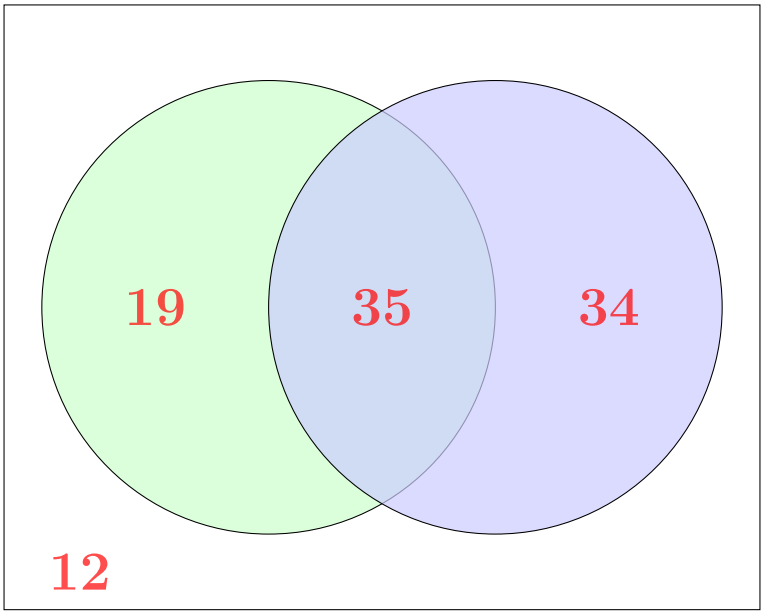
Lab 3

Exercise 1: In a high school graduating class of 100 students, 54 studied mathematics, 69 studied history, and 35 studied both mathematics and history. If one of these students is selected at random, find the probability that

(a) the student takes mathematics or history

(b) the student does not take either of these subjects

(c) the student takes history, but not mathematics



Explanation:

$$\text{Math only} = 54 - 35 = 19$$
$$\text{History only} = 69 - 35 = 34$$
$$\text{Neither} = 100 - (19 + 35 + 34) = 12$$

(a) Probability(student takes mathematics or history)

$$P(M \cup H) = P(M) + P(H) - P(M \cap H) = \frac{54}{100} + \frac{69}{100} - \frac{35}{100} = \frac{88}{100}$$

(b) Probability(student does not take either subject)

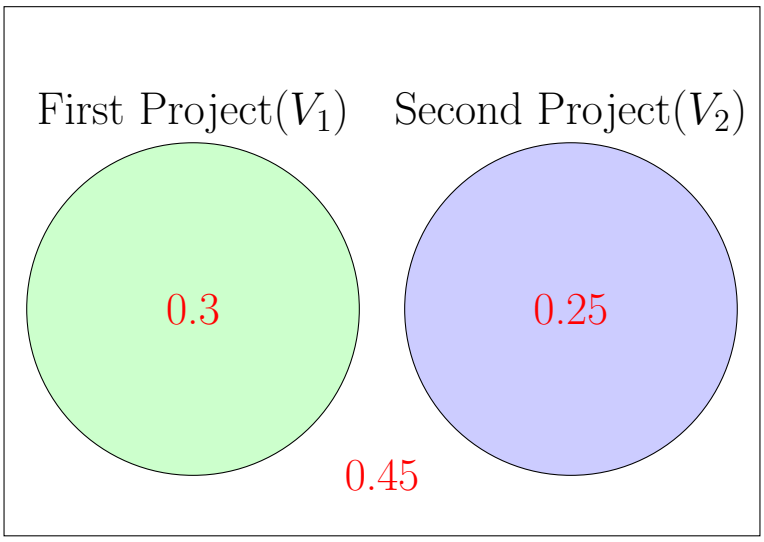
$$P(\text{Neither}) = \frac{12}{100}$$

(c) Probability(student takes history but not mathematics)

$$P(H \cup M') = \frac{34}{100}$$

Exercise 2:

A federal agency is deciding which of two waste dump projects to investigate. A top administrator estimates that the probability of federal law violations is 0.30 at the first project and 0.25 at the second project. Also, he believes the occurrences of violations in these two projects are disjoint.



Explanation:

$$\begin{aligned}
\text{First project only} &= P(V_1) = 0.30 \\
\text{Second project only} &= P(V_2) = 0.25 \\
\text{Intersection (both)} &= P(V_1 \cap V_2) = 0 \quad \text{(because disjoint)} \\
\text{Neither project} &= 1 - (0.30 + 0.25) = 0.45
\end{aligned}$$

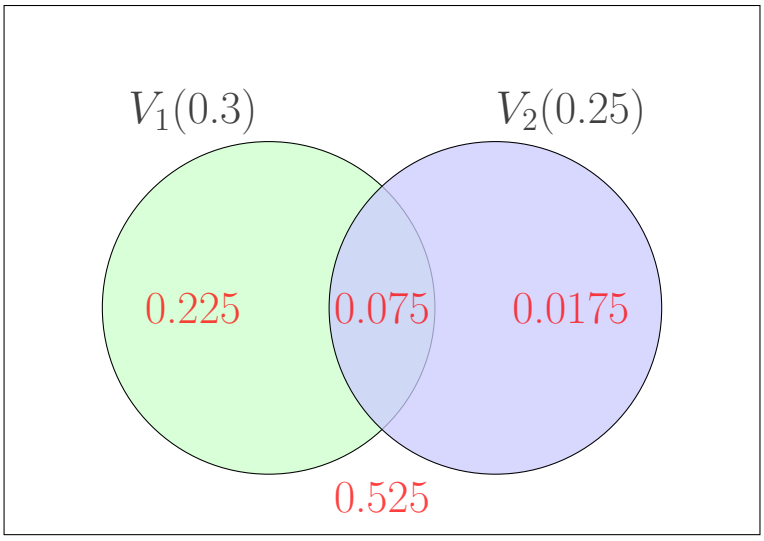
(a) What is the probability of federal law violations in the first project or in the second project?)

$$P(V_1 \cup V_2) = P(V_1) + P(V_2) = 0.55$$

(b) Given that there is not a federal law violation in the first project, find the probability that there is a federal law violation in the second project)

$$P(V_2 \mid V_1^c) = \frac{Pr(V_2 \cap V_1^c)}{Pr(V_1^c)} = \frac{Pr(V_2 \cap V_1^c)}{1 - Pr(V_1)} = \frac{0.25}{1 - 0.3} = 0.375$$

(c) In reality, the administrator confused disjoint and independent, and the events are actually independent. Answer parts a and b with the correct information)



$$\begin{aligned}
\text{Intersection (both)} &= P(V_1 \cap V_2) = P(V_1) \cdot P(V_2) = 0.075 \\
\text{First project only} &= P(V_1) - P(V_1 \cap V_2) = 0.30 - 0.075 = 0.225 \\
\text{Second project only} &= P(V_2) - P(V_1 \cap V_2) = 0.25 - 0.075 = 0.175 \\
\text{Neither project} &= 1 - (0.225 + 0.075 + 0.175) = 0.525
\end{aligned}$$

Finally,

$$P(V_1 \cup V_2) = 0.225 + 0.075 + 0.175 = 0.475$$

$$P(V_2 \mid V_1^c) = \frac{Pr(V_2 \cap V_1^c)}{Pr(V_1^c)} = \frac{0.175}{1 - 0.3} = 0.25 = Pr(V_2)$$

This is not a coincidence, as events V_1 and V_2 are independent.