Problem 1: Simulating the Exit Poll

In 2018, Tim Walz won the Minnesota gubernatorial election with 54% of the vote. He plans to run for a third term in 2026. However, since he lost the Vice Presidential race, his approval rating has dropped to 47% in September 2025. We will conduct a simulation using the actual population proportion $p = 0.54$.

Since Tim Walz is running for a 3rd term, we will use a simulation to estimate his chances of winning. One of the current approval ratings as of September 2025 is $\hat{p} = 0.47$.

To simulate an exit poll:

- Go to: `https://istats.shinyapps.io/SampDist_Prop/`
- On the left side of screen, set Population Proportion p as 0.54
- Set sample size n as 10 (Check "Enter Numerical Values for n and p")
- Click "Draw sample(s)". Observe the "Data distribution" and "Sampling distribution"
- Data distribution shows the most recent experiment result
- Sampling distribution shows the distribution of $\hat{p}$ with its mean and standard deviation

a.) Simulate a sample of size 10. What sample proportion of voters vote for Tim Walz in your simulated sample?

60% of the MN voters voted for Tim Walz. *Answers will vary.*

b.) Simulate at least 10,000 samples of size $n = 10$. To simulate 20,000 samples, set the number of simulations to 10,000 and click "Draw Samples" twice. What are the mean and standard deviation of the sampling distribution of sample proportion?

The mean is 0.54 and the standard deviation is 0.158

c.) Use a formula from Chapter 7 to predict/verify the value of the standard deviation of sample proportion that you generated in part b.

From Chapter 7, $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.54(1-0.54)}{10}} = 0.1576$

d.) (True or False) Identify each of i, ii, iii is true or false. If false, explain why. If we increase the sample size from 10 to 500:

i.) the mean of sampling distribution of sample proportion increases.

False, the mean remains the same. Because it does not depend on n

ii.) the standard deviation of sampling distribution increases.

False, the standard deviation decreases since it is divided by n.

iii.) the shape of the sampling distribution becomes approximately normal.

True, by the central limit theorem, the shape of sampling distribution of sample proportion becomes approximately normal as n increases (more specifically the expected number of successes $np = 500 \times 0.54 \geq 15$ and the expected number of failure $n(1-p) = 500(1-0.54) \geq 15$).

e.) Simulate 10,000 samples of size $n = 500$. Based on the sampling distribution of the sample proportion, give an interval where approximately the middle 95% of the distribution falls. (Use 68-95-99.7 Rule).

The mean is 0.54 and the standard deviation is 0.022.
According to the 68-95-99.7 Rule, 95% of the observations are within 2 standard deviations of the mean. Therefore, $0.54 \pm 2 \times 0.022 = 0.4954$ to 0.5846. Also, $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.54(1-0.54)}{500}} = 0.0223$.

f.) What is the probability that Tim Walz can win the election if the 47% approval rating is accurate? Assume $p = 0.47$ and determine the probability of getting at least 50% of the vote to win.

$P(\hat{p} \geq 0.50) = P\left(\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \geq \frac{0.50-0.47}{0.0158}\right) \sim P(Z \geq 1.90)$

Problem 2: Simulating Customer Expense

Zen's All You Can Eat Restaurant charges \$8.95 per customer to eat at the restaurant. Restaurant management finds that its expense per customer, based on how much the customer eats and the expense of labor, has an extremely skewed to the right distribution with a mean of \$8.20 and a standard deviation of \$3.

We will use a Web app to simulate random samples from this population:

- Go to: `https://istats.shinyapps.io/sampdist_cont/`
- Select "Skewed" under Select population distribution
- Use the default skewness "Extremely right"
- Check the empty box next to "Enter population mean and st.dev" then enter 8.20 for population mean, and 3 for population standard deviation

a.) Set n (sample size) 10. Draw one sample. What are sample mean and sample standard deviation from your sample of 10 customers? What are the population mean and population standard deviation from the population of all customers? How are they different (what are they measuring)?

Answers for sample mean and sample standard deviation vary. Based on on the picture, sample mean $\bar{x} = 9.38$ sample standard deviation is $s = 3.89$.

The population mean expense is $\mu = 8.20$ and population standard deviation $\sigma = 3$.

b.) Let $\bar{X}_{10}$ be the sample mean expense per customer based on a random sample of $n = 10$ customers. Simulate 50,000 samples with this sample size. What are the mean and the standard deviation of the sampling distribution of $\bar{X}_{10}$? Is the sampling distribution of sample mean $\bar{X}_{10}$ approximately normal?

Answers for sample mean and sample standard deviation vary.

Sample mean $\bar{x} = 8.2$
Sample standard deviation is $s = 0.95$.

Yes, the sampling distribution of the sample mean is approximately normal.

This agrees with statistical theory that $\mu_{\bar{x}} = 8.20$ and $s = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{10}} = 0.95$.

c.) Let $\bar{X}_{100}$ be the sample mean expense per customer based on a random sample of $n = 100$ customers. Find the mean and standard deviation of the sampling distribution of the sample mean expense per customer from 100 customers. Describe the shape of the sampling distribution.

The mean of the sampling distribution of the sample mean is $\mu_{\bar{x}_{100}} = 8.20$. The standard deviation of this sampling distribution is $\sigma_{\bar{x}_{100}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{100}} = 0.3$. Since n is ≥ 30, the sampling distribution is approximately normal.

d.) Suppose that on a particular day, the restaurant had 100 customers that have the characteristics of a random sample from their customer base. Find the probability that the restaurant makes a positive profit that day, with the sample mean expense being less than \$8.95.

A profit is made when the sample mean expense is less than \$8.95. Therefore, we will compute $P(\bar{x} < 8.95)$ based on a sampling distribution with $\mu_{\bar{x}_{100}} = 8.20$ and $\sigma_{\bar{x}_{100}} = 0.3$ from the previous problem. $P(\bar{X} < 8.95) = P\left(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < \frac{8.95-8.20}{0.3}\right) = P(Z < 2.5)$ pnorm(2.5) $= 0.9937903$. Therefore, the restaurant will make a profit 99.4% of the time.

Problem 3: Homework Problem

In the following exercises, indicate whether the Central Limit Theorem applies so that the sample proportions follow a normal distribution.

a.) $n = 500$ and $p = 0.2$

We can assume the normal distribution by the CLT if the expected number of successes (np) and the expected number of failures ($n(1-p)$) are both at least 15.

$\rightarrow np = 500(0.2) = 100$ and $nq = 500(0.8) = 400$.

Yes, CLT applies.

b.) $n = 20$ and $p = 0.5$

$\rightarrow np = 20(0.5) = 10$ and $nq = 20(0.5) = 10$.

NO, CLT does not apply.

c.) $n = 30$ and $p = 0.2$

$\rightarrow np = 30(0.2) = 6$ and $nq = 30(0.8) = 24$.

NO, CLT does not apply.

d.) $n = 100$ and $p = 0.8$

$\rightarrow np = 100(0.8) = 80$ and $nq = 100(0.2) = 20$.

Yes, CLT applies.