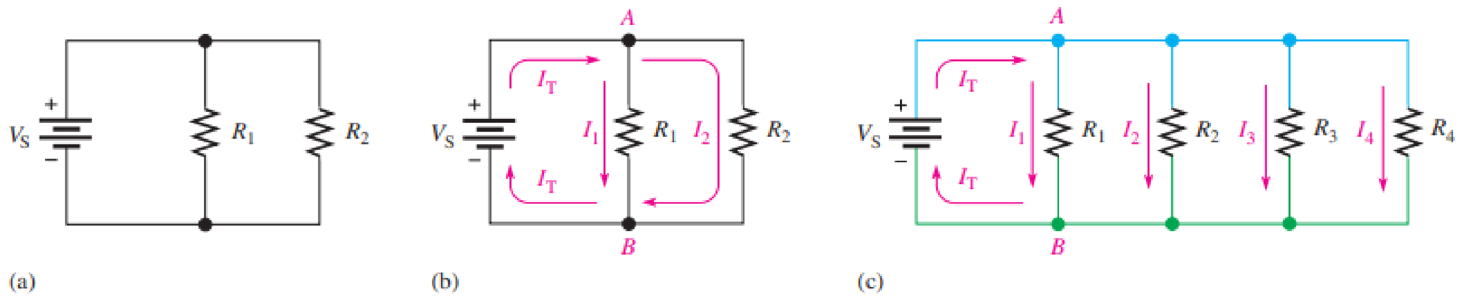
Basic Electronics

- Resistors in Parallel
- Kirchoff's Current Law

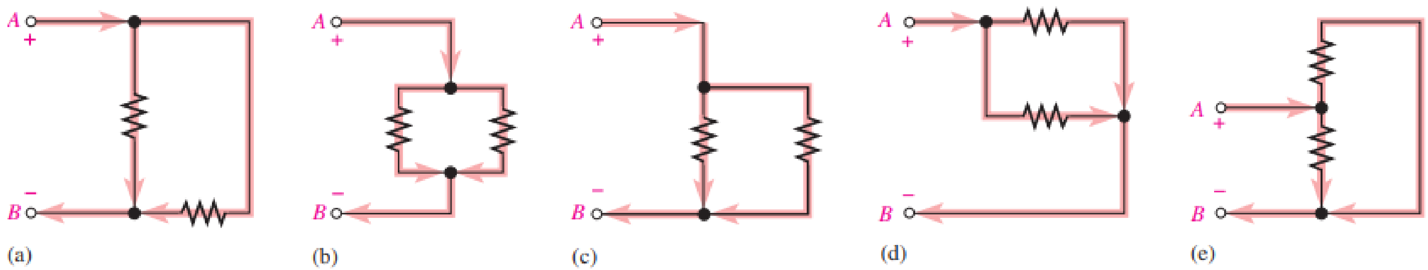
Resistors in Parallel

When two or more resistors are individually connected between two separate points (nodes) in a circuit, they are in parallel with each other. A parallel circuit provides more than one path for current



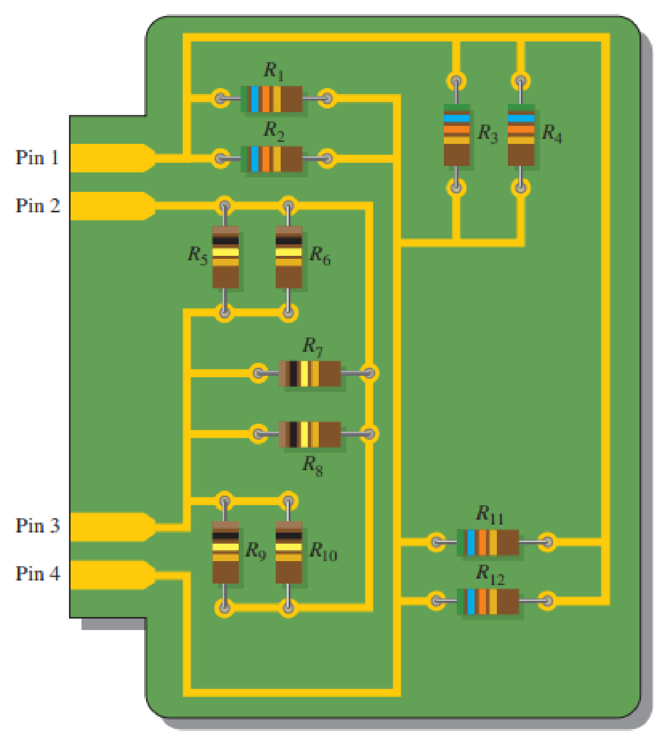
If there is more than one current path (branch) between two separate points and if the voltage between those two points also appears across each of the branches, then there is a parallel circuit between those two points.

