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Solutions to Practice Problems Chapter 9

Responses to Questions

Q9.4 Yes, a is the rate of change of the speed so it is the same for all points on the chain. $v = r_{\text{rear}}\omega_{\text{rear}} = r_{\text{front}}\omega_{\text{front}}$ so $\omega_{\text{front}} / \omega_{\text{rear}} = r_{\text{rear}} / r_{\text{front}}$. The angular acceleration is the rate of change of the angular speed ω so $\alpha_{\text{front}} / \alpha_{\text{rear}} = r_{\text{rear}} / r_{\text{front}}$. ω and α are larger for the smaller sprocket.

Q9.9 It is not possible for a body to have the same moment of inertia for all possible axes. The mass of the body cannot be distributed the same relative to all axes, including those that lie outside the body. Yes, a sphere has the same moment of inertia for all axes passing through its center.

Q9.17 Ball *A* will have more kinetic energy. Conservation of energy applied to the system of the ball and the pulley gives $mgd = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$, where *d* is the distance the ball has fallen and *m* is the mass of the ball. $\omega = v / R$. Pulley *B* has a greater moment of inertia for rotation about an axis at its center. Therefore, for a given *d*, more of the total kinetic energy of the system will reside with the pulley in case *B* and therefore the kinetic energy of the ball will be less.

Solutions to Problem

P9.1. **IDENTIFY:** $s = r\theta$, with θ in radians.

SET UP: $\pi \text{ rad} = 180°$.

EXECUTE: **(a)** $\theta = \dfrac{s}{r} = \dfrac{1.50 \text{ m}}{2.50 \text{ m}} = 0.600 \text{ rad} = 34.4°$

(b) $r = \dfrac{s}{\theta} = \dfrac{14.0 \text{ cm}}{(128°)(\pi \text{ rad}/180°)} = 6.27 \text{ cm}$

(c) $s = r\theta = (1.50 \text{ m})(0.700 \text{ rad}) = 1.05 \text{ m}$

EVALUATE: An angle is the ratio of two lengths and is dimensionless. But, when $s = r\theta$ is used, θ must be in radians. Or, if $\theta = s/r$ is used to calculate θ, the calculation gives θ in radians.

P9.6. **IDENTIFY:** $\omega_z(t) = \dfrac{d\theta}{dt}$. $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. $\omega_{\text{av}-z} = \dfrac{\Delta\theta}{\Delta t}$.

SET UP: $\omega_z = (250\text{ rad/s}) - (40.0\text{ rad/s}^2)t - (4.50\text{ rad/s}^3)t^2$. $\alpha_z = -(40.0\text{ rad/s}^2) - (9.00\text{ rad/s}^3)t$.

EXECUTE: **(a)** Setting $\omega_z = 0$ results in a quadratic in *t*. The only positive root is $t = 4.23\text{ s}$.

(b) At $t = 4.23\text{ s}$, $\alpha_z = -78.1\text{ rad/s}^2$.

(c) At $t = 4.23\text{ s}$, $\theta = 586\text{ rad} = 93.3\text{ rev}$.

(d) At $t = 0$, $\omega_z = 250\text{ rad/s}$.

(e) $\omega_{\text{av-z}} = \dfrac{586\text{ rad}}{4.23\text{ s}} = 138\text{ rad/s}$.

EVALUATE: Between $t = 0$ and $t = 4.23\text{ s}$, ω_z decreases from 250 rad/s to zero. ω_z is not linear in *t*, so $\omega_{\text{av-z}}$ is not midway between the values of ω_z at the beginning and end of the interval.

P9.11. **IDENTIFY:** Apply the constant angular acceleration equations to the motion. The target variables are *t* and $\theta - \theta_0$.

SET UP: **(a)** $\alpha_z = 1.50\text{ rad/s}^2$; $\omega_{0z} = 0$ (starts from rest); $\omega_z = 36.0\text{ rad/s}$; $t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{36.0\text{ rad/s} - 0}{1.50\text{ rad/s}^2} = 24.0\text{ s}$

(b) $\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = 0 + \frac{1}{2}(1.50\text{ rad/s}^2)(24.0\text{ s})^2 = 432\text{ rad}$

$\theta - \theta_0 = 432\text{ rad}(1\text{ rev}/2\pi\text{ rad}) = 68.8\text{ rev}$

EVALUATE: We could use $\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$ to calculate $\theta - \theta_0 = \frac{1}{2}(0 + 36.0\text{ rad/s})(24.0\text{ s}) = 432\text{ rad}$, which checks.

