Part 2: Exam Practice

We don't have any homework or quiz for Ch 6.3 (binomial distribution) and Ch 7 (sampling distributions) before Exam 1. Please use the following exercises to review and practice what you have learned in the lecture.

Exercise (a)

Explain why the following random variables do not have binomial distributions.

- X = the number of times we need to roll a fair die until we get the first "6".

  X is not the count of successes. Alternatively, the number of trials is uncertain.

- Y = the number of people in a family of size 4 who go to church on a given Sunday, when any one of them goes 50% of the time in the long run. Does Y have Bin(4, 0.5)?

  No. The independence condition is not plausible. If one member of the family is going, the others are more likely to go.

- W = the number of females in a random sample of four students from a class of size 20, when half the class is female. Does W have Bin(4, 0.5)?

  No. The trials are not independent, because the probability of the second student being female depends on the gender of the first student. Alternatively: the success probability differs for the two trials.

Exercise (b)

A balanced die with six sides is rolled 60 times. Let X = the number of 6's.

- Find the mean and standard deviation of X.

  Given:
  $n = 60, \quad p = \frac{1}{6}$

  Mean:

  $$np = (60)\left(\frac{1}{6}\right) = 10$$

  Standard Deviation:

  $$\sqrt{np(1-p)} = \sqrt{60\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)} = 2.887$$

- If you observe $x = 0$, would you be skeptical that the die is balanced?

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> dbinom(0, size = 60, prob = 1/6)
[1] 1.774701e-05
```

$P(X = 0) = 0.0000177$. It's very unlikely to have no 6's when rolling a balanced die 60 times.

Exercise (c)

Suppose that 25% of iPhone owners use a certain app. We randomly select 80 iPhone owners.

- Solve the mean and standard deviation of the sample proportion of iPhone users who use the app.

Given:
$n = 80, \quad p = 0.25$

Mean: $= p = 0.25$.

Standard Deviation:

$$= \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.25 \times 0.75}{80}} = 0.0484$$

- Does the sample proportion have a normal distribution approximately?

Expected number of successes $= np = (80)(0.25) = 20$
Expected number of failures $= n(1-p) = (80)(1-0.25) = 60$

- What is the probability that the sample proportion is above 0.5? (A large value of sample proportion provides a very misleading estimate for the true p, since it is only 0.25.)

$P(\hat{p} > 0.5) = P\left(\frac{\hat{p}-0.25}{0.0484} > \frac{0.5-0.25}{0.0484}\right) = P(Z > 5.165)$ where $Z \sim N(0,1)$.

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> pnorm(5.165, lower.tail = FALSE)
[1] 1.202195e-07
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Exercise (d)

Jane's All You Can Eat Restaurant charges \$8.95 per customer to eat at the restaurant. Restaurant management finds that its expense per customer, based on how much the customer eats and the expense of labor, has a distribution that is skewed to the right with a mean of \$8.2 and a standard deviation of \$3.

- If the 100 customers on a particular day have the characteristics of a random sample from their large customer base, find the mean and standard deviation of the sampling distribution of the restaurant's sample mean expense per customer.

Mean $= 8.2$. Standard deviation $= \frac{3}{\sqrt{100}} = 0.3$

- Find the probability that the restaurant makes a profit that day (i.e., the sample mean expense being less than \$8.95).

  Since $n = 100 > 30$ and it's a random sample from a large population of customers, the sample mean $\bar{X} \sim N(8.2, 0.3)$ approximately by the Central Limit Theorem.
  $P(\bar{X} < 8.95) = P\left(\frac{\bar{X}-8.2}{0.3} < \frac{8.95-8.2}{0.3}\right) = P(Z < 2.5) = 0.994$

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  > pnorm(2.5)
  [1] 0.9937903
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