Resistors in Parallel



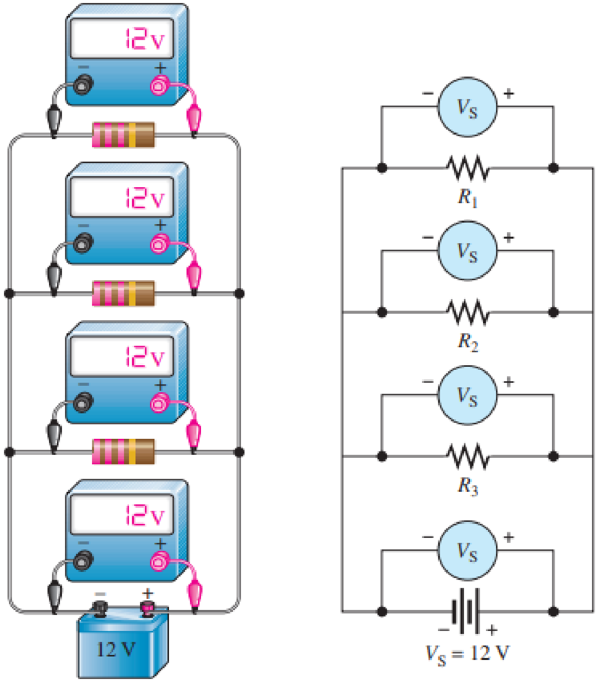
Resistors in Parallel

EXAMPLE 2 Determine the parallel groupings in Figure 5 and the value of each resistor.



Voltage in Parallel Circuits

The voltage across any given branch of a parallel circuit is equal to the voltage across each of the other branches in parallel. As you know, each current path in a parallel circuit is called a branch



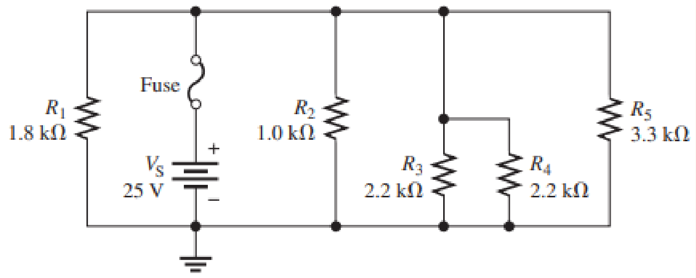
(a) Pictorial (b) Schematic

Voltage in Parallel Circuits

EXAMPLE 3

Determine the voltage across each resistor in Figure 9.

▶ FIGURE 9

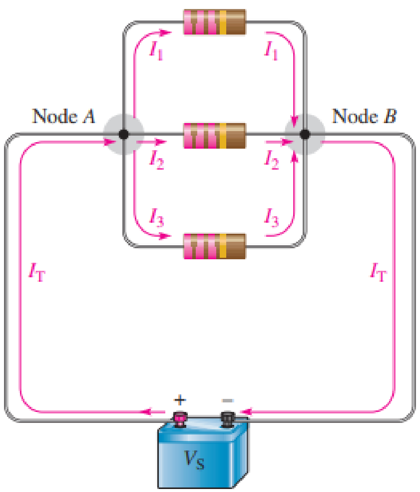


Kirchhoff's Current Law

Kirchhoff's current law, often abbreviated KCL, can be stated as follows: The sum of the currents into a node (total current in) is equal to the sum of the currents out of that node (total current out).

$$I_T = I_1 + I_2 + I_3$$

Kirchhoff's current law says that the total current into node A is equal to the total current out of node A;



Kirchhoff’s Current Law

$$I_{\mathrm{IN}(1)} + I_{\mathrm{IN}(2)} + I_{\mathrm{IN}(3)} + \cdots + I_{\mathrm{IN}(n)} = I_{\mathrm{OUT}(1)} + I_{\mathrm{OUT}(2)} + I_{\mathrm{OUT}(3)} + \cdots + I_{\mathrm{OUT}(m)}$$