P9.16. **IDENTIFY:** Apply the constant angular acceleration equations separately to the time intervals 0 to 2.00 s and 2.00 s until the wheel stops.

(a) SET UP: Consider the motion from $t = 0$ to $t = 2.00\text{ s}$:

$\theta - \theta_0 = ?$; $\omega_{0z} = 24.0\text{ rad/s}$; $\alpha_z = 30.0\text{ rad/s}^2$; $t = 2.00\text{ s}$

EXECUTE: $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (24.0\text{ rad/s})(2.00\text{ s}) + \frac{1}{2}(30.0\text{ rad/s}^2)(2.00\text{ s})^2$

$\theta - \theta_0 = 48.0\text{ rad} + 60.0\text{ rad} = 108\text{ rad}$

Total angular displacement from $t = 0$ until stops: $108\text{ rad} + 432\text{ rad} = 540\text{ rad}$

Note: At $t = 2.00\text{ s}$, $\omega_z = \omega_{0z} + \alpha_z t = 24.0\text{ rad/s} + (30.0\text{ rad/s}^2)(2.00\text{ s}) = 84.0\text{ rad/s}$; angular speed when breaker trips.

(b) SET UP: Consider the motion from when the circuit breaker trips until the wheel stops. For this calculation let $t = 0$ when the breaker trips.

$t = ?;\ \theta - \theta_0 = 432 \text{ rad};\ \omega_z = 0;\ \omega_{0z} = 84.0 \text{ rad/s}$ (from part (a))

$$\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t$$

EXECUTE: $t = \dfrac{2(\theta - \theta_0)}{\omega_{0z} + \omega_z} = \dfrac{2(432 \text{ rad})}{84.0 \text{ rad/s} + 0} = 10.3 \text{ s}$

The wheel stops 10.3 s after the breaker trips so $2.00 \text{ s} + 10.3 \text{ s} = 12.3 \text{ s}$ from the beginning.

(c) SET UP: $\alpha_z = ?;$ consider the same motion as in part (b):

$$\omega_z = \omega_{0z} + \alpha_z t$$

EXECUTE: $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{0 - 84.0 \text{ rad/s}}{10.3 \text{ s}} = -8.16 \text{ rad/s}^2$

EVALUATE: The angular acceleration is positive while the wheel is speeding up and negative while it is slowing down. We could also use $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ to calculate $\alpha_z = \dfrac{\omega_z^2 - \omega_{0z}^2}{2(\theta - \theta_0)} = \dfrac{0 - (84.0 \text{ rad/s})^2}{2(432 \text{ rad})} = -8.16 \text{ rad/s}^2$ for the acceleration after the breaker trips.

P9.20. IDENTIFY: Linear and angular velocities are related by $v = r\omega$. Use $\omega_z = \omega_{0z} + \alpha_z t$ to calculate α_z.

SET UP: $\omega = v/r$ gives ω in rad/s.

EXECUTE: **(a)** $\dfrac{1.25 \text{ m/s}}{25.0 \times 10^{-3} \text{ m}} = 50.0 \text{ rad/s},\ \dfrac{1.25 \text{ m/s}}{58.0 \times 10^{-3} \text{ m}} = 21.6 \text{ rad/s}.$

(b) $(1.25 \text{ m/s})(74.0 \text{ min})(60 \text{ s/min}) = 5.55 \text{ km}.$

(c) $\alpha_z = \dfrac{21.55 \text{ rad/s} - 50.0 \text{ rad/s}}{(74.0 \text{ min})(60 \text{ s/min})} = -6.41 \times 10^{-3} \text{ rad/s}^2.$

EVALUATE: The width of the tracks is very small, so the total track length on the disc is huge.

P9.24. IDENTIFY: Apply constant angular acceleration equations. $v = r\omega$. A point on the rim has both tangential and radial components of acceleration.

SET UP: $a_{\tan} = r\alpha$ and $a_{\text{rad}} = r\omega^2$.

