Discrete Structures

Introductions to Set Theory

Today's Lecture

- Why study set Theory
- Sets
- Operations on sets
- Memberships
- Notations
- Venn diagrams

Why study Set Theory?

Understanding set theory helps people to …

1. See things in terms of systems.

2. Organize things into groups.

3. Begin to understand logic.

Sets

A set is a gathering together into a whole of definite, distinct objects of our perception and of our thought which are called elements of the set.

The elements or members of a set can be anything: numbers, people, letters of the alphabet, other sets, and so on. Sets are conventionally denoted with capital letters.

Note: A set should be well defined and distinct.

Examples

(1) A = {tiger, lion, puma, cheetah, leopard, cougar, ocelot}

(this is a set of large species of cats).

(2) A = {a, b, c, ..., z} (this is a set consisting of the lowercase letters of the alphabet)

(3) A = {-1, -2, -3, ...} (this is a set of the negative numbers)

In all above examples each element of the sets is distinct and well defined.

Operations on sets

Union

Two sets can be "added" together. The *union* of *A* and *B*, denoted by $A \cup B$, is the set of all things which are members of either *A* or *B*.

Examples:

- $\{1, 2\} \cup \{\text{red, white}\} = \{1, 2, \text{red, white}\}$.
- $\{1, 2, \text{green}\} \cup \{\text{red, white, green}\} = \{1, 2, \text{red, white, green}\}$.
- $\{1, 2\} \cup \{1, 2\} = \{1, 2\}$.

Intersection

A new set can also be constructed by determining which members two sets have "in common".

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The *intersection of A and B*, denoted by $A \cap B$, is the set of all things which are members of both A and B. If $A \cap B = \emptyset$, then A and B are said to be *disjoint*.

Examples

{1, 2} ∩ {red, white} = ∅.

{1, 2, green} ∩ {red, white, green} = {green}.

{1, 2} ∩ {1, 2} = {1, 2}.

Compliments

Two sets can also be "subtracted". The *relative complement* of B in A *(also called the set-theoretic difference of A and B)*,

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denoted by $A \setminus B$ (or $A - B$), is the set of all elements which are members of A but not members of B.

Note: That it is valid to "subtract" members of a set that are not in the set, such as removing the element *green* from the set {1, 2, 3}; doing so has no effect.

Example:

- {1, 2} \ {red, white} = {1, 2}.
- {1, 2, green} \ {red, white, green} = {1, 2}.
- {1, 2} \ {1, 2} = ∅.
- If U is the set of integers, E is the set of even integers, and O is the set of odd integers, then $E' = O$.

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Cartesian products

A new set can be constructed by associating every element of one set with every element of another set. The *Cartesian product* of two sets *A* and *B*, denoted by $A \times B$ is the set of all Ordered pairs (a, b) such that a is a member of A and b is a member of B.

Examples

- {1, 2} × {red, white} = {(1, red), (1, white), (2, red), (2, white)}.
- {1, 2, green} × {red, white, green} = {(1, red), (1, white), (1, green), (2, red), (2, white), (2, green), (green, red), (green, white), (green, green)}.
- {1, 2} × {1, 2} = {(1, 1), (1, 2), (2, 1), (2, 2)}.

Special Sets

There are some sets which hold great mathematical importance and are referred to with such regularity that they have acquired special names and notational conventions to identify them. One of these is the empty set, denoted { } or ∅. Another is the singleton set {x} which contains exactly one element, namely x.

- **P** or $\mathbb{P}$, denoting the set of all primes **P** = {2, 3, 5, 7,11, 13, 17, ...}.
- **N** or $\mathbb{N}$, denoting the set of all natural numbers: **N** = {1, 2, 3, . . .}.
- **Z** or $\mathbb{Z}$, denoting the set of all integers (whether positive, negative or zero): **Z** = {..., −2, −1, 0, 1, 2, ...}.
- **Q** or $\mathbb{Q}$, **Q** = $\{a/b : a, b \in \mathbf{Z}, b \neq 0\}$. For example, $1/4 \in \mathbf{Q}$ and $11/6 \in \mathbf{Q}$.

