Qatar University
College of Arts and Sciences
Department of Math, Physics & Stat
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Solutions to Practice Problems Chapter 10

Responses to Questions

Q10.1 The torque is a measure of the effectiveness of rotating the bolts. The effectiveness of the force depends on the length of the wrench handle.

Q10.5 Reducing the weight of the wheels reduces the moment of inertia of the wheels and less torque is required to give them a given angular acceleration. Reducing the total mass of the bike also reduces the net horizontal force required for a given linear acceleration but for this motion it doesn't matter where on the bike the weight is removed.

Q10.19 The ball will go higher up the hill if the hill has enough friction to prevent slipping. Energy conservation: As the ball rolls on the horizontal surface at the bottom of the hill it has both translational and rotational kinetic energy. If the hill is perfectly smooth the ball continues to rotate with the same angular speed and only the initial translational kinetic energy is converted into gravitational potential energy; at the maximum height up the hill the ball is still spinning at the same rate as initially. If there is enough friction to prevent slipping then the ball has stopped rotating at the maximum height and both the initial translational and rotational kinetic energies are converted into gravitational potential energy.
Newton's 2nd law: When there is no friction the net force on the ball as it rolls up the hill is $mg \sin\alpha$ directed down the incline (α is the slope angle of the hill). With friction, there is a friction force f directed up the incline and the net force down the incline is $(mg \sin\alpha - f)$. With friction the acceleration opposing the translational motion is less and the ball travels a greater distance before coming to rest.

Solutions to Problem

P10.2. **IDENTIFY**: $\tau = Fl$ with $l = r\sin\phi$. Add the two torques to calculate the net torque.

SET UP: Let counterclockwise torques be positive.

EXECUTE: $\tau_1 = -F_1 l_1 = -(8.00\text{ N})(5.00\text{ m}) = -40.0\text{ N}\cdot\text{m}$.
$\tau_2 = +F_2 l_2 = (12.0\text{ N})(2.00\text{ m})\sin 30.0° = +12.0\text{ N}\cdot\text{m}$. $\sum\tau = \tau_1 + \tau_2 = -28.0\text{ N}\cdot\text{m}$. The net torque is $28.0\text{ N}\cdot\text{m}$, clockwise.

EVALUATE: Even though $F_1 < F_2$, the magnitude of τ_1 is greater than the magnitude of τ_2, because F_1 has a larger moment arm.

P10.8. **IDENTIFY:** Use $\tau = Fl = rF\sin\phi$ for the magnitude of the torque and the right-hand rule for the direction.

SET UP: In part (a), $r = 0.250\text{ m}$ and $\phi = 37°.$

EXECUTE: **(a)** $\tau = (17.0\text{ N})(0.250\text{ m})\sin 37° = 2.56\text{ N}\cdot\text{m}.$ The torque is counterclockwise.

(b) The torque is maximum when $\phi = 90°$ and the force is perpendicular to the wrench. This maximum torque is $(17.0\text{ N})(0.250\text{ m}) = 4.25\text{ N}\cdot\text{m}.$

EVALUATE: If the force is directed along the handle then the torque is zero. The torque increases as the angle between the force and the handle increases.

P10.9. **IDENTIFY:** Apply $\Sigma\tau_z = I\alpha_z.$

SET UP: $\omega_{0z} = 0.$ $\omega_z = (400\text{ rev/min})\left(\dfrac{2\pi\text{ rad/rev}}{60\text{ s/min}}\right) = 41.9\text{ rad/s}$

EXECUTE: $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t} = (2.50\text{ kg}\cdot\text{m}^2)\dfrac{41.9\text{ rad/s}}{8.00\text{ s}} = 13.1\text{ N}\cdot\text{m}.$

EVALUATE: In $\tau_z = I\alpha_z$, α_z must be in $\text{rad/s}^2.$

P10.17. **IDENTIFY:** Apply $\Sigma\vec{F} = m\vec{a}$ to each box and $\Sigma\tau_z = I\alpha_z$ to the pulley. The magnitude a of the acceleration of each box is related to the magnitude of the angular acceleration α of the pulley by $a = R\alpha.$

