COUNTING

Counting problems arise throughout mathematics and computer science.

- We must count the successful outcomes of experiments and all the possible outcomes of these experiments to determine probabilities of discrete events

- We need to count the number of operations used by an algorithm to study its time complexity.

- Suppose that a password on a computer system consists of six, seven, or eight characters. Each of these characters must be a digit or a letter of the alphabet. Each password must contain at least one digit. How many such passwords are there?

Basic Counting Principles

THE PRODUCT RULE

Suppose that a procedure can be broken down into a sequence of two tasks:

- If there are **m** ways to do the first task and for each of these ways of doing the first task, there are **n** ways to do the second task, then there are **mn** ways to do the procedure.

Example 1:A new company with just two employees, Sanchez and Patel, rents a floor of a building with 12 offices. How many ways are there to assign different offices to these two employees?

- The procedure of assigning offices to these two employees consists of assigning an office to Sanchez, which can be done in 12 ways, then assigning an office to Patel different from the office assigned to Sanchez, which can be done in 11 ways. By the product rule, there are

- 12 • 11 = 132 ways to assign offices to these two employees.

Example 2: The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?

- The procedure of labeling a chair consists of two tasks, namely, assigning to the seat one of the 26 uppercase English letters, and then assigning to it one of the 100 possible integers.

- The product rule shows that there are 26 • 100 = 2600 different ways that a chair can be labeled.

- Therefore, the largest number of chairs that can be labeled differently is 2600.

Example 3: There are 32 microcomputers in a computer center. Each microcomputer has 24 ports. How many different ports to a microcomputer in the center are there?

- The procedure of choosing a port consists of two tasks:
  - first picking a microcomputer and then picking a port on this microcomputer. Because there are 32 ways to choose the microcomputer
  - Secondly24 ways to choose the port no matter which microcomputer has been selected,
- the product rule shows that there are 32 • 24 = 768 ports.

Example 4: How many different bit strings of length seven are there?

- Each of the seven bits can be chosen in two ways, because each bit is either 0 or 1.

- Therefore, the product rule shows there are a total of 2^7 = 128 different bit strings of length seven.

Example 5: How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits (and no sequences of letters are prohibited)?

- There are 26 choices for each of the three uppercase English letters ten choices for each of the three digits. Hence, by the product rule there are a total of $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 17,576,000$ possible license plates.

THE SUM RULE

- If a task can be done either in one of n_1 ways or in one of n_2 ways, where none of the set of n_1 ways is the same as any of the set of n_2 ways, then there are $n_1 + n_2$ ways to do the task.

Example 1: A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 19 possible projects, respectively. No project is on more than one list. How many possible projects are there to choose from?

- The student can choose a project by selecting a project from the first list, the second list, or the third list.

- Because no project is on more than one list, by the sum rule there are 23 + 15 + 19 = 57 ways to choose a project.

Solutions in Class

There are 18 mathematics majors and 325 computer science majors at a college.

      a) In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?

      b) In how many ways can one representative be picked who is either a mathematics major or a computer science major?

An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?

A multiple-choice test contains 10 questions. There are four possible answers for each question.
a) In how many ways can a student answer the questions on the test if the student answers every question?
b) In how many ways can a student answer the questions on the test if the student can leave answers blank?

There are four major auto routes from Boston to Detroit and six from Detroit to Los Angeles. How many major auto routes are there from Boston to Los Angeles via Detroit?

A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each gender. How many different types of this shirt are made?