CURRENT AND RESISTANCE

In this chapter we shall study electric currents, i.e., of charges in motion. Example of electric currents abound, ranging from large currents that constitute lightning strokes to the tiny nerve current that regulate our muscular activity. We are familiar with currents resulting from charges flowing through solids conductors (household wiring, light bulbs), semiconductors (integrated circuits), gases (fluorescent lamps), liquids (automobile batteries), and even evacuated spaces (TV picture tubes)

32.1 ELECTRIC CURRENT

The time rate of flow of charge through a conductor is called current. If a charge 'dq' flows through any cross-section of a conductor in time 'dt', then the current 'I' is given by

$$I = \frac{dq}{dt}$$

The SI unit of current is Ampere, which can be defined as, "when one coulomb charge flows through a cross-section in one second, then the current flowing is one ampere".

Problem 1. A current of 4.82 A exist in a 12.4Ω resistor for 4.6 minutes. (a) Find out charge, (b) How many electrons pass through resistor in this time?

Solution. $I = 4.82\ A$

$R = 12.4\Omega$

$t = 4.6\ min = 4.6 \times 60\ s$

$q = ?$

$n = ?$

(a) As $I = \frac{q}{t}$

$\Rightarrow q = It = 4.82 \times 4.6 \times 60 = 1.33 \times 10^3\ C$

(b) As $q = ne \Rightarrow n = \frac{q}{e} = \frac{1.33\times10^3}{1.6\times10^{-19}} = 8.3 \times 10^{21}\ e$

Problem 2. The current in the electron beam of a typical video display terminal is $200\mu A$. How many electrons strike the screen each minute?

Solution:

$I = 200\ \mu A = 200 \times 10^{-6} A$

$n = ?$

$t = 1\ min = 60\ s$

As $I = \frac{q}{t}$

$\Rightarrow q = It = 200 \times 10^{-6} \times 60 = 12000 \times 10^{-6}\ C$

Also $q = ne \Rightarrow n = \frac{q}{e} = \frac{12000\times10^{-6}}{1.6\times10^{-19}} = 7.5 \times 10^{16}\ e$

32.2 CURRENT FLOW AND FLUID FLOW: A COMPARISON

When a steady current is flowing through idealized conducting wire, the electric current remains same for all cross-sections, even though the cross-sectional area may be different at different points. The condition of steady current flow is similar to the motion of incompressible fluid. The fluid that flows through any cross-section of the pipe is the same even if the cross-section varies. The fluid flows faster where the cross-section of the pipe is smaller and slower where it is larger, but the volume rate of flow remains constant.

32.3 DIRECTION OF CURRENT

In metals, the charge carriers are electrons. But in electrolytes, the current flow due to motion of negative and positive ions. A positive charge moving in one direction is equivalent in all external effects to a negative charge moving in the opposite the opposite direction. Hence for simplicity and algebraic consistency, we adopt the following convention:

The direction of current is the direction that positive charges would move, even if the actual charge carriers are negative. Thus, the direction of current is taken from the point of higher potential to the point of lower potential.

Even though we assign a direction, current is a scalar quantity, not a vector. The arrow that we draw to indicate the direction of current merely show the sense of charge flow through the wire and is not be taken as a vector. Current does not obey the law of vector addition. Changing the direction of wires does not change the way the currents are added.

32.4 CURRENT DENSITY

The current flowing per unit area is called the current density. It is a vector quantity and the SI unit of this quantity is Ampere per square meter ($\frac{A}{m^2}$). The electric current I can be described in terms of current density as

"The scalar product of current density '$\vec{J}$' and vector area '$\vec{A}$' is called the electric current".

Mathematically,

$$I = \vec{J}.\vec{A}$$

The electric current is macroscopic quantity, while the current density is its corresponding microscopic quantity. In general, if '$d\vec{A}$' is the small area element of conductor, then the current flowing through the whole area of conductor is.

$$I = \int \vec{J}.d\vec{A}$$

Sample Problem 1. One end of an aluminum wire whose diameter is 2.5 mm is welded to one end of copper wire whose diameter is 1.8mm. The composite wire carries a steady current of 1.3 A. What is current density in each wire?

Solution: Current flowing through composite wire $I = 1.3\ A$

For Aluminum

Diameter of wire $d = 2.5\ mm$

Radius $r = 1.25\ mm = 1.25 \times 10^{-3} m$

, , ,

Current Density in Al $J_{Al} = \frac{I}{A} = \frac{1.3}{3.14 \times (1.25 \times 10^{-3})^2} = 2.64 \times 10^5 \frac{A}{m^2}$

For Copper

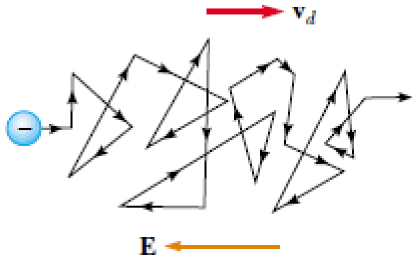
Diameter of wire $d = 1.8\ mm$

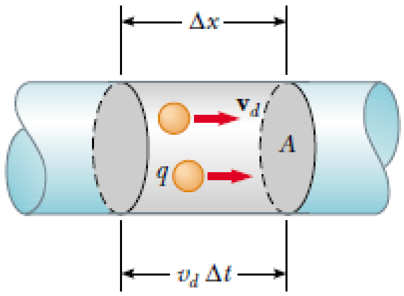
Radius $r = 0.5\ mm = 0.9 \times 10^{-3} m$

Current Density in Cu $J_{Cu} = \frac{I}{A} = \frac{1.3}{3.14 \times (0.9 \times 10^{-3})^2} = 5.1 \times 10^5 \frac{A}{m^2}$

32.5 DETERMINATION OF CURRENT DENSITY OF CONDUCTOR

Let ‘A’ is the area of the cross-section of a conductor of length ‘L’ in which the current ‘I’ is flowing. The flow of current through a conductor is due to motion of electron in the direction opposite to electric field ‘$\vec{E}$’. The force on one electron due to electric field is ‘$-e\vec{E}$’. But this force does not produce any acceleration in the motion of electrons, because the conduction electrons keep on colliding with the lattice ions of conductor. Instead of this, the electrons acquire a constant drift speed ‘v_d’ in the direction of ‘$-\mathbf{E}$’.





Let

n = Number of free electrons per unit

AL = Volume of the conductor

nAL = Number of free electrons in the conductor

e = Charge on one electron

$nALe = \Delta q$ = Total charge flowing in conductor

If the charge ‘q’ passes through conductor in time ‘Δt’, then

$$\Delta t = \frac{L}{v_d}$$

So, the current

$$I = \frac{\Delta q}{\Delta t} = \frac{nALe}{L/v_d} = nAev_d$$

The current density is

$$J = \frac{I}{A} = \frac{nAev_d}{A}$$

$$J = nev_d$$

The direction of ‘$\vec{J}$’ is opposite to the direction of flow of electrons.

$$\vec{J} = -ne\,\vec{v}_d$$

The mean drift velocity of electrons is very small i.e., of the order of 'cm/s'. While in random motion, the speed of electrons has a typical value of $10^6\ m/s$ in metals.

Sample Problem 2. What is drift speed of the conduction electron in copper wire of sample problem 1. The electron density in copper is $\mathbf{8.49 \times 10^{28} \frac{electrons}{m^3}}$

Solution: Given $n = 8.49 \times 10^{28} \frac{electrons}{m^3}$

$$J_{Cu} = 5.1 \times 10^5 \frac{A}{m^2}$$

The drift velocity can be find out by expression

$$v_d = \frac{J}{ne} = \frac{5.1 \times 10^5}{8.49 \times 10^{28} \times 1.6 \times 10^{-19}} = 0.37 \times 10^{-4} \frac{m}{s}$$

Sample problem 3. A strip of Si of width 3.2 cm and thickness $d = 250\ \mu m$ carries a current of 190 mA and $n = 8 \times 10^{21}\ m^{-3}$, (a) Find current density. (b) Find drift speed

Solution:(a) Current flowing through strip $I = 190\ mA = 190 \times 10^{-3} A$

Width $w = 3.2\ cm = 3.2 \times 10^{-2} m$

Thickness $d = 250\ \mu m = 250 \times 10^{-6} m$

Current Density in Si $J_{Si} = \frac{I}{A} = \frac{I}{w.d} = \frac{190\times10^{-3}}{3.2\times10^{-2}\times250\times10^{-6}} = 2.4 \times 10^5 \frac{A}{m^2}$

(b) Given $n = 8 \times 10^{21}\ m^{-3}$

The drift velocity can be find out by expression

$$v_d = \frac{J}{ne} = \frac{2.4 \times 10^5}{8 \times 10^{21} \times 1.6 \times 10^{-19}} = 190\ \frac{m}{s}$$

Problem 3. Suppose that we have $\mathbf{2.1 \times 10^8}$ doubly charge positive ions per cubic centimeter, all moving north with a speed of $\mathbf{1.4 \times 10^5\ m/s}$, (a) Find current density and direction. (b) Can you calculate total current? If not! What additional information is needed?

