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# Solutions to Practice Problems Chapter 7

## Responses to Questions

**Q7.3** No Friction: The kinetic energy at the bottom equals the gravitational potential energy at the top, if the potential energy is taken to be zero at the bottom. The change in gravitational potential energy depends only on the change in height and is independent of the path, so the speed at the bottom does not depend on the shape of the ramp.

Friction: $K_2 = U_1 + W_f$. The speed at the bottom depends on the amount of mechanical energy lost due to the negative work done by friction. The work done by friction depends on the path, so the speed at the bottom depends on the shape of the ramp.

**Q7.7** To increase the mechanical energy, the total work done by friction would have to be positive. Whether or not this can happen depends on the system. If you set a box on a moving conveyor belt, the friction force on the box does positive work and gives the box kinetic energy. But the friction exerted on the belt by the box does negative work on the belt. By Newton's 3rd law the friction force exerted by the belt on the box equals the magnitude of the friction force exerted by the box on the belt. If the box doesn't slip the magnitudes of the displacement of each object are the same and the total work done by friction is zero. If the box slips before reaching the same speed as the belt, the belt travels farther than the box and the total work done is negative. If the box is the system, friction has increased the mechanical energy. If the box and belt are the system, either the mechanical energy of the system has decreased or it has stayed the same. For an isolated system, friction never increases the mechanical energy, it is always a dissipative force.

**Q7.14** Gravity is a conservative force. The work done by gravity depends only on the initial and final heights of the object. It is independent of the path and can be expressed in terms of the change in a potential energy function. Friction is a non-conservative force. The work done by friction depends on the path taken between the initial and final positions. The work done by friction therefore cannot be expressed in terms of a change in a potential energy function.

## Solutions to Problem

**P7.2.** **IDENTIFY:** The change in height of a jumper causes a change in their potential energy.

**SET UP:** Use $\Delta U_{\text{grav}} = mg(y_f - y_i)$.

**EXECUTE:** $\Delta U_{\text{grav}} = (72\text{ kg})(9.80\text{ m/s}^2)(0.60\text{ m}) = 420\text{ J}$.

**EVALUATE:** This gravitational potential energy comes from elastic potential energy stored in the jumper's tensed muscles.

**P7.6.** **IDENTIFY:** The normal force does no work, so only gravity does work and Eq. (7.4) applies.

**SET UP:** $K_1 = 0$. The crate's initial point is at a vertical height of $d\sin\alpha$ above the bottom of the ramp.

**EXECUTE:** **(a)** $y_2 = 0$, $y_1 = d\sin\alpha$. $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $U_{\text{grav},1} = K_2$. $mgd\sin\alpha = \frac{1}{2}mv_2^2$ and $v_2 = \sqrt{2gd\sin\alpha}$.

**(b)** $y_1 = 0$, $y_2 = -d\sin\alpha$. $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $0 = K_2 + U_{\text{grav},2}$. $0 = \frac{1}{2}mv_2^2 + (-mgd\sin\alpha)$ and $v_2 = \sqrt{2gd\sin\alpha}$, the same as in part (a).

**(c)** The normal force is perpendicular to the displacement and does no work.

**EVALUATE:** When we use $U_{\text{grav}} = mgy$ we can take any point as $y = 0$ but we must take $+y$ to be upward.

**P7.11.** **IDENTIFY:** Apply Eq. (7.7) to the motion of the car.

**SET UP:** Take $y = 0$ at point *A*. Let point 1 be *A* and point 2 be *B*.

$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$$

**EXECUTE:** $U_1 = 0$, $U_2 = mg(2R) = 28{,}224\text{ J}$, $W_{\text{other}} = W_f$

$K_1 = \frac{1}{2}mv_1^2 = 37{,}500\text{ J}$, $K_2 = \frac{1}{2}mv_2^2 = 3840\text{ J}$

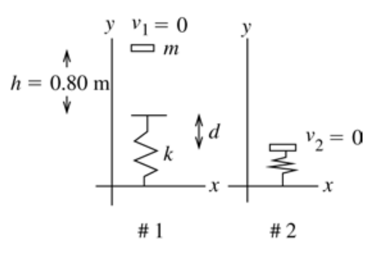
The work-energy relation then gives $W_f = K_2 + U_2 - K_1 = -5400\text{ J}$.

