Solutions to Practice Problems Chapter 3

Responses to Questions

Q3.3 $\vec{a}$ is always vertically downward and $\vec{v}$ is always tangent to the path. There is no point where $\vec{a}$ and $\vec{v}$ are parallel. At the maximum height $\vec{v}$ is horizontal so that at that point $\vec{a}$ and $\vec{v}$ are perpendicular.

Q3.10 $\boldsymbol{v}_{av} = \Delta \boldsymbol{r} / \Delta t$. For one revolution the object returns to its starting point, so $\Delta \boldsymbol{r} = \mathbf{0}$ and $\boldsymbol{v}_{av} = \mathbf{0}$. $\boldsymbol{a}_{av} = \Delta \boldsymbol{v} / \Delta t$. After one revolution the velocity vector returns to its initial value so $\Delta \boldsymbol{v} = \mathbf{0}$ and $\boldsymbol{a}$av $= \mathbf{0}$. In circular motion the directions of the velocity and acceleration are continually changing and average to zero over one complete revolution.

Q3.16 $v_x = v_0 \cos\alpha_0$, where α_0 is the launch angle, is constant throughout the motion, since $a_x = 0$. $|v_y|$ decreases to zero and then starts to increase. The speed is $\sqrt{v_x^2 + v_y^2}$. At $t = 0$, $v = v_0$. At the maximum height the speed has decreased to $v = v_0 \cos\alpha_0$, and then it increases. So, during the motion the speed reaches a minimum at the maximum height, but this minimum speed is not zero. Only graph (d) shows this behavior.

Solutions to Problem

P3.3. **(a) IDENTIFY** and **SET UP:** From $\vec{r}$ we can calculate x and y for any t.
Then use Eq. (3.2), in component form.

EXECUTE: $\vec{r} = [4.0 \text{ cm} + (2.5 \text{ cm/s}^2)t^2]\hat{i} + (5.0 \text{ cm/s})t\hat{j}$

At $t = 0$, $\vec{r} = (4.0 \text{ cm})\,\hat{i}$.

At $t = 2.0$ s, $\vec{r} = (14.0 \text{ cm})\,\hat{i} + (10.0 \text{ cm})\,\hat{j}$.

$$(v_{av})_x = \frac{\Delta x}{\Delta t} = \frac{10.0 \text{ cm}}{2.0 \text{ s}} = 5.0 \text{ cm/s}.$$

$$(v_{av})_y = \frac{\Delta y}{\Delta t} = \frac{10.0 \text{ cm}}{2.0 \text{ s}} = 5.0 \text{ cm/s}.$$

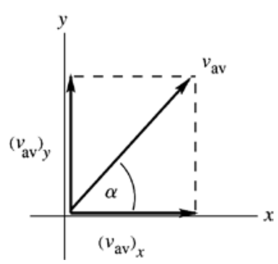

Figure 3.3a

$$v_{av} = \sqrt{(v_{av})_x^2 + (v_{av})_y^2} = 7.1 \text{ cm/s}$$

$$\tan\alpha = \frac{(v_{av})_y}{(v_{av})_x} = 1.00$$

$$\theta = 45°.$$

EVALUATE: Both x and y increase, so $\vec{v}_{av}$ is in the 1st quadrant.

(b) IDENTIFY and **SET UP:** Calculate $\vec{r}$ by taking the time derivative of $\vec{r}(t)$.

EXECUTE: $\vec{v} = \dfrac{d\vec{r}}{dt} = ([5.0\ \text{cm/s}^2]t)\hat{i} + (5.0\ \text{cm/s})\hat{j}$

$\underline{t = 0}$: $v_x = 0$, $v_y = 5.0$ cm/s; $v = 5.0$ cm/s and $\theta = 90°$

$\underline{t = 1.0\ \text{s}}$: $v_x = 5.0$ cm/s, $v_y = 5.0$ cm/s; $v = 7.1$ cm/s and $\theta = 45°$

$\underline{t = 2.0\ \text{s}}$: $v_x = 10.0$ cm/s, $v_y = 5.0$ cm/s; $v = 11$ cm/s and $\theta = 27°$

(c) The trajectory is a graph of y versus x.

$x = 4.0\ \text{cm} + (2.5\ \text{cm/s}^2)\,t^2,\quad y = (5.0\ \text{cm/s})t$

For values of t between 0 and 2.0 s, calculate x and y and plot y versus x.