Solution:

$$v_d = 1.4 \times 10^5\ m/s$$
$$n = 2.1 \times 10^8\ /cm^3 = 2.1 \times 10^{14}\ /m^3$$

$J = ?$

$I = ?$

$$J = nev_d = 2.1 \times 10^{14} \times 1.6 \times 10^{-19} \times 1.4 \times 10^5 = 9.408 \frac{A}{m^2}$$

$Now \quad J = \frac{I}{A} \Longrightarrow I = JA$

So we can't find current because the value of area is not given.

,

,

33.6 OHM'S LAW

It states that

"*The current flowing through a conductor is directly proportional to the applied potential difference if all physical states remain same*."

If 'V' is the potential difference between the ends of conductor and 'I' is the current flowing through it, then the Ohm's law is described mathematically as:

$$V \ \alpha \ I$$

$$V = R \ I$$

Where R is the constant of proportionality, called resistance of the conductor. It is described as the opposition offered by conductor to the flow of current. In system international, its unit is ohm. It is a macroscopic quantity. Its corresponding microscopic quantity is resistivity.

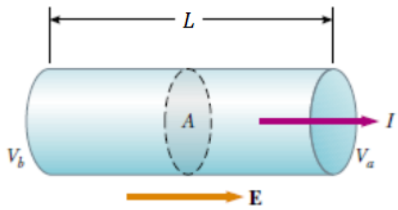
33.7 RESISTIVITY

The resistance of a meter cube of a substance is called resistivity or specific resistance. It is denoted by the symbol ρ. Its SI unit is ohm-meter $(\Omega - m)$.

Consider a conductor of length L and cross-sectional area A as shown in the figure. It is found experimentally that the resistance of conductor is directly proportional to length L of conductor and inversely proportional to cross-sectional are A:

$$R \propto L \quad ----(1)$$

$$R \propto \frac{1}{A} \quad -----(2)$$



Combining (1) and (2), we have:

$$R \propto \frac{L}{A}$$

$$R = \rho \frac{L}{A}$$

Where ρ is constant of proportionality, called resistivity or specific resistance of conductor.

$$\Rightarrow \rho = \frac{RA}{L}$$

This is the expression of resistivity for a conductor having length L and cross-sectional area A.

Problem 17. A steel trolley car-rail has a cross-sectional area of 56 cm^2. What is resistance of 11 km of rail? The resistance of steel is $3 \times 10^{-7} \Omega - m$.

Solution:

$$A = 56 \text{ cm}^2 = 56 \times 10^{-4} \text{m}^2$$

$$L = 11 \text{ km} = 11 \times 10^3 \text{m}$$

$$\rho = 3 \times 10^{-7} \Omega\text{m}$$

R =?

$$R = \frac{\rho L}{A} = \frac{3 \times 10^{-7} \times 11 \times 10^3}{56 \times 10^{-4}} = 0.58\Omega$$

Sample problem 4. A rectangular block of iron has dimension $1.2cm \times 1.2cm \times 15cm$. (a) What is resistance of block measured between two square ends? (b) What is resistance between two opposite rectangular face? The resistivity of iron at room temperature is $9.68 \times 10^{-8} \Omega m$.

Solution: (a) Area between two square faces $A = 1.2cm \times 1.2cm = 1.44\ cm^2 = 1.44 \times 10^{-4} m^2$

Length of conductor between two square ends $L = 15\ cm = 15 \times 10^2 m$

Resistance $R = \rho \frac{L}{A} = 9.68 \times 10^{-8} \times \frac{15 \times 10^2}{1.44 \times 10^{-4}} = 1 \times 10^{-4} \Omega$

(b)Area between two rectangular faces $A' = 15cm \times 1.2cm = 18\ cm^2 = 18 \times 10^{-4} m^2$

Length of conductor between two square ends $L' = 1.2\ cm = 1.2 \times 10^2 m$

Resistance $R' = \rho \frac{L}{A} = 9.68 \times 10^{-8} \times \frac{1.2 \times 10^2}{18 \times 10^{-4}} = 6.5 \times 10^{-7} \Omega$

33.8 MICROSCOPIC FORM OF OHM'S LAW

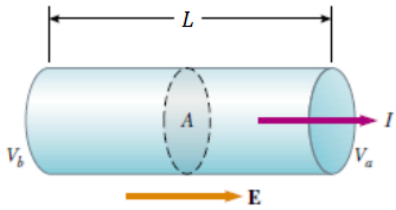
Consider a conductor of length L and cross-sectional area A as shown in the figure. The expression of resistivity ρ of such conductor is described by formula:

$$\rho = \frac{RA}{L}$$

By Ohm's law, we have $R = \frac{V}{I}$. Therefore,

$$\rho = \frac{VA}{IL}$$

$$\Rightarrow \rho = \frac{V}{\left(\frac{I}{A}\right) L}$$



where $J = \frac{I}{A}$ is the current density.

$$\rho = \frac{V}{\mathrm{J}L}$$

$$\therefore \textit{For present case, the potential difference applied and}$$
$$\textit{the electric field developed across of ends conductors is } V = EL$$

$$\Rightarrow \rho = \frac{EL}{\mathrm{J}L}$$

$$\Rightarrow \rho = \frac{E}{\mathrm{J}}$$

$$E = \rho \mathrm{J}$$

As resistivity'ρ' is reciprocal of conductivity 'σ'. So we can write:

$$\rho = \frac{1}{\sigma}$$

Therefore:

$$E = \left(\frac{1}{\sigma}\right) \mathrm{J}$$

$$\mathrm{J} = \sigma E$$

In vector form:

$$\vec{\mathrm{J}} = \sigma \vec{\mathrm{E}}$$

This is known as microscopic form of ohm's law.

Problem 33. When 115 V is applied across a 9.66 m long wire, the current density is $1.4\,\frac{A}{cm^2}$. Calculate the conductivity of the wire material.

Solution:

$$V = 115\ \mathrm{V}$$

$$L = 9.66\ \mathrm{m}$$

$$J = 1.4\,\frac{\mathrm{A}}{\mathrm{cm}^2} = 1.4 \times 10^4\,\frac{\mathrm{A}}{\mathrm{m}^2}$$

$\sigma = ?$

$$J = \sigma E \Longrightarrow \sigma = \frac{J}{E} = \frac{J}{(V/L)} = \frac{JL}{V} = \frac{1.4 \times 10^{-4} \times 9.66}{115} = 1.19 \times 10^3\ \frac{\mathrm{C}}{\mathrm{m}^2}$$

TEMPERATURE VARIATION OF RESISTIVITY

The resistivity of the conductor increases with increase in temperature. The temperature dependence of resistivity 'ρ' is shown in the figure.

Let 'ρ_0' and 'ρ' be the values of resistivity at temperature 0˚C and T˚C, respectively. The change of resistivity $(\rho - \rho_0)$ is directly proportional to the resistivity 'ρ_0' and change of temperature $(T - T_0)$. That is

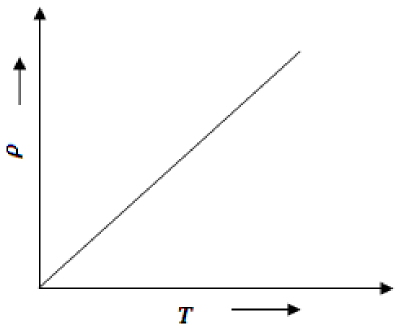
$$(\rho - \rho_0) \propto \rho_0 \quad ---- \quad (1)$$

$$(\rho - \rho_0) \propto (T - T_0) \quad ---- \quad (2)$$

Combining (1) and (2), we have:

$$(\rho - \rho_0) \propto \rho_0 (T - T_0)$$

$$\Longrightarrow (\rho - \rho_0) = \overline{\alpha}\, \rho_0 (T - T_0)$$



Where $\overline{\alpha}$ is the mean temperature coefficient of resistivity.

$$\overline{\alpha} = \frac{(\rho - \rho_0)}{\rho_0 (T - T_0)}$$

Thus we can describe resistivity as the fractional change in resistivity per unit rise in temperature. The temperature coefficient of resistivity depends upon the nature of material and it is measure in the units $°\mathrm{C}^{-1}$ or K^{-1}. The general formula for temperature coefficient of resistivity is

$$\alpha = \frac{1}{\rho}\,\frac{d\rho}{dT}$$

where $\frac{d\rho}{dT}$ is the rate of change of resistivity with respect to temperature.

ANALOGY BETWEEN THE CURRENT AND HEAT FLOW

Derivation of Expression for Current Flow

Consider a thin electrically conducting slab of thickness 'Δx' and area 'A'. When potential difference 'ΔV' is applied across the ends of a conductor, the current 'I' will flow through it.

By Ohm's law

$$I = \frac{\Delta V}{R}$$

As $R = \frac{\rho \Delta x}{A}$

$$\Rightarrow I = \frac{\Delta V}{\left(\frac{\rho \Delta x}{A}\right)} = \frac{A}{\rho}\frac{\Delta V}{\Delta x}$$

But $I = \frac{dq}{dt}$. So,

$$\frac{dq}{dt} = \frac{A}{\rho}\frac{dV}{dx} \Rightarrow \frac{dq}{dt} = A\sigma\frac{dV}{dx} \qquad \therefore \frac{1}{\rho} = \sigma$$

As the current flows in the direction of decreasing potential, so we place a minus sign to encounter this phenomenon. Therefore,

$$\frac{dq}{dt} = -A\sigma\frac{dV}{dx} \quad ----(1)$$

The equation (1) describe current flow through conductor.