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- **R** or $\mathbb{R}$, denoting the set of all real numbers. This set includes all rational numbers, together with all irrational numbers (that is, numbers which cannot be rewritten as fractions, such as π, *e*, and √2, as well as numbers that cannot be defined).
- **C** or $\mathbb{C}$, denoting the set of all complex numbers : **C** = {*a* + *bi* : *a*, *b* ∈ **R**}. For example, 1 + 2*i* ∈ **C**.

Finite and Infinite Sets

Finite Set: A set is finite if it contains a specific (finite) number of elements, i.e., If we can count the element in a set, such sets are called finite sets.

Example: Some finite numbers in a set: the number of digits on your hand, the number of seats on a bus, and the number of people on earth.

Infinite Set: If we can not count the elements in a set such sets are called infinite sets.

Example: Set of Natural numbers. Set of Whole numbers.

Cardinality: refers to the number of elements in a set.

Finite and Infinite Set Cardinality

Set Definition	*Cardinality*
A = {x | x is a lower case letter}	$\|A\| = 26$
B = {2, 3, 4, 5, 6, 7}	$\|B\| = 6$
C = {x | x is an even number < 10}	$\|C\| = 4$
A = {1, 2, 3, …}	$\|A\| = \aleph_0$
B = {x | x is a point on a line}	$\|B\| = \aleph_0$
C = {x| x is a point in a plane}	$\|C\| = \aleph_0$

Memberships

The key relation between sets is *membership* when one set is an element of another. If a is a member of B, this is denoted $a \in B$, while if c is not a member of B then $c \notin B$. For example, With respect to the sets $A = \{1,2,3,4\}$ and $B = \{\text{blue, white, red}\}$, $4 \in A$ and green $\notin B$.

- Universal Sets: The universal set is the set of all things relevant to a given discussion and is designated by the symbol U. *i.e. it contains every set.*
- Subsets: If every member of set A is also a member of set B, then A is said to be a *subset* of B, written $A \subseteq B$ (also pronounced *A is contained in B*). The relationship between sets established by $\subseteq$ is called *inclusion or containment*.

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Examples

A = {1,2,3,4}, B = {1,2,3,4,5,7}, and C = {7,9,3}, and the universal set U = {1,2,3,4,5,6,7,8,9}.

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- Super Set: if we can write $B \supseteq A$, read as *B is a superset of A*, *B includes A*, or *B contains A*.
- Proper Subset: If A is a subset of, but not equal to, B, then A is called a *proper subset* of B, written $A \subset B$ (*A is a proper subset of B*) or $B \supset A$ (*B is a proper superset of A*).

Examples

- The set of all men is a proper subset of the set of all people.
- $\{1, 3\} \subset \{1, 2, 3, 4\}$.
- $\{1, 2, 3, 4\} \subseteq \{1, 2, 3, 4\}$.

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An obvious but useful identity, which can often be used to show that two seemingly different sets are equal:

$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

Power set: The power set of a set A is the set containing all possible subsets of A including the empty subset. It contains 2^n elements where n is the number of elements in A. It is typically denoted by $P(A)$ or 2^A. For example, the power set of the set $A=\{a,b,c,d\}$ is the set $P(A)=\{\{\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$.

Subset Relationships

A = {x | x is a positive integer $\leq$ 8}

set A contains: 1, 2, 3, 4, 5, 6, 7, 8

B = {x | x is a positive even integer $<$ 10}

set B contains: 2, 4, 6, 8

C = {2, 4, 6, 8, 10}

set C contains: 2, 4, 6, 8, 10

The universal set U = {1,2,3,4,5,6,7,8,9,10}.

Subset Relationships

$A \subseteq A$	$A \not\subset B$	$A \not\subset C$
$B \subset A$	$B \subseteq B$	$B \subset C$
$C \not\subset A$	$C \not\subset B$	$C \subseteq C$

Set Equality

Two sets are *equal* if and only if they contain precisely the same elements. The order in which the elements are listed is un important. Elements may be repeated in set definitions without increasing the size of the sets.

Examples

- $A = \{1, 2, 3, 4\}$ $B = \{1, 4, 2, 3\}$

  $A \subset B$ and $B \subset A$; therefore, $A = B$ and $B = A$.