EXECUTE: **(a)** $\omega_z = \omega_{0z} + \alpha_z t\ =\ 0.250 \text{ rev/s} + (0.900 \text{ rev/s}^2)(0.200 \text{ s}) = 0.430 \text{ rev/s}$

(Note that since ω_{0z} and α_z are given in terms of revolutions, it's not necessary to convert to radians).

(b) $\omega_{\text{av-}z}\Delta t = (0.340 \text{ rev/s})(0.2 \text{ s}) = 0.068 \text{ rev}.$

(c) Here, the conversion to radians must be made to use Eq. (9.13), and

$$v = r\omega = \left(\frac{0.750 \text{ m}}{2}\right)(0.430 \text{ rev/s})(2\pi \text{ rad/rev}) = 1.01 \text{ m/s}.$$

(d) Combining Eqs. (9.14) and (9.15), $a = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2}.$

$$a = \sqrt{\left[((0.430 \text{ rev/s})(2\pi \text{ rad/rev}))^2(0.375 \text{ m})\right]^2 + \left[(0.900 \text{ rev/s}^2)(2\pi \text{ rad/rev})(0.375 \text{ m})\right]^2}.$$

$a = 3.46 \text{ m/s}^2$.

EVALUATE: If the angular acceleration is constant, $a_{\tan}$ is constant but a_{rad} increases as ω increases.

P9.30. IDENTIFY and SET UP: Use Eq. (9.16). Treat the spheres as point masses and ignore I of the light rods.

EXECUTE: The object is shown in Figure 9.30a.

(a)

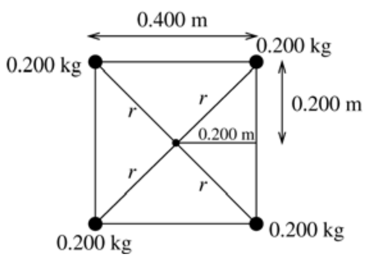

Figure 9.30a

$$r = \sqrt{(0.200 \text{ m})^2 + (0.200 \text{ m})^2} = 0.2828 \text{ m}$$

$$I = \sum m_i r_i^2 = 4(0.200 \text{ kg})(0.2828 \text{ m})^2$$

$$I = 0.0640 \text{ kg} \cdot \text{m}^2$$

(b) The object is shown in Figure 9.30b.

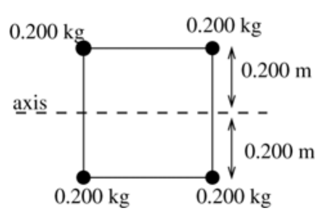

Figure 9.30b

$$r = 0.200 \text{ m}$$

$$I = \sum m_i r_i^2 = 4(0.200 \text{ kg})(0.200 \text{ m})^2$$

$$I = 0.0320 \text{ kg} \cdot \text{m}^2$$

(c) The object is shown in Figure 9.30c.

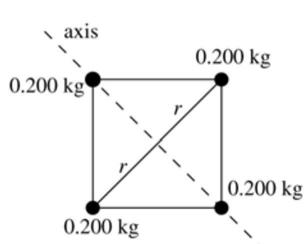

Figure 9.30c

$$r = 0.2828 \text{ m}$$

$$I = \sum m_i r_i^2 = 2(0.200 \text{ kg})(0.2828 \text{ m})^2$$

$$I = 0.0320 \text{ kg} \cdot \text{m}^2$$

P9.40. IDENTIFY: Knowing the angular acceleration of the sphere, we can use angular kinematics (with constant angular acceleration) to find its angular velocity. Then using its mass and radius, we can find its kinetic energy, the target variable.

SET UP: $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$, $K = \frac{1}{2}I\omega^2$, and $I = \frac{2}{3}MR^2$ for a uniform hollow spherical shell.

EXECUTE: $I = \frac{2}{3}MR^2 = \frac{2}{3}(8.20\text{ kg})(0.220\text{ m})^2 = 0.2646\text{ kg}\cdot\text{m}^2$. Converting the angle to radians gives $\theta - \theta_0 = (6.00\text{ rev})(2\pi\text{ rad/1 rev}) = 37.70\text{ rad}$. The angular velocity is $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$, which gives $\omega_z = \sqrt{2\alpha_z(\theta - \theta_0)} = \sqrt{2(0.890\text{ rad/s}^2)(37.70\text{ rad})} = 8.192\text{ rad/s}$.