Equation for Heat Flow

If "dQ" is the heat flows through the area 'A' in the small interval of time 'dt', then the rate of flow of heat $\frac{dQ}{dt}$ is described as:

$$\frac{dQ}{dt} = -kA\frac{dT}{dx} \quad ---- \ (2)$$

Comparison of Current Flow and Heat Flow

The rate of flow charge is given by eq. (1) and the rate of flow of heat is described in eq. (2). Hence there is a close analogy. The current flows due to the difference in the potential and heat flows due to the difference in temperature. Here

$\frac{dV}{dx}$ = Potential Gradient

$\frac{dT}{dx}$ = Temperature Gradient

Similarly, the electrical conductivity in eq. (1) has the same effect as that of thermal conductivity in eq. (2).

DERIVATION OF EXPRESSION OF RESISTIVITY OF CONDUCTOR

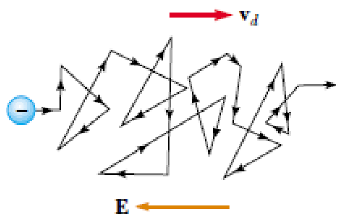
In metals, the valance electrons are not attached to the individual atoms but are free to move about within the lattice called conduction electrons. According to free electron model, the conduction electrons are assumed to move freely throughout the conducting material, somewhat like the molecules of gas in a container. These conduction electron moves randomly like the molecules of gas. The electrons make collisions with atoms and molecules during their random motion. In case of copper, the average speed of electrons in random motion is 1.6×10^6 ms^{-1}.

, ,

If the electric field is applied, then the motion of electrons slightly shifted in the direction opposite of that of $\vec{E}$. Then the force '$\vec{F}$' acting on the free electron is find out using relation

$$F = eE$$

where 'e' is the charge on electron. By using Newton's second law of motion, eq. (1) will become



$$ma = eE$$

$$\Rightarrow a = \frac{eE}{m}$$

During the collision of free electrons and the atom (or ion core), the tendency of the electron to drift is destroyed. Therefore the average drift speed of the electron will be

$$v_d = a\tau = \frac{eE\tau}{m}$$

where a is the average acceleration of electrons and τ is the mean free time.

Thus the current density J is given by

$$J = nev_d$$

Where n is the number of electrons per unit volume

$$J = ne\left(\frac{eE\tau}{m}\right)$$

$$\Rightarrow \frac{E}{J} = \frac{m}{ne^2\tau} \quad ----\ (1)$$

According to microscopic form of Ohm's law, the resistivity 'ρ' of a conductor is expressed as

$$\rho = \frac{E}{J}$$

Thus equation (1) becomes:

$$\rho = \frac{m}{ne^2\tau}$$

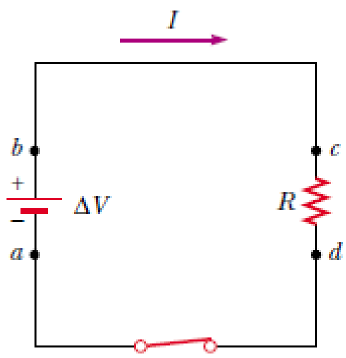
This equation gives the value of electrical resistivity.

OHMIC DEVICES

A material of a circuit element that obeys the ohm law is called the ohmic. Therefore a conducting device obeys ohm's law, if the resistance between the two points is independent of magnitude and polarity of potential difference.

ENERGY TRANSFER IN AN ELECTRIC CIRCUIT AND POWER DISSIPATION

Let a battery is connected between the terminals 'a' and 'b' of an electric circuit as shown in the figure.



Let 'V' is the potential difference applied by the battery between the points 'a' and 'b'. As the result the current 'I' flow through the circuit. During this process, energy is transfer from battery to the electrical circuit. Let a small amount of charge 'dq' during the small interval of time 'dt'.

Using the meaning of potential difference, the work done ΔW in moving ΔQ up through the potential difference V is:

$$\Delta W = V \times \Delta Q$$

This work done will be appear the energy supplied by the battery. The rate at which the battery is supplying electrical energy is called the electrical power of the battery.

$$Electrical\ Power\ = \frac{Energy\ Supplied}{Time\ Taken} = V\ \frac{\Delta Q}{\Delta t}$$

Since $I\ = \frac{\Delta Q}{\Delta t}$

$$Electrical\ Power\ = \ VI$$

By the principal of conservation of energy, the electrical power of the battery is dissipated in the resistor R. Therefore,

$$Power\ Dissipated\ (P) = \ VI$$

From Ohm's law, substituting $V\ = \ IR$ and $I = \frac{V}{R}$

$$Power\ Dissipated\ (P) = \ VI = \ IR * I\ = \ I^2 R$$

$$Power\ Dissipated\ (P) = \ VI = V * \frac{V}{R} = \frac{V^2}{R}$$

JOULE HEATING

The electrical energy consumed in a resistor appears in the form of heat, which is also called 'Joule Heating'. The heat energy produced in t interval of time is given by

Heat Energy = (Power)(Time)

$$= \ (VI)(t)$$

$$= \ VIt\ = \ I^2 R\ t = \frac{V^2}{R} t$$

Sample problem 6. You are given length of heating wire made of nickel-chromium-iron Alloy called Nichrome. It has a resistance of 72Ω. Under what circumstances, the wire will dissipates more power. (a) It is to be connected across a 120 V line. (b) The wire is cut in half pieces and two halves are connected in parallel across the line?

Solution: (a) Power dissipation in the wire:

$$R = 72\Omega$$

$$P = \frac{V^2}{R} = \frac{(120\ )^2}{72} = 200\ \text{Watt}$$

(b) The power dissipation when the wire is cut in half pieces and two halves are connected in parallel across the line:

$$R' = 36\Omega$$

$$P' = \frac{V^2}{R'} = \frac{(120\ )^2}{36} = 400\ \text{Watt}$$

The power dissipation in second case in more than 1st case

Problem 45. A student has a portable radio $9V, 7.5W$ was left on from 9:00 pm to 3:am. How much charge pass through it.

Solution: $V = 9V$

$P = 7.5W$

$t = 6h = 6 \times 3600\ s = 21600\ s$

$q = ?$

As

$$P = VI \Longrightarrow I = \frac{P}{V} \Longrightarrow \frac{q}{t} = \frac{P}{V} \Longrightarrow q = \frac{Pt}{V}$$

$$\Longrightarrow q = \frac{Pt}{V} = \frac{7.5 \times 21600}{9} = 18000\ C$$

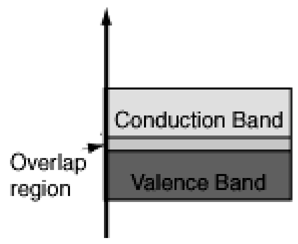
ENERGY BAND THEORY AND ELECTRICAL BEHAVIOR OF CONDUCTORS, INSULATORS AND SEMICONDUCTORS

Energy band theory

The electrons of an atom have the discrete values of energy which are also called quantized energy levels. The concept of discrete energy level is related to an isolated atoms i.e. the atom that does not interact with other atoms. But if the atom is not alone and is under the influence of its neighboring atoms, then each energy level splits into sub-levels. This group of sub-level is called the energy band. Within the energy band, there are permitted energy states, which are so close together that they are virtually continuous. But there exist an energy gap between these bands, which contains no states that an individual electron may occupy. It is also called forbidden energy gap. The electron may jump from one energy band to another by acquiring energy equal to the energy of forbidden energy gap.

Conductors

All the metals are good conductors of electricity and their resistivity is of the order of $10^{-8}\ \Omega - m$. In case of conductors, there is no forbidden energy gap between the valance and the conduction band. The valance band and conduction band are partially filled at room temperature. So the electrons can easily jump from valance band to the conduction band. Due to this reason, the current can easily pass through conductors.

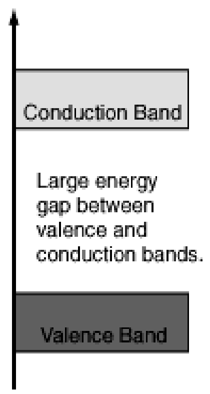


Conductor

The temperature coefficient resistivity is positive. It means that the resistance of conductors increases by increasing the temperature.

Insulators

The insulators have the very large value of resistivity which is of the order of $10^{10}\ \Omega - m$. In case of insulators, the valance band is completely filled and the conduction band is empty. The energy gap between the valance and conduction band is very large. Thus, no electron can jump from valence band to conduction band. As there are no free electrons in insulator, hence no current can pass through insulators.