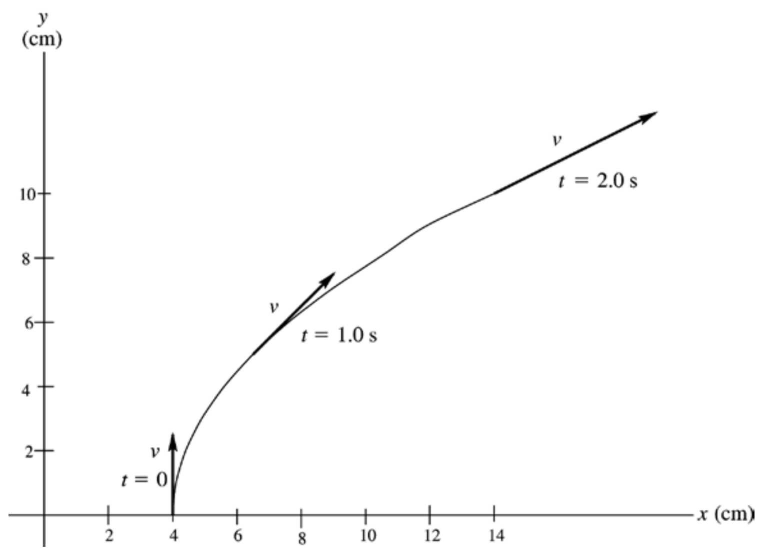

Figure 3.3b

EVALUATE: The sketch shows that the instantaneous velocity at any t is tangent to the trajectory.

P3.6. **IDENTIFY:** Use Eq. (3.8), written in component form.

SET UP: $a_x = (0.45\text{m/s}^2)\cos 31.0° = 0.39\text{m/s}^2$, $a_y = (0.45\text{m/s}^2)\sin 31.0° = 0.23\text{m/s}^2$

EXECUTE: **(a)** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$ and $v_x = 2.6\ \text{m/s} + (0.39\ \text{m/s}^2)(10.0\ \text{s}) = 6.5\ \text{m/s}$. $a_{\text{av-}y} = \dfrac{\Delta v_y}{\Delta t}$ and

$v_y = -1.8\ \text{m/s} + (0.23\ \text{m/s}^2)(10.0\ \text{s}) = 0.52\ \text{m/s}$.

(b) $v = \sqrt{(6.5\text{m/s})^2 + (0.52\text{m/s})^2} = 6.52\text{m/s}$, at an angle of $\arctan\left(\dfrac{0.52}{6.5}\right) = 4.6°$ above the horizontal.

(c) The velocity vectors $\vec{v}_1$ and $\vec{v}_2$ are sketched in Figure 3.6. The two velocity vectors differ in magnitude and direction.

EVALUATE: $\vec{v}_1$ is at an angle of $35°$ below the $+x$-axis and has magnitude $v_1 = 3.2$ m/s, so $v_2 > v_1$ and the direction of $\vec{v}_2$ is rotated counterclockwise from the direction of $\vec{v}_1$.

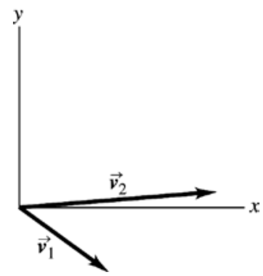

Figure 3.6

P3.8. **IDENTIFY:** Use the velocity components of a car (given as a function of time) to find the acceleration of the car as a function of time and to find the magnitude and direction of the car's velocity and acceleration at a specific time.

SET UP: $a_x = dv_x/dt$ and $a_y = dv_y/dt$; the magnitude of a vector is $A = \sqrt{(A_x^2 + A_y^2)}$.

EXECUTE: **(a)** Taking the derivatives gives $a_x(t) = (-0.0360 \text{ m/s}^3)t$ and $a_y(t) = 0.550 \text{ m/s}^2$.

(b) Evaluating the velocity components at $t = 8.00$ s gives $v_x = 3.848$ m/s and $v_y = 6.40$ m/s, which gives $v = 7.47$ m/s. The direction is $\tan\theta = \dfrac{6.40}{3.848}$ so $\theta = 59.0^{\circ}$ (counterclockwise from $+x$-axis).

