STAT 3011 Discussion 015

Lab 10: Hypothesis Testing

Talha Hamza

University of Minnesota

College of Science and Engineering

Fall 2025

Five Elements

1. **Assumptions**: Conditions that must be verified
2. **Hypotheses**: Null (H_0) and alternative (H_a)
3. **Test Statistic**: Calculated from sample data
4. **p-value**: Probability measure
5. **Conclusion**: Statistical decision

Element 1: Assumptions

For Population Proportion p

1. Random sample
2. $n \cdot p_0 \geq 15$ AND $n \cdot (1 - p_0) \geq 15$

For Population Mean μ

1. Random sample
2. Sample is drawn from a Normal population,
   OR sample is sufficient large to assume normality ($n \geq 30$).
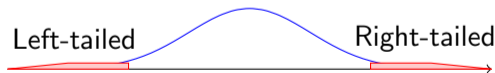
Element 2: Hypothesis Formulation

Null Hypothesis (H_0)

- Status quo claim
- Always contains "="
- Example: H_0: $\mu = \mu_0$
- The statement we want to **disprove**.

Alternative (H_a)

Three possible forms:

- $\mu < \mu_0$ (left-tailed)
- $\mu > \mu_0$ (right-tailed)
- $\mu \neq \mu_0$ (two-tailed)

Left-tailed   Right-tailed

Element 3: Test Statistic

One-Sample t-Statistic (Mean)

$$t^* = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim t_{df=n-1}$$

$df = n - 1$

- $\bar{x}$: Sample mean
- s: Sample standard deviation
- n: Sample size
- μ_0: Hypothesized population mean
- Follows a standard T-Distribution

One-Sample z-Statistic (Proportion)

$$z^* = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} \sim N(0, 1)$$

- $\hat{p}$: Sample proportion
- p_0: Hypothesized population proportion
- n: Sample size
- Follows a Standard Normal Distribution

Element 4: Understanding & Calculating p-values

What is a p-value?

- Probability of observing a test statistic *as or more extreme* than what we got,
- **assuming** the null hypothesis H_0 is true.

Means (t distribution):

- $H_a : \mu > \mu_0$:
  `pt(t*, df=n-1, lower.tail=FALSE)`

- $H_a : \mu < \mu_0$:
  `pt(t*, df=n-1, lower.tail=TRUE)`

- $H_a : \mu \neq \mu_0$:
  `2 * pt(abs(t*), df=n-1, lower.tail=FALSE)`

Proportions (Normal):

- $H_a : p > p_0$:
  `pnorm(z*, lower.tail=FALSE)`

- $H_a : p < p_0$:
  `pnorm(z*, lower.tail=TRUE)`

- $H_a : p \neq p_0$:
  `2 * pnorm(abs(z*), lower.tail=FALSE)`

Element 5: Making Conclusions

Decision Rule

Compare p-value to significance level α:

$$\text{p-value} \leq \alpha \Rightarrow \text{Reject } H_0$$

$$\text{p-value} > \alpha \Rightarrow \text{Fail to reject } H_0$$

Common α levels

- 0.01 (1%)
- 0.05 (5%)
- 0.10 (10%)

Never say "Accept H_0"

We either reject or fail to reject - never prove the null!

Conducting Hypothesis Tests in R

One-Sample t-Test:

```
t.test(x,                 # Data vector
       mu = ,             # Null value mu_0
       alternative = ,    # "two.sided", "less", "greater"
       conf.level = )     # 1 - alpha
```

One-Sample Proportion Test:

```
prop.test(x,                 # Number of successes
          n,                 # Sample size
          p = ,              # Null proportion p_0
          alternative = ,    # "two.sided", "less", "greater"
          conf.level =,      # 1 - alpha
          correct = FALSE)
```

Type I & Type II Errors

Truth	Fail to Reject H_0	Reject H_0
H_0 is true	Correct Decision	Type I Error (α)
H_a is true	Type II Error (β)	Correct Decision

Type I Error: Rejecting a true H_0
Type II Error: Failing to reject a false H_0

Note: P(Type II error) increases as P(Type I error) decreases.

Questions?