Chapter 22 Electric Fields

Key contents

Forces and fields
The electric field due to different charge distributions
A point charge in an electric field
A dipole in an electric field

22.2 The Electric Field:

Electric Field To explain the electrostatic force between two charges, we assume that each charge sets up an electric field in the space around it. The force acting on each charge is then due to the electric field set up at its location by the other charge.

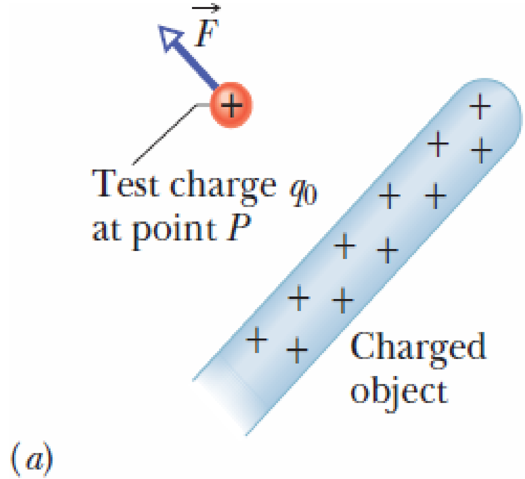
Definition of Electric Field The *electric field* $\vec{E}$ at any point is defined in terms of the electrostatic force $\vec{F}$ that would be exerted on a positive test charge q_0 placed there:

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad \text{(electric field).}$$

The SI unit for the electric field is the newton per coulomb (N/C).

22.2 The Electric Field:

The electric field is a *vector field*.



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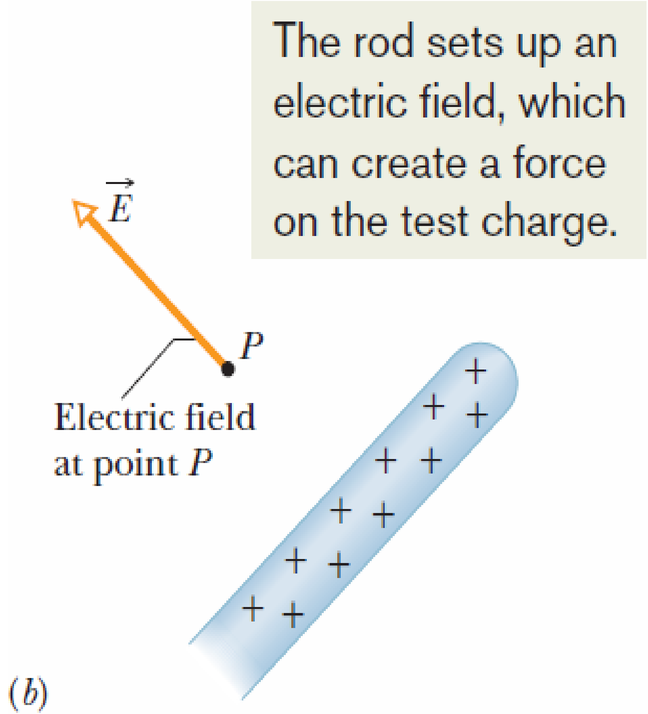


Fig. 22-1 (*a*) A positive test charge q_0 placed at point P near a charged object. An electrostatic force $\vec{F}$ acts on the test charge. (*b*) The electric field $\vec{E}$ at point P produced by the charged object.

22.2 The Electric Field:

Table 22-1

Some Electric Fields

Field Location or Situation	Value (N/C)
At the surface of a uranium nucleus	3×10^{21}
Within a hydrogen atom, at a radius of 5.29×10^{-11} m	5×10^{11}
Electric breakdown occurs in air	3×10^{6}
Near the charged drum of a photocopier	10^{5}
Near a charged comb	10^{3}
In the lower atmosphere	10^{2}
Inside the copper wire of household circuits	10^{-2}

22.3 Electric Field Lines:

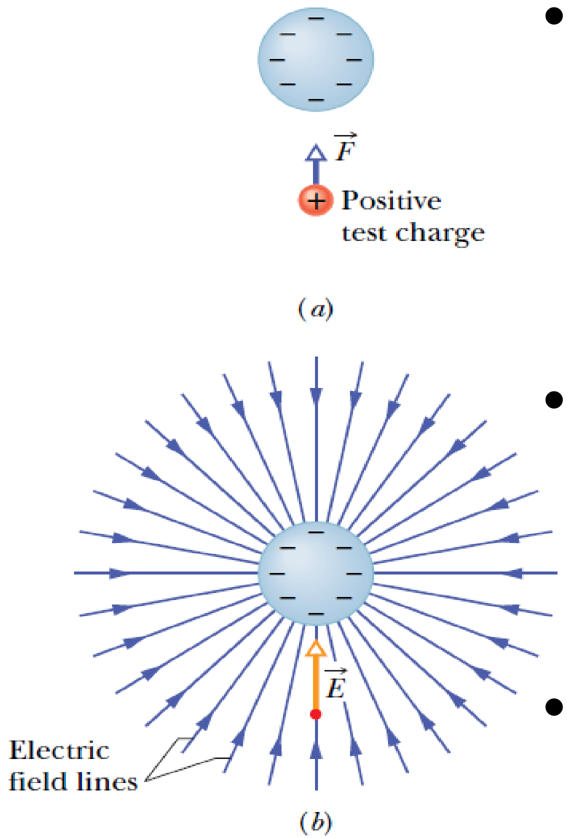


Fig. 22-2 (*a*) The electrostatic force $\vec{F}$ acting on a positive test charge near a sphere of uniform negative charge. (*b*) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sphere. The field lines extend *toward* the negatively charged sphere. (They originate on distant positive charges.)

- Electric field lines are imaginary lines which extend away from positive charge (where they originate) and toward negative charge (where they terminate).

- At any point, the direction of the tangent to a curved field line gives the direction of the electric field at that point.

- The field lines are drawn so that the number of lines per unit area, measured in a plane that is perpendicular to the lines, is proportional to the magnitude of ***E***. Thus, ***E*** is large where field lines are close together and small where they are far apart.