- $A = \{1, 2, 2, 3, 4, 1, 2\}$ $B = \{1, 2, 3, 4\}$

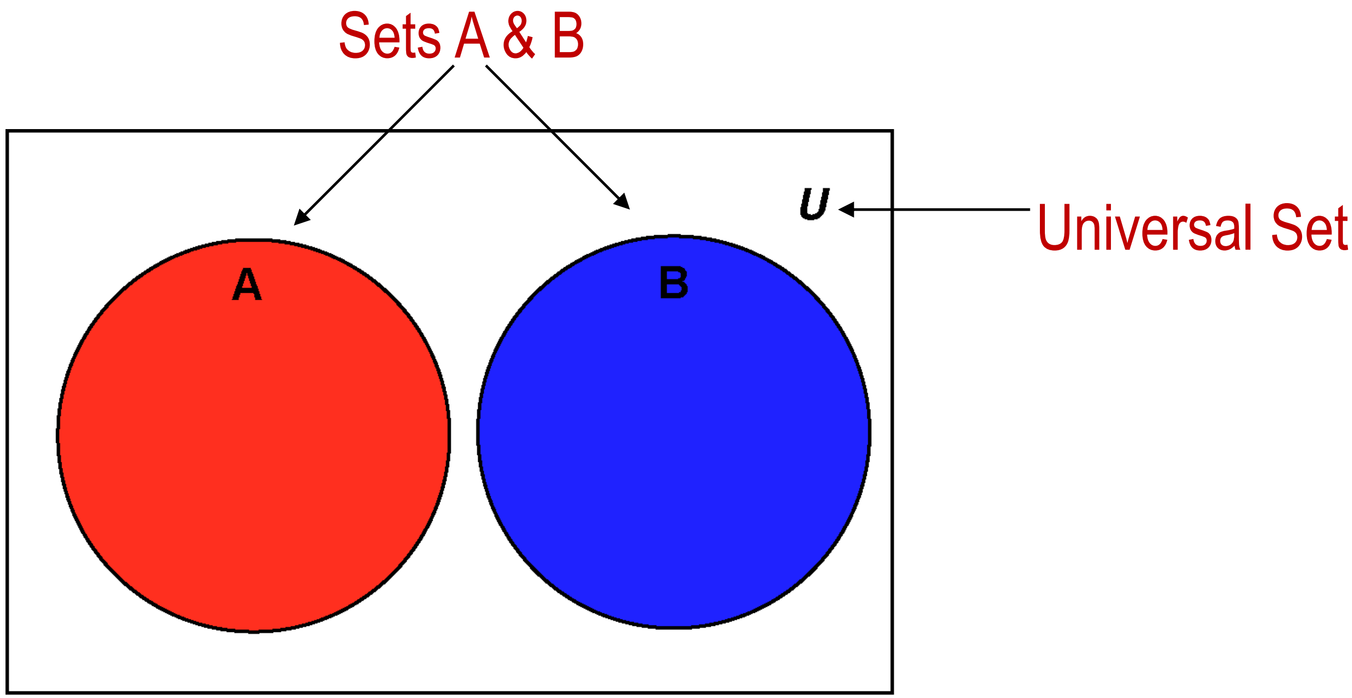
  $A \subset B$ and $B \subset A$; therefore, $A = B$ and $B = A$.

Notations

Symbol	*Meaning*
Upper case	designates set name
Lower case	designates set elements
{ }	enclose elements in set
$\in$ (or $\notin$)	is (or is not) an element of
$\subseteq$	is a subset of (includes equal sets)
$\subset$	is a proper subset of
$\not\subset$	is not a subset of
$\supset$	is a superset of
$\mid$ or :	such that (if a condition is true)
$\mid \ \mid$	the cardinality of a set

Venn Diagrams

Venn diagrams or **set diagrams** are diagrams that show all possible logical relations between a finite collection of sets. Venn diagrams were conceived around 1880 by John Venn. Venn diagrams show relationships between sets and their elements.

