STAT 3011 Discussion 015

Standard Normal Distribution: Week 6

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Spring 2025

The Binomial Distribution

Definition: A discrete random variable X that counts the number of successes in n independent trials, each with success probability p, follows a binomial distribution:

$$X \sim \text{Binom}(n, p)$$

Conditions for Binomial Distribution:

1. Each trial has two possible outcomes: success or failure.
2. The probability of success, p, remains constant for each trial.
3. Trials are independent of each other.
4. The number of trials, n, is fixed.

Examples:

- Number of heads in 5 coin tosses of a fair coin: $X \sim \text{Binom}(5, 0.5)$

Expected Value and Variance of Binomial Distribution

Expected Value:

$$E(X) = n \cdot p$$

Variance:

$$\mathrm{Var}(X) = n \cdot p \cdot (1 - p)$$

where:

- n is the number of trials.
- p is the probability of success in each trial.

Understanding the Standard Normal Distribution

- The **normal distribution** is a continuous probability distribution completely characterized by:
  - **Mean** (μ): The center of the distribution.
  - **Standard deviation** (σ): Measures data spread.
- The **standard normal distribution** is a special case where:
  - $\mu = 0$, $\sigma = 1$
  - Denoted as: $Z \sim N(0, 1)$

Z-Score and Any Normal Distribution

- Any normal distribution $X \sim N(\mu, \sigma^2)$ can be converted to the standard normal distribution using the **z-score**:
$$Z = \frac{X - \mu}{\sigma}$$
- This transformation allows us to use standard normal tables or R's `pnorm()` function for probability calculations.
- **Key Idea:** The z-score tells us how far a value is from the mean, making different normal distributions comparable.

Using pnorm() in R

To find the probability under the standard normal curve:

- $P(Z < z)$: **pnorm(z)**
- $P(Z > z)$: **1 - pnorm(z)**
- $P(z_a < Z < z_b)$: **pnorm(z_b) - pnorm(z$_a$)**

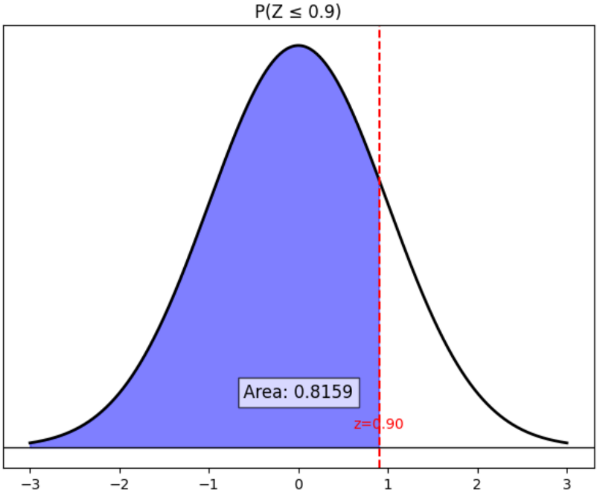
Note: `pnorm()` can also be used for any normal distribution $X \sim N(\mu, \sigma^2)$ by specifying the mean and standard deviation:

- $P(X < x)$: `pnorm(x, mean =` μ`, sd =` σ`)`
- $P(X > x)$: `1 - pnorm(x, mean =` μ`, sd =` σ`)`

Examples

Example 1: Find $P(Z < 0.9)$

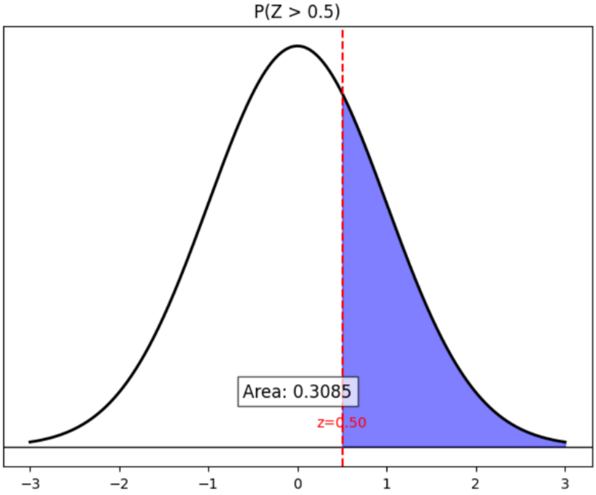
```
pnorm(0.9)
# Output: 0.8159
```



Examples

Example 2: Find $P(Z > 0.5)$

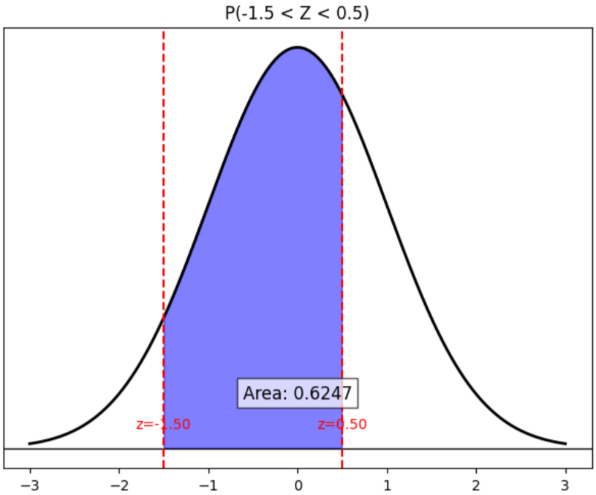
```
1 - pnorm(0.5) OR
pnorm(0.5, LOWER.TAIL = FALSE)

# Output: 0.3085
```



Examples

Example 3: Find $P(-1.5 < Z < 0.5)$

```
pnorm(0.5) - pnorm(-1.5)
# Output: 0.6247
```



Using qnorm() in R

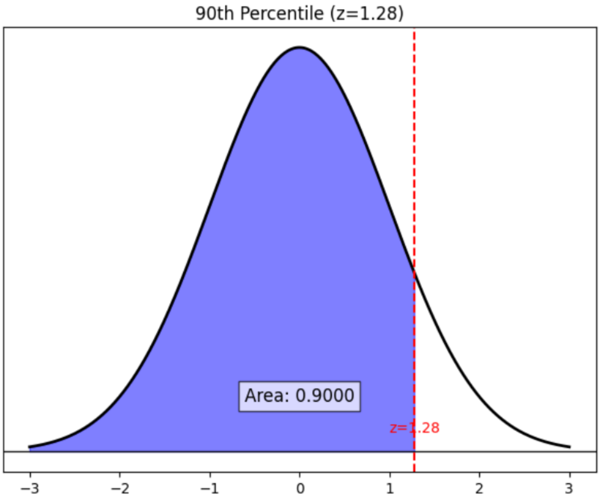
To find the z-value corresponding to a given percentile:

- z such that $P(Z < z) = p$: ‘qnorm(p)‘
- z such that $P(Z > z) = p$: ‘qnorm(1 - p)‘

Example

Example 4: Find the z-value that marks the 90th percentile of the standard normal distribution.

```
qnorm(0.9)
# Output: 1.2816
```



`pnorm()` **vs.** `qnorm()`

- **pnorm(z)** $\rightarrow$ Returns the probability (area under the curve) below z.
  "Given a z-score, what proportion of values are smaller?"

- **qnorm(p)** $\rightarrow$ Returns the z-score that corresponds to a given probability.
  "Given a probability, what z-score separates the lower area?"

Conclusion: pnorm() finds the area give a z-score, **qnorm()** finds the z-score given the area.

Questions? Let's Discuss!