22.3 Electric Field Lines:

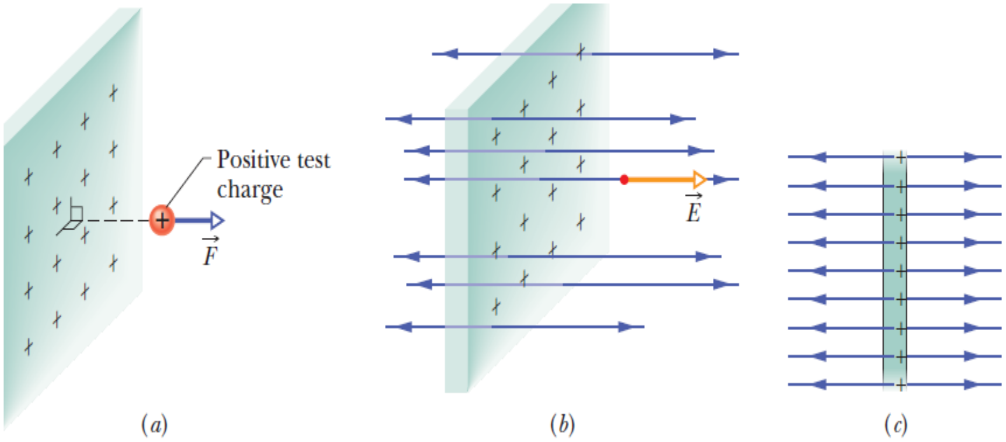


Fig. 22-3 (*a*) The electrostatic force $\vec{F}$ on a positive test charge near a very large, nonconducting sheet with uniformly distributed positive charge on one side. (*b*) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sheet. The field lines extend *away from* the positively charged sheet. (*c*) Side view of (*b*).

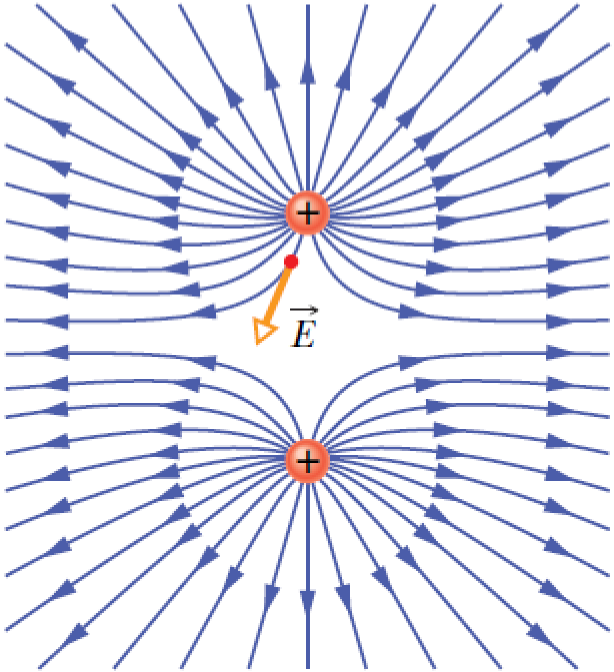
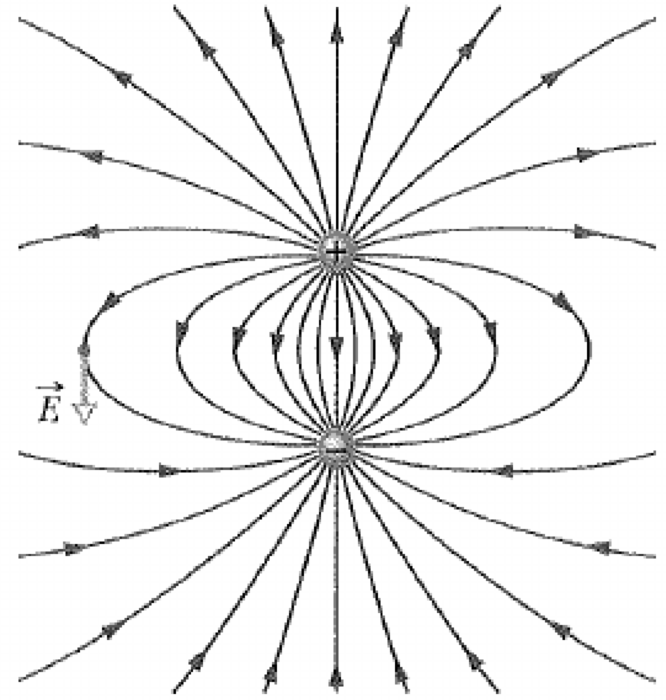


Fig. 22-4 Field lines for two equal positive point charges. The charges repel each other. (The lines terminate on distant negative charges.) To "see" the actual three-dimensional pattern of field lines, mentally rotate the pattern shown here about an axis passing through both charges in the plane of the page. The three-dimensional pattern and the electric field it represents are said to have *rotational symmetry* about that axis. The electric field vector at one point is shown; note that it is tangent to the field line through that point.

Fig. 22-5 Field lines for a positive point charge and a nearby negative point charge that are equal in magnitude. The charges attract each other. The pattern of field lines and the electric field it represents have rotational symmetry about an axis passing through both charges in the plane of the page. The electric field vector at one point is shown; the vector is tangent to the field line through the point.



22.4 The Electric Field due to a Point:

- To find the electric field due to a point charge q (or charged particle) at any point a distance r from the point charge, we put a positive test charge q_o at that point.

The electrostatic force acting on q_o is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\,\hat{\mathrm{r}}\,.$$

- The electric field E at a point is the electric force experienced by a test charge q_o divided by test charge $q_{o.}$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{\mathrm{r}} \qquad \text{(point charge).}$$

Example: The Electric Field Magnitude for a Point Charge:

What is the magnitude of the electric field at a field point 2.0 m from a point charge q = 4.0nC? (The point charge could represent any small charged object with this value of q, provided the dimensions of the object are much less than the distance from the object to the field point.)

Solution

This problem uses the expression for the electric field due to a point charge.

