

Chapter 11 Angular Momentum; General Rotation

11.1 Conceptual Questions

- 1) State the law of conservation of angular momentum for a rotating object.

Answer: The total angular momentum of a rotating object remains constant if the net external torque acting on it is zero.

Diff: 1 Page Ref: Sec. 11-1

- 2) State the law of conservation of angular momentum for a system.

Answer: The total angular momentum of a system remains constant if the net external torque acting on the system is zero.

Diff: 1 Page Ref: Sec. 11-6

- 3) A child is riding near the outer rim of a freely-rotating playground merry-go-round. What happens if the child walks towards the center?

Answer: The merry-go-round rotates more rapidly because the moment of inertia has decreased while the angular momentum is conserved.

Diff: 1 Page Ref: Sec. 11-6

- 4) A child runs and jumps tangentially onto an initially stationary playground merry-go-round. As a result, the merry-go-round begins to turn. Explain in terms of the child-merry-go-round system.

Answer: The child has an initial angular momentum about the center of the merry-go-round. The total angular momentum of the system is conserved, so the merry-go-round must rotate when the child is on it.

Diff: 1 Page Ref: Sec. 11-6

- 5) A stunt motorcyclist rides his motorcycle over a short ramp so that the motorcycle loses contact with the ground. As he does this, he guns his engine and the motorcycle's front wheel is observed to rise. Explain.

Answer: There is no external torque on the system, so the angular momentum is conserved. If the motorcycle is traveling from left to right, the increased clockwise angular momentum of the drive wheel requires a compensating counterclockwise angular momentum for the body of the motorcycle, thus raising the front end.

Diff: 1 Page Ref: Sec. 11-6

- 6) For angular momentum to be conserved, the net torque must be zero.

Answer: TRUE

Diff: 1 Page Ref: Sec. 11-1

- 7) For angular momentum to be conserved, the net force must be zero.

Answer: FALSE

Diff: 1 Page Ref: Sec. 11-1

- 8) If you cross a vector with itself, the resultant will be a unit vector.

Answer: FALSE

Diff: 1 Page Ref: Sec. 11-2

- 9) Vector cross products are not commutative.

Answer: TRUE

Diff: 1 Page Ref: Sec. 11-2

- 10) The time rate of change of the total angular momentum of a system of particles equals the resultant external torque on the system.

Answer: TRUE

Diff: 1 Page Ref: Sec. 11-4

- 11) The total force on an object is zero. The angular momentum of the object

- A) is zero.
- B) may be changing.
- C) can only be changing direction.
- D) is constant.
- E) can only be changing magnitude.

Answer: B

Diff: 1 Page Ref: Sec. 11-1

- 12) Angular momentum cannot be conserved if

- A) the angular acceleration changes.
- B) the angular velocity changes.
- C) there is a net force on the system.
- D) the moment of inertia changes.
- E) the net torque is not zero.

Answer: E

Diff: 1 Page Ref: Sec. 11-1

- 13) A figure skater is spinning slowly with arms outstretched. She brings her arms in close to her body and her angular speed increases dramatically. The speed increase is a demonstration of

- A) conservation of kinetic energy: her moment of inertia is decreased, and so her angular speed must increase to conserve energy.
- B) conservation of total energy: her moment of inertia is decreased, and so her angular speed must increase to conserve energy.
- C) conservation of angular momentum: her moment of inertia is decreased, and so her angular speed must increase to conserve angular momentum.
- D) Newton's second law for rotational motion: she exerts a torque and so her angular speed increases.
- E) This has nothing to do with mechanics, it is simply a result of her natural ability to perform.

Answer: C

Diff: 1 Page Ref: Sec. 11-1

- 14) A person sits on a freely spinning lab stool (no friction). When this person extends her arms,

- A) her moment of inertia decreases and her angular velocity increases.
- B) her moment of inertia decreases and her angular velocity decreases.
- C) her moment of inertia increases and her angular velocity increases.
- D) her moment of inertia increases and her angular velocity decreases.
- E) her moment of inertia increases and her angular velocity remains the same.

