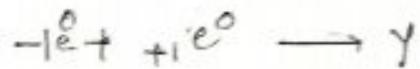
- (4) Positron Decay: When a radioactive nucleus contains an excess of protons, positron (positive beta) decay occurs, converting a proton into a neutron in the process



In positron decay, the daughter nucleus has one less proton than the parent nucleus. Therefore, to maintain the electric charge neutrality of the atom, one orbital

(20)

electron has to be released. This orbital electron then combines with one emitted positron so that producing γ energy to the sum of their rest masses



Radioactive decay: It has been observed that the emission of the particles in the forms of alpha, beta or gamma radiations is not an instantaneous process, for various elements, the decay time is different, which follows certain law.

The law states: The rate of decay is a function only of the number of radioactive nuclei present at a time provided that the number is large. It does not depend on temperature, pressure or the physical & chemical states of matter i.e. whether it is in solid, liquid or gaseous phase or in chemical combination with other atoms.

If N be number of radioactive nuclei of an species at any time t , the rate of decay

$$\frac{dN}{dt} = -\lambda N \quad \text{where } \lambda \rightarrow \text{proportionality const or the radioactive decay constant of the material}$$

Negative sign represents that during disintegration the number of the nuclei is decreasing

Now integrate the above eqⁿ

$$\int_{N_0}^N \frac{dN}{N} = - \int_0^t dt$$

where $N_0 \rightarrow$ Initial number of such nuclei
 $N \rightarrow$ Number of radioactive nuclei present at any time t

$$[\log_e N]_{N_0}^N = -\lambda t$$

$(-\frac{dN}{dt})$ is called activity

$$\log_e N - \log_e N_0 = -\lambda t$$

$$\log_e \left(\frac{N}{N_0} \right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t} \Rightarrow$$

$$N = N_0 e^{-\lambda t}$$

Half life: Half-life represents the rate of decay of one radioactive isotopes. The half-life is the time required for half of the parent nuclei to decay or to disintegrate.

$$N = \frac{N_0}{2} \quad \text{if } t = t_{1/2}$$

we know

$$N = N_0 e^{-\lambda t}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}$$

$$\frac{1}{2} = \frac{1}{e^{\lambda t_{1/2}}}$$

$$e^{\lambda t_{1/2}} = 2 \Rightarrow \lambda t_{1/2} = \ln 2$$

$$t_{1/2} = \frac{\ln 2}{\lambda} \Rightarrow t_{1/2} = \frac{0.693}{\lambda}$$

Ex: The half-life of radium 226 (atomic mass = 226.095) is 1620 years. Calculate decay constant.

$$\text{Soln: } t_{1/2} = \frac{\ln 2}{\lambda} \Rightarrow \lambda = \frac{0.693}{t_{1/2}}$$

$$\lambda = \frac{0.693}{1620} = \frac{0.6932}{1620 \times 365 \times 24 \times 3600}$$

$$\lambda = 1.3566 \times 10^{-11} \text{ s}^{-1}$$

Nuclear fusion is the process of combining two or more lighter nuclei which are unstable to form a heavier nucleus which is more stable, in this process large amount of energy is released because the mass of product nucleus collectively is less than their sum of original nucleus

(22)

nuclear fission is the process in which a heavier unstable nucleus splits into unequal lighter nucleus. In this process neutron and gamma rays are also emitted.

Nuclear Fusion : Nuclear Fusion is the process of combining or fusing two lighter nuclei into a stable & heavier nuclei. In this process large amount of energy is released because mass of the product nucleus is less than the masses of the two nuclei which are fused.

Nuclear Fission : Fission is the process that occurs when a neutron collides with one nucleus of certain of the heavy atoms, causing the original nucleus to split into or more unequal fragments which carry off most of the energy of fission as kinetic energy. This process is accompanied by the emission of neutron and gamma rays.

The chain reaction: A chain reaction is that process in which the number of neutrons keeps on multiplying rapidly during fission till whole of the fissionable material is disintegrated. The chain reaction will become self-sustaining or self-propagating only if, for every neutron absorbed, at least one fission neutron becomes ~~available~~ available for causing fission of another nucleus. This condition can be conveniently expressed in the form of multiplication factor or reproduction factor (K) of the system which may be defined as

$$K = \frac{\text{No. of neutrons in any particular generation}}{\text{No. of neutrons in the preceding generation}}$$

If $K > 1$, chain reaction will continue (increased power off)
 $K < 1$, chain reaction cannot be maintained

$$K = \frac{\text{No. of neutrons released during present cycle}}{\text{No. of neutrons released during previous cycle}}$$