**EVALUATE:** Friction does negative work. The final mechanical energy $(K_2 + U_2 = 32{,}064\text{ J})$ is less than the initial mechanical energy $(K_1 + U_1 = 37{,}500\text{ J})$ because of the energy removed by friction work.

**P7.19.** **IDENTIFY** and **SET UP:** Use energy methods. There are changes in both elastic and gravitational potential energy; elastic; $U = \frac{1}{2}kx^2$, gravitational: $U = mgy$.

**EXECUTE:** **(a)** $U = \frac{1}{2}kx^2$ so $x = \sqrt{\dfrac{2U}{k}} = \sqrt{\dfrac{2(3.20\text{ J})}{1600\text{ N/m}}} = 0.0632\text{ m} = 6.32\text{ cm}$

**(b)** Points 1 and 2 in the motion are sketched in Figure 7.19.




$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$W_{\text{other}} = 0$ (Only work is that done by gravity and spring force)

$K_1 = 0$, $K_2 = 0$

$y = 0$ at final position of book

$U_1 = mg(h + d)$, $U_2 = \frac{1}{2}kd^2$

**Figure 7.19**

$0 + mg(h+d) + 0 = \frac{1}{2}kd^2$

The original gravitational potential energy of the system is converted into potential energy of the compressed spring.

$\frac{1}{2}kd^2 - mgd - mgh = 0$

$$d = \frac{1}{k}\left(mg \pm \sqrt{(mg)^2 + 4\left(\frac{1}{2}k\right)(mgh)}\right)$$

$d$ must be positive, so $d = \frac{1}{k}\left(mg + \sqrt{(mg)^2 + 2kmgh}\right)$

$$d = \frac{1}{1600\ \text{N/m}}(1.20\ \text{kg})(9.80\ \text{m/s}^2) +$$
$$\sqrt{((1.20\ \text{kg})(9.80\ \text{m/s}^2))^2 + 2(1600\ \text{N/m})(1.20\ \text{kg})(9.80\ \text{m/s}^2)(0.80\ \text{m})}$$

$d = 0.0074\ \text{m} + 0.1087\ \text{m} = 0.12\ \text{m} = 12\ \text{cm}$

**EVALUATE:** It was important to recognize that the total displacement was $h+d;$ gravity continues to do work as the book moves against the spring. Also note that with the spring compressed 0.12 m it exerts an upward force (192 N) greater than the weight of the book (11.8 N). The book will be accelerated upward from this position.

**P7.23.** **IDENTIFY:** Only the spring does work and Eq. (7.11) applies. $a = \frac{F}{m} = \frac{-kx}{m},$ where $F$ is the force the spring exerts on the mass.

**SET UP:** Let point 1 be the initial position of the mass against the compressed spring, so $K_1 = 0$ and $U_1 = 11.5\ \text{J}.$ Let point 2 be where the mass leaves the spring, so $U_{\text{el},2} = 0.$

**EXECUTE:** **(a)** $K_1 + U_{\text{el},1} = K_2 + U_{\text{el},2}$ gives $U_{\text{el},1} = K_2.$ $\frac{1}{2}mv_2^2 = U_{\text{el},1}$ and

$$v_2 = \sqrt{\frac{2U_{\text{el},1}}{m}} = \sqrt{\frac{2(11.5\ \text{J})}{2.50\ \text{kg}}} = 3.03\ \text{m/s}.$$

$K$ is largest when $U_{\text{el}}$ is least and this is when the mass leaves the spring. The mass achieves its maximum speed of 3.03 m/s as it leaves the spring and then slides along the surface with constant speed.