Moving the terms on the right side to the left side and changing the sign results in the following equivalent equation:

$$I_{\mathrm{IN}(1)} + I_{\mathrm{IN}(2)} + I_{\mathrm{IN}(3)} + \cdots + I_{\mathrm{IN}(n)} - I_{\mathrm{OUT}(1)} - I_{\mathrm{OUT}(2)} - I_{\mathrm{OUT}(3)} - \cdots - I_{\mathrm{OUT}(m)} = 0$$

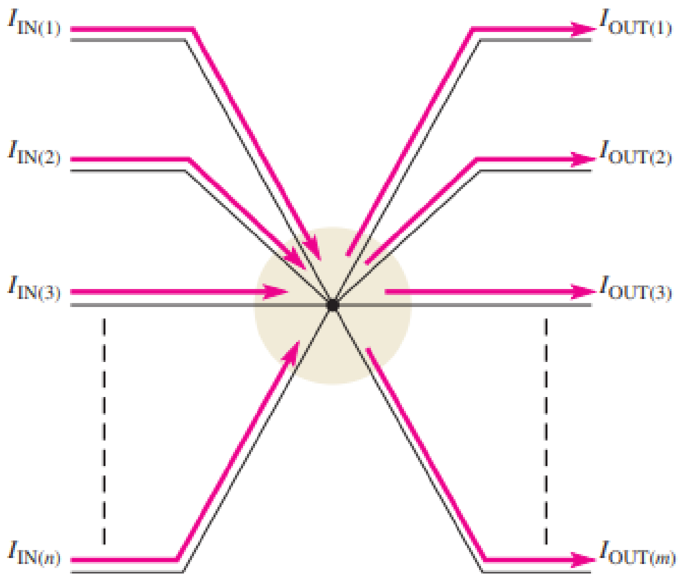
This equation shows that all current into and out of the junction sums to zero and can be stated as

The algebraic sum of all of the currents entering and leaving a node is equal to zero.

An equivalent way of writing Kirchhoff’s current law can be expressed using mathematical summation shorthand.

$$\sum_{i=1}^{n} I_i = 0$$

Kirchhoff’s Current Law

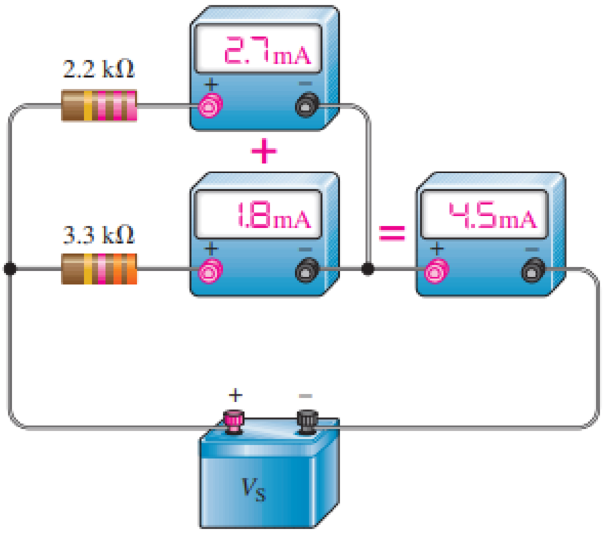


$$I_{IN(1)} + I_{IN(2)} + I_{IN(3)} + \cdots + I_{IN(n)} = I_{OUT(1)} + I_{OUT(2)} + I_{OUT(3)} + \cdots I_{OUT(m)}$$

Verification of Kirchhoff's Law

▶ FIGURE 14

An illustration of Kirchhoff's current law.

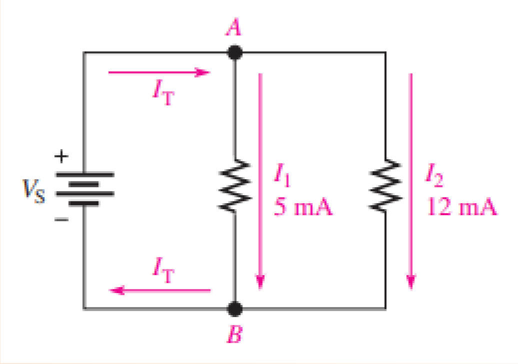


Verification of Kirchhoff's Law

EXAMPLE 4

The branch currents are shown in the circuit of Figure 15. Determine the total current entering node *A* and the total current leaving node *B*.