(c) Evaluating the acceleration components at $t = 8.00$ s gives $a_x = 2\ 0.288 \text{ m/s}^2$ and $a_y = 0.550 \text{ m/s}^2$, which gives $a = 0.621 \text{ m/s}^2$. The angle with the $+y$ axis is given by $\tan\theta = \dfrac{0.288}{0.550}$, so $\theta = 27.6^{\circ}$. The direction is therefore 118° counterclockwise from $+x$-axis.

EVALUATE: The acceleration is not constant, so we cannot use the standard kinematics formulas.

P3.19. **IDENTIFY:** Take the origin of coordinates at the point where the quarter leaves your hand and take positive y to be upward. The quarter moves in projectile motion, with $a_x = 0$, and $a_y = -g$. It travels vertically for the time it takes it to travel horizontally 2.1 m.

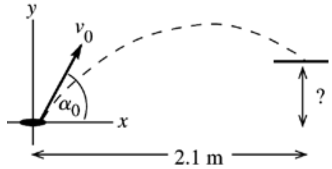

Figure 3.19

$$v_{0x} = v_0 \cos\alpha_0 = (6.4 \text{ m/s}) \ \cos 60°$$
$$v_{0x} = 3.20 \text{ m/s}$$
$$v_{0y} = v_0 \sin\alpha_0 = (6.4 \text{ m/s}) \ \sin 60°$$
$$v_{0y} = 5.54 \text{ m/s}$$

(a) SET UP: Use the horizontal (*x*-component) of motion to solve for *t*, the time the quarter travels through the air:

$t = ?,\quad x - x_0 = 2.1\text{ m},\quad v_{0x} = 3.2\text{ m/s},\quad a_x = 0$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = v_{0x}t,$ since $a_x = 0$

EXECUTE: $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{2.1\text{ m}}{3.2\text{ m/s}} = 0.656\text{ s}$

SET UP: Now find the vertical displacement of the quarter after this time:

$y - y_0 = ?,\quad a_y = -9.80\text{ m/s}^2,\quad v_{0y} = +5.54\text{ m/s},\quad t = 0.656\text{ s}$

$y - y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $y - y_0 = (5.54\text{ m/s})(0.656\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(0.656\text{ s})^2 = 3.63\text{ m} - 2.11\text{ m} = 1.5\text{ m}.$

(b) SET UP: $v_y = ?,\quad t = 0.656\text{ s},\quad a_y = -9.80\text{ m/s}^2,\quad v_{0y} = +5.54\text{ m/s}\quad v_y = v_{0y} + a_y t$

EXECUTE: $v_y = 5.54\text{ m/s} + (-9.80\text{ m/s}^2)(0.656\text{ s}) = -0.89\text{ m/s}.$

EVALUATE: The minus sign for v_y indicates that the *y*-component of $\vec{v}$ is downward. At this point the quarter has passed through the highest point in its path and is on its way down. The horizontal range if it returned to its original height (it doesn't!) would be 3.6 m. It reaches its maximum height after traveling horizontally 1.8 m, so at $x - x_0 = 2.1\text{ m}$ it is on its way down.

P3.23. IDENTIFY and **SET UP:** The stone moves in projectile motion. Its initial velocity is the same as that of the balloon. Use constant acceleration equations for the *x* and *y* components of its motion. Take $+y$ to be downward.

EXECUTE: **(a)** Use the vertical motion of the rock to find the initial height.

$t = 6.00\text{ s},\quad v_{0y} = +20.0\text{ m/s},\quad a_y = +9.80\text{ m/s}^2,\quad y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = 296\text{ m}$

(b) In 6.00 s the balloon travels downward a distance $y - y_0 = (20.0\text{ m/s})(6.00\text{ s}) = 120\text{ m}.$ So, its height above ground when the rock hits is $296\text{ m} - 120\text{ m} = 176\text{ m}.$

(c) The horizontal distance the rock travels in 6.00 s is 90.0 m. The vertical component of the distance between the rock and the basket is 176 m, so the rock is $\sqrt{(176\text{ m})^2 + (90\text{ m})^2} = 198\text{ m}$ from the basket when it hits the ground.

(d) (i) The basket has no horizontal velocity, so the rock has horizontal velocity 15.0 m/s relative to the
basket. Just before the rock hits the ground, its vertical component of velocity is $v_y = v_{0y} + a_y t = 20.0\text{ m/s} + (9.80\text{ m/s}^2)(6.00\text{ s}) = 78.8\text{ m/s},$ downward, relative to the ground. The basket is moving downward at 20.0 m/s, so relative to the basket the rock has a downward component of velocity 58.8 m/s.

