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PHYS 191, Fall 2014, Dr. H. Merabet

Solutions to Practice Problems Chapter 1

Responses to Questions

Q1.10 The SI units of volume are m^3 . The quantity $\pi\, r^3\, h$ has dimensions of $(\text{length})^4$ so can't be a volume.

Q1.13 It is not possible for two vectors with different length to have a vector sum of zero. The vector sum has its smallest magnitude when the two vectors are antiparallel, and in that case the magnitude of the vector sum is the difference in the lengths of the two vectors, and this is zero only when the two vectors have the same length. If three vectors are to have a vector sum of zero the length of any one can't be greater than the sum of the lengths of the other two.

Q1.19 No. $\vec{A}.\vec{B} = 0$ only if $\boldsymbol{A}$ and $\boldsymbol{B}$ are perpendicular. $\boldsymbol{A}\times\boldsymbol{B} = \boldsymbol{0}$ only if $\boldsymbol{A}$ and $\boldsymbol{B}$ are either parallel or antiparallel.

Solutions to Problems

P.1.8. **IDENTIFY:** Apply the given conversion factors.

SET UP: $1 \text{ furlong} = 0.1250 \text{ mi}$ and $1 \text{ fortnight} = 14 \text{ days.}$ $1 \text{ day} = 24 \text{ h.}$

EXECUTE: $(180{,}000 \text{ furlongs/fortnight})\left(\dfrac{0.125 \text{ mi}}{1 \text{ furlong}}\right)\left(\dfrac{1 \text{ fortnight}}{14 \text{ days}}\right)\left(\dfrac{1 \text{ day}}{24 \text{ h}}\right) = 67 \text{ mi/h}$

EVALUATE: A furlong is less than a mile and a fortnight is many hours, so the speed limit in mph is a much smaller number.

P.1.10. **IDENTIFY:** Convert units.

SET UP: Use the unit conversions given in the problem. Also, $100 \text{ cm} = 1 \text{ m}$ and $1000 \text{ g} = 1 \text{ kg}$.

EXECUTE: **(a)** $\left(60 \dfrac{\text{mi}}{\text{h}}\right)\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right)\left(\dfrac{5280 \text{ ft}}{1 \text{ mi}}\right) = 88 \dfrac{\text{ft}}{\text{s}}$

(b) $\left(32 \dfrac{\text{ft}}{\text{s}^2}\right)\left(\dfrac{30.48 \text{ cm}}{1 \text{ ft}}\right)\left(\dfrac{1 \text{ m}}{100 \text{ cm}}\right) = 9.8 \dfrac{\text{m}}{\text{s}^2}$

(c) $\left(1.0 \dfrac{\text{g}}{\text{cm}^3}\right)\left(\dfrac{100 \text{ cm}}{1 \text{ m}}\right)^3\left(\dfrac{1 \text{ kg}}{1000 \text{ g}}\right) = 10^3 \dfrac{\text{kg}}{\text{m}^3}$

EVALUATE: The relations $60\text{ mi/h} = 88\text{ ft/s}$ and $1\text{ g/cm}^3 = 10^3\text{ kg/m}^3$ are exact. The relation $32\text{ ft/s}^2 = 9.8\text{ m/s}^2$ is accurate to only two significant figures.

P.1.15. IDENTIFY: Use your calculator to display $\pi \times 10^7$. Compare that number to the number of seconds in a year.

SET UP: $1\text{ yr} = 365.24\text{ days},\ 1\text{ day} = 24\text{ h},$ and $1\text{ h} = 3600\text{ s}.$

EXECUTE: $(365.24\text{ days/1 yr})\left(\dfrac{24\text{ h}}{1\text{ day}}\right)\left(\dfrac{3600\text{ s}}{1\text{ h}}\right) = 3.15567\ldots\times 10^7\text{ s};\ \pi\times 10^7\text{ s} = 3.14159\ldots\times 10^7\text{ s}$

The approximate expression is accurate to two significant figures. The percent error is 0.45%.

EVALUATE: The close agreement is a numerical accident.

P.1.26. IDENTIFY: The displacements must be added as vectors and the magnitude of the sum depends on the relative orientation of the two displacements.

SET UP: The sum with the largest magnitude is when the two displacements are parallel and the sum with the smallest magnitude is when the two displacements are antiparallel.

EXECUTE: The orientations of the displacements that give the desired sum are shown in Figure 1.26.

EVALUATE: The orientations of the two displacements can be chosen such that the sum has any value between 0.6 m and 4.2 m.

