STAT 3011 Discussion 007

Week 5

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Logistics

Announcement

No Lab
Next Week

Office Hours

Tuesday, March 3rd
2:30 – 3:30 PM
Ford 495

Stop by for help with:

- Extra Midterm Practice
- Cheat Sheet Review
- Last Minute Questions
- Concept Clarification

Binomial Distribution

Definition: Number of successes in n independent trials with probability p of success.

Conditions

- Fixed number of trials (n)
- Constant probability of success (p)
- Two outcomes: success or failure
- Independent trials

Notation: $X \sim \text{Binomial}(n, p)$

Binomial Distribution

What is a "Success"?

- The outcome being counted
- Not necessarily "good" or desirable

Examples

- Defective item in quality control
- Patient experiencing side effects
- Loan default in finance
- Machine breakdown in reliability

Key Properties

- Mean: $\mu = np$
- Variance: $\sigma^2 = np(1-p)$
- Standard deviation: $\sigma = \sqrt{np(1-p)}$

Summation Operator

The summation operator $\sum$ means we are **adding up a sequence of values**.

Example:

$$\sum_{i=1}^{4} i = 1 + 2 + 3 + 4 = 10$$

- The bottom number ($i = 1$) tells us where to start.
- The top number (4) tells us where to stop.
- Each term is added together step by step.

Summations without R

dbinom(k, n, p) gives the probability of **exactly** k successes:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

pbinom(k, n, p) gives the **cumulative** probability up to k successes:

$$P(X \leq k) = \sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n-i}$$

Summations with R

$$\underbrace{\texttt{dbinom}(k,\ n,\ p)}_{\text{probability of exactly } k \text{ successes out of } n \text{ trials with success probability } p} = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\underbrace{\texttt{pbinom}(k,\ n,\ p)}_{\text{probability of up to } k \text{ successes out of } n \text{ trials with success probability } p} = \sum_{i=0}^{k} \binom{n}{i} p^i (1-p)^{n}$$

$$\boxed{\texttt{pbinom(k, n, p)} = \sum_{i=0}^{k} \texttt{dbinom(i, n, p)}}$$

Using pbinom() in R

To find cumulative probabilities for binomial distribution:

- $P(X \leq k)$: `pbinom(k, size = n, prob = p)`
- $P(X > k)$: `1 - pbinom(k, size = n, prob = p)`
- $P(X < k)$: `pbinom(k-1, size = n, prob = p)`
- $P(X \geq k)$: `1 - pbinom(k-1, size = n, prob = p)`

For exact probabilities:

- $P(X = k)$: `dbinom(k, size = n, prob = p)`

Binomial Distribution Examples in R

Example: Coin flipping

- What's the probability of getting 7 or fewer heads in 10 flips of a fair coin?

```r
# P(X <= 7) where X ~ Binomial(10, 0.5)
pbinom(7, size = 10, prob = 0.5)
# Output: 0.9453
```

```r
# P(X = 5) exactly 5 heads
dbinom(5, size = 10, prob = 0.5)
# Output: 0.2461
```

Sampling Distribution Cheat Sheet

Sample Mean: $\bar{X}$

- Mean: $\mu_{\bar{X}} = \mu$
- Standard error: $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$
- CLT condition: $n \geq 30$ or population normal $\Rightarrow \bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$ approximately

Sample Proportion: $\hat{p}$

- Mean: $\mu_{\hat{p}} = p$
- Standard error: $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$
- CLT condition: $np \geq 15$, $n(1-p) \geq 15 \Rightarrow \hat{p} \sim N(p, \frac{\sqrt{p(1-p)}}{n})$ approximately

Statistical Notation Cheat Sheet

Symbol	Name / Type	Meaning
X	Population variable	Value from the population (e.g., expense per customer)
μ	Population mean	Average of all X values
σ	Population standard deviation	Variability of X values
$\bar{X}$	Sample mean	Average of sampled X values
$\sigma_{\bar{X}}$	Standard error of $\bar{X}$	$\sigma/\sqrt{n}$, variability of sample mean
p	Population proportion	True proportion of "successes" in population
$\hat{p}$	Sample proportion	Proportion of successes in sample
$\sigma_{\hat{p}}$	Standard error of $\hat{p}$	$\sqrt{p(1-p)/n}$, variability of sample proportion
Z	Standardized value	(statistic − mean)/SE, for normal approximation

Questions?