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{4.0 \times 10^{-9}\ \mathrm{C}}{(2.0\ \mathrm{m})^2}$$
$$= 9.0\ \mathrm{N/C}$$

To check our result, we use the definition of electric field as the electric force per unit charge. We can first use Coulomb's law, to find the magnitude F_o of the force on a test charge q_o placed 2.0 m from q:

$$F_0 = \frac{1}{4\pi\epsilon_0}\frac{|qq_0|}{r^2} = (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{4.0 \times 10^{-9}\ \mathrm{C}|q_0|}{(2.0\ \mathrm{m})^2}$$
$$= (9.0\ \mathrm{N/C})|q_0|$$

$$E = \frac{F_0}{|q_0|} = 9.0\ \mathrm{N/C}$$

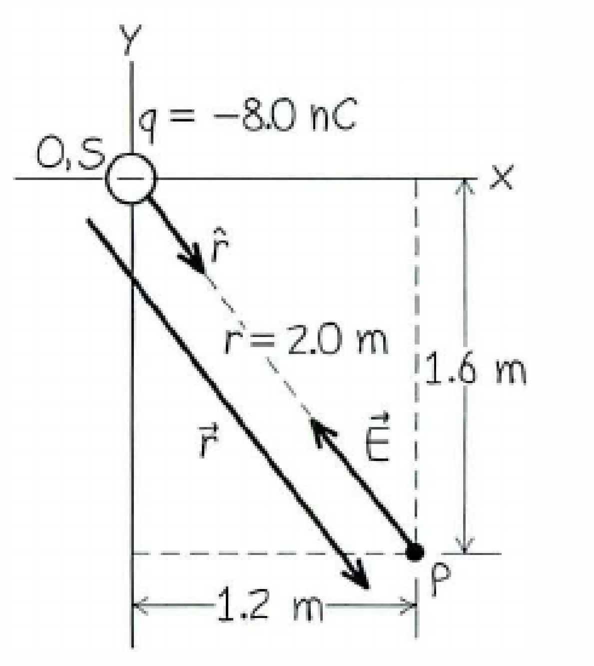
Example: The Electric Field Vector for a point Charge:

A point charge q = -8.0 n C is located at the origin. Find the electric-field vector at the field point x = l.2 m, y= -1.6m.

Solution:

In this problem we are asked to find the electric-field vector $\vec{E}$ due to a point charge. Hence we need to find either the components of $\vec{E}$ or its magnitude and direction.

Our sketch for this problem.



$$r = \sqrt{x^2 + y^2} = \sqrt{(1.2\ \mathrm{m})^2 + (-1.6\ \mathrm{m})^2} = 2.0\ \mathrm{m}$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{\imath} + y\hat{\jmath}}{r}$$

$$= \frac{(1.2\ \mathrm{m})\hat{\imath} + (-1.6\ \mathrm{m})\hat{\imath}}{2.0\ \mathrm{m}} = 0.60\hat{\imath} - 0.80\hat{\jmath}$$

Hence the electric-field vector is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$= (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(-8.0 \times 10^{-9}\ \mathrm{C})}{(2.0\ \mathrm{m})^2}(0.60\hat{\imath} - 0.80\hat{\jmath})$$

$$= (-11\ \mathrm{N/C})\hat{\imath} + (14\ \mathrm{N/C})\hat{\jmath}$$

Net Electric Field

We can quickly find the net, or resultant, electric field due to more than one point charge. If we place a positive test charge q_o near n point charges $q_1, q_2, \ldots, q_n$, then,

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0}$$

$$= \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n.$$

Example, The net electric field due to three charges:

Figure 22-7*a* shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?

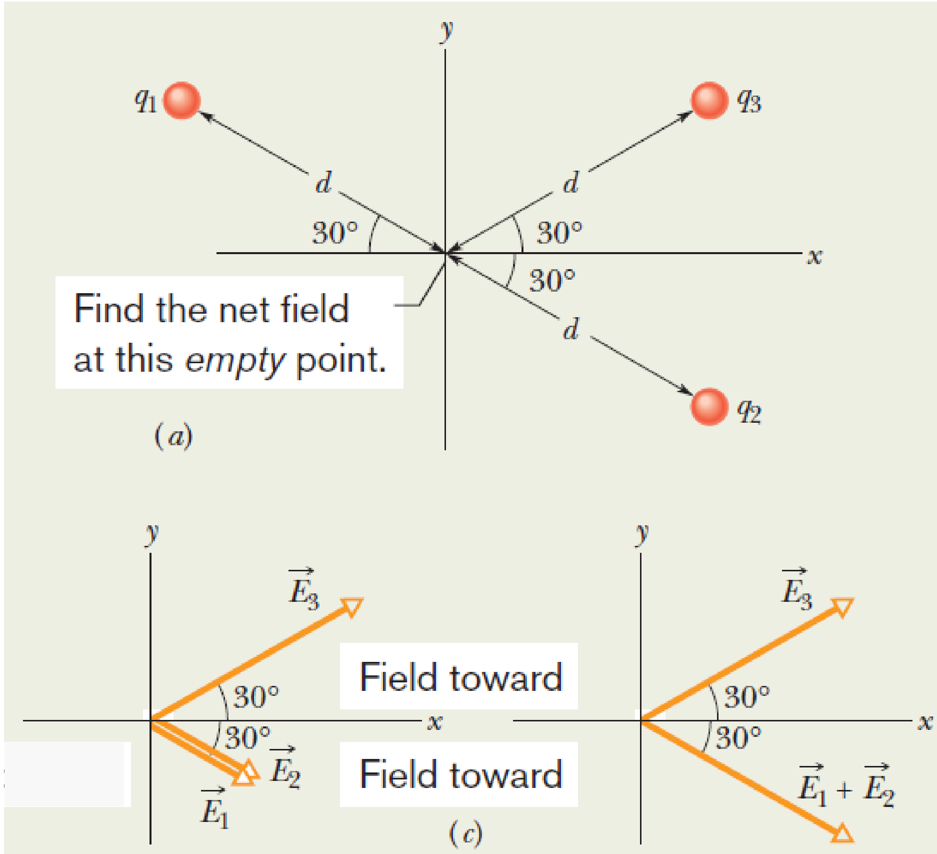


Fig. 22-7 (*a*) Three particles with charges q_1, q_2, and q_3 are at the same distance d from the origin. (*b*) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. (*c*) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

From the symmetry of Fig. 22-7c, we realize that the equal y components of our two vectors cancel and the equal x components add.