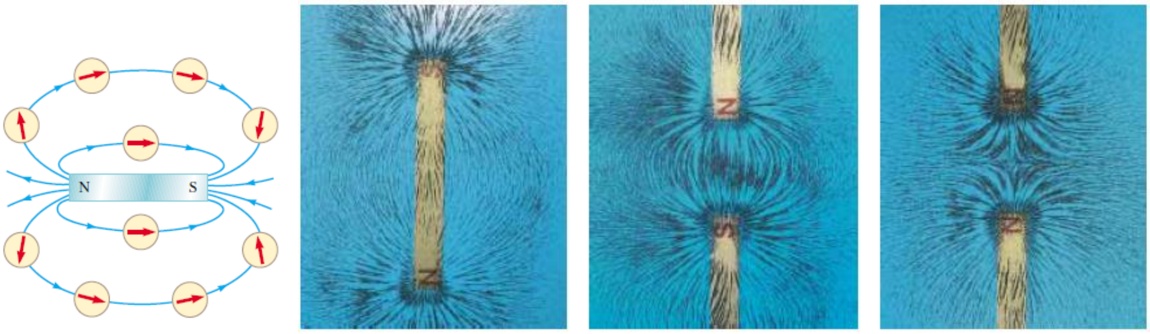
Insulator

MAGNETIC FIELD EFFECTS

34.1 Magnetic Field

Magnetic field is the region or space around any charge within which its influence can be felt by other magnetic substances.

The magnetic field around any magnet is considered as closely spaced magnetic field lines. The magnetic field lines of a bar magnet can be traced with the aid of a compass as shown in the figure below:



In addition to a bar magnet, a moving charge or a current creates a magnetic field in the surrounding space (in addition to its electric field). The magnetic field exerts a force F on any other moving charge or current that is present in the field.

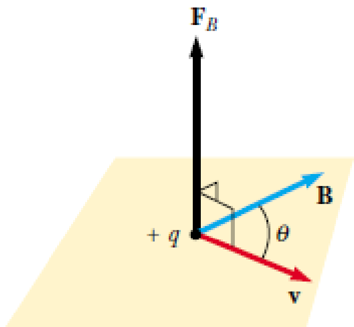
34.2 Magnetic Force on a Charged Particle

Consider a charged object having charge q is projected a uniform magnetic field of flux density **B** with velocity **v**. The magnetic force **F** acting on the object can be expressed as:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

$$\mathbf{F} = qvB \sin\theta\, \hat{n} \qquad \text{------------------------} \quad (1)$$

Here $\hat{n}$ is the unit vector that is perpendicular to the plane of **v** and **B**, and is used to describe the direction of magnetic force **F** on the charged object.



It is clear from equation (1) that the maximum magnetic force will act on the charged object when it will be projected perpendicular to the magnetic field. The maximum magnetic force on the charged object will be

$$F = qvB$$

$$\Rightarrow B = \frac{F}{qv}$$

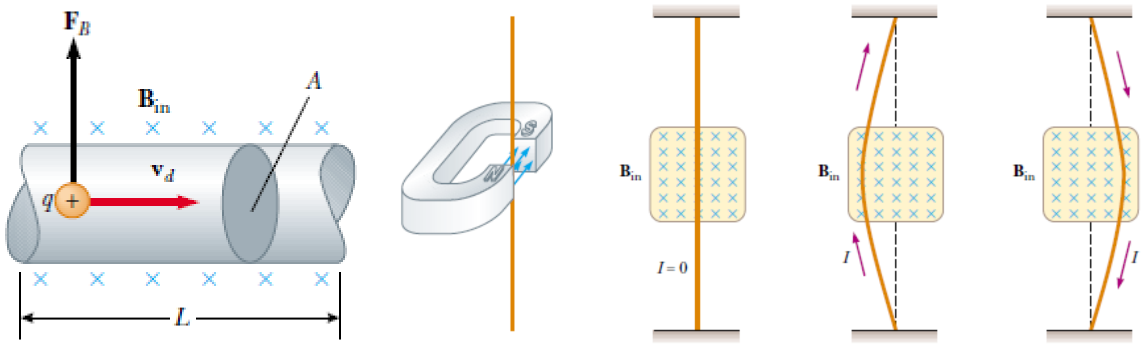
B is also called the magnetic field induction which can be defined as:

"*The force on a unit positive charge moving perpendicular to the magnetic field with uniform velocity*". The SI unit of magnetic field induction **B** is tesla, while in cgs system of units **B** is measured in gauss.

$$1\ tesla = 10^4\ gauss$$

34.3 Magnetic Force on a Current Carrying Conductor

Consider a current carrying wire of length L and cross-sectional area A, is placed in a uniform magnetic field of flux density **B** as shown in the figure below:



If n is the number of free charges per unit volume of a conductor (each having charge e), then the total charge flowing through the wire is $q = nALe$.

Suppose the charges are moving with drift velocity $\mathbf{v_d}$ and cover length L in t seconds, then

$$t = \frac{L}{v_d}$$

If I is the current flowing through the conductor, then

$$I = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d$$

$$\Rightarrow v_d = \frac{I}{nAe}$$

Now

Force on one charge $F' = evB_\perp$

Force on nAL charges $F = nALF' = nALevB_\perp$

Putting value of v_d, we get

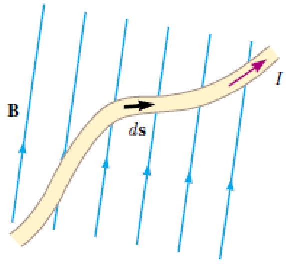
$$F = nALe\left(\frac{I}{nAe}\right)B_\perp = ILB_\perp$$

In vector form:

$$\mathbf{F} = I(\mathbf{L} \times \mathbf{B})$$

The direction of F will be perpendicular to the plane of **L** and **B**.

If the wire is not straight or field is not uniform, then we divide the wire into small elements of length ‘d**s**’. Then the force on each segment is written as:



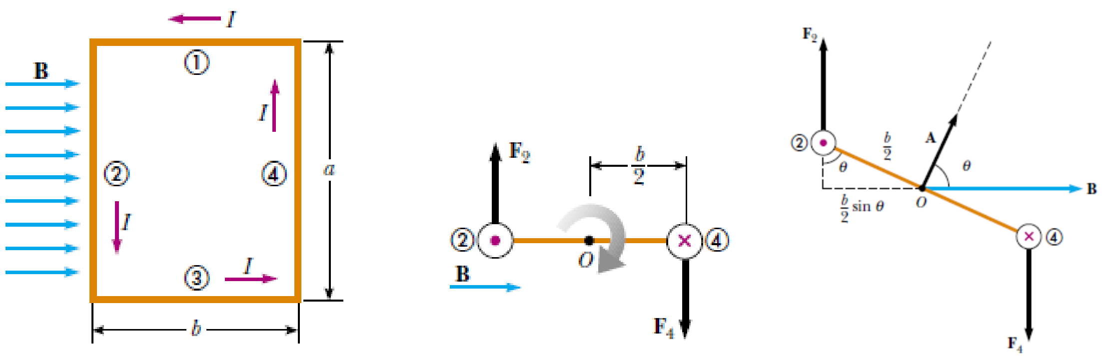
$$d\mathbf{F} = I(d\mathbf{s} \times \mathbf{B})$$

Then the force on whole wire is obtained by integrating over the whole length L.

$$\mathbf{F} = \int d\mathbf{F} = \int I(d\mathbf{s} \times \mathbf{B})$$

34.4 Torque on a Current Loop in Magnetic Field

Consider a loop of wire carrying current suspended in a uniform magnetic field of flux density **B** as shown in the figure below:



We want to find out the expression of torque produced in current carrying coil due to the effect of magnetic field.

The side 1 and 3 of coil are oriented parallel, while the sides 2 and 4 are held perpendicular to the magnetic field. No magnetic forces act on sides 1 and 3 because these wires are parallel to the field; hence $\mathbf{L} \times \mathbf{B} = 0$, for these sides. However, magnetic forces do act on sides 2 and 4 because these sides are oriented perpendicular to the field. The magnitude of these forces is

$$F_2 = F_4 = F = ILB \sin\theta = IaB \sin 90^{\circ} = IaB \qquad \because L = a$$

The direction of F_2, the magnetic force exerted on wire 2, is out of the page and that of F_4, the magnetic force exerted on wire 4, is into the page. These two anti parallel forces are separated by a small distance form a couple. The couple tends to rotate the loop about point O. The magnitude of torque of this couple will be:

$$Torque = (Force)\,(Perpendicular\ Distance\ between\ line\ of\ action\ of\ Force)$$

$$\tau = (F)(b \sin\theta) = (IaB)(b \sin\theta) = IabB \sin\theta$$

where θ is the angle between the magnetic field **B** and vector area **A** of loop.

$\tau = IAB \sin\theta$ $\qquad\qquad\qquad \because A = ab = area\ of\ loop$

This is the expression of torque produced in a loop with single turn. For a loop with N turns, the torque will be:

$\tau = NIAB \sin\theta$

This result shows that the torque has its maximum value IAB when the field is perpendicular to the normal to the plane of the loop $(\theta = 90°)$, and is zero when the field is parallel to the normal to the plane of the loop $(\theta = 0°)$.

In vector form:

$\boldsymbol{\tau} = NI(\mathbf{A} \times \mathbf{B})$

This is the expression of the torque on a current carrying loop in a magnetic field.

34.5 The Magnetic Dipole

The electric dipole placed in an electric field will experience a torque, which is expressed as:

$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$

where $p = qd$ is the electric dipole moment. The magnitude of the torque produce in an electric dipole in an electric field is expressed as:

$\tau = pE \sin\theta$

where θ is the angle between **p** and **E**.

