STAT 3011 Discussion 015

Week 11: Two-Sample Comparisons

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Two-Sample Comparison Framework

Independent Samples	**Matched Pairs**
Compare μ_1 vs μ_2 $\bar{x}_1 - \bar{x}_2$	Compare μ_D (pair differences) $\bar{x}_D$ (mean of differences)
Example: Test scores from two different classrooms	**Example**: Twins assigned to different treatments

Key Distinction

- **Independent**: Two completely separate groups with no pairing
- **Paired**: Individuals paired by characteristics (age, weight, etc.)

Note

Matched pairs could be the same subject measured twice

Assumptions

Independent Samples	**Matched Pairs**
1. Two independent random samples 2. Both populations normal OR $n_1, n_2 \geq 30$	1. Random sample of pairs 2. Differences normally distributed OR large enough sample size

Confidence Intervals for Two-Sample Comparisons

Independent Samples	Matched Pairs
$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$	$\bar{x}_D \pm t_{\alpha/2}\frac{s_D}{\sqrt{n_D}}$
$\text{df} = \min(n_1 - 1,\ n_2 - 1)$	$\text{df} = n_D - 1$

Intuitive Interpretation

- **CI excludes 0**: The true difference is unlikely to be zero ("statistically significant")
  - Example: If 95% CI (1.2, 3.4) kg for weight loss $\implies$ Effective treatment
- **CI includes 0**: No evidence of difference ("null plausible")
  - Example: If 95% CI (-0.5, 1.5) kg $\implies$ Might just be random variation

Hypothesis Testing for Two Samples

Independent Samples

Hypotheses:

- $H_0 : \mu_1 = \mu_2$
- $H_a : \mu_1 \neq \mu_2$ (two-tailed)
- $H_a : \mu_1 < \mu_2$ (left-tailed)
- $H_a : \mu_1 > \mu_2$ (right-tailed)

Test Statistic:

$$t^* = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Matched Pairs

Hypotheses:

- $H_0 : \mu_D = 0$
- $H_a : \mu_D \neq 0$ (two-tailed)
- $H_a : \mu_D < 0$ (left-tailed)
- $H_a : \mu_D > 0$ (right-tailed)

Test Statistic:

$$t^* = \frac{\bar{x}_D}{s_D/\sqrt{n_D}}$$

p-values and Conclusions

Calculating p-values

- Left-tailed: `pt(t*, df, lower.tail=TRUE)`
- Right-tailed: `pt(t*, df, lower.tail=FALSE)`
- Two-tailed: `2*pt(abs(t*), df, lower.tail=FALSE)`

Conclusion

- if p-value $\leq \alpha \implies$ Reject H_0 and Accept H_a
- if p-value $> \alpha \implies$ Fail to reject H_0

Crucial Nuance

- We never **accept** H_0 - we only fail to reject it

R Commands

Independent Samples

```
t.test(group1, group2, var.equal=FALSE)
```

Paired Samples

```
t.test(group1, group2, paired=TRUE)
```

Output Includes

- Confidence interval
- Test statistic
- p-value

Questions?

Problem 1

Scenario:
A researcher randomly samples four patients with high blood pressure and measures their blood pressure initially. The researcher then assigned the patients to walk briskly for half an hour a day. A month later, those patients' blood pressure was measured again.

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Answer: Matched Pairs

Explanation:
Each patient's blood pressure is measured before and after treatment. Same subjects measured twice = matched pairs design.

Problem 2

Scenario:
A STAT 3011 student compares textbook prices at Amazon vs. Barnes Noble. She randomly selects 10 textbooks used that term and records the price of each book at both sites.

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Answer: Matched Pairs

Explanation:
Each book has two prices — one from each site — so prices can be directly matched by textbook. If she had sampled different books from each site, it would be independent.

Problem 3

Scenario:
A researcher is studying smoking and lung capacity. Participants are paired — one smoker and one non-smoker — matched on age, gender, and BMI. Each participant's lung capacity is measured.

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Answer: Matched Pairs

Explanation:
Participants are deliberately paired based on similar characteristics. This creates matched pairs suitable for paired analysis.

Problem 4

Scenario:
In an experiment, researchers either stared or didn't stare at drivers at a stop sign, then timed how long it took the drivers to cross. They want to test whether staring affects crossing time.

Data:
No-stare group: 14 observations
Stare group: 13 observations

Problem 4

Scenario:
In an experiment, researchers either stared or didn't stare at drivers at a stop sign, then timed how long it took the drivers to cross. They want to test whether staring affects crossing time.

Data:
No-stare group: 14 observations
Stare group: 13 observations

Answer: Independent Samples

Explanation:
There's no way to meaningfully match drivers between the two groups. Also, sample sizes differ. This is a classic two-sample independent design.