Thus, the net electric field at the origin is in the positive direction of the x axis and has the magnitude

$$E = 2E_{3x} = 2E_3 \cos 30^\circ$$

$$= (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}.$$

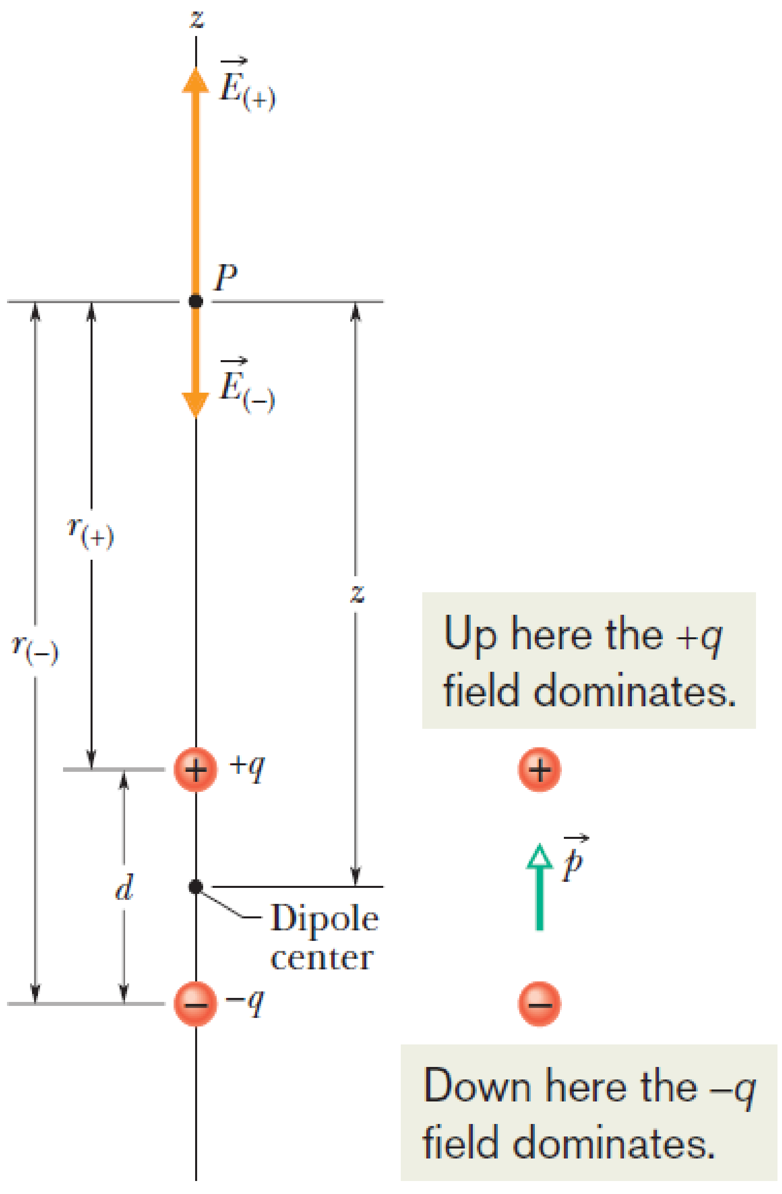
22.5 The Electric Field due to an Electric Dipole: Home Work

- Figure a shows two charged particles of magnitude q but of opposite sign,separated by a distance d. We call this configuration an electric dipole.

- Let us find the electric field due to the dipole of Figur a at a point P, a distance z from the mid point of the dipole and on the axis through the particles, which is called the dipole axis.

- From symmetry, the electric field E at point P and also the fields $E_{(+)}$ and $E_{(-)}$ due to the separate charges that make up the dipole must lie along the dipole axis, which we have taken to be a z axis.

- Applying the superposition principle for electric fields, we find that the magnitude E of the electric field at P is



(a) (b)

22.5 The Electric Field due to an Electric Dipole: Home Work

$$E = E_{(+)} - E_{(-)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2}$$

$$= \frac{q}{4\pi\varepsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z + \frac{1}{2}d)^2}.$$

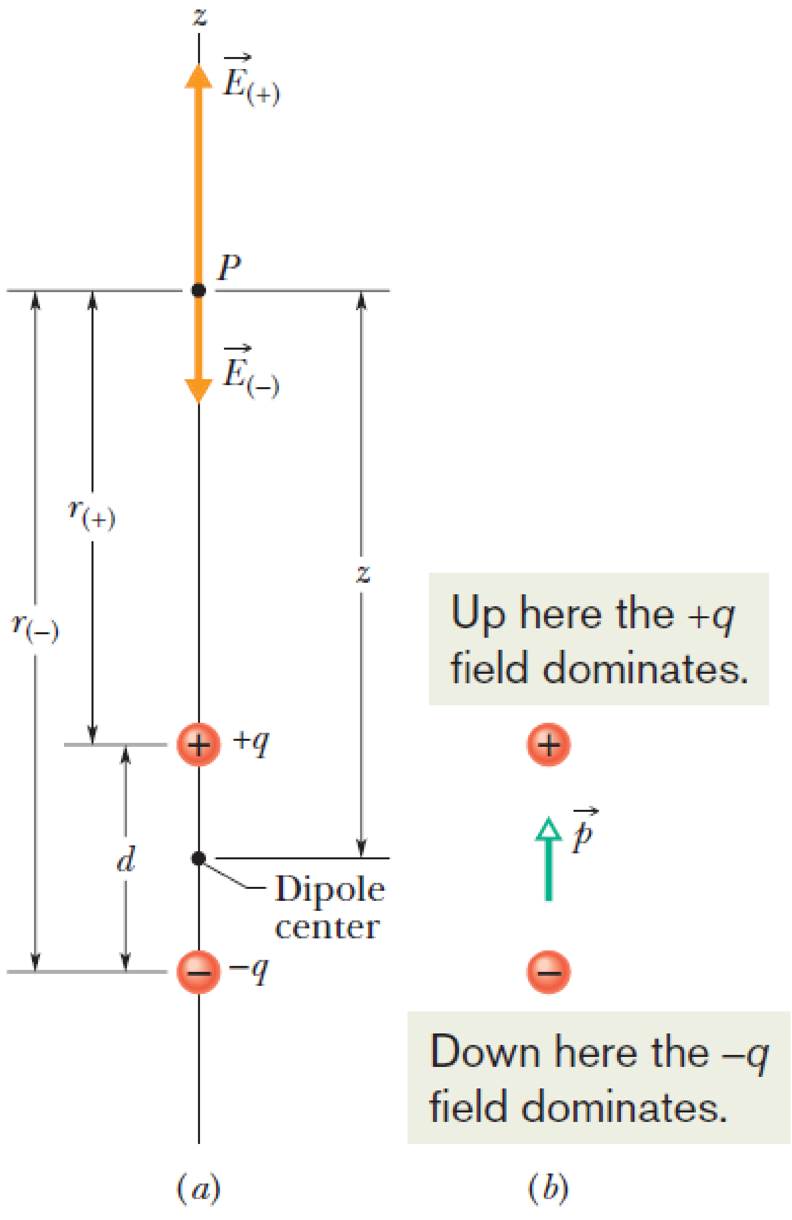
$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2}\right).$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d/z}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\varepsilon_0 z^3}\frac{d}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2}.$$