Similarly, the expression of torque acting on a magnetic dipole placed in a magnetic field of flux density **B** is described as:

$\boldsymbol{\tau} = NI(\mathbf{A} \times \mathbf{B})$

The magnitude of the torque is

$\tau = NIAB \sin\theta$ ------------------------ (1)

By analogy with the electrical case, we define a vector **μ**, the magnetic dipole moment, to have the magnitude:

$\mu = NIA$ ------------------------ (2)

Thus, the equation (1) becomes

$\tau = \mu B \sin\theta$

In vector form,

$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$ ------------------------ (3)

Equation (3) gives the torque on a current carrying loop in a magnetic field of flux density **B**.

, ,

The magnetic dipole consist of two opposite magnetic poles (north and south), which are separated by a small distance. And the magnetic dipole moment is a vector associated with a magnet or a current loop, whose cross product with magnetic field strength is equal to the torque exerted on the system by the field.

The work done on a magnetic dipole to change its orientation in the magnetic field is stored as potential energy of magnetic dipole which is given as:

$$P.E = \text{Work Done}$$

$$P.E = \int \tau \, d\theta$$

$$= \int \mu B \sin\theta \, d\theta$$

$$= \mu B \int \sin\theta \, d\theta$$

$$= \mu B(-\cos\theta)$$

$$= -\mu B \cos\theta$$

$$\Rightarrow P.E = -\boldsymbol{\mu}.\mathbf{B} \quad \text{------------------------} \quad (4)$$

This is the expression for P.E. of magnetic dipole.

The equation (4) tells us that the unit of magnetic dipole moment is obtained by dividing by energy unit by the unit of magnetic induction. Therefore,

Unit of Dipole Moment $\mu = \frac{joule}{tesla} = \frac{J}{T} = JT^{-1}$

The other unit of dipole moment is described by using the equation (2):

$$\mu = NIA$$

Unit of Dipole Moment $\mu = (ampere) \times (meter)^2 = A - m^2$

Question. Prove that $\boldsymbol{\frac{joule}{tesla} = (ampere) \times (meter)^2}$

$$\text{L.H.S.} = \frac{joule}{tesla} = \frac{(newton) \times (meter)}{tesla}$$

$$\because work\ W = (force\ F) \times (displacement\ d)$$

$$= \frac{(ampere \times meter \times tesla) \times (meter)}{tesla}$$

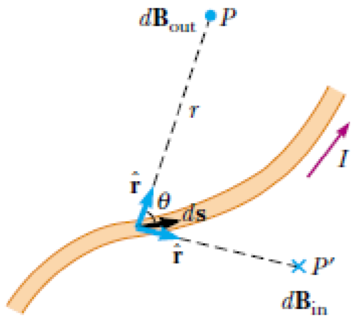
$$\because force\ F = (current\ I) \times (length\ L) \times (magnetic\ induction\ B)$$

$$= \frac{ampere \times (meter)^2 \times tesla}{tesla} = ampere \times (meter)^2 = \text{R.H.S.}$$

AMPERE'S LAW

35.1 The Biot–Savart Law

Shortly after Oersted's discovery in 1819 that a compass needle is deflected by a current-carrying conductor, Biot (1774–1862) and Savart (1791–1841) performed quantitative experiments on the force exerted by an electric current on a nearby magnet. The experimental observations of Biot and Savart about the magnetic field produced by a current carrying conductor at some point in space are as follows:



- The vector $d\mathbf{B}$ is perpendicular both to $d\mathbf{s}$ (which points in the direction of the current) and to the unit vector $\hat{\mathbf{r}}$ directed from $d\mathbf{s}$ toward P.
- The magnitude of $d\mathbf{B}$ is inversely proportional to r^2, where r is the distance from $d\mathbf{s}$ to P.
- The magnitude of $d\mathbf{B}$ is proportional to the current and to the magnitude $|d\mathbf{s}|$ of the length element $d\mathbf{s}$.
- The magnitude of $d\mathbf{B}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\mathbf{s}$ and $\hat{\mathbf{r}}$.

These observations are summarized in the mathematical expression known today as the Biot–Savart law:

$$|d\mathbf{B}| \ \alpha \ \frac{I d\mathbf{s} \sin\theta}{r^2}$$

$$\Longrightarrow |d\mathbf{B}| = \frac{\mu_0}{4\pi} \frac{I\, ds \sin\theta}{r^2}$$

Where $\frac{\mu_0}{4\pi}$ is the constant of proportionality and $\mu_0 = 4\pi \times 10^{-7} \ \frac{Wb}{A-m}$ is called the permeability of free space.

In vector form:

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, ds \sin\theta}{r^2} \hat{\mathbf{n}} \qquad \text{------------------------} \ (1)$$

Where $\hat{\mathbf{n}}$ is the unit vector showing the direction of field and is determined by right band rule.

If $\hat{\mathbf{i}}$ is a unit vector along current element and $\hat{\mathbf{r}}$ is the unit vector along position vector $\mathbf{r}$ of point P, then

$$\hat{\mathbf{i}} \times \hat{\mathbf{r}} = |\hat{\mathbf{i}}||\hat{\mathbf{r}}| \sin\theta\, \hat{\mathbf{n}} = (1)(1)\sin\theta\, \hat{\mathbf{n}} = \sin\theta\, \hat{\mathbf{n}}$$

The equation (1) will become:

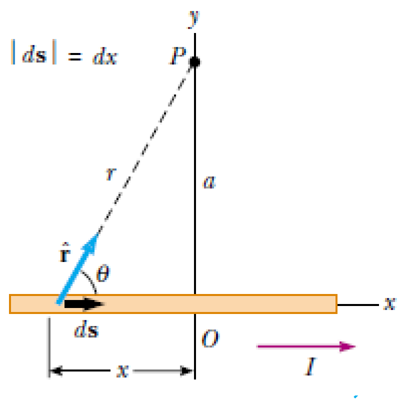
$$d\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, ds \sin\theta}{r^2} \hat{\mathbf{n}} = \frac{\mu_0}{4\pi} \frac{I\, ds\, (\hat{\mathbf{i}}\times\hat{\mathbf{r}})}{r^2} = \frac{\mu_0}{4\pi} \frac{I\, ds\, \hat{\mathbf{i}}\times\hat{\mathbf{r}}}{r^2}$$

As $\hat{\mathbf{i}}$ is a unit vector along current element, therefore, $d\mathbf{s} = ds\,\hat{\mathbf{i}}$

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$

$$\Longrightarrow d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{s}\times\left(\frac{\mathbf{r}}{r}\right)}{r^2} \qquad \because\ \hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$$

$$\Longrightarrow d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{s}\times\mathbf{r}}{r^3} \qquad \text{------------------------} \quad (2)$$



The magnetic induction **B** at point P due to whole wire is obtained by integrating eq. (2):

$$\mathbf{B} = \int d\mathbf{B} = \frac{\mu_0}{4\pi}\int\frac{I\,d\mathbf{s}\times\mathbf{r}}{r^3}$$

This is known as Biot–Savart law.

35.2 Application of Biot–Savart law

35.2.1 Magnetic Field due to Current in a Straight Conductor

Consider a long straight conductor carries current I as shown in the figure below:

We want to find out magnetic field strength at point P due to this current carrying conductor.

The perpendicular distance of point P from the wire is 'a'.

For this we consider a small length element ds, which is at the distance x from point O (taken as the origin). The magnetic field strength due to length element ds by Biot–Savart law will be:

$$dB = \frac{\mu_0}{4\pi}\frac{I\,ds\sin\theta}{r^2} \qquad \text{------------------------} \quad (1)$$

$$\because\ r^2 = x^2 + a^2 \quad and \quad ds = dx$$

Thus equation (1) will become

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dx\sin\theta}{x^2+a^2} \qquad \text{------------------------} \quad (2)$$

From figure:

$$\tan\theta = \frac{a}{-x} \Longrightarrow x = -\frac{a}{\tan\theta}$$

$\because$ negative sign is necessary because ds is located at a negative value of x

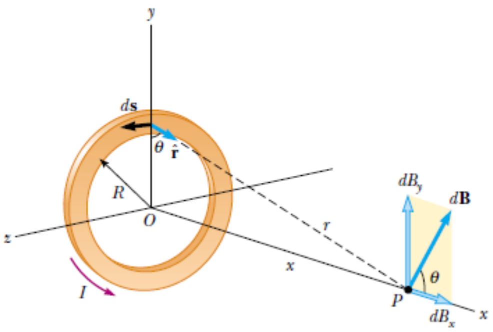
$$\Longrightarrow x = -a\cot\theta$$

And $dx = a\csc^2\theta\ d\theta$

The equation (2) will become:

$$dB = \frac{\mu_0}{4\pi}\frac{I\,(a\csc^2\theta\ d\theta)\sin\theta}{a^2\cot^2\theta + a^2} = \frac{\mu_0}{4\pi}\frac{I\,(a\csc^2\theta\ d\theta)\sin\theta}{a^2\,(1+\cot^2\theta)} = \frac{\mu_0}{4\pi}\frac{I\,(a\csc^2\theta\ d\theta)\sin\theta}{a^2\csc^2\theta}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I \sin\theta d\theta}{a} \qquad \text{------------------------} \quad (3)$$



To find out the field due to the whole wire can be obtained by integrating equation (3):

$$B = \int_{-\infty}^{+\infty} dB = \frac{\mu_0}{4\pi} \int_{-\infty}^{+\infty} \frac{I \sin\theta d\theta}{a} = \frac{\mu_0 I}{4\pi a} \int_{-\infty}^{+\infty} \sin\theta \, d\theta$$

As $x = -a \cot\theta$

$when\, x \to -\infty: \qquad \theta \to 0$

$when\, x \to \infty: \qquad \theta \to \pi$

$$B = \frac{\mu_0 I}{4\pi a} \int_0^{\pi} \sin\theta \, d\theta = \frac{\mu_0 I}{4\pi a} \left| -\cos\theta \right|_0^{\pi}$$

$$B = -\frac{\mu_0 I}{4\pi a} (\cos\pi - \cos 0) = -\frac{\mu_0 I}{4\pi a} (-2)$$

$$B = \frac{\mu_0 I}{2\pi a}$$

This is the expression of magnetic field induction due to a current carrying conductor.

