Question 1:

Code:

```
% Define the matrix (A) to be solved
A = [4, -3; -7, 8];

% Compute eigenvalues and eigenvectors of matrix A
eigenvalues = eig(A);
[eigenvectors, diagonal_matrix] = eig(A);

% Display the eigenvectors
disp('Eigenvectors (P):');
disp(eigenvectors);

% Display the diagonal matrix containing the eigenvalues
disp('Diagonal Matrix (D):');
disp(diagonal_matrix);

% Solve the intital value problem for constants
c = eigenvectors\[10; -5];
disp('Solution constants c:');
disp(c);

% Create the diagonal matrix R using the values in vector c
R = [c(1) 0; 0 c(2)];

% Compute the matrix E = P*R
E = eigenvectors * R;

% Display the resulting matrix E
disp('Resulting Matrix E:');
disp(E);
```

Output:

Eigenvectors (P):
  -0.7071   0.3939
  -0.7071   -0.9191

Diagonal Matrix (D):
    1   0
    0   11

Solution constants c:
  -7.7782
  11.4237

Resulting Matrix E:
   5.5000   4.5000
   5.5000 -10.5000

Solution:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 5.5 & 4.5 \\ 5.5 & -10.5 \end{bmatrix} * \begin{bmatrix} e^{t} \\ e^{11t} \end{bmatrix}$$

Explanation:

This code computes the eigenvalues and eigenvectors of a given matrix A, then utilizes these eigenvectors to solve an initial value problem and construct a resulting matrix E. Finally, it presents the solution in terms of a matrix multiplication with exponential terms.

Question 2:

Code:

```
% Define the matrix (A) to be solved
A = [12, -7, 4; 6, -1, 1; -12, 12, -7];

% Compute eigenvalues and eigenvectors of matrix A
eigenvalues = eig(A);
[eigenvectors, diagonal_matrix] = eig(A);

% Display the eigenvectors
disp('Eigenvectors (P):');
disp(eigenvectors);

% Display the diagonal matrix containing the eigenvalues
disp('Diagonal Matrix (D):');
disp(diagonal_matrix);

% Solve the inital value problem for constants
c = eigenvectors\[-10;20;25];
disp('Solution constants c:');
disp(c);

% Create the diagonal matrix R using the values in vector c
R = [c(1) 0 0; 0 c(2) 0; 0 0 c(3)];

% Compute the matrix E = P*R
E = eigenvectors * R;

% Display the resulting matrix E
disp('Resulting Matrix E:');
disp(E);
```

Output:

Eigenvectors (P):

0.2024	0.7071	0.2617
-0.1215	0.7071	0.7328
-0.9717	0.0000	0.6281

Diagonal Matrix (D):

```
   -3.0000         0         0
         0    5.0000         0
         0         0    2.0000
```

Solution constants c:

```
   27.7855
  -52.7384
   82.7882
```

Resulting Matrix E:

```
    5.6250  -37.2917   21.6667
   -3.3750  -37.2917   60.6667
  -27.0000   -0.0000   52.0000
```

Solution:

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} 5.625 & -37.2917 & 21.667 \\ -3.375 & -37.2917 & 60.667 \\ -27 & 0 & 52 \end{bmatrix} * \begin{bmatrix} e^{-3t} \\ e^{5t} \\ e^{2t} \end{bmatrix}$$

Explanation:

Exactly the same code as Question 1, just different input matrices.

Question 3:

Code:

```
% Define the matrix (A) to be solved
A = [0, 0, 1, 0; 0, 0, 0, 1; 25, 18, 0, 0; 12, 40, 0, 0];

% Compute eigenvalues and eigenvectors of matrix A
eigenvalues = eig(A);
[eigenvectors, diagonal_matrix] = eig(A);

% Display the eigenvectors
disp('Eigenvectors (P):');
disp(eigenvectors);

% Display the diagonal matrix containing the eigenvalues
disp('Diagonal Matrix (D):');
disp(diagonal_matrix);

% Solve the inital value problem
c = eigenvectors\[23;16;69;0];
disp('Solution constants c:');
disp(c);

% Create the diagonal matrix R using the values in vector c
R = [c(1) 0 0 0; 0 c(2) 0 0; 0 0 c(3) 0; 0 0 0 c(4)];
```

```
% Compute the matrix E = P*R
E = eigenvectors * R;

% Display the resulting matrix E
disp('Resulting Matrix E:');
disp(E);
```

Output:

Eigenvectors (P):

0.0849	0.2169	0.0849	0.2169
0.1131	-0.1085	0.1131	-0.1085
0.5940	0.8677	-0.5940	-0.8677
0.7920	-0.4339	-0.7920	0.4339

Diagonal Matrix (D):

7.0000	0	0	0
0	4.0000	0	0
0	0	-7.0000	0
0	0	0	-4.0000

Solution constants c:

104.2294
47.3549
72.5473
-10.4768

Resulting Matrix E:

8.8442	10.2727	6.1558	-2.2727
11.7922	-5.1364	8.2078	1.1364
61.9091	41.0909	-43.0909	9.0909
82.5455	-20.5455	-57.4545	-4.5455

Solution:

$$\begin{bmatrix} w(t) \\ x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} 8.8442 & 10.2727 & 6.1558 & -2.2727 \\ 11.7922 & -5.1364 & 8.2078 & 1.1364 \\ 61.9091 & 41.0909 & -43.0909 & 9.0909 \\ 82.5455 & -20.5455 & -57.4545 & -4.5455 \end{bmatrix} * \begin{bmatrix} e^{7t} \\ e^{4t} \\ e^{-7t} \\ e^{-4t} \end{bmatrix}$$

Explanation:

The same code as Question 1, just different input matrices.