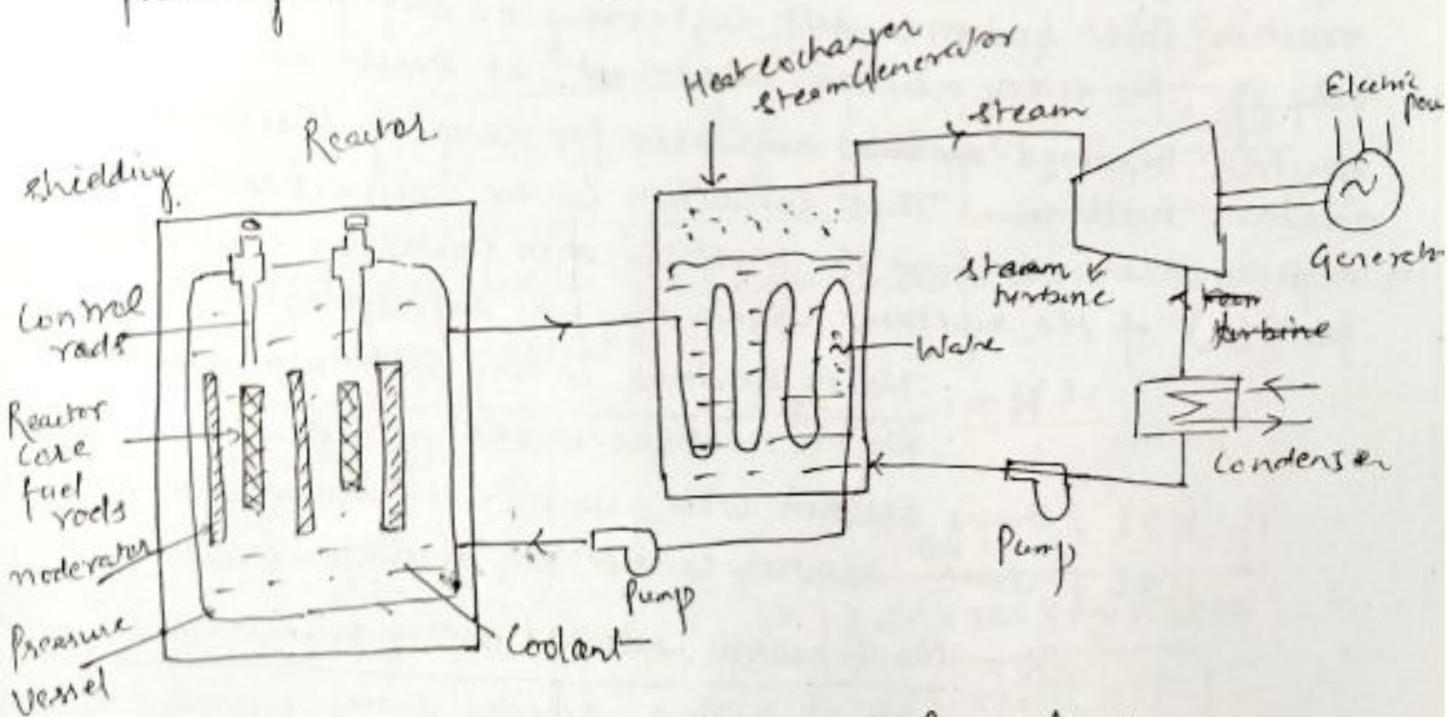
$K=1$ - Critical chain reaction
= const power off.

Nuclear Reactor Power Plant :

The main components of a nuclear power plant are:

- (1) Nuclear reactor
- (2) Heat exchanger (steam generator)
- (3) Steam turbine
- (4) Condenser
- (5) Electric generator

In a nuclear power plant the reactor performs the same function as that of the furnace of steam power plant (ie produces heat). The heat liberated in the reactor as a result of the nuclear fission of the fuel is taken up by the coolant circulating through the reactor core. Hot coolant leaves the reactor at the top and then flows through the tubes of steam generator and passes on its heat to the feed water. The steam so produced expands in the steam turbine, producing work which is thereafter is condensed in condenser. The steam turbine in turn runs an electric generator thereby producing electrical energy.



Schematic of a Nuclear Power Plant

Made of steel (in which fission reaction takes place)

(24).

Essential Components of a Nuclear Reactor:

- (1) Reactor Core (2) Control Rods (3) Coolant
- (4) Moderator (5) Shielding

(1) Reactor Core: The reactor core is one part of a nuclear power plant where fission chain reaction is made to occur and where fission energy is liberated in the form of heat for operating power conversion equipment. The core of reactor consists of an assemblage of fuel elements, control rods, coolant & moderator. Reactor cores generally have a shape approximating to a right circular cylinder with diameters ranging from 0.5m to 15m. The pressure vessels which houses the reactor core is also considered a part of the core. The fuel elements are made of plates or rods of Uranium metal.

(2) Control Rods: Control rods are used to absorb the excess neutron produced in chain reaction in order to control the rate of heat production. The control rods made of high neutron capture material are inserted into the core.

The control rod material is : Cadmium, Boron

To start the reactor from shutdown, the control rods are partially withdrawn until k is greater than 1.

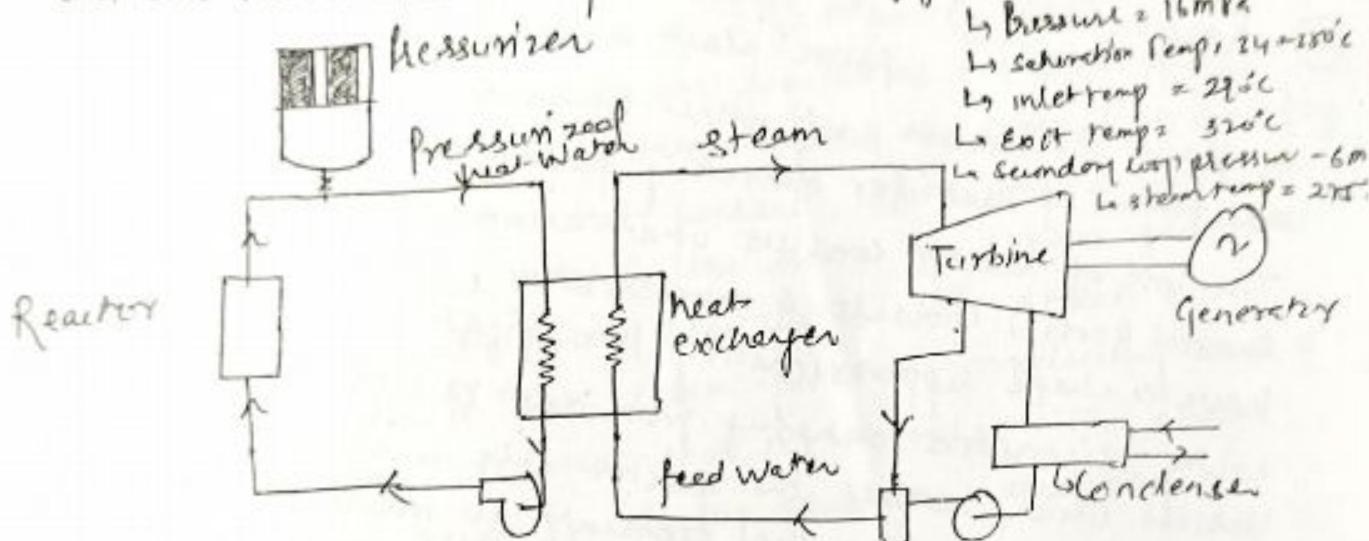
Neutron flux and heat output rate begins to rise when desired rate is achieved the control rods are quickly reversed to keep k at unity.

