# STAT 3011 Discussion 015

## Week 6

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# What is a Sampling Distribution?

- A **sampling distribution** is the distribution of a statistic (like $\hat{p}$ or $\bar{X}$) over many random samples.
- It shows how sample results vary from sample to sample.
- Helps us understand the concept of **sampling variability**.

# Key Symbols and Meanings

| Symbol | Meaning |
|---|---|
| $p$ | Population proportion (true, fixed value) |
| $\hat{p}$ | Sample proportion (statistic, varies by sample) |
| $\mu_{\hat{p}}$ | Mean of the sampling distribution of $\hat{p}$ |
| $\sigma_{\hat{p}}$ | Standard deviation of the sampling distribution of $\hat{p}$ |
| $\mu$ | Population mean (true, fixed value) |
| $\bar{X}$ | Sample mean (statistic, varies by sample) |
| $\mu_{\bar{X}}$ | Mean of the sampling distribution of $\bar{X}$ |
| $\sigma_{\bar{X}}$ | Standard deviation of the sampling distribution of $\bar{X}$ |
| $n$ | Sample size |
| $N(\mu, \sigma)$ | Normal distribution with mean $\mu$ and standard deviation $\sigma$ |

# Sample Proportion ($\hat{p}$)

- For a population proportion $p$, the sample proportion is:

$$\hat{p} = \frac{\text{of successes}}{n}$$

- Mean of sampling distribution: $\mu_{\hat{p}} = p$
- Standard deviation:

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

- As $n$ increases:
  - $\sigma_{\hat{p}}$ decreases (less variability)
  - Shape becomes more **normal** (by CLT)

## Sampling Distribution of the Sample Mean ($\bar{X}$)

- Population mean: $\mu$
- Population standard deviation: $\sigma$
- Then:

$$\mu_{\bar{X}} = \mu, \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

- As $n$ increases:
  - $\bar{X}$ gets closer to $\mu$
  - Distribution becomes more normal (CLT)

# Central Limit Theorem (CLT)

**Key Idea**

When sample size $n$ is large enough:

$$\text{Sampling distribution of } \hat{p} \text{ or } \bar{X} \approx N(\text{mean}, \text{SD})$$

# Central Limit Theorem (CLT)

**Key Idea**

When sample size $n$ is large enough:

$$\text{Sampling distribution of } \hat{p} \text{ or } \bar{X} \approx N(\text{mean}, \text{SD})$$

- For proportions: approximately normal if

$$np \geq 15 \quad \text{AND} \quad n(1-p) \geq 15$$

- For means: approximately normal if

$$n \geq 30 \text{ (or if population is already normal)}$$

# Empirical Rule

For a normal distribution:

$$\begin{array}{lll} 68\% & \text{within} & \pm 1\sigma \\ 95\% & \text{within} & \pm 2\sigma \\ 99.7\% & \text{within} & \pm 3\sigma \end{array}$$

Used to approximate probability intervals for $\hat{p}$ or $\bar{X}$.

# Putting It Together: Interpretation Tips

- The **mean** of the sampling distribution equals the population mean ($\mu$).
- The **standard deviation** (standard error) gets smaller with larger $n$.
- The **shape** becomes more normal as $n$ grows (CLT).
- Use the normal model to find probabilities and intervals for sample statistics.