STAT 3011 Discussion 015

Week 9

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Confidence Intervals: Overview

- A confidence interval provides a range of plausible values for a population parameter such as the population mean μ.
- It is calculated as:

$$\bar{x} \pm t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$$

- Where:
  - $\bar{x}$ = Sample mean
  - s = Sample standard deviation
  - n = Sample size
  - $t_{\alpha/2}$ = t-multiplier from t-distribution

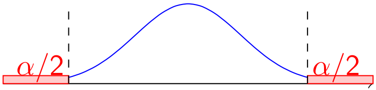
Why We Need the Student's t-Distribution

- **Problem with Normal Distribution**: When population standard deviation (σ) is unknown (common in real-world scenarios), using sample standard deviation (s) introduces extra uncertainty.
- **Solution**: The t-distribution:
  - Has heavier tails than the normal distribution
  - Accounts for additional uncertainty when estimating σ with s
  - Shape depends on degrees of freedom ($df = n - 1$)
- **Key Properties**:
  - Symmetric and bell-shaped (like normal)
  - Approaches N(0,1) as $df \to \infty$
  - For small n, gives wider CIs than normal distribution

What is α in $t_{\alpha/2}$?

- α **(alpha)** represents the total error probability for the confidence interval
  - For a 95% CI: $\alpha = 0.05$
  - For a 99% CI: $\alpha = 0.01$
- **Why α /2?** The error is split equally between both tails:

$$\text{Confidence} = 1 - \alpha$$



- **Example**: For 95% CI (α =0.05), we use $t_{0.025}$ because $0.05/2 = 0.025$

Calculating the t-Multiplier

Formula: $t_{\alpha/2,df}$ where:

- $df = n - 1$ (degrees of freedom)
- $\alpha/2 =$ right-tailed probability

R Commands:

```
# For 95% CI with n=10 (df=9):
qt(0.025, df=9, lower.tail=FALSE)  # Returns 2.262
# Equivalent alternatives:
qt(0.975, df=9)                    # Also returns 2.262
```

Assumptions for Confidence Intervals

- **Random Sample:** Data should be randomly selected.
- **Normality:**
  - If $n \geq 30$, the Central Limit Theorem ensures normality.
  - If $n < 30$, check normality using a Q-Q plot:

```
qqnorm(x)
qqline(x)
```

Standard Error and Margin of Error

- Standard error (SE) estimates how much the sample mean varies:

$$SE = \frac{s}{\sqrt{n}}$$

- Margin of error (MOE):

$$MOE = t_{\alpha/2,n-1} \times SE$$

Using t.test() to Construct Confidence Intervals

- In R, use `t.test()` to compute a confidence interval:

```
t.test(x, conf.level=0.96, alternative="two.sided")
```

- Output includes:
  - Confidence interval
  - Sample mean estimate

Interpreting 95% Confidence Intervals

Three Valid Interpretations

1. **Long-run frequency**:
   "If we repeated this procedure many times, 95% of calculated CIs would contain the true parameter."
2. **Single sample probability**:
   "There's a 95% probability that this procedure produces an interval containing the true parameter."
3. **Statistical significance**:
   "Values within the 95% CI are not significantly different from our estimate at α=0.05."

Note: The probability statement is about the **method**, *not any particular interval.*

What Confidence Intervals Don't Mean

Frequent Misconceptions

- **Not**: "There's a 95% chance the true value is in this interval"
  (The interval either contains it or doesn't - no probability after calculation)
- **Not**: "95% of our data points are in this interval"
  (It's about the parameter estimate, not individual observations)
- **Not**: "If we repeat the study, there's 95% chance new estimate will be in this interval"
  (New intervals will center on new estimates)

Weather Forecast Analogy

- "95% accurate forecasts" doesn't mean today's forecast has 95% chance of being correct
- Different forecast types (rain/sun) have different accuracy rates
- Similarly, CIs describe the **method's reliability**, not a specific interval's certainty

Questions? Let's Discuss!