SET UP: The free-body diagrams for each object are shown in Figure 10.17a–c. For the pulley, $R = 0.250\text{ m}$ and $I = \frac{1}{2}MR^2.$ T_1 and T_2 are the tensions in the wire on either side of the pulley. $m_1 = 12.0\text{ kg},$ $m_2 = 5.00\text{ kg}$ and $M = 2.00\text{ kg}.$ $\vec{F}$ is the force that the axle exerts on the pulley. For the pulley, let clockwise rotation be positive.

EXECUTE: **(a)** $\Sigma F_x = ma_x$ for the 12.0 kg box gives $T_1 = m_1 a.$ $\Sigma F_y = ma_y$ for the 5.00 kg weight gives $m_2 g - T_2 = m_2 a.$ $\Sigma\tau_z = I\alpha_z$ for the pulley gives $(T_2 - T_1)R = \left(\frac{1}{2}MR^2\right)\alpha.$ $a = R\alpha$ and $T_2 - T_1 = \frac{1}{2}Ma.$ Adding these three equations gives $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ and

$a = \left(\dfrac{m_2}{m_1 + m_2 + \frac{1}{2}M}\right)g = \left(\dfrac{5.00\text{ kg}}{12.0\text{ kg} + 5.00\text{ kg} + 1.00\text{ kg}}\right)(9.80\text{ m/s}^2) = 2.72\text{ m/s}^2.$ Then

$T_1 = m_1 a = (12.0\text{ kg})(2.72\text{ m/s}^2) = 32.6\text{ N}.$ $m_2 g - T_2 = m_2 a$ gives

$T_2 = m_2(g - a) = (5.00\text{ kg})(9.80\text{ m/s}^2 - 2.72\text{ m/s}^2) = 35.4\text{ N}.$ The tension to the left of the pulley is 32.6 N and below the pulley it is 35.4 N.

(b) $a = 2.72\text{ m/s}^2$

(c) For the pulley, $\Sigma F_x = ma_x$ gives $F_x = T_1 = 32.6\text{ N}$ and $\Sigma F_y = ma_y$ gives

$F_y = Mg + T_2 = (2.00\text{ kg})(9.80\text{ m/s}^2) + 35.4\text{ N} = 55.0\text{ N}.$

EVALUATE: The equation $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ says that the external force $m_2 g$ must accelerate all three objects.

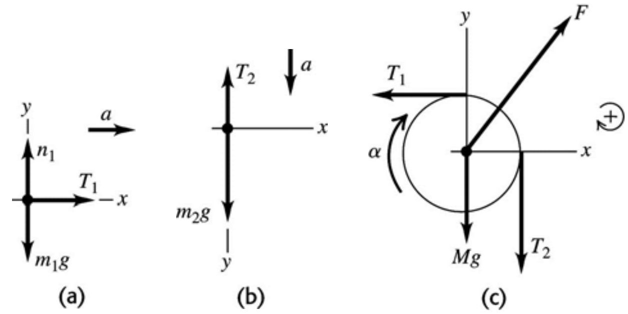

Figure 10.17

P10.21. IDENTIFY: Apply Eq. (10.8).

SET UP: For an object that is rolling without slipping, $v_{cm} = R\omega$.

EXECUTE: The fraction of the total kinetic energy that is rotational is

$$\frac{(1/2)I_{cm}\omega^2}{(1/2)Mv_{cm}^2 + (1/2)I_{cm}\omega^2} = \frac{1}{1+(M/I_{cm})v_{cm}^2/\omega^2} = \frac{1}{1+(MR^2/I_{cm})}$$

(a) $I_{cm} = (1/2)MR^2$, so the above ratio is 1/3.

(b) $I_{cm} = (2/5)MR^2$ so the above ratio is $2/7$.

(c) $I_{cm} = (2/3)MR^2$ so the ratio is $2/5$.

