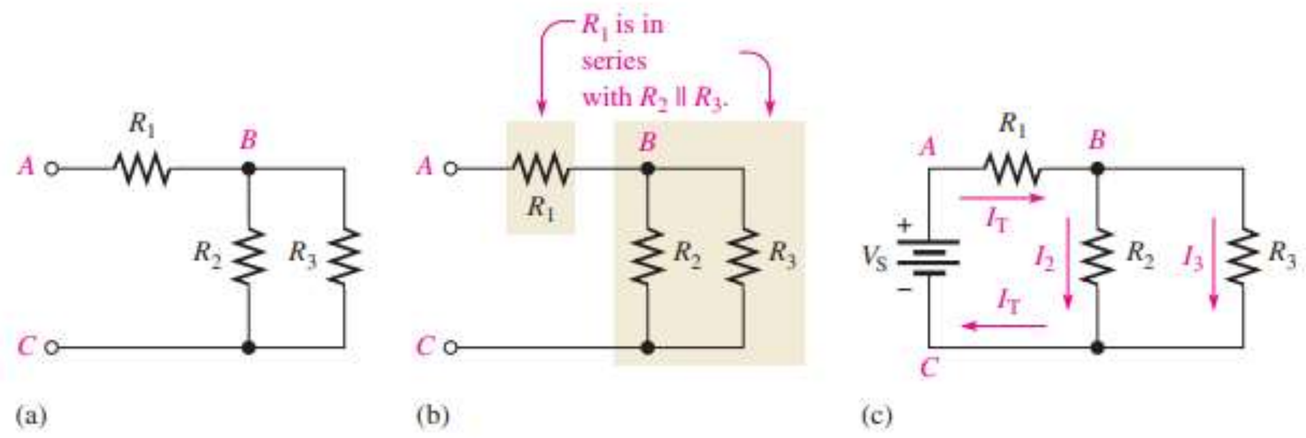
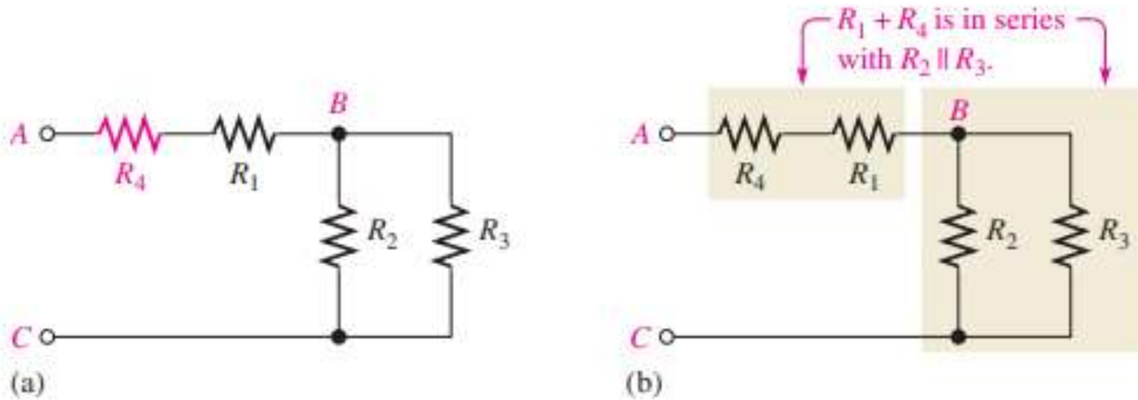
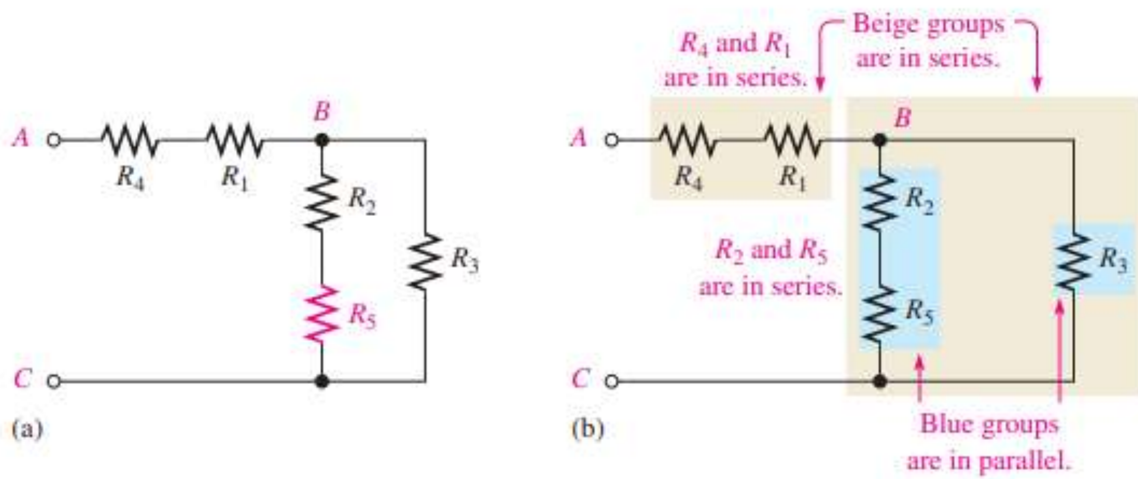
Identifying Series Parallel Relationship