▶ FIGURE 15

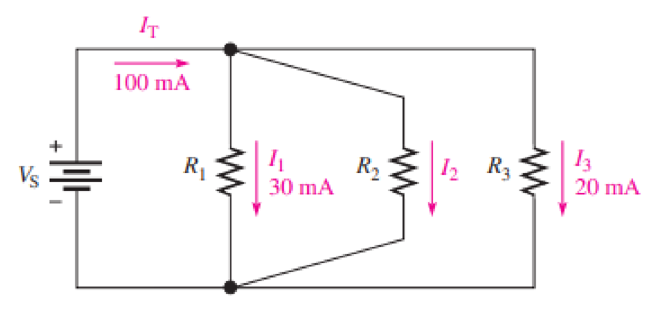


Verification of Kirchhoff's Law

EXAMPLE 5

Determine the current I_2 through R_2 in Figure 16.

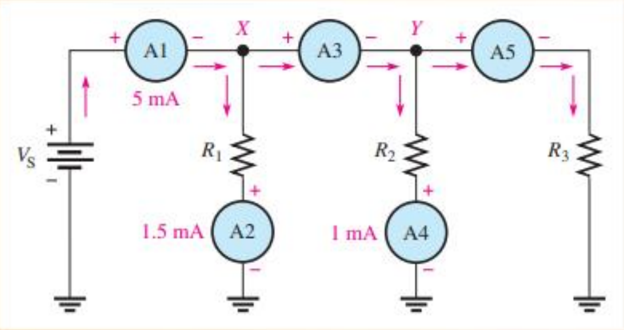
▶ FIGURE 16



Verification of Kirchhoff's Law

EXAMPLE 6 Use Kirchhoff's current law to find the current measured by ammeters A3 and A5 in Figure 17.

▶ FIGURE 17



Solution The total current into node X is 5 mA. Two currents are out of node X: 1.5 mA through resistor R_1 and the current through A3. Kirchhoff's current law applied at node X gives

$$5\text{ mA} - 1.5\text{ mA} - I_{A3} = 0$$

Solving for I_{A3} yields

$$I_{A3} = 5\text{ mA} - 1.5\text{ mA} = \mathbf{3.5\text{ mA}}$$

The total current into node Y is $I_{A3} = 3.5$ mA. Two currents are out of node Y: 1 mA through resistor R_2 and the current through A5 and R_3. Kirchhoff's current law applied at node Y gives

$$3.5\text{ mA} - 1\text{ mA} - I_{A5} = 0$$

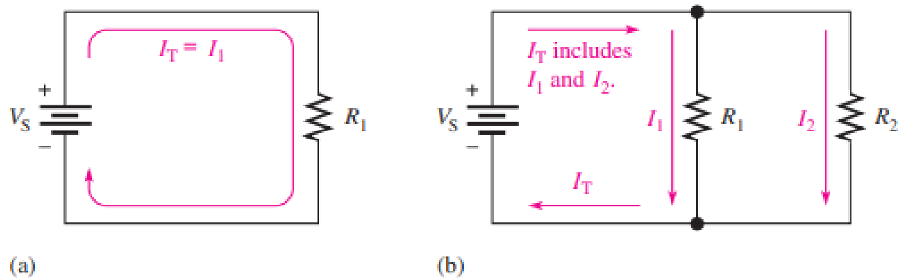
Solving for I_{A5} yields

$$I_{A5} = 3.5\text{ mA} - 1\text{ mA} = \mathbf{2.5\text{ mA}}$$

Total Parallel Resistance

When resistors are connected in parallel, the total resistance of the circuit decreases. The total resistance of a parallel circuit is always less than the value of the smallest resistor.