To shut down the reactor the control rods are inserted completely.

(26)

These reactors are again classified according to the type of fuel used, the coolant, and the moderator.

① Pressurized Water Reactor (PWR): A PWR power plant is composed of two loops in series, the coolant loop, called primary loop and the water steam loop as shown in fig.



The coolant picks up heat in the reactor and transfers it to the working fluid in the heat exchanger (steam generator).

The steam is then used in a Rankine type cycle to produce electricity.

The fuel in PWRs is slightly enriched uranium in the form of thin rods.

The coolant leaving the reactor enters the steam generator which can be either shell & tube type with U-tube or ~~pressurizer~~, enriched fuel is used.

~~pressurizer~~ The coolant in the PWR primary loop is maintained at a pressure (about 15 bar) greater than the saturation pressure corresponding to the maximum coolant temperature in the reactor to prevent bulk boiling. Because liquids are incompressible, small change in volume due to changes in coolant temperature because of either load variation or sudden nuclear reactivity

③ Coolant: The heat produced in the reactor is absorbed by coolant and this heat is used for conversion of water into steam in heat exchanger (steam generator) & this heat is used for power generation.

Used Coolant: Water, Liquid sodium, heavy water.

④ Moderator: It is used to reduce the speed of neutron without absorbing them.

Ordinary water is used as moderator in the enriched uranium power plant. In other power plant materials like heavy water, graphite & beryllium (Be).

⑤ Shielding: Shielding is necessary in order to:

- ① protect the walls of the reactor vessel from radiation damage.
- ② protect the operating personnel from exposure to radiation.

Types of Reactors: Reactors can be heterogeneous or homogeneous. A heterogeneous reactor has a large number of fuel rods with the coolant circulating around them and carrying the heat released by nuclear fission.

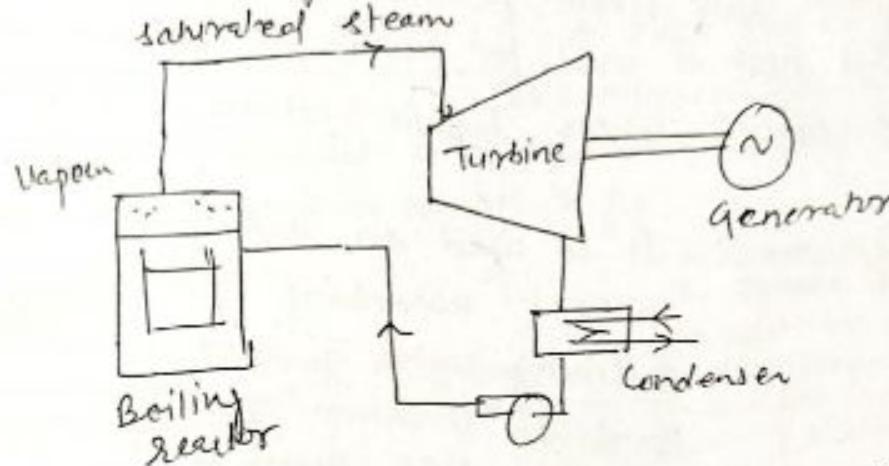
In homogeneous reactor, the fuel and moderator are mixed e.g. a fissionable salt of uranium like uranium sulphate dissolved in the moderator like H_2O & D_2O .

- * Sulphate dissolved in the water. Due to difficulties in component maintenance, erosion & corrosion, homogeneous reactors are not common.

Present day nuclear reactors are of the heterogeneous type.

20

fuel used → enriched Uranium
 coolant → light Water (H_2O)
 moderator → light Water

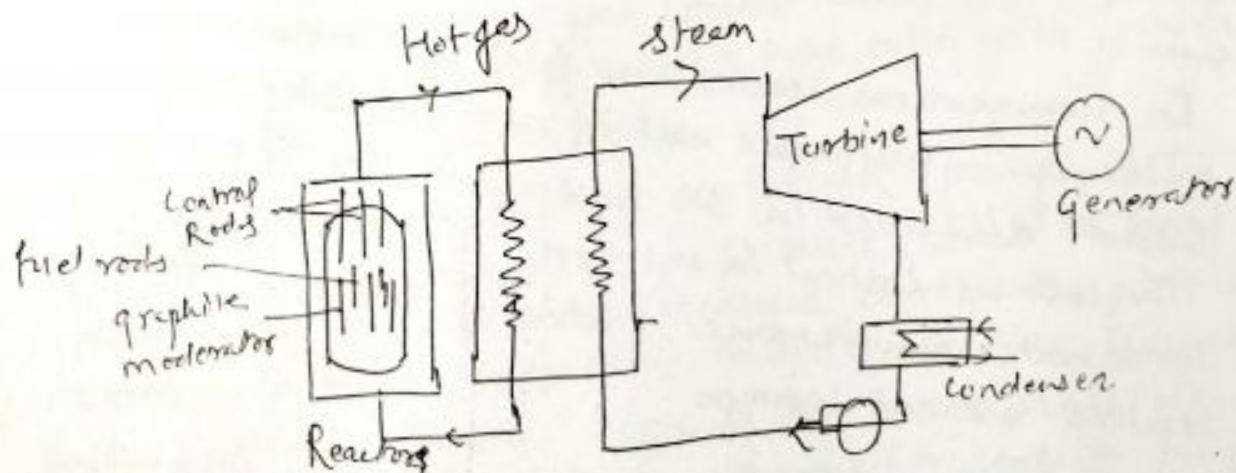


③ CANDU (Canadian - Deuterium - Uranium) Reactor:
 fuel used → Natural uranium ($0.7\% U^{235}$)