**(b)** The acceleration is greatest when the force on the mass is the greatest, and this is when the spring has its maximum compression. $U_{\text{el}} = \frac{1}{2}kx^2$ so

$x = -\sqrt{\frac{2U_{\text{el}}}{k}} = 2\sqrt{\frac{2(11.5\ \text{J})}{2500\ \text{N/m}}} = -0.0959\ \text{m}.$ The minus sign indicates compression. $F = -kx = ma_x$

and $a_x = -\frac{kx}{m} = -\frac{(2500\ \text{N/m})(-0.0959\ \text{m})}{2.50\ \text{kg}} = 95.9\ \text{m/s}^2.$

**EVALUATE:** If the end of the spring is displaced to the left when the spring is compressed, then $a_x$ in part (b) is to the right, and vice versa.

**P7.27. IDENTIFY:** Apply $W_{f_k} = f_k s\cos\phi$. $f_k = \mu_k n$.

**SET UP:** For a circular trip the distance traveled is $d = 2\pi r$. At each point in the motion the friction force and the displacement are in opposite directions and $\phi = 180°$. Therefore, $W_{f_k} = -f_k d = -f_k(2\pi r)$. $n = mg$ so $f_k = \mu_k mg$.

**EXECUTE:** **(a)** $W_{f_k} = -\mu_k mg 2\pi r = -(0.250)(10.0\text{ kg})(9.80\text{ m/s}^2)(2\pi)(2.00\text{ m}) = -308\text{ J}$.

**(b)** The distance along the path doubles so the work done doubles and becomes $-616$ J.
**(c)** The work done for a round trip displacement is not zero and friction is a nonconservative force.
**EVALUATE:** The direction of the friction force depends on the direction of motion of the object and that is why friction is a nonconservative force.

**P7.30. IDENTIFY** and **SET UP:** The force is not constant so we must use Eq. (6.14) to calculate *W*. The properties of work done by a conservative force are described in Section 7.3.

$$W = \int_1^2 \vec{F}\cdot d\vec{l}, \quad \vec{F} = -\alpha x^2 \hat{i}$$

**EXECUTE:** **(a)** $d\vec{l} = dy\hat{j}$ (*x* is constant; the displacement is in the $+y$-direction)
$\vec{F}\cdot d\vec{l} = 0$ (since $\hat{i}\cdot\hat{j} = 0$) and thus $W = 0$.
**(b)** $d\vec{l} = dx\hat{i}$
$\vec{F}\cdot d\vec{l} = (-\alpha x^2\hat{i})\cdot(dx\hat{i}) = -\alpha x^2\,dx$

$$W = \int_{x_1}^{x_2}(-\alpha x^2)\,dx = -\tfrac{1}{3}\alpha x^3\big|_{x_1}^{x_2} = -\tfrac{1}{3}\alpha\,(x_2^3 - x_1^3) = -\frac{12\text{ N/m}^2}{3}\,((0.300\text{ m})^3 - (0.10\text{ m})^3) = -0.10\text{ J}$$

**(c)** $d\vec{l} = dx\hat{i}$ as in part (b), but now $x_1 = 0.30$ m and $x_2 = 0.10$ m

$$W = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = +0.10\text{ J}$$

**(d) EVALUATE:** The total work for the displacement along the *x*-axis from 0.10 m to 0.30 m and then back to 0.10 m is the sum of the results of parts (b) and (c), which is zero. The total work is zero when the starting and ending points are the same, so the force is conservative.
**EXECUTE:** $W_{x_1\to x_2} = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = \tfrac{1}{3}\alpha x_1^3 - \tfrac{1}{3}\alpha x_2^3$
The definition of the potential energy function is $W_{x_1\to x_2} = U_1 - U_2$. Comparison of the two expressions for *W* gives $U = \tfrac{1}{3}\alpha x^3$. This does correspond to $U = 0$ when $x = 0$.
**EVALUATE:** In part **(a)** the work done is zero because the force and displacement are perpendicular. In part **(b)** the force is directed opposite to the displacement and the work done is negative. In part **(c)** the force and displacement are in the same direction and the work done is positive.