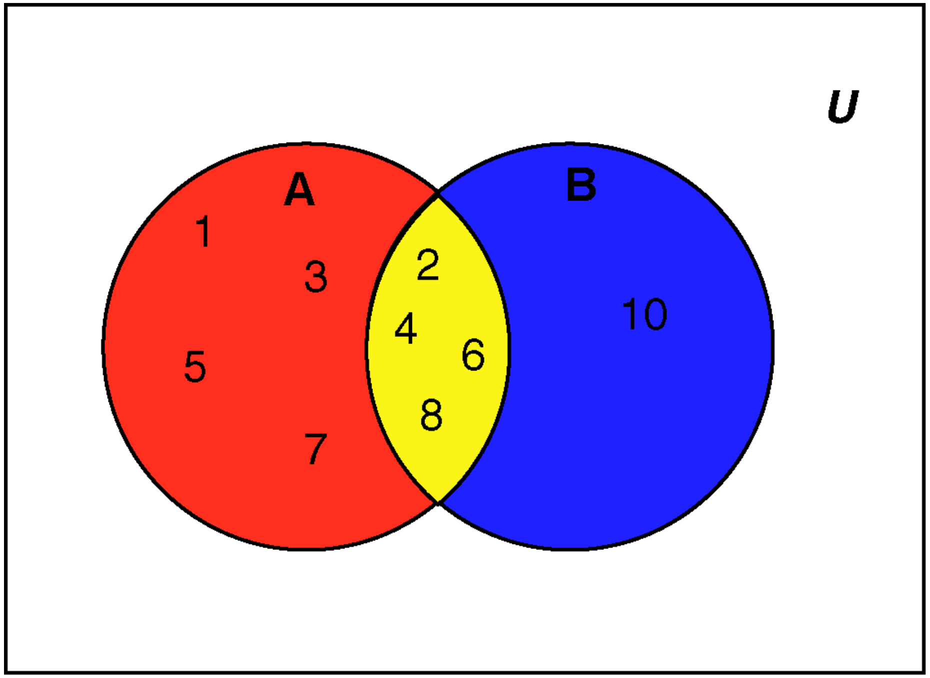


Examples

Set Definition	*Elements*
$A = \{x \mid x \in Z^+ \text{ and } x \leq 8\}$	$\{1, 2, 3, 4, 5, 6, 7, 8\}$
$B = \{x \mid x \in Z^+, x \text{ is even and } \leq 10\}$	$\{2, 4, 6, 8, 10\}$

$A \not\subset B$

$B \not\subset A$



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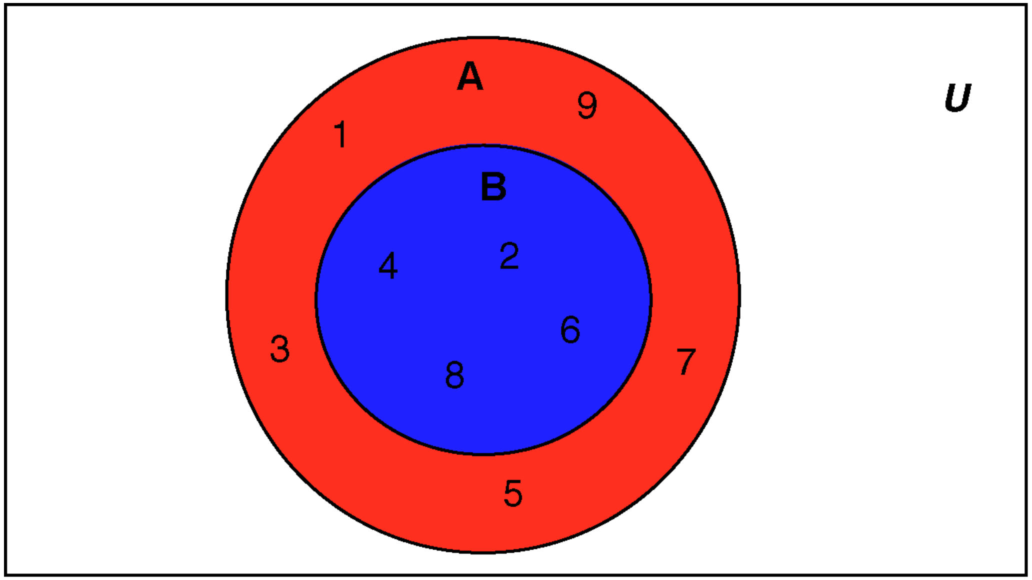
Set Definition	*Elements*
$A = \{x \mid x \in Z^+ \text{ and } x \leq 9\}$	{1, 2 ,3, 4, 5, 6 ,7, 8,9}
$B = \{x \mid x \in Z^+ \text{ ; x is even and } \leq 8\}$	{2, 4, 6, 8,}