(D_2O) Heavy water is used as coolant & moderator.
 These reactors are more economical to those countries which do not produce enriched Uranium, as the enrichment of Uranium is very costly.

In PWR; one same water serves as both moderator & coolant whereas in the CANDU reactor, one moderator and coolant are kept separate.

④ Gas cooled reactor (GCR): The coolant is gas (CO_2). The moderator is graphite.



(27)

insertions cause severe or oscillatory pressure changes, due to which pressure may increase or decrease. If the pressure increases, some water will flash into steam and it will affect the reactor performance. If the pressure decreases there may be cavitation. It is necessary to provide a surge chamber that will accommodate the coolant volume changes while maintaining pressure within permissible limits. Such a chamber is called a pressurizer.

② Pressurized Heavy Water Reactor: The difference b/w PHWR & PWR is

mainly in the ~~use~~ of use of coolant / moderator i.e. Heavy Water OR light Water.

In PHWR the heavy water is used as coolant plus moderator. While in PWR the light water is used as coolant plus moderator.

Enriched fuel is used.

③ Boiling Water Reactor (BWR): A BWR differs from the PWR in that the steam

flowing to the turbine produced directly in the reactor core. Steam is separated & dried by mechanical devices located in the upper part of the pressure vessel assembly. The dried steam is sent directly to the high pressure turbine thus eliminating the need for steam generators. The coolant thus serves in triple function of - coolant, moderator & working fluid. Since the coolant boils in the reactor itself, its pressure is much less than that in a PWR & it is maintained at about 7 bar with steam temperature around 205°C .

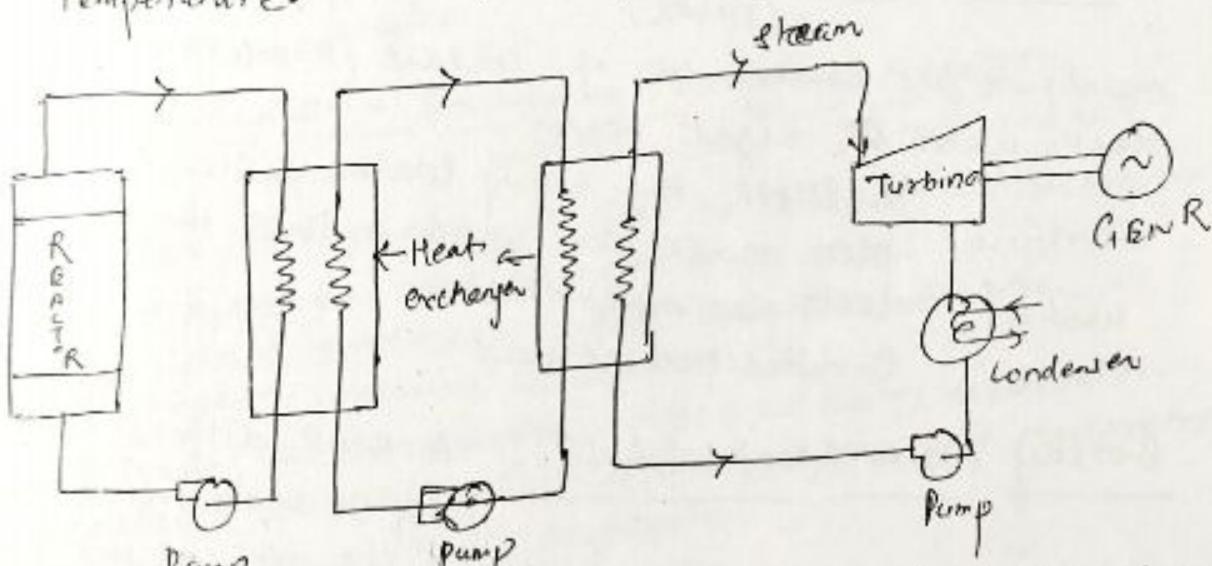
↳ much lower pressure level - 7mpa allowing boiling of coolant
↳ no secondary loop, same fluid acts as both coolant & moderator working fluid.

↳ High thermal efficiency compared to PWR.

(28)

The gas is circulated through the reactor core (instead of water). The gas is safer & effective as a coolant and easy to handle. It can be operated at high temperature because of very high melting point of Graphite (C).

- ⑤ Liquid Metal Fast ~~Booster~~ Reactor: In this reactor sodium works as a coolant & graphite works as moderator. The liquid metal coolant can be operated at high operating temperature and lower pressure (650°C , 1 bar). The thermal efficiency is higher by due to higher temperature.



first heat exchanger → Primary sodium loop to Secondary sodium loop

Second heat exchanger → Secondary sodium loop to Water / Steam loop

→ Sodium boils at 800°C under atmospheric pressure and freezes at 95°C . Hence sodium is first melted by electric heating system and is pressurised to about 7 bar. The liquid Sodium is then circulated by the circulation pump. The reactor have two coolant circuits or loops.