$K = \frac{1}{2}(0.2646\text{ kg}\cdot\text{m}^2)(8.192\text{ rad/s})^2 = 8.88\text{ J}$.

EVALUATE: The angular velocity must be in radians to use the formula $K = \frac{1}{2}I\omega^2$.

P9.49. IDENTIFY: With constant acceleration, we can use kinematics to find the speed of the falling object. Then we can apply the work-energy expression to the entire system and find the moment of inertia of the wheel. Finally, using its radius we can find its mass, the target variable.

SET UP: With constant acceleration, $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$. The angular velocity of the wheel is related to the linear velocity of the falling mass by $\omega_z = \frac{v_y}{R}$. The work-energy theorem is $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, and the moment of inertia of a uniform disk is $I = \frac{1}{2}MR^2$.

EXECUTE: Find v_y, the velocity of the block after it has descended 3.00 m.

$y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$ gives $v_y = \frac{2(y - y_0)}{t} = \frac{2(3.00\text{ m})}{2.00\text{ s}} = 3.00\text{ m/s}$. For the wheel,

$\omega_z = \frac{v_y}{R} = \frac{3.00\text{ m/s}}{0.280\text{ m}} = 10.71\text{ rad/s}$. Apply the work-energy expression: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, giving $mg(3.00\text{ m}) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. Solving for I gives $I = \frac{2}{\omega^2}\left[mg(3.00\text{ m}) - \frac{1}{2}mv^2\right]$.

$I = \frac{2}{(10.71\text{ rad/s})^2}\left[(4.20\text{ kg})(9.8\text{ m/s}^2)(3.00\text{ m}) - \frac{1}{2}(4.20\text{ kg})(3.00\text{ m/s})^2\right]$. $I = 1.824\text{ kg}\cdot\text{m}^2$. For a solid disk, $I = \frac{1}{2}MR^2$ gives $M = \frac{2I}{R^2} = \frac{2(1.824\text{ kg}\cdot\text{m}^2)}{(0.280\text{ m})^2} = 46.5\text{ kg}$.

EVALUATE: The gravitational potential of the falling object is converted into the kinetic energy of that object and the rotational kinetic energy of the wheel.

P9.54. **IDENTIFY:** Apply Eq. (9.19), the parallel-axis theorem.

SET UP: The center of mass of the hoop is at its geometrical center.

EXECUTE: In Eq. (9.19), $I_{cm} = MR^2$ and $d = R^2$, so $I_P = 2MR^2$.

EVALUATE: I is larger for an axis at the edge than for an axis at the center. Some mass is closer than distance R from the axis but some is also farther away. Since I for each piece of the hoop is proportional to the square of the distance from the axis, the increase in distance has a larger effect.

P9.60. **IDENTIFY:** Apply Eq. (9.20).

SET UP: For this case, $dm = \gamma\, dx$.

EXECUTE: **(a)** $M = \int dm = \int_0^L \gamma x\, dx = \gamma \left.\frac{x^2}{2}\right|_0^L = \frac{\gamma L^2}{2}$

(b) $I = \int_0^L x^2(\gamma x)dx = \gamma \left.\frac{x^4}{4}\right|_0^L = \frac{\gamma L^4}{4} = \frac{M}{2}L^2$. This is larger than the moment of inertia of a uniform rod of the same mass and length, since the mass density is greater farther away from the axis than nearer the axis.

(c) $I = \int_0^L (L-x)^2 \gamma x dx = \gamma \int_0^L (L^2 x - 2Lx^2 + x^3)dx = \gamma \left.\left(L^2 \frac{x^2}{2} - 2L\frac{x^3}{3} + \frac{x^4}{4}\right)\right|_0^L = \gamma \frac{L^4}{12} = \frac{M}{6}L^2$.

This is a third of the result of part (b), reflecting the fact that more of the mass is concentrated at the right end.

EVALUATE: For a uniform rod with an axis at one end, $I = \frac{1}{3}ML^2$. The result in (b) is larger than this and the result in (c) is smaller than this.