(ii) horizontal: 15.0 m/s; vertical: 78.8 m/s

EVALUATE: The rock has a constant horizontal velocity and accelerates downward

P3.28. **IDENTIFY:** Each planet moves in a circular orbit and therefore has acceleration $a_{\text{rad}} = v^2/R.$

SET UP: The radius of the earth's orbit is $r = 1.50 \times 10^{11}$ m and its orbital period is $T = 365 \text{ days} = 3.16 \times 10^7$ s. For Mercury, $r = 5.79 \times 10^{10}$ m and $T = 88.0 \text{ days} = 7.60 \times 10^6$ s.

EXECUTE: **(a)** $v = \dfrac{2\pi r}{T} = 2.98 \times 10^4$ m/s

(b) $a_{\text{rad}} = \dfrac{v^2}{r} = 5.91 \times 10^{-3}$ m/s^2.

(c) $v = 4.79 \times 10^4$ m/s, and $a_{\text{rad}} = 3.96 \times 10^{-2}$ m/s^2.

EVALUATE: Mercury has a larger orbital velocity and a larger radial acceleration than earth.

3.40. **IDENTIFY:** As the runner runs around the track, his speed stays the same but the direction of his velocity changes so he has acceleration.

SET UP: $(v_x)_{\text{av}} = \dfrac{\Delta x}{\Delta t}$, $(a_x)_{\text{av}} = \dfrac{\Delta v_x}{\Delta t}$ (and likewise for the y components). The coordinates of each point are: A, $(-50 \text{ m}, 0)$; B, $(0, +50 \text{ m})$; C, $(+50 \text{ m}, 0)$; D, $(0, -50 \text{ m})$. At each point the velocity is tangent to the circular path, as shown in Figure 3.40. The components (v_x, v_y) of the velocity at each point are: A, $(0, +6.0 \text{ m/s})$; B, $(+6.0 \text{ m/s}, 0)$; C, $(0, -6.0 \text{ m/s})$; D, $(-6.0 \text{ m/s}, 0)$.

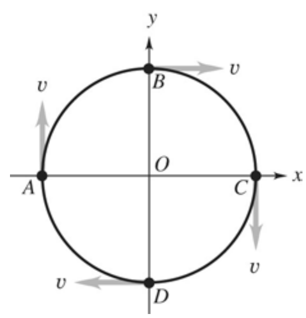

Figure 3.40

EXECUTE: **(a)** A to B: The time for one full lap is $t = \dfrac{2\pi r}{v} = \dfrac{2\pi(50 \text{ m})}{6.0 \text{ m/s}} = 52.4$ s. A to B is one-quarter lap and takes $\frac{1}{4}(52.4 \text{ s}) = 13.1$ s. $(v_x)_{\text{av}} = \dfrac{\Delta x}{\Delta t} = \dfrac{0 - (-50 \text{ m})}{13.1 \text{ s}} = 3.8$ m/s;

$(v_y)_{\text{av}} = \dfrac{\Delta y}{\Delta t} = \dfrac{+50 \text{ m} - 0}{13.1 \text{ s}} = 3.8$ m/s.

$(a_x)_{\text{av}} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{6.0 \text{ m/s} - 0}{13.1 \text{ s}} = 0.46$ m/s^2; $(a_y)_{\text{av}} = \dfrac{\Delta v_y}{\Delta t} = \dfrac{0 - 6.0 \text{ m/s}}{13.1 \text{ s}} = -0.46$ m/s^2

(b) *A* to *C*: $t = \frac{1}{2}(52.4\text{ s}) = 26.2\text{ s}$. $(v_x)_{\text{av}} = \dfrac{\Delta x}{\Delta t} = \dfrac{+50\text{ m} - (-50\text{ m})}{26.2\text{ s}} = 3.8\text{ m/s}$; $(v_y)_{\text{av}} = \dfrac{\Delta y}{\Delta t} = 0$.

$(a_x)_{\text{av}} = \dfrac{\Delta v_x}{\Delta t} = 0$; $(a_y)_{\text{av}} = \dfrac{\Delta v_y}{\Delta t} = \dfrac{-6.0\text{ m/s} - 6.0\text{ m/s}}{26.2\text{ s}} = -0.46\text{ m/s}^2$.