- (i) The primary circuit has liquid sodium which circulate through the fuel core and gets heated by the fissioning of the fuel. This liquid sodium gets cooled in the intermediate

(3e)

	BWR	PWR	CANDU	SIGR	FBR
fuel	enriched	Enriched	Natural	Enriched	Enriched
Coolant	Water	Water	Heavy Water	Liquid sodium	Liquid sodium
Moderator	Water	Water	Heavy Water	graphite	-
control Rod	Cadmium	Cadmium	-	Boron	Boron

fuel's \rightarrow Enriched uranium - $U_{92}^{235} \rightarrow 99.3\%$.

Natural Uranium - $U_{92}^{238} \rightarrow 0.7\%$.

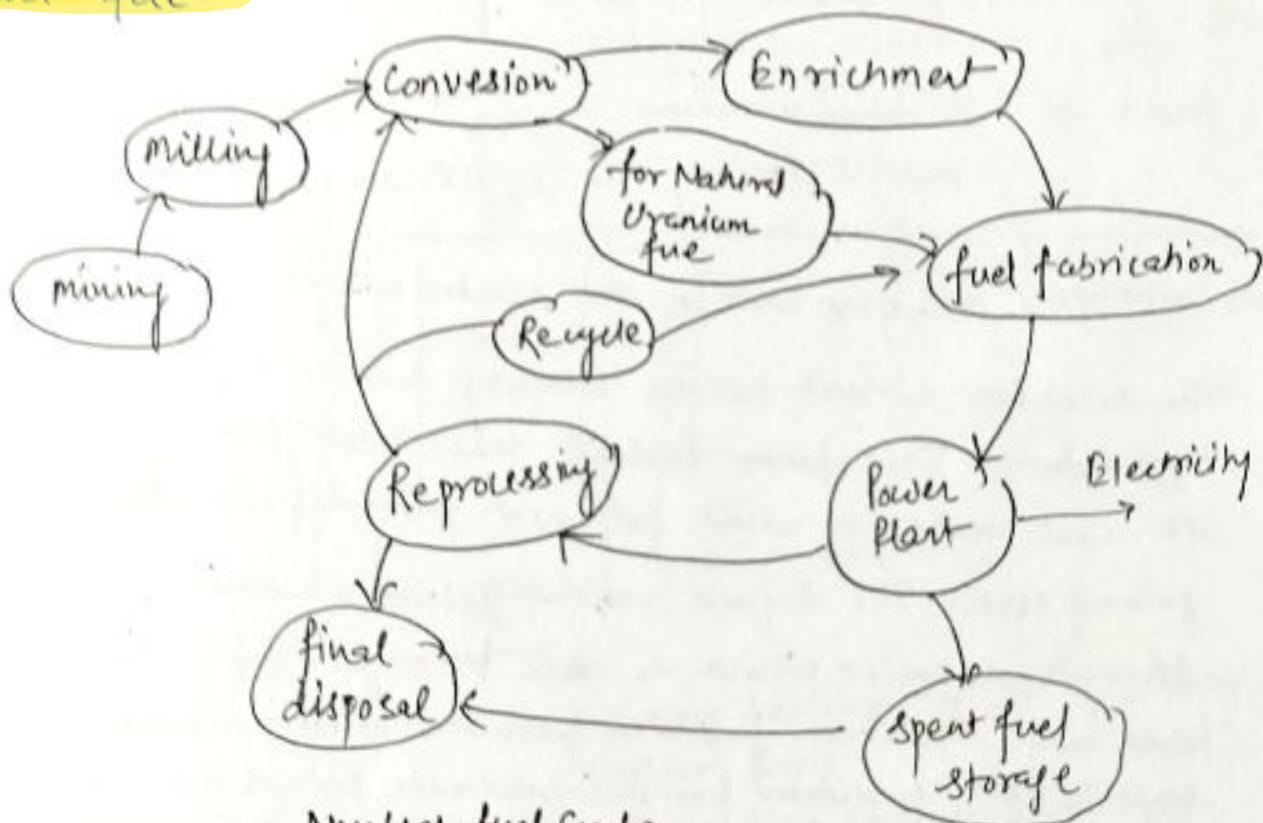
heat exchanger and goes back to the reactor vessel.

- i) The secondary circuit has an alloy of sodium & potassium in liquid form. This ~~form~~ coolant takes heat from the intermediate heat exchanger which gets heat from liquid sodium of primary circuit. The liquid sodium potassium then passes through a boiler which is once through type having tubes only. The steam generated from this boiler will be superheated. Feedwater from the condenser enters the boiler, the heated sodium potassium passing through the tubes gives it heat-to-cooler thus converting it into steam.
- ii) The reactor vessel, primary loop, and intermediate heat exchanger are to be shielded for radioactivity. The liquid metal handled under the cover of an inert gas, such as helium, to prevent contact with air while charging or draining the primary or secondary circuit.

Nuclear Fuel Cycle

Nuclear power plant use uranium as fuel to generate electricity. Uranium is a mildly radioactive metal that is very common on the earth's crust. Before it can be used as nuclear fuel uranium must be extensively processed.

The various stages that the Uranium goes through in the process of generating electricity are known as the nuclear fuel cycle.



Nuclear fuel cycle

Mining: Uranium mining is done with either traditional mining techniques such as underground or open pit (or) more commonly using a mining technique known as in situ leaching (ISL). ISL involves circulating oxygenated ground water through the porous ore deposit, dissolving the uranium oxide and bringing it to the surface.

The solution is then extracted and taken to a mill for further processing.

Milling: The ore or solution is then taken to a mill to be processed into uranium oxide concentrate, known as yellow cake (U_3O_8) which contains more than 80% uranium. Yellow cake is packed into 200-liter drums and shipped to a conversion facility.