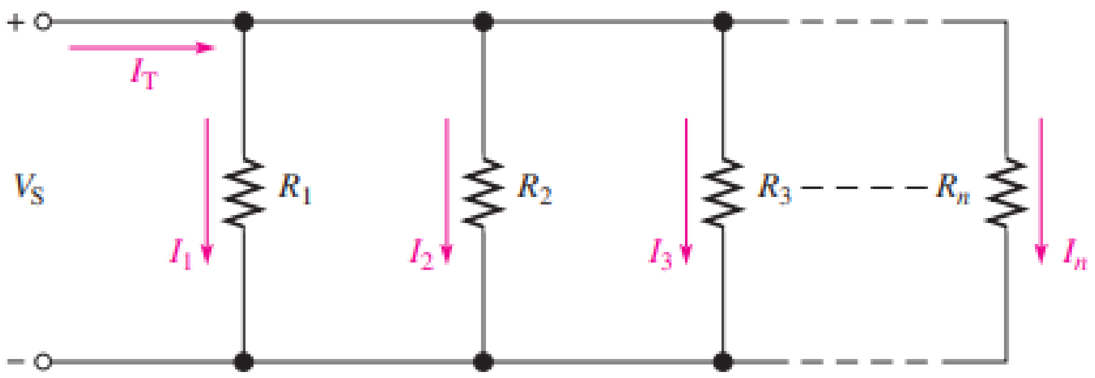
For example, if a resistor and a resistor are connected in parallel, the total resistance is less than .



Formula of Total Parallel Resistance

The circuit in Figure 19 shows a general case of n resistors in parallel (n can be any integer greater than 1). From Kirchhoff's current law, the equation for current is

$$I_T = I_1 + I_2 + I_3 + \cdots + I_n$$



▲ **FIGURE 19**

Circuit with n resistors in parallel.

Formula of Total Parallel Resistance

Since V_S is the voltage across each of the parallel resistors, by Ohm's law, $I_1 = V_S/R_1$, $I_2 = V_S/R_2$, and so on. By substitution into the equation for current,

$$\frac{V_S}{R_T} = \frac{V_S}{R_1} + \frac{V_S}{R_2} + \frac{V_S}{R_3} + \cdots + \frac{V_S}{R_n}$$

The term V_S can be factored out of the right side of the equation and canceled with V_S on the left side, leaving only the resistance terms.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

Recall that the reciprocal of resistance ($1/R$) is called *conductance*, which is symbolized by G. The unit of conductance is the siemens (S). The equation for $1/R_T$ can be expressed in terms of conductance as

$$G_T = G_1 + G_2 + G_3 + \cdots + G_n$$

Formula of Total Parallel Resistance

$$G_T = G_1 + G_2 + G_3 + \cdots + G_n$$

Solve for R_T by taking the reciprocal of (that is, by inverting) both sides of the equation for $1/R_T$.

$$R_T = \frac{1}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3}\right) + \cdots + \left(\frac{1}{R_n}\right)}$$

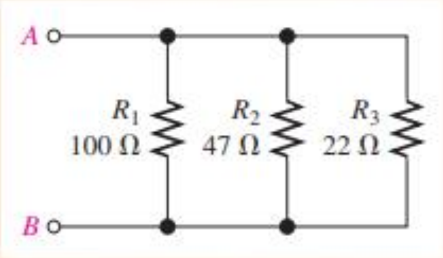
Equation 2 shows that to find the total parallel resistance, add all the $1/R$ (or conductance, G) terms and then take the reciprocal of the sum.

$$R_T = \frac{1}{G_T}$$

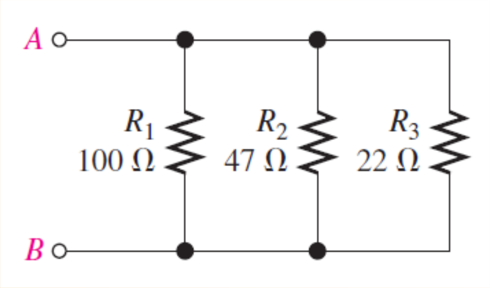
Formula of Total Parallel Resistance

EXAMPLE 8 Calculate the total parallel resistance between points *A* and *B* of the circuit in Figure 20.

▶ **FIGURE 20**



▶ FIGURE 20



Solution To calculate the total parallel resistance when you know the individual resistances, first find the conductance, which is the reciprocal of the resistance, of each of the three resistors.

$$G_1 = \frac{1}{R_1} = \frac{1}{100\ \Omega} = 10\ \text{mS}$$

$$G_2 = \frac{1}{R_2} = \frac{1}{47\ \Omega} = 21.3\ \text{mS}$$

$$G_3 = \frac{1}{R_3} = \frac{1}{22\ \Omega} = 45.5\ \text{mS}$$

Next, calculate R_T by adding G_1, G_2, and G_3 and taking the reciprocal of the sum.

$$R_T = \frac{1}{G_T} = \frac{1}{10\ \text{mS} + 21.3\ \text{mS} + 45.5\ \text{mS}} = \frac{1}{76.8\ \text{mS}} = \mathbf{13.0\ \Omega}$$

For a quick accuracy check, notice that the value of R_T (13.0 Ω) is smaller than the smallest value in parallel, which is R_3 (22 Ω), as it should be.