A series-parallel circuit consists of combinations of both series and parallel current paths.
It is important to be able to identify how the components in a circuit are arranged in terms of their series and parallel relationships



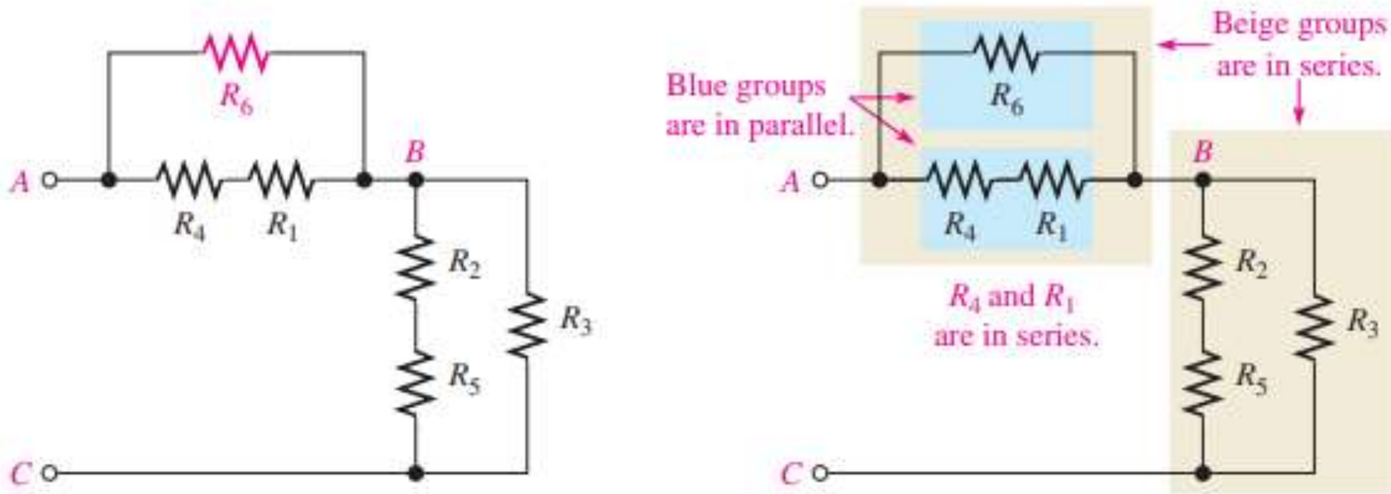
Identifying Series Parallel Relationship





Identifying Series Parallel Relationship

In Figure 4(a), R_6 is connected in parallel with the series combination of R_1 and R_4. The series-parallel combination of R_1, R_4, and R_6 is in series with the series-parallel combination of R_2, R_3, and R_5, as indicated in Figure 4(b).



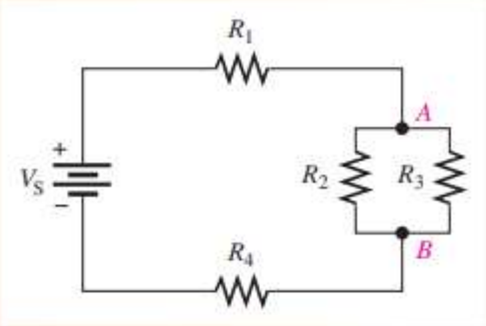
(a) (b)

Identifying Series Parallel Relationship

EXAMPLE 1

Identify the series-parallel relationships in Figure 5.