$$d/2z \ll 1$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}.$$

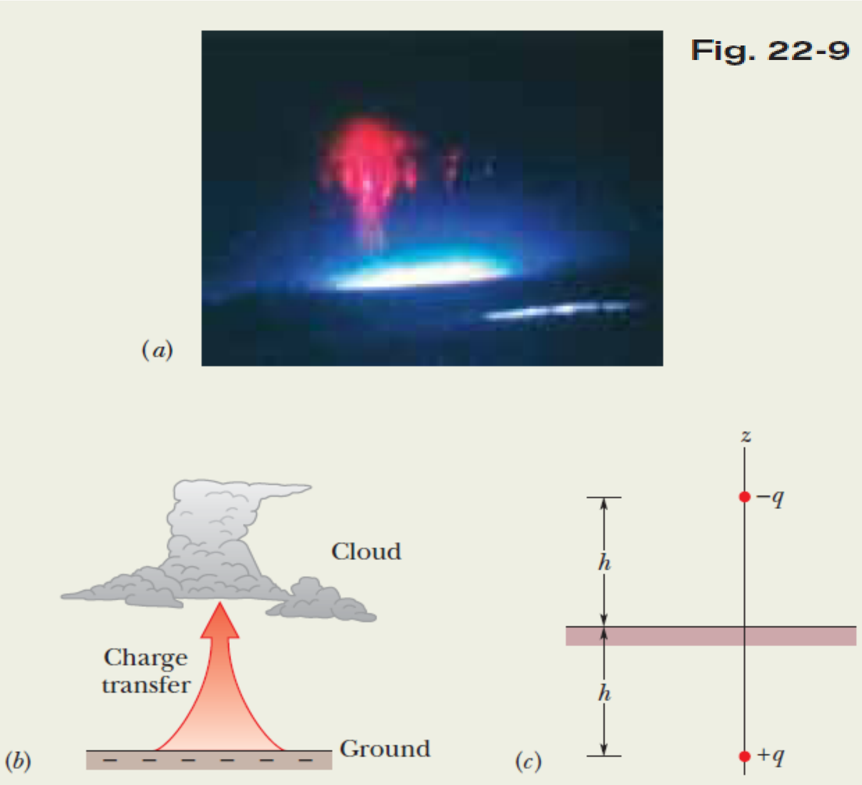
$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \qquad \text{(electric dipole).}$$



$\vec{p} = q\vec{d}$, a vector quantity known as the ***electric dipole moment*** of the dipole

The direction of $\vec{P}$ is taken to be from the negative to the positive end of the dipole. We can use the direction of $\vec{P}$ to specify the orientation of a dipole.

Example, Electric Dipole and Atmospheric Sprites:



Sprites (Fig. 22-9*a)* are huge flashes that occur far above a large thunderstorm. They are still not well understood but are believed to be produced when especially powerful lightning occurs between the ground and storm clouds, particularly when the lightning transfers a huge amount of negative charge -*q* from the ground to the base of the clouds (Fig. 22-9*b).*

We can model the electric field due to the charges in the clouds and the ground
by assuming a vertical electric dipole that has charge -*q* at cloud height *h* and charge +*q* at below-ground depth *h (Fig*. 22-9*c)*. If $q = 200\ C$ and $h = 6.0$ km, what is the magnitude of the dipole's electric field at altitude $z_1 = 30\ km$ somewhat above the clouds and altitude $z_2 = 60$ km somewhat above the stratosphere?

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q(2h)}{z^3},$$

where $2h$ is the separation between $-q$ and $+q$ in Fig. 22-9c. For the electric field at altitude $z_1 = 30$ km, we find

$$E = \frac{1}{2\pi\varepsilon_0}\frac{(200\ \text{C})(2)(6.0\times10^3\ \text{m})}{(30\times10^3\ \text{m})^3}$$

$$= 1.6\times10^3\ \text{N/C}. \qquad \text{(Answer)}$$

Similarly, for altitude $z_2 = 60$ km, we find

$$E = 2.0\times10^2\ \text{N/C}. \qquad \text{(Answer)}$$

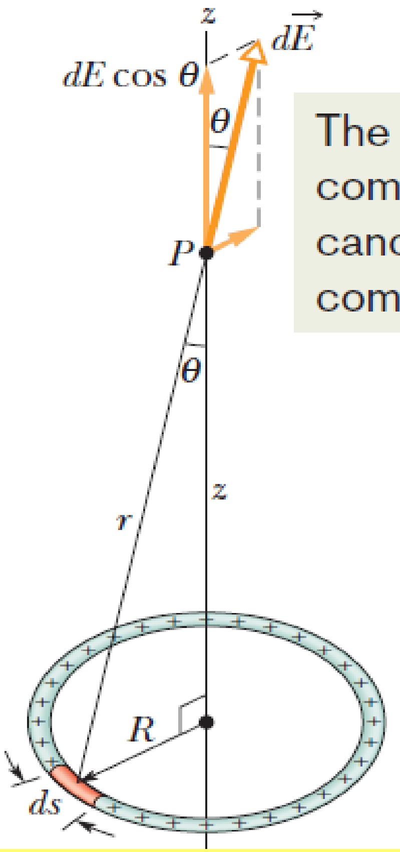
22.6 The Electric Field due to a Continuous Charge Distribution:

Table 22-2

Some Measures of Electric Charge

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

22.6 The Electric Field due to a Line Charge: Home Work



The perpendicular components just cancel but the parallel components add.