The Case of Two Resistors are in Parallel

$$R_T = \frac{1}{\left(\frac{1}{R_1}\right) + \left(\frac{1}{R_2}\right)}$$

Combining the terms in the denominator yields

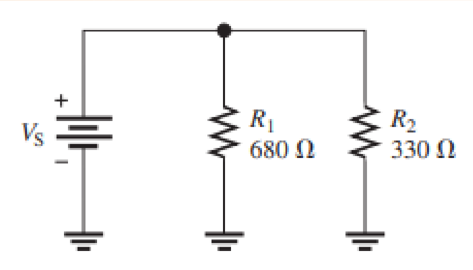
$$R_T = \frac{1}{\left(\frac{R_1 + R_2}{R_1 R_2}\right)}$$

which can be rewritten as follows:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

The Case of Two Resistors are in Parallel

EXAMPLE 9 Calculate the total resistance connected to the voltage source of the circuit in Figure 21.



The Case of Equal Value Resistors are in Parallel

Another special case of parallel circuits is the parallel connection of several resistors each having the same resistance value. There is a shortcut method of calculating R_T when this case occurs.

If several resistors in parallel have the same resistance, they can be assigned the same symbol R. For example, $R_1 = R_2 = R_3 = \cdots = R_n = R$. Starting with Equation 2, you can develop a special formula for finding R_T.

$$R_T = \frac{1}{\left(\frac{1}{R}\right) + \left(\frac{1}{R}\right) + \left(\frac{1}{R}\right) + \cdots + \left(\frac{1}{R}\right)}$$

Notice that in the denominator, the same term, $1/R$, is added n times (n is the number of equal-value resistors in parallel). Therefore, the formula can be written as

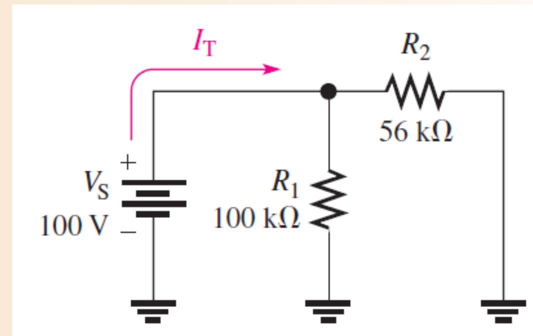
$$R_T = \frac{1}{n/R}$$

or

$$R_T = \frac{R}{n}$$

EXAMPLE 12 Find the total current produced by the battery in Figure 25.

▶ **FIGURE 25**



Solution The battery "sees" a total parallel resistance that determines the amount of current that it generates. First, calculate R_T.

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(100\ \text{k}\Omega)(56\ \text{k}\Omega)}{100\ \text{k}\Omega + 56\ \text{k}\Omega} = \frac{5600\ \text{k}\Omega^2}{156\ \text{k}\Omega} = 35.9\ \text{k}\Omega$$

The battery voltage is 100 V. Use Ohm's law to find I_T.