Answer: D

Diff: 1 Page Ref: Sec. 11-1

- 15) Two nonparallel vectors determine a plane. Relative to the plane determined by the position vector where the force is applied and the force vector, in which direction is the torque vector about the origin?

- A) perpendicular to the plane
- B) in the plane, but perpendicular to both the position vector and the velocity vector
- C) in the plane, but perpendicular to the velocity vector
- D) in a direction dependent on the magnitudes of the velocity and position vectors
- E) in the plane, but perpendicular to the position vector

Answer: A

Diff: 1 Page Ref: Sec. 11-2

- 16) Two nonparallel vectors determine a plane. Relative to the plane determined by the position vector and velocity vector of a particle, in which direction is the angular momentum vector about the origin?
- A) in the plane, but perpendicular to the velocity vector
 - B) in a direction dependent on the magnitudes of the velocity and position vectors
 - C) perpendicular to the plane
 - D) in the plane, but perpendicular to both the position vector and the velocity vector
 - E) in the plane, but perpendicular to the position vector

Answer: C

Diff: 1 Page Ref: Sec. 11-3

- 17) You are walking holding on to the axle of a spinning bicycle wheel with one hand on either side of the wheel. The top part of the wheel is moving away from you and the bottom is moving toward you and the axle is horizontal. As you start to turn left, you feel the right side of the axle
- A) push on your right hand toward the left.
 - B) push on your right hand toward you.
 - C) pull on your right hand away from you.
 - D) push on your right hand vertically up.
 - E) push on your right hand vertically down.

Answer: E

Diff: 2 Page Ref: Sec. 11-5

- 18) You are holding a finishing sander with your right hand. The sander has a flywheel which spins counterclockwise as seen from behind the handle. You are sanding a wall in front of you. As you turn the sander towards the right, you feel a tendency in the sander to
- A) turn towards the left.
 - B) turn upward.
 - C) turn downward.
 - D) push toward you.
 - E) pull away from you.

Answer: C

Diff: 2 Page Ref: Sec. 11-5

- 19) A merry-go-round spins freely when Janice moves quickly to the center along a radius of the merry-go-round. It is true to say that
- A) the moment of inertia of the system decreases and the angular speed increases.
 - B) the moment of inertia of the system decreases and the angular speed decreases.
 - C) the moment of inertia of the system decreases and the angular speed remains the same.
 - D) the moment of inertia of the system increases and the angular speed increases.
 - E) the moment of inertia of the system increases and the angular speed decreases.

Answer: A

Diff: 1 Page Ref: Sec. 11-6

- 20) In general, precession results when
- A) torque acts on a body with zero angular momentum.
 - B) the direction of torque acting on an object is parallel to the direction of angular momentum of the body.
 - C) there is no torque acting on a body.
 - D) the direction of torque acting on an object has a component perpendicular to the direction of angular momentum of the body.
 - E) the direction of torque acting on an object is opposite to the direction of angular momentum of the body.

Answer: D

Diff: 1 Page Ref: Sec. 11-7

11.2 Quantitative Problems

- 1) A flywheel with a moment of inertia $6.00 \times 10^6 \text{ kg}\cdot\text{m}^2$ is spinning with an angular speed of 120 rev/s about a vertical axis while it sits at the north pole. The disk is turning counterclockwise when viewed from above. The disk is brought to a rest. Treat the earth as a uniform sphere with a mass $5.976 \times 10^{24} \text{ kg}$ and a radius $6.374 \times 10^6 \text{ m}$.

- (a) What will be the magnitude of the change in angular momentum of the earth?
(b) What will be the change in length of a day on Earth?

Answer: (a) $4.52 \times 10^9 \text{ kg}\cdot\text{m}^2/\text{s}$

(b) $5.53 \times 10^{-20} \text{ s}$

Diff: 1 Page Ref: Sec. 11-6

- 2) An ice skater has a moment of inertia of $5.0 \text{ kg}\cdot\text{m}^2$ when her arms are outstretched. At this time she is spinning at 3.0 revolutions per second (rps). If she pulls in her arms and decreases her moment of inertia to $2.0 \text{ kg}\cdot\text{m}^2$, how fast will she be spinning?