$$dq = \lambda\, ds.$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}.$$

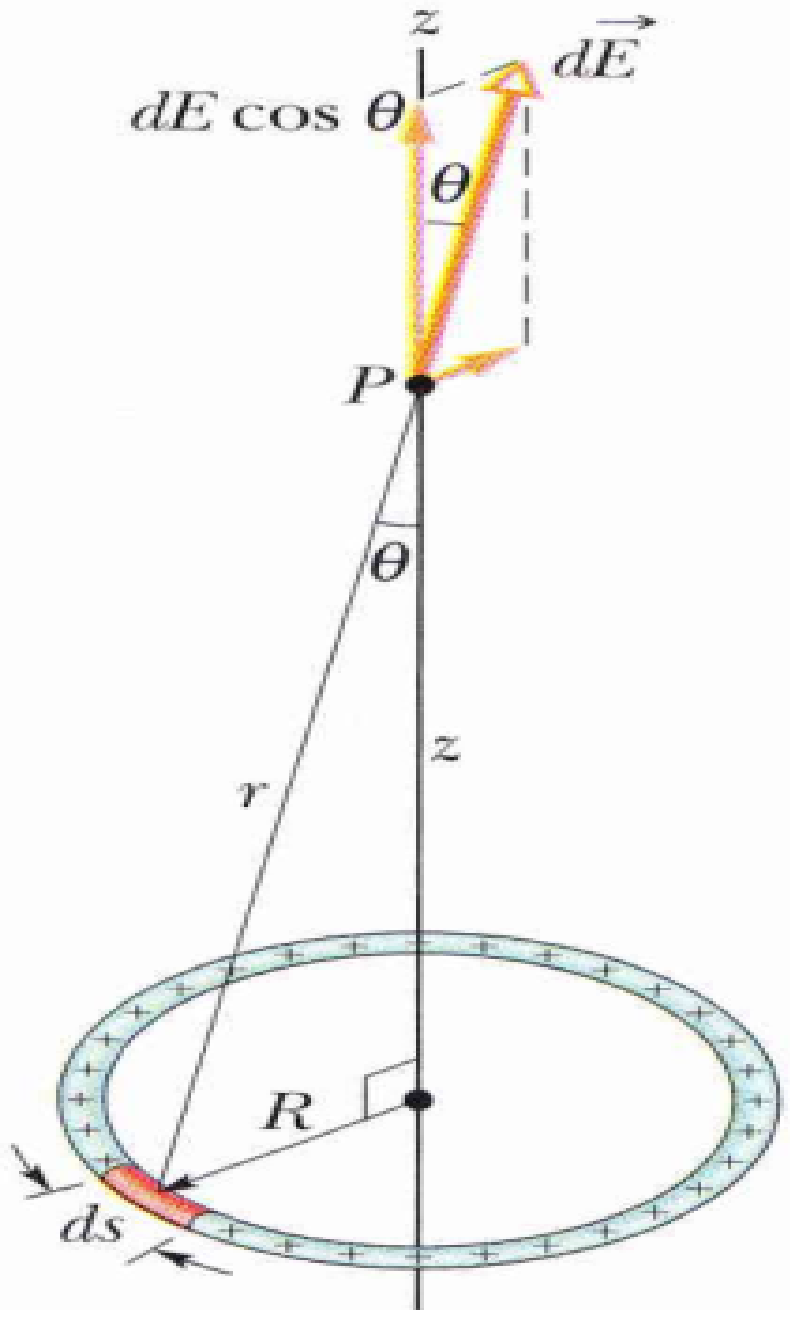
$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}.$$

$$dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\, ds.$$

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\int_0^{2\pi R} ds$$

$$= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}.$$

$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \qquad \text{(charged ring)}$$



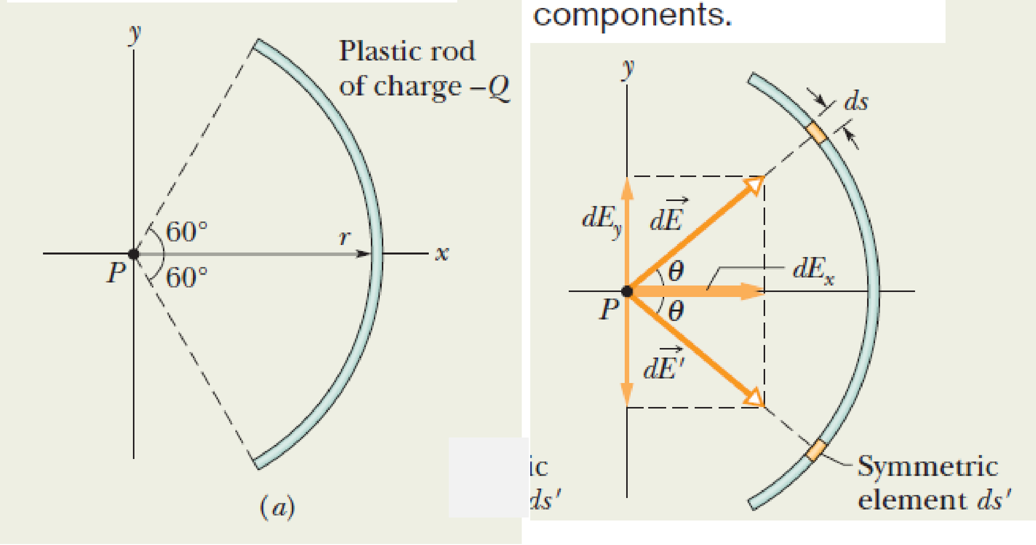
z
$d\vec{E}$
$dE \cos\theta$
θ
P
θ
z
r
R
ds

Example, Electric Field of a Charged Circular Rod

Figure 22-11a shows a plastic rod having a uniformly distributed charge $-Q$. The rod has been bent in a 120° circular arc of radius r. We place coordinate axes such that the axis of symmetry of the rod lies along the x axis and the origin is at the center of curvature P of the rod. In terms of Q and r, what is the electric field $\vec{E}$ due to the rod at point P?

This negatively charged rod is obviously not a particle.

These x components add. Our job is to add all such components.



$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2\pi r/3} = \frac{0.477Q}{r}.$$