(c) *C* to *D*: $t = \frac{1}{4}(52.4\text{ s}) = 13.1\text{ s}$. $(v_x)_{\text{av}} = \dfrac{\Delta x}{\Delta t} = \dfrac{0 - 50\text{ m}}{13.1\text{ s}} = -3.8\text{ m/s}$;

$(v_y)_{\text{av}} = \dfrac{\Delta y}{\Delta t} = \dfrac{-50\text{ m} - 0}{13.1\text{ s}} = -3.8\text{ m/s}$. $(a_x)_{\text{av}} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{-6.0\text{ m/s} - 0}{13.1\text{ s}} = -0.46\text{ m/s}^2$;

$(a_y)_{\text{av}} = \dfrac{\Delta v_y}{\Delta t} = \dfrac{0 - (-6.0\text{ m/s})}{13.1\text{ s}} = 0.46\text{ m/s}^2$.

(d) *A* to *A*: $\Delta x = \Delta y = 0$ so $(v_x)_{\text{av}} = (v_y)_{\text{av}} = 0$, and $\Delta v_x = \Delta v_y = 0$ so $(a_x)_{\text{av}} = (a_y)_{\text{av}} = 0$.

(e) For *A* to *B*: $v_{\text{av}} = \sqrt{(v_x)_{\text{av}}^2 + (v_y)_{\text{av}}^2} = \sqrt{(3.8\text{ m/s})^2 + (3.8\text{ m/s})^2} = 5.4\text{ m/s}$. The speed is constant so the average speed is 6.0 m/s. The average speed is larger than the magnitude of the average velocity because the distance traveled is larger than the magnitude of the displacement.

(f) Velocity is a vector, with both magnitude and direction. The magnitude of the velocity is constant but its direction is changing.

EVALUATE: For this motion the acceleration describes the rate of change of the direction of the velocity, not the rate of change of the speed.

P3.42. **IDENTIFY:** Use Eqs. (2.17) and (2.18).

SET UP: At the maximum height $v_y = 0$.

EXECUTE: **(a)** $v_x = v_{0x} + \dfrac{\alpha}{3}t^3,\ v_y = v_{0y} + \beta t - \dfrac{\gamma}{2}t^2$, and $x = v_{0x}t + \dfrac{\alpha}{12}t^4,\ y = v_{0y}t + \dfrac{\beta}{2}t^2 - \dfrac{\gamma}{6}t^3$.

(b) Setting $v_y = 0$ yields a quadratic in t, $0 = v_{0y} + \beta t - \dfrac{\gamma}{2}t^2$, which has as the positive solution

$t = \dfrac{1}{\gamma}\left[\beta + \sqrt{\beta^2 + 2v_{0y}\gamma}\right] = 13.59\text{ s}$. Using this time in the expression for *y*(*t*) gives a maximum height of 341 m.

(c) The path of the rocket is sketched in Figure 3.42.

(d) $y = 0$ gives $0 = v_{0y}t + \dfrac{\beta}{2}t^2 - \dfrac{\gamma}{6}t^3$ and $\dfrac{\gamma}{6}t^2 - \dfrac{\beta}{2}t - v_{0y} = 0$. The positive solution is *t* = 20.73 s. For this *t*, $x = 3.85\times 10^4\text{ m}$.

EVALUATE: The graph in part (c) shows the path is not symmetric about the highest point and the time to return to the ground is less than twice the time to the maximum height.

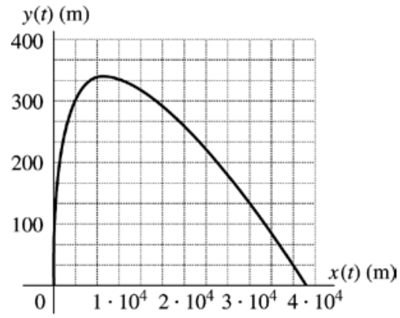

Figure 3.42

P3.56. **IDENTIFY:** The equipment moves in projectile motion. The distance *D* is the horizontal range of the equipment plus the distance the ship moves while the equipment is in the air.

