

Exercise 1:

Code:

```
% Before Modification
h=2/5; k=zeros(1,6); m=zeros(1,6); k(1)=0; m(1)=1;
for n=1:5
    k(n+1)=k(n)+h;
    m(n+1)=m(n)+(-m(n)+k(n))*h;
end

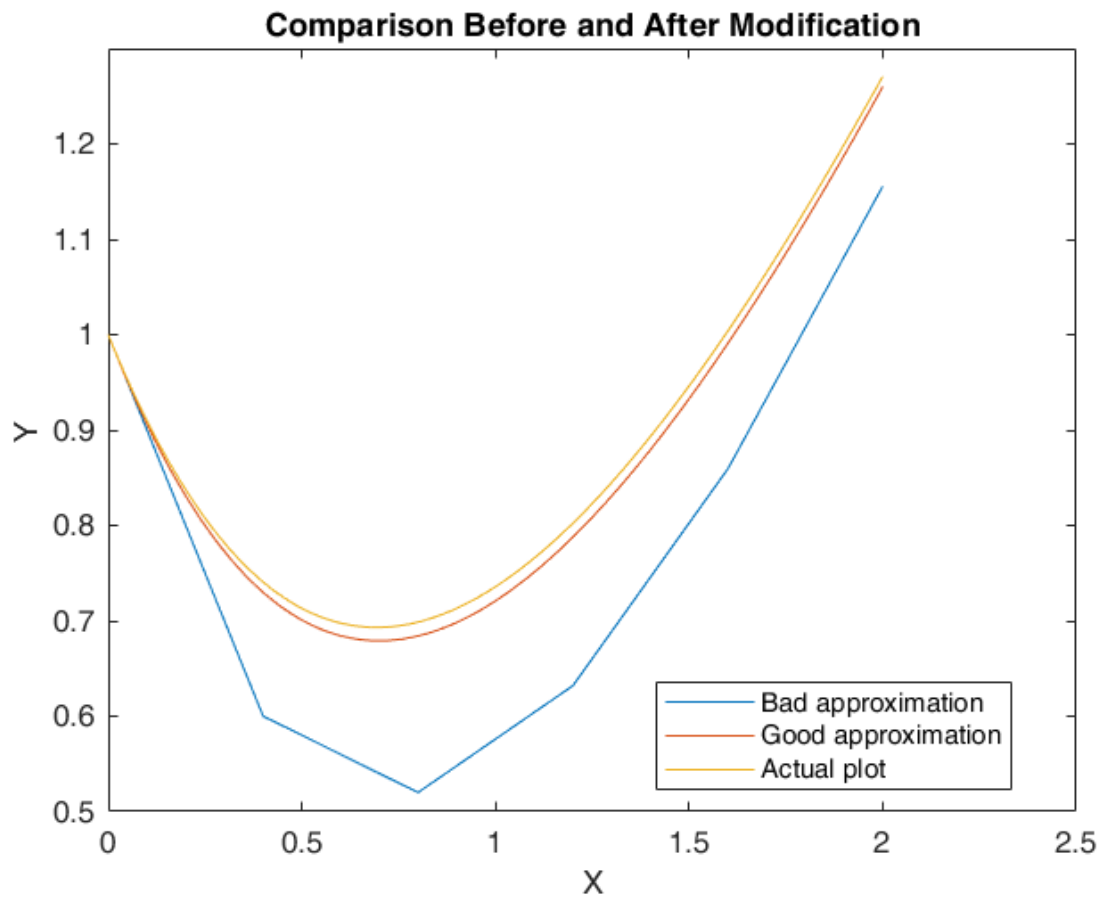
% After Modification
h=2/50; t=zeros(1,6); y=zeros(1,6); t(1)=0; y(1)=1;
for n=1:50
    t(n+1)=t(n)+h;
    y(n+1)=y(n)+(-y(n)+t(n))*h;
end

a=0:0.01:2;
b=2*exp(-a)+a-1;
plot(k,m, 'DisplayName', 'Bad approximation'); hold on;
plot(t,y, 'DisplayName', 'Good approximation'); hold on;
plot(a,b, 'DisplayName', 'Actual plot'); hold off;
legend('Location','best');
xlabel('X');
ylabel('Y');
title('Comparison Before and After Modification');
```

Explanation:

This code first initializes two sets of arrays `k` and `m` or `t` and `y`, representing points on a graph. Then it iterates through a loop, updating these arrays based on a mathematical formula. In the modified version, the step size `h` is decreased, resulting in a more accurate approximation. Finally, the code plots the original and modified approximations along with the actual plot for comparison.

Output:



Exercise 2:

Code:

```
N = 5;

for i=1:5

    h=2/N; t=zeros(1,N+1); y=zeros(1,N+1); t(1)=0; y(1)=1;
    for n=1:N
        t(n+1)=t(n)+h;
        y(n+1)=y(n)+(-0.5*y(n)+2-t(n))*h;
    end
    plot(t, y, 'DisplayName', ['N = ' num2str(N)]), hold on;
    % Calculating error at t = 2
    z = 8 - 2 * 2 - 7 * exp(-2 / 2); % Exact solution at t=2
    error = y(N + 1) - z; % Difference between Euler approximation and exact solution
at t=2
    errors(i) = error;
    N = N*2;
end

a = 0:0.01:2;
b = -7 * exp(-a / 2) - (2 * a) + 8;
plot(a, b, 'DisplayName', 'Exact Solution');

hold off;
xlabel('Time (t)');
ylabel('Value of y');
title('Approximation of y(t) with Varying N');
legend('show');

disp('Errors for different N values:');
disp(errors);
```

Explanation:

This code iteratively applies Euler's method with decreasing step sizes (h) controlled by N , plotting the approximations for increasing N values alongside the exact solution, and calculates and displays errors at $t = 2$ for each N value, highlighting the convergence of the method as step sizes decrease.

Output:

Errors for different N values:

0.2814 0.1344 0.0658 0.0325 0.0162