Conversion: At the conversion facility yellow cake - uranium is turned into uranium hexafluoride (UF_6) gas. This is necessary in order to enrich the uranium. During the conversion process impurities are removed and uranium is combined with fluorine to produce the UF_6 gas. The gas is then pressurized, cooled to a liquid, and drained into 14-ton cylinders where it solidifies. The cylinder is then shipped to an enrichment facility.

Enrichment: The UF_6 gas coming from the conversion facility is known as natural UF_6 because it still contains the natural concentration of uranium isotopes; which is about 99.27 percent uranium-238, 0.72% U-235 and less than 0.01% U-234. The fuel for most nuclear reactors must have a higher concentration of one fissionable U-235 in order to start & sustain a nuclear reaction. The UF_6 gas can be enriched by three different processes: gaseous diffusion, gas centrifuge or laser separation. The final product is an enriched UF_6 gas with a U-235 concentration of 4 to 5%, which is stored in canister, allowed to cool & solidify, and shipped to a fuel fabrication facility.

Fuel fabrication: The fuel fabrication facilities receive the UF₆ gas in solid form, they then heat it to gaseous form, and chemically process it to turn it into uranium dioxide (UO₂) powder. The powder is then compressed into pellets and sintered to a ceramic form. Sintering is the process of bonding metal particles by using heat and pressure. The pellets are then stacked and sealed into long zirconium alloy tubes about 1 centimeter in diameter to form a fuel rod. Fuel rods are then bundled into fuel assemblies, which depending on the reactor design, can have been 179 and 264 fuel rods per fuel assembly.

Power Plant (Service Period): The fuel assemblies are shipped by truck to the reactor sites where they stored in 'fresh fuel' storage bins until needed by the reactor operator. At this stage the fuel assemblies are only mildly radioactive, all the radiation is contained within the tubes. When needed for service the fuel assemblies are inserted into the reactor core which is itself a cylindrical arrangement of fuel bundles. Just by placing the fuel bundles next to other fuel bundles & adding water a nuclear reaction is initiated. Typically about one third of the fuel assemblies (80 to 90) in a reactor core are replaced by every 12 to 24 months.

Interim storage: Fuel assemblies will spend 10 to 36 months inside the reactor core, after which they are removed from the reactor and stored in special pools in the reactor building. Inside the reactor the fuel assemblies become highly radioactive.

and even though the fission process has stopped they continue to emit heat from the radioactive elements. It is necessary to store them in pools both to cool the fuel and protect the workers from the radiation. The fuel assemblies must be stored at a depth of at least 20 feet to provide sufficient radiation protection. The fuel assemblies will typically be stored in the spent fuel pools for about five years to allow them to cool. When they have cooled enough, the fuel assemblies are moved to a dry cask for further on-site storage. Each cask is designed to hold from 24 to 72 spent fuel assemblies.

Reprocessing: After removing from the reactor, the irradiated fuel elements are reprocessed with great care and precaution against the radioactive irradiation. The irradiated fuel elements taken out of the Nuclear Reactor are highly radioactive.

They are reprocessed to recover the following:

(a) Special elements and metals: Used in fabrication of fuel elements & Reactor construction.

(b) Recovered U-235 and Pu-239: - Used as feedback to ~~de~~ enrichment plant

(c) Fission Product (PP): These are separated by further processing into:

— Useful fission products: They are used in scientific, industrial, agricultural

— Nonusable fission product: They are radioactive and take short/medium/

long time for radioactive decay to ~~reach~~ reach safe level.

Disposal: Disposal of radioactive waste material & spent fuel is a major & important technology (which has become a controversial topic as environmentalists have opposed the long term storage).