- A) 2.0 rps B) 3.3 rps C) 7.5 rps D) 8.4 rps E) 10 rps

Answer: C

Diff: 1 Page Ref: Sec. 11-1

- 3) A figure skater rotating at 5.00 rad/s with arms extended has a moment of inertia of $2.25 \text{ kg}\cdot\text{m}^2$. If the arms are pulled in so the moment of inertia decreases to $1.80 \text{ kg}\cdot\text{m}^2$, what is the final angular speed?

- A) 2.25 rad/s B) 4.60 rad/s C) 6.25 rad/s D) 1.76 rad/s E) 0.81 rad/s

Answer: C

Diff: 2 Page Ref: Sec. 11-1

- 4) What is the vector product of $\vec{A} = 2.00 \hat{i} + 3.00 \hat{j} + 1.00 \hat{k}$ and $\vec{B} = 1.00 \hat{i} - 3.00 \hat{j} - 2.00 \hat{k}$?

A) $-3.00 \hat{i} + 5.00 \hat{j} - 9.00 \hat{k}$

B) $-5.00 \hat{i} + 2.00 \hat{j} - 6.00 \hat{k}$

C) $-9.00 \hat{i} - 3.00 \hat{j} - 3.00 \hat{k}$

D) $-4.00 \hat{i} + 3.00 \hat{j} - 1.00 \hat{k}$

E) $2.00 \hat{i} - 9.00 \hat{j} - 2.00 \hat{k}$

Answer: A

Diff: 1 Page Ref: Sec. 11-2

- 5) What is the vector product of $\vec{A} = 5.00 \hat{i} + 3.00 \hat{j}$ and $\vec{B} = -2.00 \hat{i} + 4.00 \hat{j}$?

A) $20.0 \hat{i} - 6.00 \hat{j}$

B) $-10.0 \hat{i} + 12.00 \hat{j}$

C) $26.0 \hat{k}$

D) 26.0

E) $20.0 \hat{i} + 6.00 \hat{j}$

Answer: C

Diff: 1 Page Ref: Sec. 11-2

6) What is the vector product of $\vec{A} = (4 \hat{i} - 3 \hat{j} - 5 \hat{k})$ and $\vec{B} = (5 \hat{i} - 4 \hat{j} + 2 \hat{k})$?

A) $(-26 \hat{i} - 33 \hat{j} - 1 \hat{k})$

B) 22

C) $(20 \hat{i} + 12 \hat{j} - 10 \hat{k})$

D) $(9 \hat{i} - 7 \hat{j} - 3 \hat{k})$

E) $(14 \hat{i} - 17 \hat{j} - 1 \hat{k})$

Answer: A

Diff: 1 Page Ref: Sec. 11-2

7) What is the magnitude of the vector product of a vector of magnitude 20.0 pointing east and a vector of magnitude 30.0 pointing 30.0° north of east?

A) 470

B) 520

C) 300

D) 340

E) 600

Answer: C

Diff: 1 Page Ref: Sec. 11-2

8) A force of $\vec{F} = 3.00 \text{ N} \hat{i} + 2.00 \text{ N} \hat{j}$ acts at a location $\vec{r} = 1.00 \text{ m} \hat{i} + 2.00 \text{ m} \hat{j}$ on an object. What is the torque that this force applies about an axis through the origin perpendicular to the xy plane?

A) $-1.00 \text{ N} \cdot \text{m} \hat{k}$

B) $7.00 \text{ N} \cdot \text{m} \hat{k}$

C) $-3.00 \text{ N} \cdot \text{m} \hat{k}$

D) $5.00 \text{ N} \cdot \text{m} \hat{k}$

E) $-8.00 \text{ N} \cdot \text{m} \hat{k}$

Answer: E

Diff: 1 Page Ref: Sec. 11-2

9) A force at $\vec{F} = 4.00 \text{ N} \hat{i} - 3.00 \text{ N} \hat{j}$ is applied to an object at position $\vec{r} = 2.00 \text{ m} \hat{i} + 3.00 \text{ m} \hat{j}$. What is the torque about the origin?