(d) $I_{cm} = (5/8)MR^2$ so the ratio is $5/13$.

EVALUATE: The moment of inertia of each object takes the form $I = \beta MR^2$. The ratio of rotational kinetic energy to total kinetic energy can be written as $\frac{1}{1+1/\beta} = \frac{\beta}{1+\beta}$. The ratio increases as β increases.

P10.30. IDENTIFY: Apply $P = \tau\omega$ and $W = \tau\Delta\theta$.

SET UP: P must be in watts, $\Delta\theta$ must be in radians, and ω must be in rad/s. $1 \text{ rev} = 2\pi$ rad. $1 \text{ hp} = 746$ W. π rad/s $= 30$ rev/min.

EXECUTE: **(a)** $\tau = \frac{P}{\omega} = \frac{(175 \text{ hp})(746 \text{ W/hp})}{(2400 \text{ rev/min})\left(\frac{\pi \quad \text{rad/s}}{30 \text{ rev/min}}\right)} = 519 \text{ N}\cdot\text{m}.$

(b) $W = \tau\Delta\theta = (519 \text{ N}\cdot\text{m})(2\pi \text{ rad}) = 3260 \text{ J}$

EVALUATE: $\omega = 40$ rev/s, so the time for one revolution is 0.025 s. $P = 1.306\times10^5$ W, so in one revolution, $W = Pt = 3260$ J, which agrees with our result.

P10.34. IDENTIFY: Apply $\sum\tau_z = I\alpha_z$ to the motion of the propeller and then use constant acceleration equations to analyze the motion. $W = \tau\Delta\theta$.

SET UP: $I = \frac{1}{12}mL^2 = \frac{1}{12}(117 \text{ kg})(2.08 \text{ m})^2 = 42.2 \text{ kg}\cdot\text{m}^2$.

EXECUTE: **(a)** $\alpha = \dfrac{\tau}{I} = \dfrac{1950\ \text{N}\cdot\text{m}}{42.2\ \text{kg}\cdot\text{m}^2} = 46.2\ \text{rad/s}^2.$

(b) $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives $\omega = \sqrt{2\alpha\theta} = \sqrt{2(46.2\ \text{rad/s}^2)(5.0\ \text{rev})(2\pi\ \text{rad/rev})} = 53.9\ \text{rad/s}.$

(c) $W = \tau\theta = (1950\ \text{N}\cdot\text{m})(5.00\ \text{rev})(2\pi\ \text{rad/rev}) = 6.13\times10^4\ \text{J}.$

(d) $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{53.9\ \text{rad/s}}{46.2\ \text{rad/s}^2} = 1.17\ \text{s}.$ $P_{\text{av}} = \dfrac{W}{\Delta t} = \dfrac{6.13\times10^4\ \text{J}}{1.17\ \text{s}} = 52.5\ \text{kW}.$

EVALUATE: $P = \tau\omega.$ τ is constant and ω is linear in *t*, so P_{av} is half the instantaneous power at the end of the 5.00 revolutions. We could also calculate *W* from $W = \Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}(42.2\ \text{kg}\cdot\text{m}^2)(53.9\ \text{rad/s})^2 = 6.13\times10^4\ \text{J}.$

P10.40. IDENTIFY: $\omega_z = d\theta/dt.$ $L_z = I\omega_z$ and $\tau_z = dL_z/dt.$

SET UP: For a hollow, thin-walled sphere rolling about an axis through its center, $I = \frac{2}{3}MR^2.$ $R = 0.240\ \text{m}.$

EXECUTE: **(a)** $A = 1.50\ \text{rad/s}^2$ and $B = 1.10\ \text{rad/s}^4,$ so that $\theta(t)$ will have units of radians.