In vector form:

$$\mathbf{B} = \frac{\mu_0 I}{2\pi a} \hat{\mathbf{n}}$$

Where $\hat{n}$ is the unit vector which represents the direction of tangent to the circle at point P.

35.2.2 Magnetic Field due to a Circular Current Loop

Consider a circular current loop of radius R carrying current I as shown in the figure below:

We want to find out the value of magnetic field strength at point P on at the distance x from the center of the loop. For this we consider a small element of length $d\mathbf{s}$ such that the angle between $\mathbf{r}$ and $d\mathbf{s}$ is 90°. Therefore,

$$dB = \frac{\mu_0}{4\pi} \frac{I\, ds \sin\theta}{r^2}$$

By Biot–Savart law, we find that magnetic induction $d\mathbf{B}$ is perpendicular to $\mathbf{r}$ and $d\mathbf{s}$. So by putting $\theta = 90^\circ$, we get:

$$dB = \frac{\mu_0}{4\pi} \frac{I\, ds}{r^2} \qquad \text{------------------------} \quad (1)$$

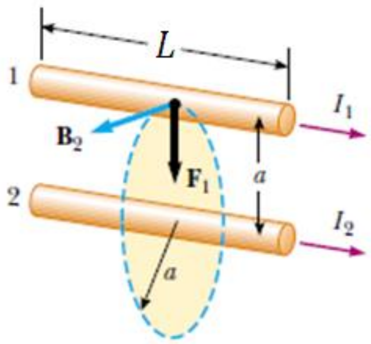
We resolve the $d\mathbf{B}$ into rectangular components dB_x and dB_y. From the symmetry, we find that dB_y has no contribution to the field at point P, because dB_y components of element will cancel out each other. Only the x-components are added up to give the magnetic field strength at point P. Therefore,

$$B = \oint dB_x = \oint dB\cos\theta = \oint \frac{\mu_0}{4\pi}\frac{I\,ds}{r^2}\cos\theta = \frac{\mu_0 I}{4\pi}\oint \frac{ds}{r^2}\cos\theta$$

From figure $r^2 = x^2 + R^2$

And

$\cos\theta = \frac{R}{r} = \frac{R}{\sqrt{x^2+R^2}}$



$$B = \frac{\mu_0 I}{4\pi}\oint \frac{ds}{x^2+R^2}\quad \frac{R}{\sqrt{x^2+R^2}} = \frac{\mu_0 IR}{4\pi\,(x^2+R^2)^{3/2}}\oint ds$$

$$B = \frac{\mu_0 IR}{4\pi\,(x^2+R^2)^{3/2}}\oint ds \qquad \because$$

$$\oint ds \quad = 2\pi R$$

$$B = \frac{\mu_0 IR}{4\pi\,(x^2+R^2)^{3/2}}(2\pi R)$$

$$B = \frac{\mu_0 IR^2}{2\,(x^2+R^2)^{3/2}} \qquad \text{------------------------} \quad (2)$$

This is the expression of magnetic induction at point P due to circular loop of current.

Special Cases.

Case 1. *Magnetic field induction at center of circular current carrying loop*

At the center of the loop $x = 0$. Putting value in equation (2), we get

$$B = \frac{\mu_0 IR^2}{2\,(R^2)^{3/2}} = \frac{\mu_0 IR^2}{2\,R^3}$$

$$B = \frac{\mu_0 I}{2\,R}$$

Case 2. *Magnetic field induction at very large distance from loop*

At very large distance from the center of the current loop i.e., $x \gg R$, the magnetic field induction will be:

$$B = \frac{\mu_0 IR^2}{2\,(x^2+R^2)^{3/2}} \qquad \because x \gg R;\quad x^2 + R^2 \approx x^2$$

$$B = \frac{\mu_0 IR^2}{2\,(x^2)^{3/2}} = \frac{\mu_0 IR^2}{2\,x^3}$$

$$B = \frac{\mu_0 I\pi R^2}{2\pi\,x^3} = \frac{\mu_0 IA}{2\pi\,x^3} \qquad \because \textit{Multipying and Dividing by } \pi$$

This is the expression of magnetic induction for a current carrying coil of one loop. For a current carrying loop having N turns:

$$B = \frac{\mu_0 NIA}{2\pi\,x^3} = \frac{\mu_0 \mu}{2\pi\,x^3} \qquad \because \text{Dipole Moment } \mu = NIA$$

35.2.3 Force between Long Parallel Current Carrying Conductor

Consider two long, straight, parallel wires separated by a small distance ‘a’ carrying currents I_1 and I_2, respectively. The current passing through each wire produces a magnetic field around it and each wire is placed in the magnetic field produced by the other.

The wire 2, which carries a current I_2 creates a magnetic field B_2 at the location of wire 1, which can be find out by using the expression:

$$B_2 = \frac{\mu_0 I_2}{2\pi a} \qquad \text{------------------------} \quad (1)$$

The direction of B_2 is perpendicular to wire 1, as shown in Figure. The magnetic force on wire 1, due to magnetic force of wire 2, will be:

$$\mathbf{F_1} = I_1 \mathbf{L} \times \mathbf{B_2}$$

The magnitude of magnetic force on wire 1 is given by:

$$\Rightarrow F_1 = I_1 L B_2 \sin 90^{\circ} \qquad \because \mathbf{L} \text{ and } \mathbf{B_2} \text{ are perpendicular to each other}$$

$$\Rightarrow F_1 = I_1 L B_2$$

$$\Rightarrow F_1 = I_1 L \left(\frac{\mu_0 I_2}{2\pi a}\right) = \frac{\mu_0 \, I_1 \, I_2}{2 \, \pi \, a} L$$

where L is the length of conductor. The direction of force F_1 is towards wire 2.

Similarly, the force on wire 2, due to the magnetic force of wire 1, carrying current I_2 is expressed as:

$$F_2 = \frac{\mu_0 \, I_1 \, I_2}{2 \, \pi \, a} L$$

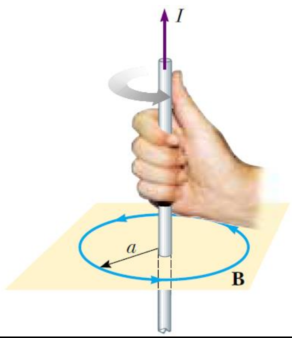
The force F_2 is directed towards wire 1. The forces are equal and opposite, so, will attract each other.

If the direction of element in one wire is opposite to that in the other, the wires will repel each other.

35.3 Ampere’s Circuital Law

The ampere law is stated as,

“The line integral of magnetic induction **B** *due to current I around any close loop is $\mu_0$times the current enclosed”*



35.3.1 Integral Form of Ampere’s Law

Consider a straight current carrying wire. The magnetic field will produced around the conductor as the current flow through it.

The magnetic field strength due to this current carrying wire at any point at the distance a is expressed as:

$$\mathbf{B} = \frac{\mu_0 I}{2\pi R}\,\hat{\mathbf{k}} \qquad \text{------------------------} \quad (1)$$

The symmetry shows that the magnetic induction is same everywhere on the circumference of circle. The equation (1) can be expressed in more general form as:

$$\oint \mathbf{B}.d\mathbf{l} = \oint \frac{\mu_0 I}{2\pi a}\hat{\mathbf{k}}.d\mathbf{l}$$

$$= \frac{\mu_0 I}{2\pi a}\oint \hat{\mathbf{k}}.d\mathbf{l}$$

$$\because \hat{\mathbf{k}} \text{ is partallel to } d\mathbf{l}$$

$$= \frac{\mu_0 I}{2\pi a}\oint dl \qquad \because dl = ad\theta$$

$$\Rightarrow \oint \mathbf{B}.d\mathbf{l} = \frac{\mu_0 I}{2\pi a}\oint ad\theta = \frac{\mu_0 I a}{2\pi a}\oint d\theta = \frac{\mu_0 I}{2\pi}(2\pi)$$

$$\Rightarrow \oint \mathbf{B}.d\mathbf{l} = \mu_0 I$$

This is called Ampere's circuital law.

As $I = \int \mathbf{J}.d\mathbf{s}$, therefore

$$\oint \mathbf{B}.d\mathbf{l} = \mu_0 \int \mathbf{J}.d\mathbf{s}$$

This is integral form of Ampere's law.