A) $8.00 \text{ N} \cdot \text{m} \hat{i} - 9.00 \text{ N} \cdot \text{m} \hat{j}$

B) $-1.00 \text{ N} \cdot \text{m} \hat{k}$

C) $8.00 \text{ N} \cdot \text{m} \hat{i} + 9.00 \text{ N} \cdot \text{m} \hat{j}$

D) $17.0 \text{ N} \cdot \text{m} \hat{k}$

E) $-18.0 \text{ N} \cdot \text{m} \hat{k}$

Answer: E

Diff: 1 Page Ref: Sec. 11-2

10) What is the torque about the origin on a particle located at $\vec{r} = (3 \hat{i} + 4 \hat{j} - 2 \hat{k}) \text{ m}$ exerted by a force $\vec{F} = (5 \hat{i} - 2 \hat{j} + 3 \hat{k}) \text{ N}$?

A) $(8 \hat{i} + 1 \hat{j} - 26 \hat{k}) \text{ N} \cdot \text{m}$

B) $(8 \hat{i} - 19 \hat{j} - 26 \hat{k}) \text{ N} \cdot \text{m}$

C) $(8 \hat{i} + 2 \hat{j} + 1 \hat{k}) \text{ N} \cdot \text{m}$

D) $(16 \hat{i} - 19 \hat{j} - 26 \hat{k}) \text{ N} \cdot \text{m}$

E) $(8 \hat{i} - 2 \hat{j} + 1 \hat{k}) \text{ N} \cdot \text{m}$

Answer: B

Diff: 1 Page Ref: Sec. 11-2

- 11) A 2.00-kg mass is located at $\vec{r} = 2.00 \text{ m } \hat{i} + 3.00 \text{ m } \hat{j}$ and has velocity $\vec{v} = 4.00 \text{ m/s } \hat{i} - 1.00 \text{ m/s } \hat{j}$. What is the angular momentum of the mass?
- A) $20.0 \text{ kg}\cdot\text{m}^2/\text{s } \hat{k}$
 - B) $-28.0 \text{ kg}\cdot\text{m}^2/\text{s } \hat{k}$
 - C) $10.0 \text{ kg}\cdot\text{m}^2/\text{s}$
 - D) $10.0 \text{ kg}\cdot\text{m}^2/\text{s } \hat{k}$
 - E) $16.0 \text{ kg}\cdot\text{m}^2/\text{s } \hat{i} - 6.00 \text{ kg}\cdot\text{m}^2/\text{s } \hat{j}$

Answer: B

Diff: 1 Page Ref: Sec. 11-3

- 12) What is the angular momentum about the origin of a particle with a mass of 500 g when it is located at $\vec{r} = (4 \hat{i} + 3 \hat{j} - 2 \hat{k}) \text{ m}$ and moving at $\vec{v} = (5 \hat{i} - 2 \hat{j} + 4 \hat{k}) \text{ m/s}$?
- A) $(24 \hat{i} - 6 \hat{j} - 8 \hat{k}) \text{ kg}\cdot\text{m}^2/\text{s}$
 - B) $(12 \hat{i} - 3 \hat{j} - 4 \hat{k}) \text{ kg}\cdot\text{m}^2/\text{s}$
 - C) $(8 \hat{i} + 14 \hat{j} - 13 \hat{k}) \text{ kg}\cdot\text{m}^2/\text{s}$
 - D) $(10 \hat{i} + 1 \hat{j} + 2 \hat{k}) \text{ kg}\cdot\text{m}^2/\text{s}$
 - E) $(4 \hat{i} - 13 \hat{j} - 11.5 \hat{k}) \text{ kg}\cdot\text{m}^2/\text{s}$

Answer: E

Diff: 1 Page Ref: Sec. 11-3

- 13) A bicycle is traveling north at 5.0 m/s. The mass of the wheel, 2.0 kg, is uniformly distributed along the rim, which has a radius of 20 cm. What are the magnitude and direction of the angular momentum of the wheel about its axle?
- A) $2.0 \text{ kg}\cdot\text{m}^2/\text{s}$ towards the west
 - B) $5.0 \text{ kg}\cdot\text{m}^2/\text{s}$ vertically upwards
 - C) $2.0 \text{ kg}\cdot\text{m}^2/\text{s}$ towards the east
 - D) $5.0 \text{ kg}\cdot\text{m}^2/\text{s}$ towards the east
 - E) $5.0 \text{ kg}\cdot\text{m}^2/\text{s}$ towards the west