(b) (i) $\omega_z = \dfrac{d\theta}{dt} = 2At + 4Bt^3.$ At $t = 3.00\ \text{s},$

$\omega_z = 2(1.50\ \text{rad/s}^2)(3.00\ \text{s}) + 4(1.10\ \text{rad/s}^4)(3.00\ \text{s})^3 = 128\ \text{rad/s}.$

$L_z = (\frac{2}{3}MR^2)\omega_z = \frac{2}{3}(12.0\ \text{kg})(0.240\ \text{m})^2(128\ \text{rad/s}) = 59.0\ \text{kg}\cdot\text{m}^2/\text{s}.$

(ii) $\tau_z = \dfrac{dL_z}{dt} = I\dfrac{d\omega_z}{dt} = I(2A + 12Bt^2)$ and

$\tau_z = \frac{2}{3}(12.0\ \text{kg})(0.240\ \text{m})^2(2[1.50\ \text{rad/s}^2] + 12[1.10\ \text{rad/s}^4][3.00\ \text{s}]^2) = 56.1\ \text{N}\cdot\text{m}.$

EVALUATE: The angular speed of rotation is increasing. This increase is due to an acceleration α_z that is produced by the torque on the sphere. When *I* is constant, as it is here, $\tau_z = dL_z/dt = Id\omega_z/dt = I\alpha_z$ and Eqs. (10.29) and (10.7) are identical.

P10.43. IDENTIFY: Apply conservation of angular momentum to the motion of the skater.

SET UP: For a thin-walled hollow cylinder $I = mR^2.$ For a slender rod rotating about an axis through its center, $I = \frac{1}{12}ml^2.$

EXECUTE: $L_\text{i} = L_\text{f}$ so $I_\text{i}\omega_\text{i} = I_\text{f}\omega_\text{f}.$

$I_\text{i} = 0.40\ \text{kg}\cdot\text{m}^2 + \frac{1}{12}(8.0\ \text{kg})(1.8\ \text{m})^2 = 2.56\ \text{kg}\cdot\text{m}^2.$ $I_\text{f} = 0.40\ \text{kg}\cdot\text{m}^2 + (8.0\ \text{kg})(0.25\ \text{m})^2 = 0.90\ \text{kg}\cdot\text{m}^2.$

$\omega_\text{f} = \left(\dfrac{I_\text{i}}{I_\text{f}}\right)\omega_\text{i} = \left(\dfrac{2.56\ \text{kg}\cdot\text{m}^2}{0.90\ \text{kg}\cdot\text{m}^2}\right)(0.40\ \text{rev/s}) = 1.14\ \text{rev/s}.$

EVALUATE: $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega.$ ω increases and *L* is constant, so *K* increases. The increase in kinetic energy comes from the work done by the skater when he pulls in his hands.

P10.45. IDENTIFY and SET UP: There is no net external torque about the rotation axis so the angular momentum $L = I\omega$ is conserved.

EXECUTE: **(a)** $L_1 = L_2$ gives $I_1\omega_1 = I_2\omega_2$, so $\omega_2 = (I_1/I_2)\omega_1$

$I_1 = I_{tt} = \frac{1}{2}MR^2 = \frac{1}{2}(120\text{ kg})(2.00\text{ m})^2 = 240\text{ kg}\cdot\text{m}^2$

$I_2 = I_{tt} + I_p = 240\text{ kg}\cdot\text{m}^2 + mR^2 = 240\text{ kg}\cdot\text{m}^2 + (70\text{ kg})(2.00\text{ m})^2 = 520\text{ kg}\cdot\text{m}^2$

$\omega_2 = (I_1/I_2)\omega_1 = (240\text{ kg}\cdot\text{m}^2/520\text{ kg}\cdot\text{m}^2)(3.00\text{ rad/s}) = 1.38\text{ rad/s}$

(b) $K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}(240\text{ kg}\cdot\text{m}^2)(3.00\text{ rad/s})^2 = 1080\text{ J}$

$K_2 = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}(520\text{ kg}\cdot\text{m}^2)(1.38\text{ rad/s})^2 = 495\text{ J}$

EVALUATE: The kinetic energy decreases because of the negative work done on the turntable and the parachutist by the friction force between these two objects.
The angular speed decreases because *I* increases when the parachutist is added to the system.