35.3.2 Differential Form of Ampere's Law

The Ampere's law is expressed in integral form as:

$$\oint \mathbf{B}.d\mathbf{l} = \mu_0 \int_s \mathbf{J}.d\mathbf{s} \qquad \text{------------------------} \quad (2)$$

Using Stoke's theorem on L.H.S., we get

$$\oint \mathbf{B}.d\mathbf{l} = \int_s curl\,\mathbf{B}\,.d\mathbf{s}$$

Thus equation (2) will become:

$$\int_s curl\,\mathbf{B}\,.d\mathbf{s} = \mu_0 \int_s \mathbf{J}.d\mathbf{s}$$

$$\int_s curl\,\mathbf{B}\,.d\mathbf{s} - \mu_0 \int_s \mathbf{J}.d\mathbf{s} = 0$$

$$\int_s (curl\,\mathbf{B} - \mu_0 \mathbf{J}).d\mathbf{s} = 0$$

$$\Rightarrow curl\,\mathbf{B} - \mu_0 \mathbf{J} = 0$$

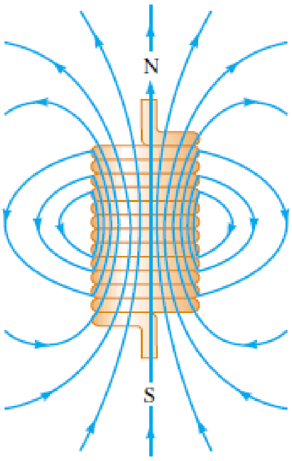
$$\Longrightarrow curl\ \mathbf{B} = \mu_0 \mathbf{J}$$

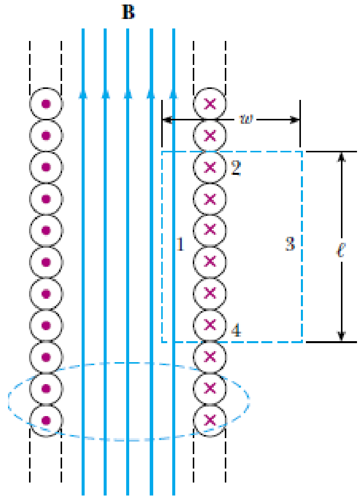
This is differential form of Ampere's law.

35.4 Applications of Ampere's Law

35.4.1 Magnetic Field due to a Solenoid

A solenoid is a cylindrical frame tightly wounded by an insulated wire. The magnetic field produced by the current carrying solenoid is like the field of a bar magnet. The magnetic field strength outside the solenoid is negligible as compared to the field inside it.





We want to find out magnetic field intensity **B** at point *P* inside a current carrying solenoid. For this we consider an Amperian loop. The symmetry shows that the close path can be divided into four elements1, 2, 3 and 4. By Ampere's law:

$$\oint \mathbf{B}.d\mathbf{l} \;=\; \mu_0 \times (Current\ Enclosed)$$

The integral $\oint \mathbf{B}.d\mathbf{l}$ can be written as the sum of four integrals:

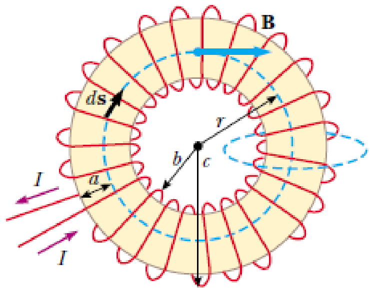
$$\int_{\mathbf{1}} \mathbf{B}.d\mathbf{l} + \int_{\mathbf{2}} \mathbf{B}.d\mathbf{l} + \int_{\mathbf{3}} \mathbf{B}.d\mathbf{l} + \int_{\mathbf{4}} \mathbf{B}.d\mathbf{l} = \mu_0 \times (Current\ Enclosed)$$

Here $\int_{\mathbf{2}} \mathbf{B}.d\mathbf{l} = \int_{\mathbf{4}} \mathbf{B}.d\mathbf{l} = 0$ because the angle between **B** and $d\mathbf{l}$ is 90°.

Also $\int_{\mathbf{3}} \mathbf{B}.d\mathbf{l} = 0$ as the field outside the solenoid is negligible.

Therefore,

$$\int_{\mathbf{1}} \mathbf{B}.d\mathbf{l} = \mu_0 \times (Current\ Enclosed)$$



$$\Rightarrow \int_{1} B\ dl\ \cos 0^{\circ} = \mu_0 \times (Current\ Enclosed)$$

$$\Rightarrow \int_{1} B\ dl = \mu_0 \times (Current\ Enclosed)$$

Since the magnetic field strength **B** for the loop element 1 is constant inside the solenoid. So,

$$B \int_{1} dl = \mu_0 \times (Current\ Enclosed)$$

$$B\ l = \mu_0 \times (Current\ Enclosed) \quad \text{------------------------} \quad (1)$$

where l is the length of element 1 of Amperian loop.

To find out the current enclosed by the loop, we consider that there are n turns per unit length of a solenoid. Then

$$Number\ of\ turns\ in\ length\ l\ of\ solinoid = n\ l$$

If I is the current flowing through solenoid, then

$$Current\ Enclosed\ by\ the\ Amperian\ Loop = n\ l\ I$$

Hence the equation (1) will become:

$$B\ l = \mu_0 \times (n\ l\ I)$$

$$B = \mu_0 n\ I$$

This is the expression of magnetic field strength due to a current carrying solenoid. This relation shows that the magnetic field inside a solenoid depend only on current I and number of turns per unit length n.

35.4.2 Magnetic field due to a Toroid

A toroid is used to create an almost uniform magnetic field in some enclosed area. The device consists of a conducting wire wrapped around a ring (a *torus*) made of a nonconducting material as shown in the figure.

We want to find out magnetic field strength at any point inside a toroid. For this we consider circular Amperian loop of radius r. By symmetry, we see that the magnitude of the field is constant on this circle and tangent to it, therefore:

$$\mathbf{B}.d\mathbf{s} = B\ ds$$

Where $d\mathbf{s}$ is the small element of amperian loop.

By applying Ampere's law:

$$\oint \mathbf{B}.d\mathbf{s} = \mu_0 \times (Current\ Enclosed)$$

, ,

$$\Rightarrow \oint B\, ds \; = \; \mu_0 \times (Current\ Enclosed)$$

$$\Rightarrow B \oint \; ds \; = \; \mu_0 \times (Current\ Enclosed)$$

$$\because \oint \; ds \; = 2\pi r$$

$$\Rightarrow B \times 2\pi r \; = \; \mu_0 \times (Current\ Enclosed) \quad \text{------------------------} \quad (1)$$

If N are the number of turns of the toroid and I is the current in the toroid, then

$$Current\ Enclosed\ by\ the\ Amperian\ Loop = N\ I$$

The equation (1) will become:

$$B \times 2\pi r \; = \; \mu_0 \times (N\ I\ )$$

$$B \; = \; \frac{\mu_0 N\ I}{2\pi r}$$

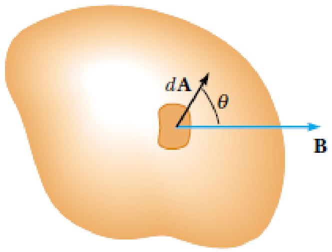
This is the expression of magnetic field strength inside a current carrying toroid.

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

36.1 Magnetic Flux

The number of magnetic lines of force passing normally through certain area is called magnetic flux. It is denoted by Φ_B. It is a scalar quantity and its SI unit is weber (Wb). It is measured by the product of magnetic field strength and the component of vector area parallel to magnetic field.

If d**A** is the vector area element of the surface placed in uniform magnetic field of magnetic field strength **B** as shown in the figure below.



The magnetic flux $d\Phi_B$ through d**A** is given by:

$$d\Phi_B = \mathbf{B}.\,\mathrm{d}\mathbf{A}$$

$$\Phi_B = \int \mathbf{B}.\,\mathrm{d}\mathbf{A}$$

$$\Phi_B = \int B\,dA\cos\theta$$

where θ is the angle between magnetic field strength and vector area element.

36.2 Faraday's Law of Electromagnetic Induction

Statement:

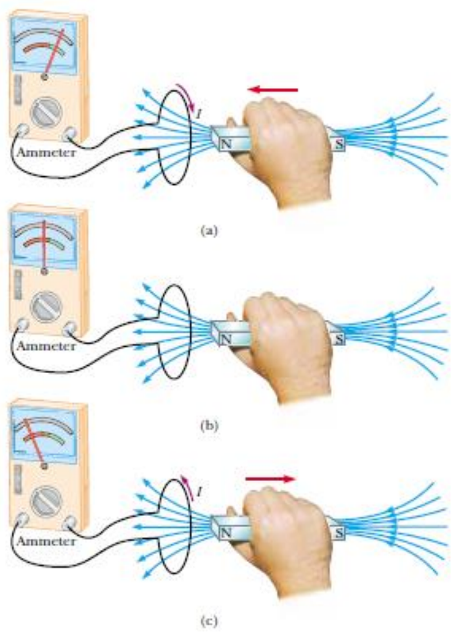
The induced emf in a circuit is equal to the negative of rate at which the magnetic flux through the circuit is changing with time.

Or

The magnitude of induced emf in a circuit is directly proportional to the rate of change of magnetic flux.