$A \not\subset B$

$B \subset A$

$A \supset B$



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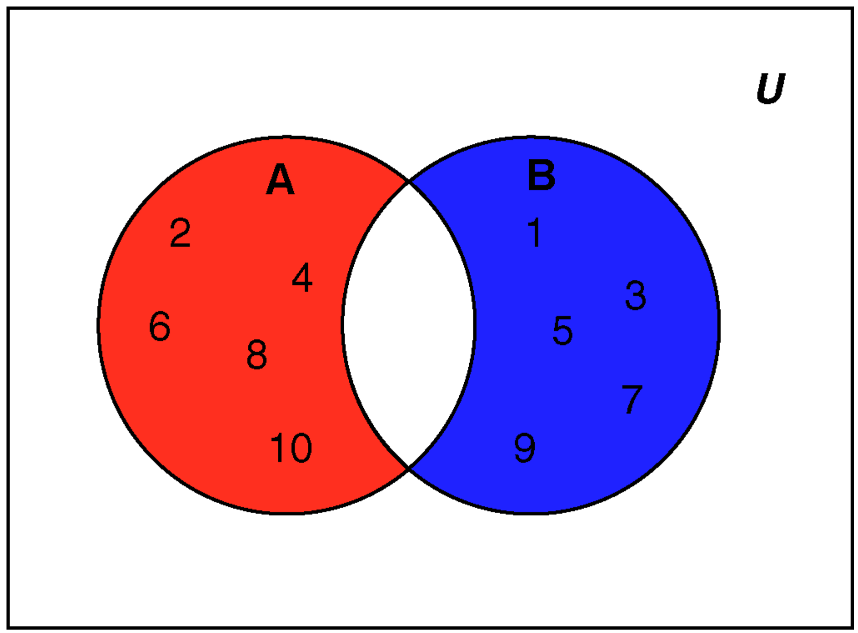
Set Definition *Elements*

$A = \{x \mid x \in Z^+ \; x \text{ is even and } x \leq 10\}$ $\{2, 4, 6, 8, 10\}$

$B = \{x \mid x \in Z^+ ; x \text{ is odd and} \leq 10\}$ $\{1, 3, 5, 7, 9\}$

$A \not\subset B$
$B \not\subset A$



Set operations and Venn diagram

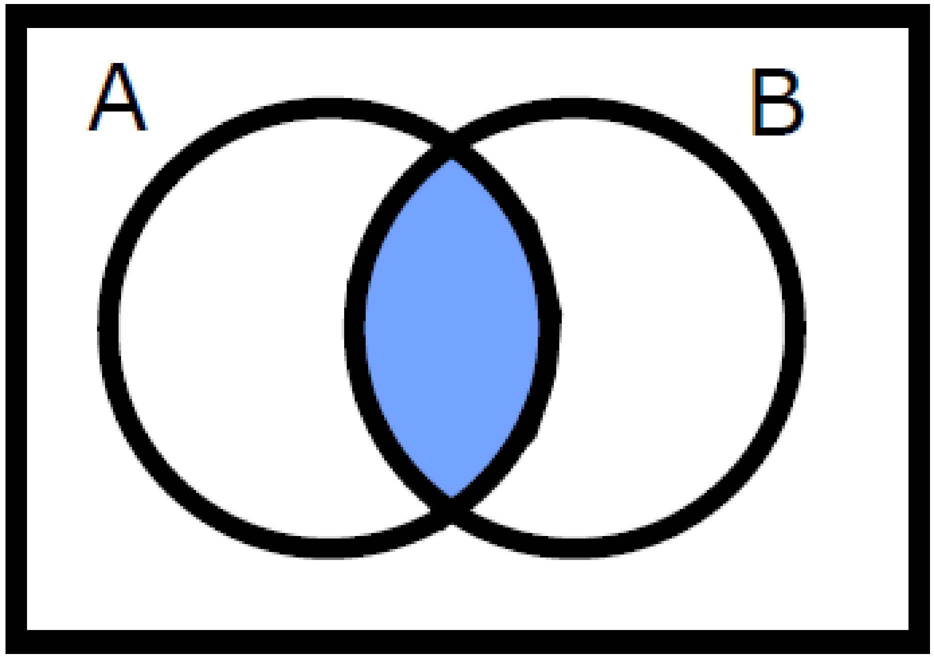
$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$