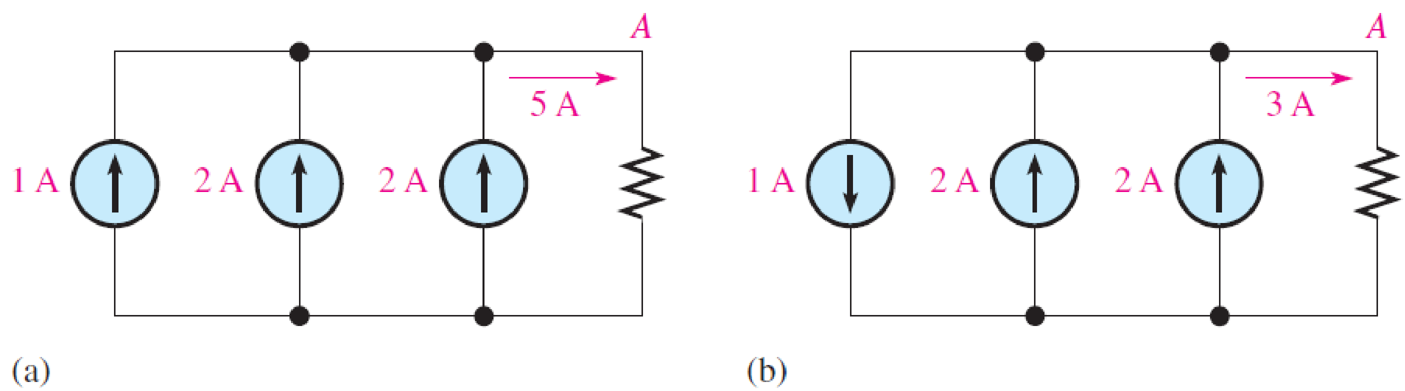
$$I_T = \frac{V_S}{R_T} = \frac{100\ \text{V}}{35.9\ \text{k}\Omega} = \mathbf{2.79\ mA}$$

In general, the total current produced by current sources in parallel is equal to the algebraic sum of the individual current sources. The algebraic sum means that you must consider the direction of current when you combine the sources in parallel. For example, in Figure 29(a), the three current sources in parallel provide current in the same direction (into node *A*). So the total current into node *A* is

$$I_T = 1\text{ A} + 2\text{ A} + 2\text{ A} = 5\text{ A}$$

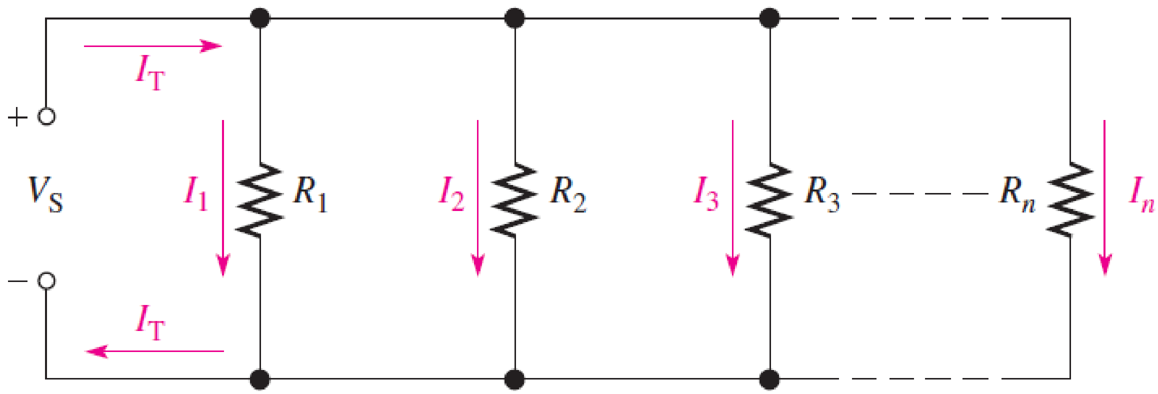
In Figure 29(b), the 1 A source provides current in a direction opposite to the other two. The total current into node *A* in this case is

$$I_T = 2\text{ A} + 2\text{ A} - 1\text{ A} = 3\text{ A}$$



Current-Divider Formula

You can develop a formula for determining how currents divide among any number of parallel resistors as shown in Figure 34, where n is the total number of resistors.



▲ **FIGURE 34**

A parallel circuit with n branches.

The current through any one of the parallel resistors is I_x, where x represents the number of a particular resistor (1, 2, 3, and so on). By Ohm's law, you can express the current through any one of the resistors in Figure 34 as follows:

$$I_x = \frac{V_S}{R_x}$$

The source voltage, V_S, appears across each of the parallel resistors, and R_x represents any one of the parallel resistors. The total source voltage, V_S, is equal to the total current times the total parallel resistance.