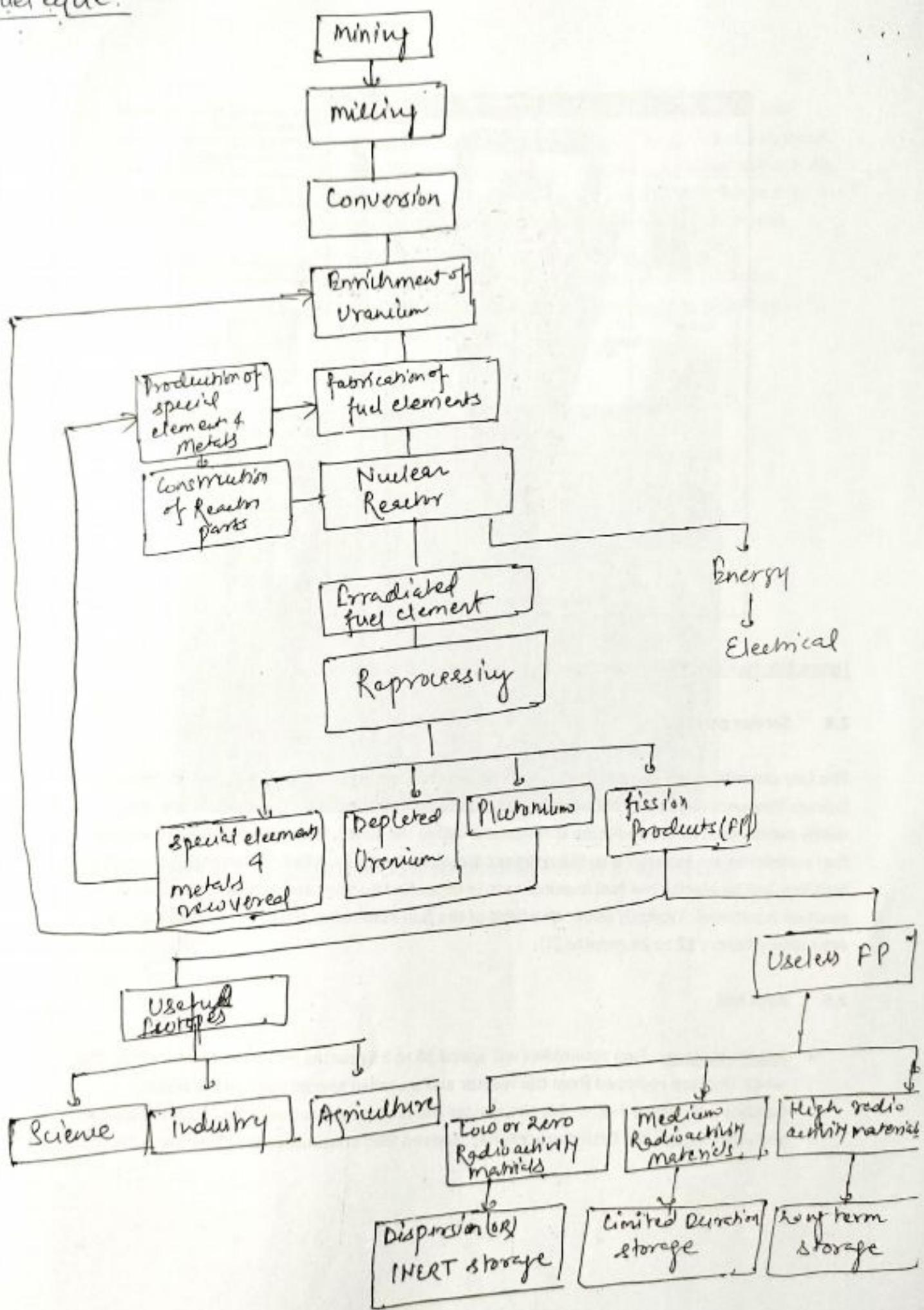
The waste radioactive material is segregated in three categories:

(a) Zero or low Radioactivity materials — material is stored without elaborate shielding

(b) Medium Radioactivity — material is stored for short duration of about 5 years to allow decay of radioactivity

(c) High Radioactive materials — They are stored in water for several months to permit radioactive decay to a acceptable low level.

Fuel Cycle.



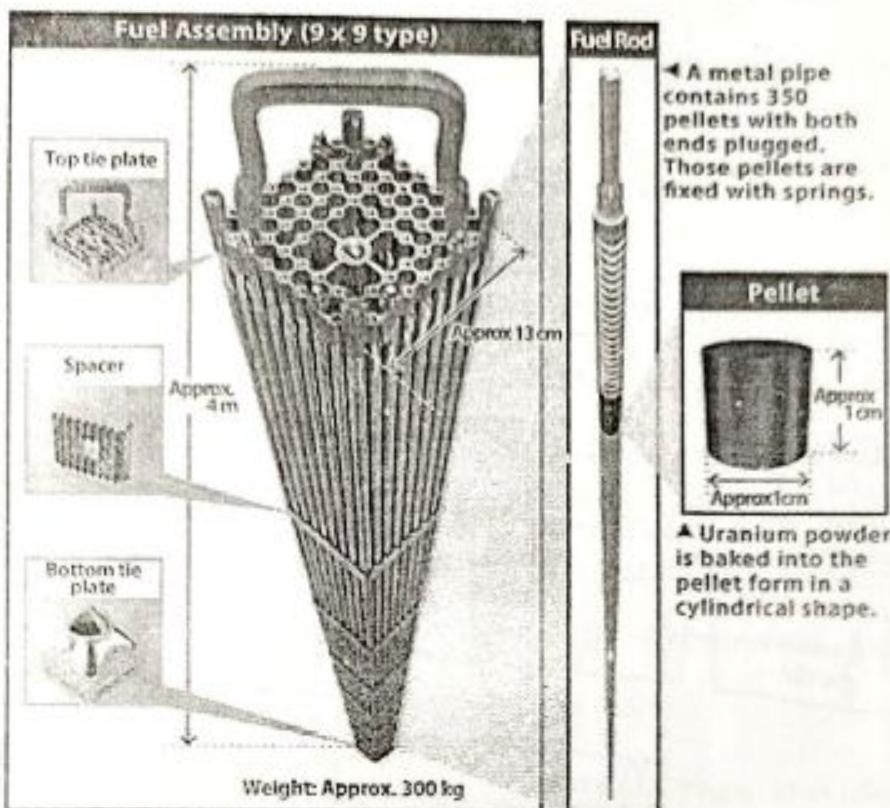


Figure 2-5: Fuel assembly (Source: Nuclear Fuel Industries, Ltd [14])

2.8 Service period

The fuel assemblies are shipped by truck to the reactor sites where they are stored in "fresh fuel" storage bins until needed by the reactor operator. At this stage the fuel assemblies are only mildly radioactive; all the radiation is contained within the tubes. When needed for service the fuel assemblies are inserted into the reactor core which is itself a cylindrical arrangement of fuel bundles. Just by placing the fuel bundles next to other fuel bundles and adding water a nuclear reaction is initiated. Typically about one third of the fuel assemblies (40 to 90) in a reactor core are replaced every 12 to 24 months [7].