$$dq = \lambda\, ds.$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}$$

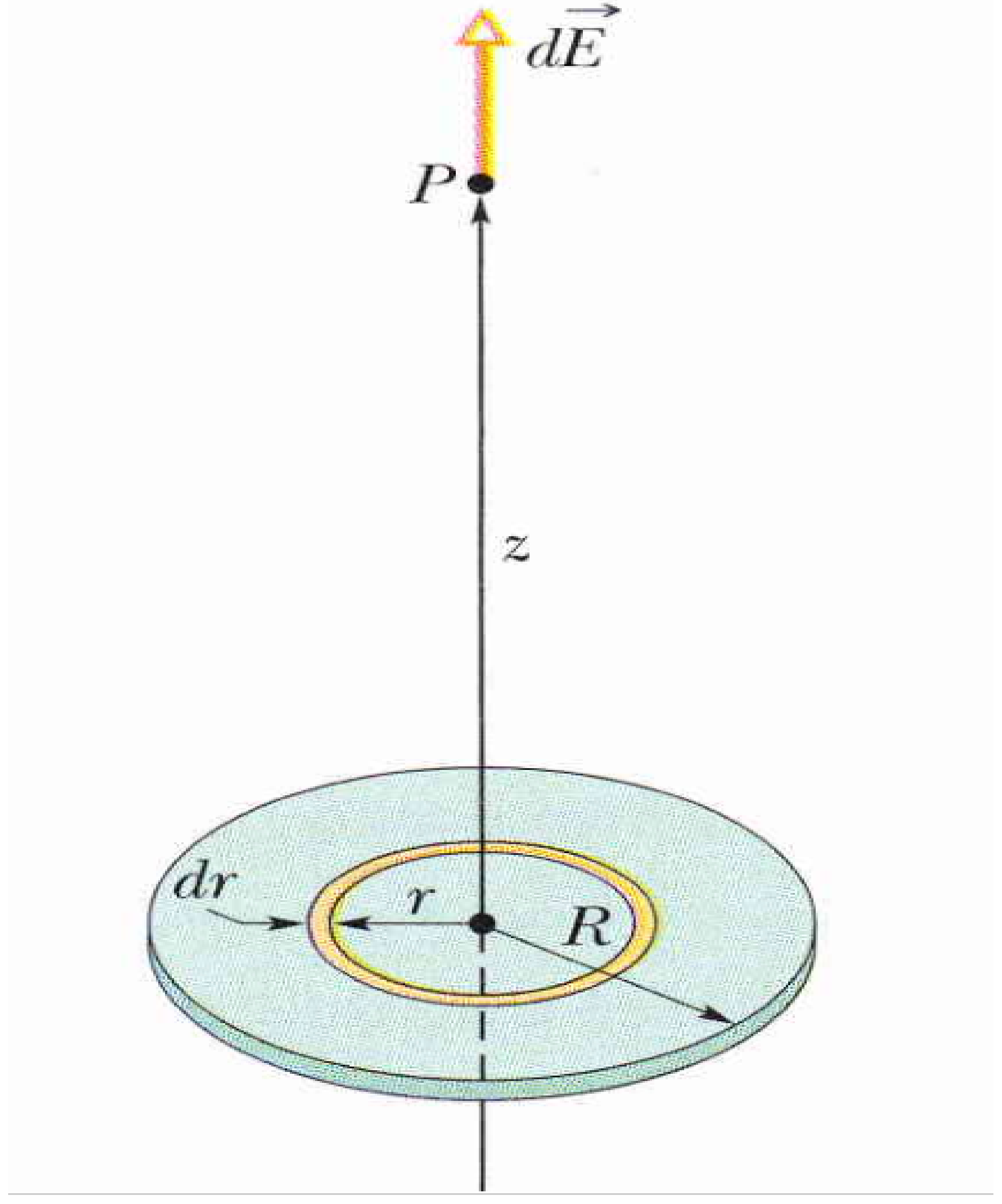
$$E = \int dE_x = \int_{-60°}^{60°} \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, r\, d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}\int_{-60°}^{60°}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-60°}^{60°}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}[\sin 60° - \sin(-60°)]$$

$$= \frac{1.73\lambda}{4\pi\varepsilon_0 r}. = \frac{0.83Q}{4\pi\varepsilon_0 r^2}.$$

Fig. 22-11 (a) A plastic rod of charge Q is a circular section of radius r and central angle 120°; point P is the center of curvature of the rod. (b) The field components from symmetric elements from the rod.



$d\vec{E}$
P
z
dr
r
R

22.6 The Electric Field due to a Charged Disk: Home Work

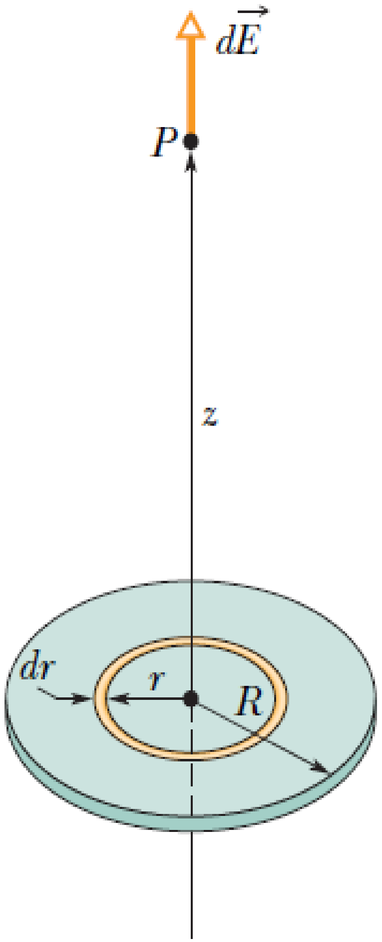
$$dq = \sigma\, dA = \sigma(2\pi r\, dr),$$

$$dE = \frac{z\sigma 2\pi r\, dr}{4\pi\varepsilon_0(z^2 + r^2)^{3/2}} = \frac{\sigma z}{4\varepsilon_0}\frac{2r\, dr}{(z^2 + r^2)^{3/2}}$$

$$E = \int dE = \frac{\sigma z}{4\varepsilon_0}\int_0^R (z^2 + r^2)^{-3/2}(2r)\, dr$$

$$E = \frac{\sigma z}{4\varepsilon_0}\left[\frac{(z^2 + r^2)^{-1/2}}{-\frac{1}{2}}\right]_0^R$$

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$



If we let $R \to \infty$, while keeping z finite, the second term in the parentheses in the above equation approaches zero, and this equation reduces to

$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{(infinite sheet).}$$

22.8: A Point Charge in an Electric Field

$$\boxed{\vec{F} = q\vec{E},}$$

When a charged particle, of charge *q*, is in an electric field, ***E*,** set up by other stationary or slowly moving charges, an electrostatic force, ***F*,** acts on the charged particle as given by the above equation.

22.9: A Dipole in an Electric Field

Although the net force on the dipole from the field is zero, and the center of mass of the dipole does not move, the forces on the charged ends do produce a net torque $\boldsymbol{\tau}$ on the dipole about its center of mass.

The center of mass lies on the line connecting the charged ends, at some distance x from one end and a distance $d - x$ from the other end.