Answer: A

Diff: 1 Page Ref: Sec. 11-3

- 14) The position vector of an object of mass 0.50 kg subject to a constant force is given by $\vec{r} = (at^2+bt) \hat{i} + (ct^2+dt) \hat{j} + (et^2+ft) \hat{k}$, where $a = 2.0 \text{ m/s}^2$, $b = 3.0 \text{ m/s}$, $c = 2.5 \text{ m/s}^2$, $d = -2.0 \text{ m/s}$, $e = 1.0 \text{ m/s}^2$, and $f = 4.0 \text{ m/s}$. What is the angular momentum of the object about the origin at $t = 2.0 \text{ s}$?

- A) $(24 \hat{i} - 122 \hat{j} - 89 \hat{k}) \text{ kg}\cdot\text{m}^2/\text{s}$
- B) $(-24 \hat{i} + 10 \hat{j} + 23 \hat{k}) \text{ kg}\cdot\text{m}^2/\text{s}$
- C) $(25 \hat{i} - 14 \hat{j} + 20 \hat{k}) \text{ kg}\cdot\text{m}^2/\text{s}$
- D) $(25 \hat{i} + 14 \hat{j} + 20 \hat{k}) \text{ kg}\cdot\text{m}^2/\text{s}$
- E) $(24 \hat{i} + 122 \hat{j} + 23 \hat{k}) \text{ kg}\cdot\text{m}^2/\text{s}$

Answer: B

Diff: 3 Page Ref: Sec. 11-3

- 15) A horizontal, 2.00 m long, 3.00 kg uniform beam that lies along the east-west direction is acted on by two forces. At the east end of the beam, a 200 N force pushes downward. At the west end of the beam, a 200 N force pushed upward. What is the torque about the center of mass of the beam?
- A) 800 N·m north
 - B) zero
 - C) 400 N·m north
 - D) 800 N·m south
 - E) 400 N·m south

Answer: C

Diff: 1 Page Ref: Sec. 11-5

- 16) A horizontal 2.00-m long, 5.00-kg uniform beam that lies along the east-west direction is acted on by two forces. At the east end of the beam, a 200-N forces pushes downward. At the west end of the beam, a 200-N force pushed upward. What is the angular acceleration of the beam?
- A) 240 rad/s² north
 - B) 1.33×10^2 rad/s² north
 - C) zero
 - D) 240 rad/s² south
 - E) 1.33×10^2 rad/s² south

Answer: A

Diff: 1 Page Ref: Sec. 11-5

- 17) The boom of a crane is 20 m long and is inclined 20° from the horizontal. A bucket of cement with a mass of 3000 kg is suspended from the upper end of the boom. What is the torque acting on the point where the boom is attached to the crane?
- A) 5.5×10^5 N m
 - B) 3.5×10^5 N m
 - C) 4.5×10^5 N m
 - D) 2.0×10^5 N m
 - E) 2.0×10^4 N m

Answer: A

Diff: 1 Page Ref: Sec. 11-5

- 18) An object is rotating with an angular momentum of magnitude 20.0 kg·m²/s in a direction directly east. A torque of magnitude 10.0 N·m acts in a direction 30° north of east acts on the object for 5.00 s. What is the angular momentum after the torque has acted?
- A) 45.0 kg·m²/s, 51.3° north of east
 - B) 68.1 kg·m²/s, 21.5° north of east
 - C) 25.0 kg·m²/s, 51.3° north of east
 - D) 63.3 kg·m²/s east
 - E) 32.0 kg·m²/s, 51.3° north of east

Answer: B

Diff: 1 Page Ref: Sec. 11-5

- 19) A uniform 2.00-kg circular disk of radius 20.0 cm is rotating clockwise about an axis through its center with an angular speed 30.0 revolutions per second. A second uniform 1.50-kg circular disk of radius 15.0 cm that is not rotating is dropped onto the first disk so that the axis of rotation of the first disk passes through the center of the second disk. What is the final angular speed of the two disks when they are rotating together?
- A) 16.9 revolutions per second
 - B) 19.2 revolutions per second
 - C) 21.1 revolutions per second
 - D) 17.1 revolutions per second
 - E) 12.7 revolutions per second