2.9 Back end

- Interim storage: Fuel assemblies will spend 18 to 36 months inside the reactor core, after which they are removed from the reactor and stored in special pools in the reactor building (see Figure 2-6). Inside the reactor the fuel assemblies become highly radioactive, and even though the fission process has stopped they continue to emit heat from the

radioactive elements. It is necessary to store them in pools both to cool the fuel and protect the workers from any radiation. The fuel assemblies must be stored at a depth of at least 20 feet to provide sufficient radiation protection. The fuel assemblies will typically be stored in the spent fuel pools for about five years to allow them to cool. When they have cooled enough, the fuel assemblies are moved to a dry cask for further on-site storage (see Figure 2-7). Each cask is designed to hold from 24 to 72 spent fuel assemblies. When the fuel assemblies are placed in the casks the air and water are removed, they are filled with inert gas, and sealed shut by either welding or bolting [1, 7, 8, and 9].

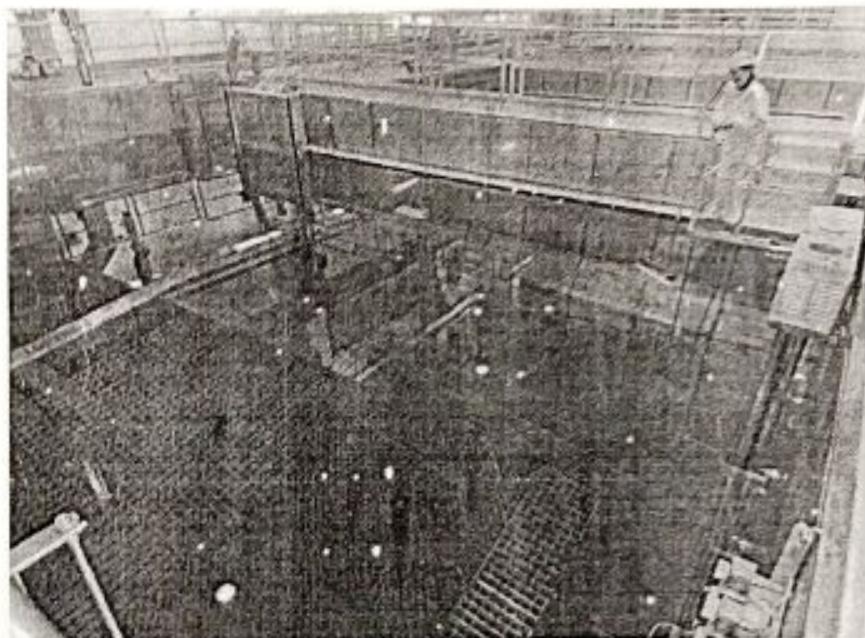


Figure 2-6: Spent fuel storage pool (Source: International Atomic Energy Agency [15])

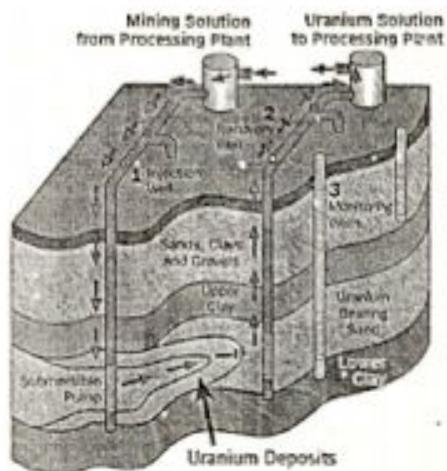


Figure 2-3: Mining via in situ leaching (Source: Nuclear Regulatory Commission [12])

- Milling: The ore or solution is then taken to a mill to be processed into uranium oxide concentrate, known as 'yellow cake' (U_3O_8) (see Figure 2-4), which contains more than 80 percent uranium. Yellow cake is packed into 200-liter drums and shipped to a conversion facility. The milling process creates a waste byproduct known as uranium tailings, which is the residual ore material. These tailings contain radon, which decays into the radioactive gas radon, and other toxic materials such as heavy metals [3]. Tailings pose a potential safety and public health hazard, and for that reason they are isolated in engineered facilities near the mine (usually the open pit) [1].



Figure 2-4: Uranium oxide concentrate (U_3O_8) (Source: World nuclear news [13])

Because fast reactors use Pu-239 as their main fuel they are being considered as a means of dealing with the nuclear waste generated by thermal reactors. When spent nuclear fuel is removed from thermal reactors about one percent is Pu-239. Since Pu-239 is highly radioactive and has a half-life of 24,000 years [21], storing it safely is very expensive. The idea is to remove the Pu-239 from the spent fuel and use it as fuel for fast reactors. This is seen as beneficial not only because it gets rid of the most hazardous waste, but it also lowers the cost of storing the remaining fuel and reduces the possibility of nuclear materials proliferation [19].

2.6 Nuclear fuel cycle

Nuclear power plants use uranium as fuel to generate electricity. Uranium is a mildly radioactive metal that is very common on the earth's crust. Before it can be used as nuclear fuel uranium must be extensively processed. The various stages that uranium goes through in the process of generating electricity are known as the nuclear fuel cycle. At the front end of the cycle are the activities related to the production of nuclear fuel; the service period is when the nuclear fuel is being used to generate electricity; and the back end of the cycle refers to the storage, reprocessing and final disposition of the spent nuclear fuel.

2.7 Front end

- Mining: Uranium mining is done with either traditional mining techniques, such as underground or open pit, or more commonly using a mining technique known as in situ leaching (ISL). ISL involves circulating oxygenated ground water through the porous ore deposit; dissolving the uranium oxide and bringing it to the surface (see Figure 2-3). The solution is then extracted and taken to a mill for further processing [1]. In 2010, U.S. operators of civilian nuclear reactors purchased the equivalent of 47 million pounds of uranium, most of which was imported. Eight percent came from the U.S.; 41 percent came from Kazakhstan, Russia and Uzbekistan; 37 percent came from Australia and Canada; and 14 percent came from Namibia, Niger and other countries [2]. While the U.S. has sufficient uranium resources to increase domestic production, deposits in other countries are larger and cheaper to mine.