This is the intersection of A and B.

$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$

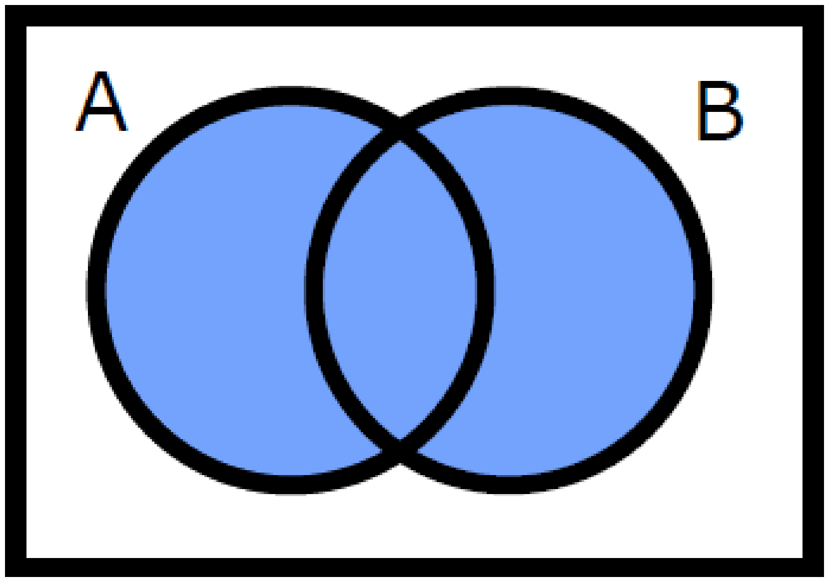
This is the union of A and B.

Set operations and Venn diagram



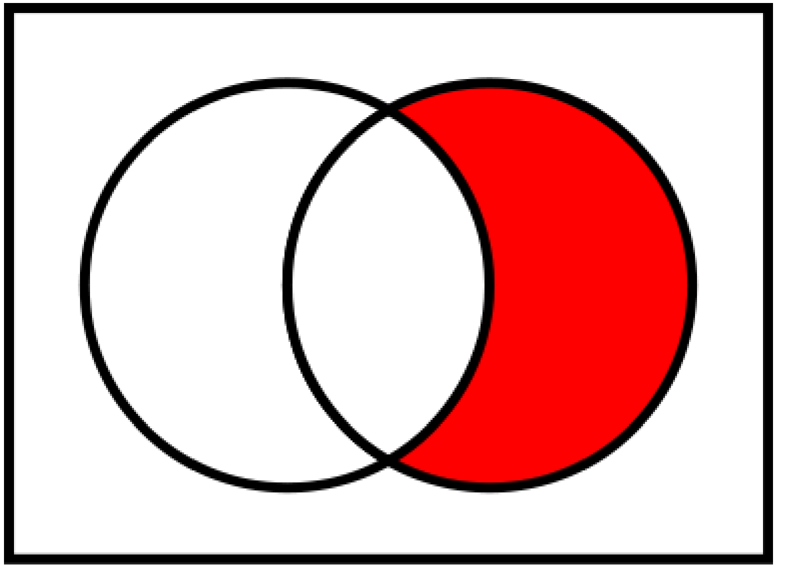
$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