SET UP: For the motion of the equipment take $+x$ to be to the right and $+y$ to be upward. Then $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$, $v_{0x} = v_0 \cos\alpha_0 = 7.50\ \text{m/s}$ and $v_{0y} = v_0 \sin\alpha_0 = 13.0\ \text{m/s}.$ When the equipment lands in the front of the ship, $y - y_0 = -8.75\ \text{m}.$

EXECUTE: Use the vertical motion of the equipment to find its time in the air: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \frac{1}{9.80}\left(13.0 \pm \sqrt{(-13.0)^2 + 4(4.90)(8.75)}\right)\ \text{s}.$ The positive root is $t = 3.21\ \text{s}.$ The horizontal range of the equipment is $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (7.50\ \text{m/s})(3.21\ \text{s}) = 24.1\ \text{m}.$ In 3.21 s the ship moves a horizontal distance $(0.450\ \text{m/s})(3.21\ \text{s}) = 1.44\ \text{m},$ so $D = 24.1\ \text{m} + 1.44\ \text{m} = 25.5\ \text{m}.$

EVALUATE: The equation $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$ from Example 3.8 can't be used because the starting and ending points of the projectile motion are at different heights.

P3.67. **(a) IDENTIFY:** Projectile motion.

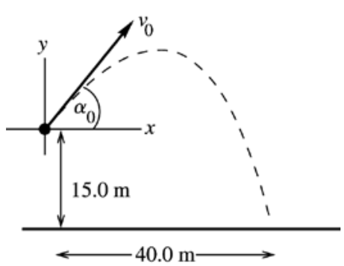

Figure 3.67

Take the origin of coordinates at the top of the ramp and take $+y$ to be upward.
The problem specifies that the object is displaced 40.0 m to the right when it is 15.0 m below the origin.

We don't know *t*, the time in the air, and we don't know v_0. Write down the equations for the horizontal and vertical displacements. Combine these two equations to eliminate one unknown.

SET UP: <u>*y*-component:</u>

$y-y_0=-15.0\text{ m},\quad a_y=-9.80\text{ m/s}^2,\quad v_{0y}=v_0\sin 53.0°$

$y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$

EXECUTE: $-15.0\text{ m}=(v_0\sin 53.0°)\,t-(4.90\text{ m/s}^2)\,t^2$

SET UP: <u>*x*-component:</u>

$x-x_0=40.0\text{ m},\quad a_x=0,\quad v_{0x}=v_0\cos 53.0°$

$x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$

EXECUTE: $40.0\text{ m}=(v_0t)\cos 53.0°$

The second equation says $v_0t=\dfrac{40.0\text{ m}}{\cos 53.0°}=66.47\text{ m}.$

Use this to replace v_0t in the first equation:

$-15.0\text{ m}=(66.47\text{ m})\sin 53°-(4.90\text{ m/s}^2)\,t^2$

$$t=\sqrt{\frac{(66.46\text{ m})\sin 53°+15.0\text{ m}}{4.90\text{ m/s}^2}}=\sqrt{\frac{68.08\text{ m}}{4.90\text{ m/s}^2}}=3.727\text{ s}.$$

Now that we have *t* we can use the *x*-component equation to solve for v_0:

$$v_0=\frac{40.0\text{ m}}{t\cos 53.0°}=\frac{40.0\text{ m}}{(3.727\text{ s})\cos 53.0°}=17.8\text{ m/s}.$$

EVALUATE: Using these values of v_0 and *t* in the $y=y_0=v_{0y}+\frac{1}{2}a_yt^2$ equation verifies that $y-y_0=-15.0\text{ m}.$

(b) IDENTIFY: $v_0=(17.8\text{ m/s})/2=8.9\text{ m/s}$

This is less than the speed required to make it to the other side, so he lands in the river. Use the vertical motion to find the time it takes him to reach the water:

SET UP: $y-y_0=-100\text{ m};\quad v_{0y}=+v_0\sin 53.0°=7.11\text{ m/s};\quad a_y=-9.80\text{ m/s}^2$

$y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $-100=7.11t-4.90t^2$

EXECUTE: $4.90t^2-7.11t-100=0$ and $t=\frac{1}{9.80}\left(7.11\pm\sqrt{(7.11)^2-4\,(4.90)(-100)}\right)$

$t=0.726\text{ s}\pm 4.57\text{ s}$ so $t=5.30\text{ s}.$

The horizontal distance he travels in this time is

$$x-x_0=v_{0x}t=(v_0\cos 53.0°)\,t=(5.36\text{ m/s})(5.30\text{ s})=28.4\text{ m}.$$

He lands in the river a horizontal distance of 28.4 m from his launch point.

EVALUATE: He has half the minimum speed and makes it only about halfway across.