**P7.35. IDENTIFY:** Apply Eq. (7.16).
**SET UP:** The sign of $F_x$ indicates its direction.

**EXECUTE:** $F_x = -\dfrac{dU}{dx} = -4\alpha x^3 = -(4.8\text{ J/m}^4)x^3.$ $F_x(-0.800\text{ m}) = -(4.8\text{ J/m}^4)(-0.80\text{ m})^3 = 2.46\text{ N}.$ The force is in the $+x$-direction.

**EVALUATE:** $F_x > 0$ when $x < 0$ and $F_x < 0$ when $x > 0,$ so the force is always directed towards the origin.

**P7.42.** **IDENTIFY:** Apply Eq. (7.14).

**SET UP:** Only the spring force and gravity do work, so $W_{\text{other}} = 0.$ Let $y = 0$ at the horizontal surface.

**EXECUTE:** **(a)** Equating the potential energy stored in the spring to the block's kinetic energy, $\frac{1}{2}kx^2 = \frac{1}{2}mv^2,$ or $v = \sqrt{\dfrac{k}{m}}x = \sqrt{\dfrac{400\text{ N/m}}{2.00\text{ kg}}}(0.220\text{ m}) = 3.11\text{ m/s}.$

**(b)** Using energy methods directly, the initial potential energy of the spring equals the final gravitational potential energy, $\frac{1}{2}kx^2 = mgL\sin\theta,$ or

$$L = \frac{\frac{1}{2}kx^2}{mg\sin\theta} = \frac{\frac{1}{2}(400\text{ N/m})(0.220\text{ m})^2}{(2.00\text{ kg})(9.80\text{ m/s}^2)\sin 37.0^\circ} = 0.821\text{ m}.$$

**EVALUATE:** The total energy of the system is constant. Initially it is all elastic potential energy stored in the spring, then it is all kinetic energy and finally it is all gravitational potential energy.

**P7.45.** **IDENTIFY:** The mechanical energy of the roller coaster is conserved since there is no friction with the track. We must also apply Newton's second law for the circular motion.

**SET UP:** For part (a), apply conservation of energy to the motion from point $A$ to point $B$: $K_B + U_{\text{grav},B} = K_A + U_{\text{grav},A}$ with $K_A = 0.$ Defining $y_B = 0$ and $y_A = 13.0\text{ m},$ conservation of energy becomes $\frac{1}{2}mv_B{}^2 = mgy_A$ or $v_B = \sqrt{2gy_A}.$ In part (b), the free-body diagram for the roller coaster car at point $B$ is shown in Figure 7.45. $\Sigma F_y = ma_y$ gives $mg + n = ma_{\text{rad}},$ where $a_{\text{rad}} = v^2/r.$ Solving for the normal force gives $n = m\left(\dfrac{v^2}{r} - g\right).$

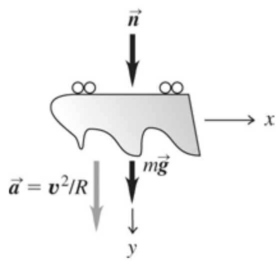



**Figure 7.45**

**EXECUTE:** **(a)** $v_B = \sqrt{2(9.80\text{ m/s}^2)(13.0\text{ m})} = 16.0\text{ m/s}.$

**(b)** $n = (350\ \text{kg})\left[\dfrac{(16.0\ \text{m/s})^2}{6.0\ \text{m}} - 9.80\ \text{m/s}^2\right] = 1.15 \times 10^4\ \text{N}.$

**EVALUATE:** The normal force $n$ is the force that the tracks exert on the roller coaster car. The car exerts a force of equal magnitude and opposite direction on the tracks.