Explanation

When a magnet is moved toward the loop, the ammeter needle deflects in one direction, as shown in the figure (a). When the magnet is brought to rest and held stationary relative to the loop figure (b), no deflection is observed. When the magnet is moved away from the loop, the needle deflects in the opposite direction, as shown in figure (c). Finally, if the magnet is held stationary and the loop is moved either toward or away from it, the needle deflects.



From these observations, we conclude that the loop detects that the magnet is moving relative to it and we relate this detection to a change in magnetic field. Thus, it seems that a relationship exists between current and changing magnetic field.

These results are quite remarkable in view of the fact that a current is set up even though no batteries are present in the circuit. We call such a current the induced current which is produced by an induced emf. This phenomenon is called electromagnetic induction.

36.2.1 Integral form of Faraday's Law of Electromagnetic Induction

If ε is the induce emf due to change in flux $d\Phi_B$ in time dt, then we can describe the Faraday's law of electromagnetic induction as:

$$\varepsilon \quad \alpha \quad -\frac{d\Phi_B}{dt}$$

$$\varepsilon \quad = -\,\text{constant}\ \frac{d\Phi_B}{dt}$$

$$\varepsilon \quad = -\ \frac{Nd\Phi_B}{dt}$$

Where N is constant and called number of turns in a coil.

Now as $\Phi_B = \int_s \mathbf{B}.\,\mathrm{d}\mathbf{A}$, therefore

$$\varepsilon \quad = -\ N\,\frac{d}{dt}\ \int_s \mathbf{B}.\,\mathrm{d}\mathbf{A}$$

For a single loop coil, $N = 1$, we have:

$$\varepsilon \quad = -\frac{d}{dt}\ \int_s \mathbf{B}.\,\mathrm{d}\mathbf{A} \qquad \text{----------------}\ (1)$$

Also,

$$\varepsilon \quad = \int \mathbf{E}.\,d\mathbf{r} \qquad \text{----------------}\ (2)$$

Combining equation (1) and (2), we get:

$$\int \mathbf{E}.\,d\mathbf{r} = -\ \frac{d}{dt}\ \int_s \mathbf{B}.\,\mathrm{d}\mathbf{A} \qquad \text{----------------}\ (3)$$

This is the integral form of Faraday's law of electromagnetic induction.

The negative sign is due to the fact that the direction of induced current is such that it opposes the cause producing it.

36.2.2 Differential form of Faraday's Law

Applying the Stoke's theorem on L.H.S of equation (3),

$$\int_s curl\ \mathbf{E}.\,d\mathbf{A} = -\ \frac{d}{dt}\ \int_s \mathbf{B}.\,\mathrm{d}\mathbf{A}$$

$$\int_s\ (curl\ \mathbf{E} + \ \frac{d\mathbf{B}}{dt}).\,d\mathbf{A}$$

$$\Rightarrow curl\ \mathbf{E} + \frac{d\mathbf{B}}{dt} = 0$$

$$\Rightarrow curl\ \mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

Or $$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

This is differential form of Faraday's law of electromagnetism induction.

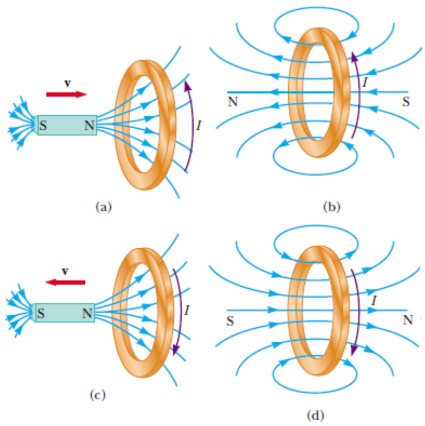
36.3 Lenz law

Statement:

The direction of induced current is such that it opposes it own cause.

Explanation:

Consider a bar magnet moves toward a stationary metal loop, as in figure (a). As the magnet moves to the right toward the loop, the external magnetic flux through the loop increases with time. As the result, the induced current set up in the loop which produces magnetic field, as illustrated in figure (b). Knowing that like magnetic poles repel each other, we conclude that the left face of the current loop acts like a north pole and that the right face acts like a south pole.



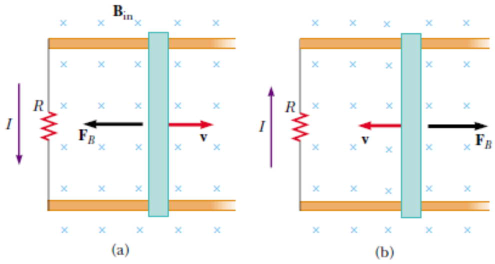
If the magnet moves to the left, as in figure (c), its flux through the area enclosed by the loop decreases in time. Now the induced current in the loop produces the magnetic field as shown in figure (d). In this case, the left face of the loop is a south pole and the right face is a north pole.

36.3.1 Lenz Law and Conservation of Energy

"*Lenz law is the statement of law of conservation of energy for the circuit involving induced current*"

To understand this statement, consider a conducting bar moving to the right on two parallel rails in the presence of a uniform magnetic field as shown in the figure below. As the bar moves to the right, the magnetic flux through the area enclosed by the circuit increases with time because the area increases. As the result, the induced current must be directed counterclockwise when the bar moves to the right. Since the current carrying bar is moving in the magnetic field, it will experience a magnetic force $\mathbf{F_B}$. By using right hand rule, the direction of $\mathbf{F_B}$ is opposite to that of $\mathbf{v}$, that tends to stop the rod. An external dragging force must be applied to keep the rod moving in the magnetic field.

This dragging force provides the energy for the induced currents to flow. This energy is the source of induced current. Thus the electromagnetic induction is exactly according to the law of conservation of energy.



If the bar is moving to the left, as in figure (b), the external magnetic flux through the area enclosed by the loop decreases with time. Because the field is directed into the page, the direction of the induced current must be clockwise.

36.4 Motional Induction and Motional emf

The emf induced in a loop by moving it a magnetic field is called motional emf.
Consider a straight conductor of length l is placed in a magnetic field which is directed into the plane of paper. For simplicity, we assume that the conductor is moving in a direction perpendicular to the field with constant velocity under the influence of some external agent.

As the bar is pulled to the right with a velocity **v** under the influence of an applied force $\mathbf{F_{app}}$, free charges in the bar experience a magnetic force directed along the length of the bar. This force sets up an induced current because the charges are free to move in the closed conducting path. The magnetic flux passing through the circuit is:

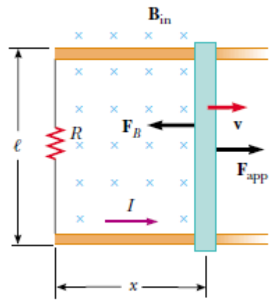
$$\Phi_B = B\,(Area) = B\,(l)\,(x)$$

The induced emf can be determined using Faraday's law of electromagnetic induction:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d\,(B\,l\,x)}{dt} = -B\,l\left(\frac{dx}{dt}\right)$$

$$\varepsilon = -B\,l\,v$$

This is the expression of induced emf in moving conductor. Due to this induced emf, the current induce in the conductor and is given by:



$$I = \frac{\varepsilon}{R} = \frac{-B\,l\,v}{R} \qquad \text{----------------} \quad (1)$$

Where R is the resistance of the loop.

This induced current gives rise to the magnetic forces $\mathbf{F_B} = I\,(\boldsymbol{l} \times \mathbf{B})$ acting on the conductor. The magnitude of magnetic force will be:

$$F_B = I\,l\,B$$

Putting the values of I from equation (1):

$$F_B = \left(\frac{-B\ l\ v}{R}\right) l\ B = \frac{-B^2\ l^2\ v}{R}$$

The negative sign is due to the fact that this magnetic force tends to stop the motion of conductor. In order to move the conductor in this uniform magnetic field with constant velocity, the applied force F_{app} and F_B must be equal and opposite. Therefore,

$$F_{app} = \frac{B^2\ l^2\ v}{R}$$

The power expended in moving the conductor can be find out by the expression:

$$P = F_{app}\ v$$

$$P = \left(\frac{B^2\ l^2\ v}{R}\right) v = \frac{B^2\ l^2\ v^2}{R}$$

This is the expression of power delivered to the conductor t move in magnetic field.

Power Dissipation

The power dissipated due to joule heating can be find out using expression:

$$P_{dissipated} = I^2 R$$

Putting value of I, we get:

$$P_{dissipated} = \left(\frac{-B\ l\ v}{R}\right)^2 R = \left(\frac{B^2\ l^2\ v^2}{R^2}\right) R$$

$$P_{dissipated} = \frac{B^2\ l^2\ v^2}{R}$$

So in case of motional induction, the power delivered and power dissipation is equal. Therefore the work done by the external agent is dissipated as joule heating.

36.5 Induced Electric Field

A straight current carrying conductor surrounded by a magnetic field. This magnetic field is surrounded by magnetic lines of force which are found to be concentric circles having their centers on the wire.

Similarly a changing magnetic field is surrounded by an electric field called induced electric field. This induced electric field is represented by electric lines of force which are found to be concentric circles.

We want to find out the expression of induced electric field due to changing magnetic field. For this, we consider a loop of conducting wire placed in a uniform magnetic field. This magnetic field may be applied by an electromagnet. By varying the current in the electromagnet, we can change the strength of magnetic field.