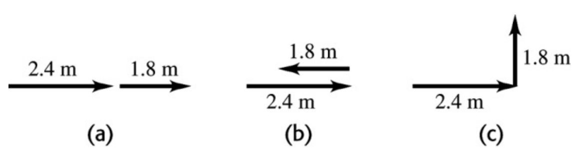

Figure 1.26

P.1.35. IDENTIFY: If $\vec{C} = \vec{A} + \vec{B}$, then $C_x = A_x + B_x$ and $C_y = A_y + B_y$. Use C_x and C_y to find the magnitude and direction of $\vec{C}$.

SET UP: From Figure E1.28 in the textbook, $A_x = 0,\ A_y = -8.00\text{ m}$ and

$B_x = +B\sin 30.0° = 7.50\text{ m},\ \ B_y = +B\cos 30.0° = 13.0\text{ m}.$

EXECUTE: **(a)** $\vec{C} = \vec{A} + \vec{B}$ so $C_x = A_x + B_x = 7.50\text{ m}$ and $C_y = A_y + B_y = +5.00\text{ m}.\ C = 9.01\text{ m}.$

$\tan\theta = \dfrac{C_y}{C_x} = \dfrac{5.00\text{ m}}{7.50\text{ m}}$ and $\theta = 33.7°.$

(b) $\vec{B} + \vec{A} = \vec{A} + \vec{B}$, so $\vec{B} + \vec{A}$ has magnitude 9.01 m and direction specified by $33.7°.$

(c) $\vec{D} = \vec{A} - \vec{B}$ so $D_x = A_x - B_x = -7.50 \text{ m}$ and $D_y = A_y - B_y = 2\ 21.0 \text{ m}.$ $D = 22.3 \text{ m}.$

$\tan\phi = \dfrac{D_y}{D_x} = \dfrac{2\ 21.0 \text{ m}}{2\ 7.50 \text{ m}}$ and $\phi = 70.3°.$ $\vec{D}$ is in the 3^{rd} quadrant and the angle θ counterclockwise from the $+x$ axis is $180° + 70.3° = 250.3°.$

(d) $\vec{B} - \vec{A} = -(\vec{A} - \vec{B}),$ so $\vec{B} - \vec{A}$ has magnitude 22.3 m and direction specified by $\theta = 70.3°.$

EVALUATE: These results agree with those calculated from a scale drawing in Problem 1.28.

P.1.43. **IDENTIFY:** Use trig to find the components of each vector. Use Eq. (1.11) to find the components of the vector sum. Eq. (1.14) expresses a vector in terms of its components.

SET UP: Use the coordinates in the figure that accompanies the problem.

EXECUTE: **(a)** $\vec{A} = (3.60 \text{ m})\cos 70.0°\hat{\boldsymbol{i}} + (3.60 \text{ m})\sin 70.0°\hat{\boldsymbol{j}} = (1.23 \text{ m})\hat{\boldsymbol{i}} + (3.38 \text{ m})\hat{\boldsymbol{j}}$

$\vec{B} = -(2.40 \text{ m})\cos 30.0°\hat{\boldsymbol{i}} - (2.40 \text{ m})\sin 30.0°\hat{\boldsymbol{j}} = (-2.08 \text{ m})\hat{\boldsymbol{i}} + (-1.20 \text{ m})\hat{\boldsymbol{j}}$

(b) $\vec{C} = (3.00)\,\vec{A} - (4.00)\,\vec{B} = (3.00)(1.23 \text{ m})\hat{\boldsymbol{i}} + (3.00)(3.38 \text{ m})\hat{\boldsymbol{j}} - (4.00)(-2.08 \text{ m})\hat{\boldsymbol{i}} - (4.00)(-1.20 \text{ m})\hat{\boldsymbol{j}}$

$$= (12.01 \text{ m})\hat{\boldsymbol{i}} + (14.94)\hat{\boldsymbol{j}}$$

(c) From Equations (1.7) and (1.8),

$C = \sqrt{(12.01 \text{ m})^2 + (14.94 \text{ m})^2} = 19.17 \text{ m},\ \arctan\left(\dfrac{14.94 \text{ m}}{12.01 \text{ m}}\right) = 51.2°$

EVALUATE: C_x and C_y are both positive, so θ is in the first quadrant.

P1.52. **IDENTIFY:** Use Eq. (1.27) for the components of the vector product.