Answer: C

Diff: 1 Page Ref: Sec. 11-6

- 20) A 40.0-kg child running at a speed 3.00 m/s jumps on a stationary playground merry-go-round at a distance 1.50 m from the axis of rotation of the merry-go-round. The child is traveling tangential to the edge of the merry-go-round which has a $600 \text{ kg}\cdot\text{m}^2$ moment of inertia about its axis of rotation as she is running. What is the angular speed of the merry-go-round after the child jumps on it?
- A) 0.788 rad/s
 - B) 0.261 rad/s
 - C) 6.28 rev/s
 - D) 2.00 rad/s
 - E) 3.14 rev/s

Answer: B

Diff: 1 Page Ref: Sec. 11-6

- 21) A playground merry-go-round with a radius of 2.0 m and a rotational inertia of $100 \text{ kg}\cdot\text{m}^2$ is rotating at 3.0 rad/s. A child with a mass of 22 kg jumps onto the edge of the merry-go-round, traveling radially inward. What is the new angular speed of the merry-go-round?
- A) 1.6 rad/s
 - B) 1.2 rad/s
 - C) 2.4 rad/s
 - D) 2.0 rad/s
 - E) 3.4 rad/s

Answer: A

Diff: 1 Page Ref: Sec. 11-6

- 22) A playground merry-go-round with a radius of 1.8 m and a rotational inertia of $120 \text{ kg}\cdot\text{m}^2$ is stationary. A child with a mass of 25 kg gets on and walks around the edge of the merry-go-round. How many revolutions around the merry-go-round must the child make in order for the merry-go-round to make one full revolution?
- A) 2.5 revolutions
 - B) 1.5 revolutions
 - C) 3.0 revolutions
 - D) 1.0 revolutions
 - E) 2.0 revolutions

Answer: B

Diff: 2 Page Ref: Sec. 11-6

- 23) A 5.0-m radius playground merry-go-round with a moment of inertia of $2000 \text{ kg}\cdot\text{m}^2$ is rotating freely with an angular speed of 1.0 rad/s. Two people, each having a mass of 60 kg are standing right outside the edge of the merry-go-round and step on it with negligible speed. What is the angular speed of the merry-go-round right after the two people have stepped on?
- A) 0.20 rad/s
 - B) 0.40 rad/s
 - C) 0.60 rad/s
 - D) 0.80 rad/s
 - E) 0.67 rad/s

Answer: B

Diff: 2 Page Ref: Sec. 11-6

- 24) A 2.00-kg sphere is rotating about an axis through its center at 40.0 rev/s with the angular velocity in the +z direction. A torque 10.0 N·m acts on the sphere about the center of the sphere in the +x direction. What is the rate of change of the angular momentum of the sphere?
- A) $-20.0 \text{ kg}\cdot\text{m}^2/\text{s}^2 \hat{i}$
 - B) $20.0 \text{ kg}\cdot\text{m}^2/\text{s}^2 \hat{i}$
 - C) $80.0 \text{ kg}\cdot\text{m}^2/\text{s}^2 \hat{i}$
 - D) $8.00 \text{ kg}\cdot\text{m}^2/\text{s}^2 \hat{i}$
 - E) $10.0 \text{ kg}\cdot\text{m}^2/\text{s}^2 \hat{i}$

Answer: E

Diff: 1 Page Ref: Sec. 11-7

- 25) An object is rotating with an angular momentum $4.00 \text{ kg}\cdot\text{m}^2/\text{s} \hat{k}$ while being acted on by a constant torque $3.00 \text{ N}\cdot\text{m} \hat{i}$. What is the angular speed of precession of the angular velocity of the object?
- A) 1.33 rad/s
 - B) 0.750 rad/s
 - C) 12.0 rad/s
 - D) zero
 - E) It depends on the moment of inertia of the object.

Answer: B

Diff: 1 Page Ref: Sec. 11-7