$$V_S = I_T R_T$$

Substituting $I_T R_T$ for V_S in the expression for I_x results in

$$I_x = \frac{I_T R_T}{R_x}$$

Rearranging terms yields

$$I_x = \left(\frac{R_T}{R_x}\right) I_T$$

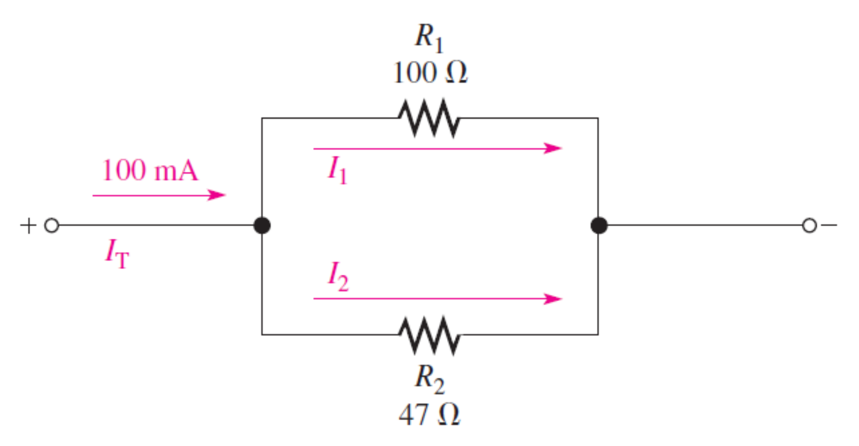
where $x = 1, 2, 3$, etc. Equation 6 is the general current-divider formula and applies to a parallel circuit with any number of branches.

The current (I_x) through any branch equals the total parallel resistance (R_T) divided by the resistance (R_x) of that branch, and then multiplied by the total current (I_T) into the junction of parallel branches.

MPLE 18

Find I_1 and I_2 in Figure 37.

GURE 37



Solution Use Equation 7 to determine I_1.

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I_T = \left(\frac{47\ \Omega}{147\ \Omega}\right)100\ \text{mA} = \mathbf{32.0\ mA}$$

Use Equation 8 to determine I_2.

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right)I_T = \left(\frac{100\ \Omega}{147\ \Omega}\right)100\ \text{mA} = \mathbf{68.0\ mA}$$

$$P_T = P_1 + P_2 + P_3 + \cdots + P_n$$

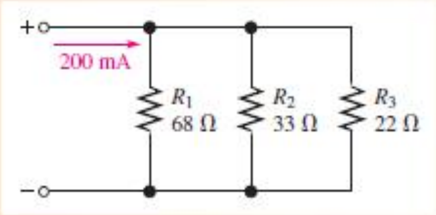
where P_T is the total power and P_n is the power in the last resistor in parallel. As you can see, the powers are additive, just as in a series circuit.

Power formulas are directly applicable to parallel circuits. The following formulas are used to calculate the total power P_T:

$$P_T = VI_T$$
$$P_T = I_T^2 R_T$$
$$P_T = \frac{V^2}{R_T}$$

where V is the voltage across the parallel circuit, I_T is the total current into the parallel circuit, and R_T is the total resistance of the parallel circuit. Examples 19 and 20 show how total power can be calculated in a parallel circuit.

EXAMPLE 19 Determine the total amount of power in the parallel circuit in Figure 40.



▲ FIGURE 40

Solution The total current is 200 mA. The total resistance is

$$R_T = \frac{1}{\left(\frac{1}{68\ \Omega}\right) + \left(\frac{1}{33\ \Omega}\right) + \left(\frac{1}{22\ \Omega}\right)} = 11.1\ \Omega$$

The easiest power formula to use is $P_T = I_T^2 R_T$ because you know both I_T and R_T.

$$P_T = I_T^2 R_T = (200\ \text{mA})^2(11.1\ \Omega) = \mathbf{444\ mW}$$

Let's demonstrate that if you determine the power in each resistor and if you add all of these values together, you will get the same result. First, find the voltage across each branch of the circuit.

$$V = I_T R_T = (200\ \text{mA})(11.1\ \Omega) = 2.22\ \text{V}$$

Remember that the voltage across all branches is the same.

Next, use $P = V^2/R$ to calculate the power for each resistor.

$$P_1 = \frac{(2.22\ \text{V})^2}{68\ \Omega} = 72.5\ \text{mW}$$

$$P_2 = \frac{(2.22\ \text{V})^2}{33\ \Omega} = 149\ \text{mW}$$

$$P_3 = \frac{(2.22\ \text{V})^2}{22\ \Omega} = 224\ \text{mW}$$

Add these powers to get the total power.

$$P_T = 72.5\ \text{mW} + 149\ \text{mW} + 224\ \text{mW} = 446\ \text{mW}$$