STAT 3011 Discussion 015

Week 10: Hypothesis Testing Fundamentals

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The Five Essential Elements

1. **Assumptions**: Conditions that must be verified
2. **Hypotheses**: Null (H_0) and alternative (H_a)
3. **Test Statistic**: Calculated from sample data
4. **p-value**: Probability measure
5. **Conclusion**: Statistical decision

Element 1: Assumptions

Assumptions for one population mean μ

- **Random sampling**: a random sample from a distribution with unknown μ and σ.
- **Normality:** Population distribution is (approximately) normal.

R Commands

`qqnorm()`, `hist()`

Element 2: Hypothesis Formulation

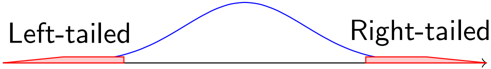
Null Hypothesis (H_0)

- Status quo claim
- Always contains "$=$"
- Example: H_0: $\mu = \mu_0$

Alternative (H_a)

Three possible forms:

- $\mu < \mu_0$ (left-tailed)
- $\mu > \mu_0$ (right-tailed)
- $\mu \neq \mu_0$ (two-tailed)



Element 3: Test Statistic

One-Sample t-Statistic

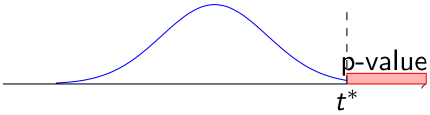
$$t^* = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim df = n - 1$$

- $\bar{x}$: Sample mean
- s: Sample standard deviation
- n: Sample size
- μ_0: Hypothesized population mean

Element 4: Understanding p-values

Formal Definition

Probability of observing a test statistic *as or more extreme* than the sample result, assuming H_0 is true.



- Small p-value $\Rightarrow$ Strong evidence against H_0
- Calculated using `pt()` in R

Calculating p-values in R

Right-tailed test ($H_a : \mu > \mu_0$)

```
pt(t*, df=n-1, lower.tail=FALSE)
```

Left-tailed test ($H_a : \mu < \mu_0$)

```
pt(t*, df=n-1, lower.tail=TRUE)
```

Two-tailed test ($H_a : \mu \neq \mu_0$)

```
2*pt(abs(t*), df=n-1, lower.tail=FALSE)
```

Element 5: Making Conclusions

Decision Rule

Compare p-value to significance level α:

$$\text{p-value} \leq \alpha \Rightarrow \text{Reject } H_0$$

$$\text{p-value} > \alpha \Rightarrow \text{Fail to reject } H_0$$

Common α levels

- 0.01 (1%)
- 0.05 (5%)
- 0.10 (10%)

Never say "Accept H_0"

We either reject or fail to reject - never prove the null!

Conducting t-tests in R

One-Sample t-Test Function

```
t.test(x,                      # Data vector
       mu = ,                  # Null value mu_0
       alternative = ,         # "two.sided", "less", "greater"
       conf.level = )          # 1 - alpha
```

- Output includes: test statistic, p-value, and CI
- Automatically checks assumptions

Potential Errors in Hypothesis Testing

	H_0 True	H_a True
Fail to Reject H_0	Correct	Type II Error (β)
Reject H_0	Type I Error (α)	Correct (Power)

Key Relationships

- $\alpha = P(\text{Reject } H_0 | H_0 \text{ true})$ a.k.a Type 1 Error
- $\beta = P(\text{Fail to reject } H_0 | H_a \text{ true})$ a.k.a Type 2 Error
- Note: As you reduce the Type 1 Error, the chances of a Type 2 Error occurring increase. (Inverse relationship)

Questions?

Lab 10 - Question 1a

Problem: The Ford Motor Company claims that its 2017 model of the Ford Escape averages 30 miles per gallon for highway driving. A group of owners of the 2017 Ford Escape model suspects that the company is exaggerating the highway mile per gallon (mpg) and decides to conduct a test. After 50 test drives, they found a sample mean of 27 mpg.

Solution: $H_0 : \mu = 30$
$H_a = \mu < 30$

Lab 10 - Question 1b

Problem: One study has suggested that students in college study an average of 11 hours per week. Suppose you want to test whether college students who plan to go to graduate school study more than 11 hours per week on average. So you randomly selected 20 students who planned to go to graduate schools and asked about their average study hours.

Solution:
$H_0 : \mu = 11$
$H_\alpha : \mu > 11$