SET UP: Use coordinates with the $+x$-axis to the right, $+y$-axis toward the top of the page, and $+z$-axis out of the page. $A_x = 0,$ $A_y = 0$ and $A_z = -3.50 \text{ cm}.$ The page is 20 cm by 35 cm, so $B_x = -20 \text{ cm}$ and $B_y = 35 \text{ cm}.$

EXECUTE: $(\vec{A} \times \vec{B})_x = 122 \text{ cm}^2, (\vec{A} \times \vec{B})_y = 70 \text{ cm}^2, (\vec{A} \times \vec{B})_z = 0.$

EVALUATE: From the components we calculated the magnitude of the vector product is $141 \text{ cm}^2.$ $B = 40.3 \text{ cm}$ and $\phi = 90°,$ so $AB\sin\phi = 141 \text{ cm}^2,$ which agrees.

P.1.66. **IDENTIFY:** Let $\vec{D}$ be the fourth force. Find $\vec{D}$ such that $\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0,$ so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C}).$

SET UP: Use components and solve for the components D_x and D_y of $\vec{D}.$

EXECUTE: $A_x = +A\cos 30.0° = +86.6 \text{ N},\ A_y = +A\sin 30.0° = +50.00 \text{ N}.$

$B_x = -B\sin 30.0° = -40.00 \text{ N},\ B_y = +B\cos 30.0° = +69.28 \text{ N}.$

$C_x = -C\cos 53.0° = -24.07 \text{ N},\ C_y = -C\sin 53.0° = -31.90 \text{ N}.$

Then $D_x = -22.53 \text{ N},\ D_y = -87.34 \text{ N}$ and $D = \sqrt{D_x^2 + D_y^2} = 90.2 \text{ N}.$ $\tan\alpha = |D_y/D_x| = 87.34/22.53.$

$\alpha = 75.54°.$ $\phi = 180° + \alpha = 256°,$ counterclockwise from the $+x$-axis.

EVALUATE: As shown in Figure 1.66, since D_x and D_y are both negative, $\vec{\boldsymbol{D}}$ must lie in the third quadrant.

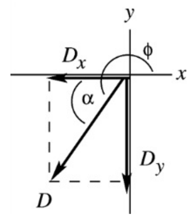

Figure 1.66

P.1.92. IDENTIFY: We know the magnitude of two vectors and the magnitude of their vector product, and we want to find the possible values of their scalar product.

SET UP: The vector product is $|\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}| = AB\sin\theta$ and the scalar product is $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = AB\cos\theta$.

EXECUTE: $|\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}}| = AB\sin\theta = 12.0\ \text{m}^2$, so $\sin\theta = \dfrac{12.0\ \text{m}^2}{(6.00\ \text{m})(3.00\ \text{m})} = 0.6667,$ which gives two possible values: $\theta = 41.81°$ or $\theta = 138.19°.$ Therefore the two possible values of the scalar product are $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = AB\cos\theta = 13.4\ \text{m}^2$ or $-13.4\ \text{m}^2.$

EVALUATE: The two possibilities have equal magnitude but opposite sign because the two possible angles are supplementary to each other. The sines of these angles are the same but the cosines differ by a factor of □1. See Figure 1.92.

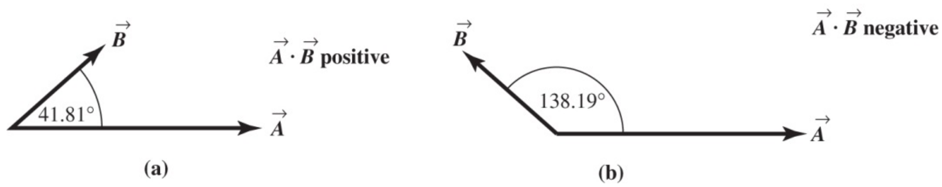

Figure 1.92

P.1.95. IDENTIFY and SET UP: The target variables are the components of $\vec{\boldsymbol{C}}$. We are given $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$. We also know $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{C}}$ and $\vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{C}}$, and this gives us two equations in the two unknowns C_x and C_y.

EXECUTE: $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{C}}$ are perpendicular, so $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{C}} = 0.$ $A_xC_x + A_yC_y = 0,$ which gives

$5.0C_x - 6.5C_y = 0.$

$\vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{C}} = 15.0,$ so $-3.5C_x + 7.0C_y = 15.0$

We have two equations in two unknowns C_x and C_y. Solving gives $C_x = 8.0$ and $C_y = 6.1.$

EVALUATE: We can check that our result does give us a vector $\vec{\boldsymbol{C}}$ that satisfies the two equations $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{C}} = 0$ and $\vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{C}} = 15.0.$