Set operations and Venn diagram



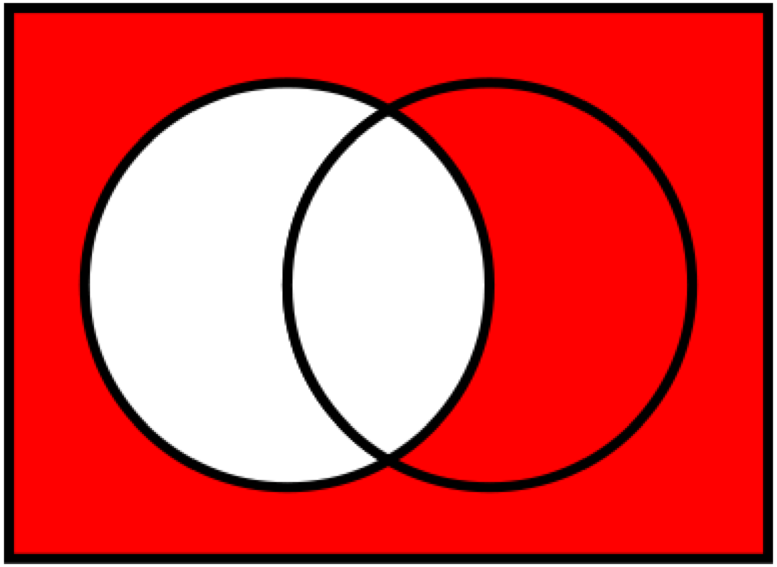
$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$

Set operations and Venn diagram



$$B \setminus A = \{ x \mid x \notin A \text{ and } x \in B \}$$

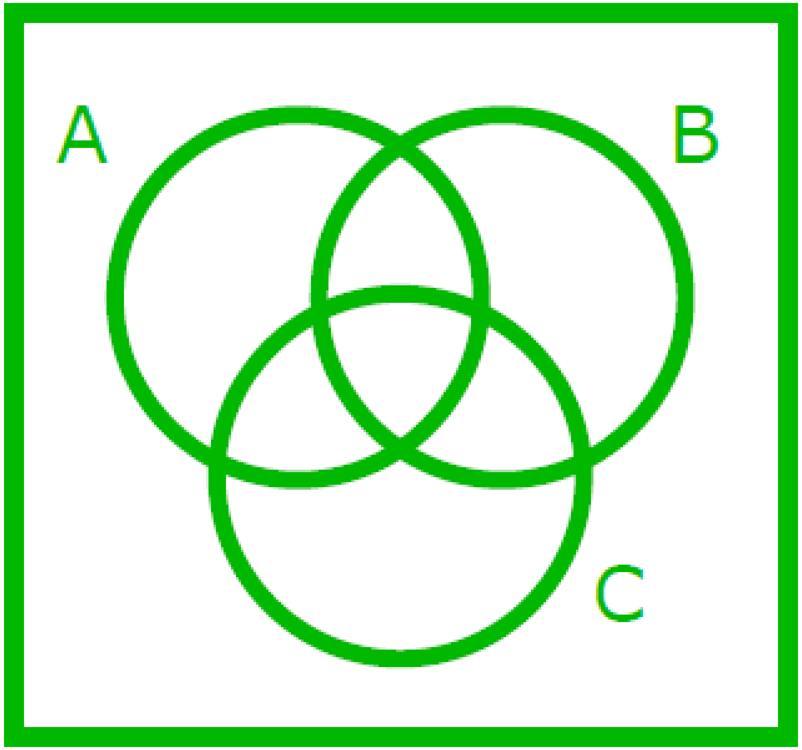
Set operations and Venn diagram



$A^c = U - A = \{ x \mid x \notin A \text{ and } x \in U \}$

Sets and Universal Set

A = {1,2,3,4}, B = {1,3,5,7}, and C = {7,9,3}, and the universal set U = {1,2,3,4,5,6,7,8,9}. Locate all this information appropriately in a Venn diagram.



Sets and Universal Set

A = {1,2,3,4}, B = {1,3,5,7}, and C = {7,9,3}, and the universal set U={1,2,3,4,5,6,7,8,9}. Locate all this information appropriately in a Venn diagram.