Problem 1

A university administration is deciding which of two buildings (Art Museum, Biology Library) on campus to use for a new coffee shop. Suppose that the probability of building a coffee shop in the Art Museum is $P(A) = 0.30$ and in the Biology Library is $P(B) = 0.25$. Additionally, building a coffee shop in these two buildings are disjoint events.

1. **What is the probability that a coffee shop is built in either the Art Museum or the Biology Library?**

   $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.25 = 0.55$

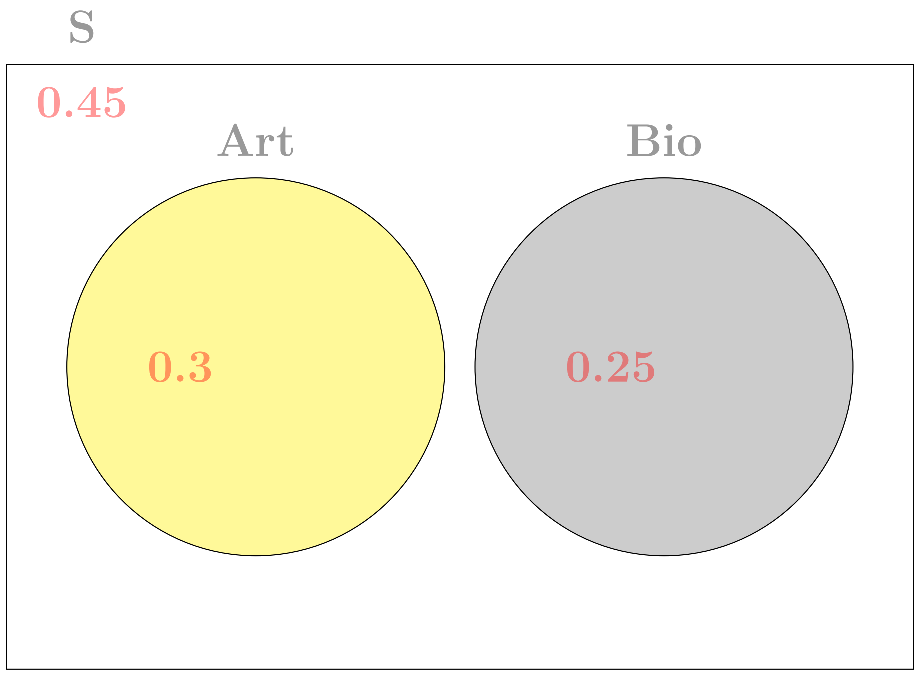
2. **Given that there will be NO coffee shop in the Art Museum, what is the probability that the Biology Library will have a coffee shop?**

   $$P(B \mid A^c) = \frac{P(B \cap A^c)}{P(A^c)} = \frac{P(B)}{1 - P(A)} = \frac{0.25}{0.7}$$

3. **Suppose that in reality, the administrator confused disjoint and independent, and the two events are actually independent. Answer parts (a) and (b) with this correct information.**

   (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A) \cdot P(B) = 0.3 + 0.25 - 0.3 \cdot 0.25$

   (b) $P(B \mid A^c) = \dfrac{P(B \cap A^c)}{P(A^c)} = \dfrac{P(B) \cdot P(A^c)}{P(A^c)} = P(B)$



Law of Total Probability: P(S) = 1

1 - 0.25 - 0.3 = 0.45

Problem 2

On a pop quiz in an introductory statistics class, there are two problems. Each problem is a multiple-choice question with four options (A, B, C, D), and only one is the correct answer. An unprepared student is randomly guessing. We are interested in whether the student's answer is correct or not for each question, denoted as C for correct and I for incorrect. For instance, an outcome CI means the student answered the first question correctly but the second question incorrectly.

1. **What is the sample space?**

$$S = \{CC, CI, IC, II\}$$

2. **What is the probability that the student answers at least one question correctly?**

$$P(\text{At least one correct}) = P(\{CC, CI, IC\}) = 1 - P(\{II\})$$

We know that the probability of answering each question correctly is $P(C) = \frac{1}{4}$, and the probability of answering incorrectly is $P(I) = \frac{3}{4}$.

Individual Probabilities:

$$P(CC) = P(\text{correct on the first, correct on the second}) = P(C)P(C) = \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{16}$$
$$P(CI) = P(\text{correct on the first, incorrect on the second}) = P(C)P(I) = \left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = \frac{3}{16}$$
$$P(IC) = P(\text{incorrect on the first, correct on the second}) = P(I)P(C) = \left(\frac{3}{4}\right)\left(\frac{1}{4}\right) = \frac{3}{16}$$
$$P(II) = P(\text{incorrect on the first, incorrect on the second}) = P(I)P(I) = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{9}{16}$$

Probability of at least one correct answer:

$$P(\text{At least one correct}) = P(CC) + P(CI) + P(IC) = \frac{1}{16} + \frac{3}{16} + \frac{3}{16} = \frac{7}{16}$$

Alternatively, you can compute the probability of getting no correct answers (i.e., II) and subtract it from 1:

$$P(\text{At least one correct}) = 1 - P(II) = 1 - \frac{9}{16} = \frac{7}{16}$$