The net torque is:

$$\tau = Fx \sin\theta + F(d - x)\sin\theta = Fd\sin\theta.$$

$$= pE\sin\theta.$$

$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on a dipole).}$$

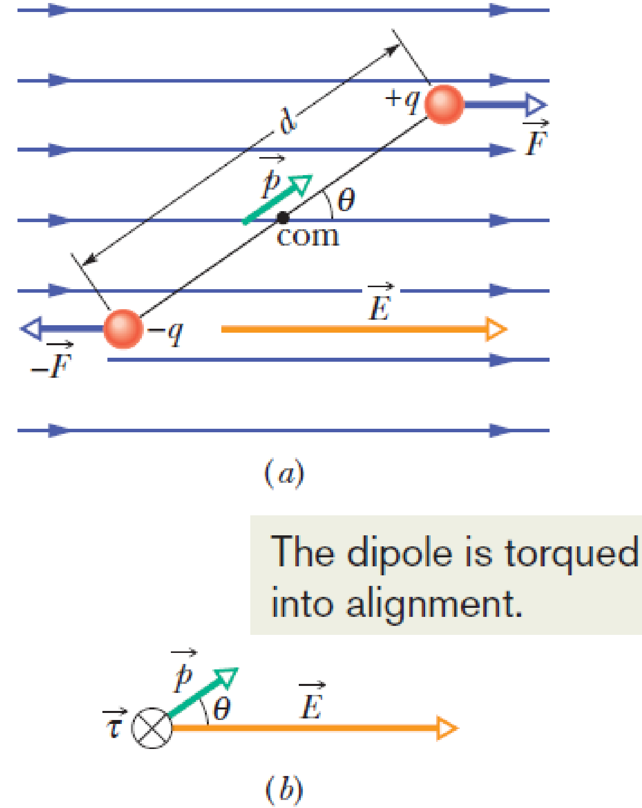


Fig. 22-19 (a) An electric dipole in a uniform external electric field $\vec{E}$. Two centers of equal but opposite charge are separated by distance d. The line between them represents their rigid connection. (b) Field $\vec{E}$ causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol $\otimes$.

22.9: A Dipole in an Electric Field: Potential Energy

Potential energy can be associated with the orientation of an electric dipole in an electric field.

The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment ***p*** is lined up with the field ***E*.**

The expression for the potential energy of an electric dipole in an external electric field is simplest if we choose the potential energy to be zero when the angle θ (Fig.22-19) is 90°.

The potential energy U of the dipole at any other value of θ can be found by calculating the work W done by the field on the dipole when the dipole is rotated to that value of θ from 90°.

$$\boxed{U = -W = -\int_{90^\circ}^{\theta} \tau \, d\theta = \int_{90^\circ}^{\theta} pE \sin\theta \, d\theta.} = -pE\cos\theta.$$

$$U = -\vec{p}\cdot\vec{E} \quad \text{(potential energy of a dipole).}$$

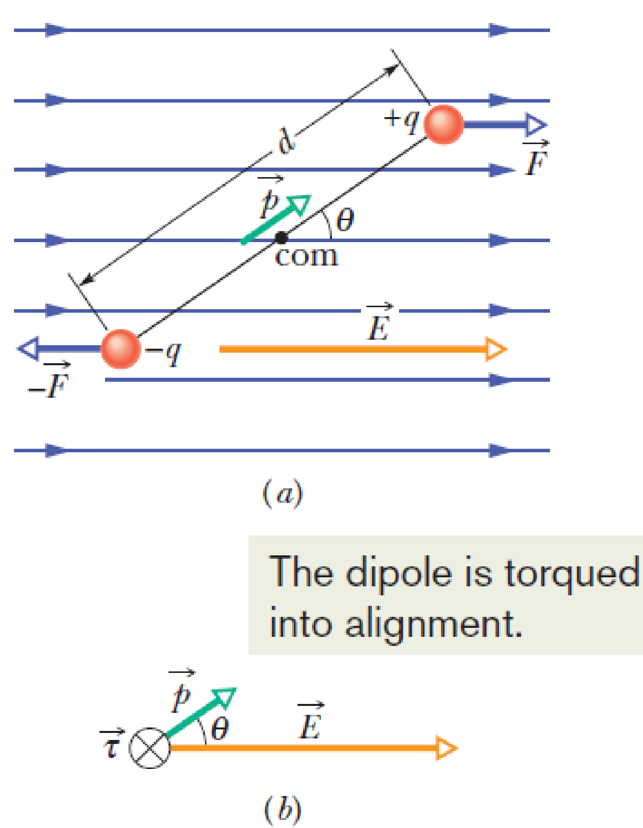


Fig. 22-19 (*a*) An electric dipole in a uniform external electric field $\vec{E}$. Two centers of equal but opposite charge are separated by distance d. The line between them represents their rigid connection. (*b*) Field $\vec{E}$ causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol ⊗.

Example, Torque, Energy of an Electric Dipole in an Electric Field

A neutral water molecule (H_2O) in its vapor state has an electric dipole moment of magnitude 6.2×10^{-30} C·m.

(a) How far apart are the molecule's centers of positive and negative charge?

KEY IDEA

A molecule's dipole moment depends on the magnitude q of the molecule's positive or negative charge and the charge separation d.

Calculations: There are 10 electrons and 10 protons in a neutral water molecule; so the magnitude of its dipole moment is

$$p = qd = (10e)(d),$$

in which d is the separation we are seeking and e is the elementary charge. Thus,

$$d = \frac{p}{10e} = \frac{6.2 \times 10^{-30}\ \text{C}\cdot\text{m}}{(10)(1.60 \times 10^{-19}\ \text{C})}$$
$$= 3.9 \times 10^{-12}\ \text{m} = 3.9\ \text{pm}. \qquad \text{(Answer)}$$

This distance is not only small, but it is also actually smaller than the radius of a hydrogen atom.

(b) If the molecule is placed in an electric field of 1.5×10^4 N/C, what maximum torque can the field exert on it? (Such a field can easily be set up in the laboratory.)

KEY IDEA

The torque on a dipole is maximum when the angle θ between $\vec{p}$ and $\vec{E}$ is 90°.

Calculation: Substituting $\theta = 90°$ in Eq. 22-33 yields

$$\tau = pE \sin\theta$$
$$= (6.2 \times 10^{-30}\ \text{C}\cdot\text{m})(1.5 \times 10^4\ \text{N/C})(\sin 90°)$$
$$= 9.3 \times 10^{-26}\ \text{N}\cdot\text{m}. \qquad \text{(Answer)}$$

(c) How much work must an *external agent* do to rotate this molecule by 180° in this field, starting from its fully aligned position, for which $\theta = 0$?

KEY IDEA

The work done by an external agent (by means of a torque applied to the molecule) is equal to the change in the molecule's potential energy due to the change in orientation.

Calculation: From Eq. 22-40, we find

$$W_a = U_{180°} - U_0$$
$$= (-pE\cos 180°) - (-pE\cos 0)$$
$$= 2pE = (2)(6.2 \times 10^{-30}\ \text{C}\cdot\text{m})(1.5 \times 10^4\ \text{N/C})$$
$$= 1.9 \times 10^{-25}\ \text{J}. \qquad \text{(Answer)}$$