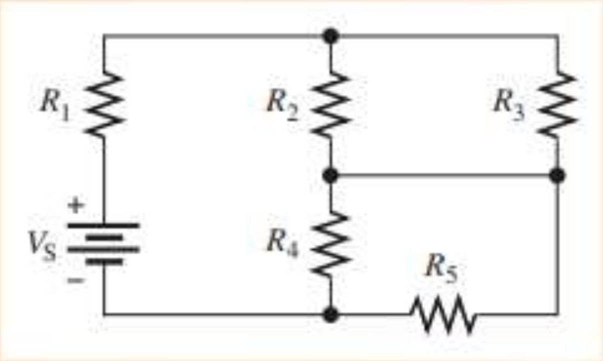
▶ **FIGURE 5**



Identifying Series Parallel Relationship

EXAMPLE 2

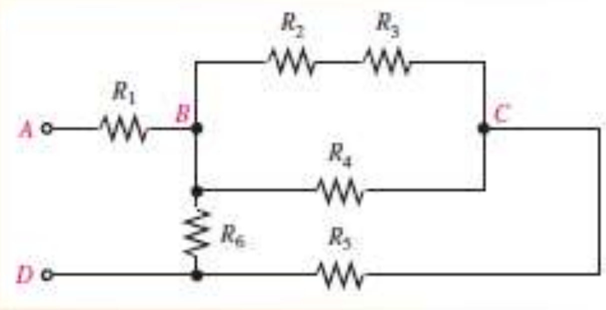
Identify the series-parallel relationship



Identifying Series Parallel Relationship

EXAMPLE 3 Describe the series-parallel combination between terminals A and D in Figure 9.

▶ FIGURE 9



Solution Between nodes B and C, there are two parallel paths. The lower path consists of R_4, and the upper path consists of a series combination of R_2 and R_3. This parallel combination is in series with R_5. The R_2, R_3, R_4, R_5 combination is in parallel with R_6. Resistor R_1 is in series with this entire combination as stated by the following expression:

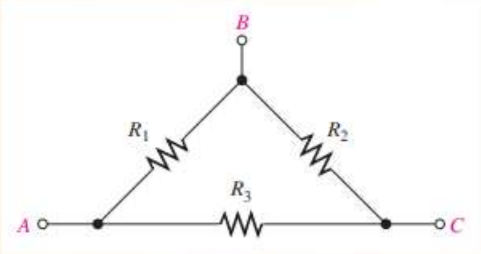
$$R_1 + R_6 \parallel (R_5 + R_4 \parallel (R_2 + R_3))$$

Related Problem If a resistor is connected from C to D in Figure 9, describe its parallel relationship.

Identifying Series Parallel Relationship

EXAMPLE 4 Describe the total resistance between each pair of terminals in Figure 10.

▶ **FIGURE 10**

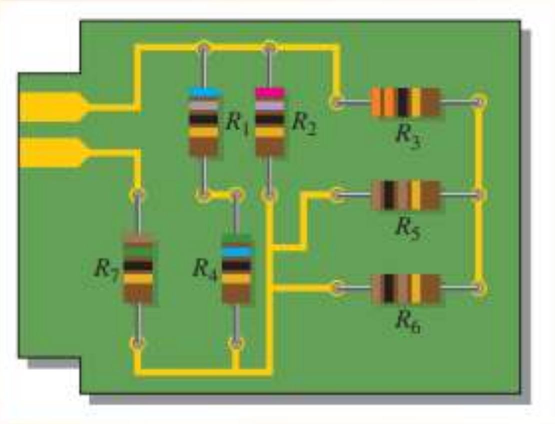


Identifying Series Parallel Relationship

EXAMPLE 5

Determine the relationships of the resistors on the PC board in Figure 11.

▶ FIGURE 11



Analysis of Series Parallel Resistive Circuits

2 ANALYSIS OF SERIES-PARALLEL RESISTIVE CIRCUITS

The analysis of series-parallel circuits can be approached in many ways, depending on what information you need and what circuit values you know. The examples in this section do not represent an exhaustive coverage, but they give you an idea of how to approach series-parallel circuit analysis.

After completing this section, you should be able to

- **Analyze series-parallel circuits**
  - Determine total resistance
  - Determine all the currents
  - Determine all the voltage drops

Analysis of Series Parallel Resistive Circuits

If you know Ohm's law, Kirchhoff's laws, the voltage-divider formula, and the current divider formula, and if you know how to apply these laws, you can solve most resistive circuit analysis problems. The ability to recognize series and parallel combinations

Total Resistance

EXAMPLE 7

Find the total resistance between the positive and negative terminals of the battery in Figure 17.

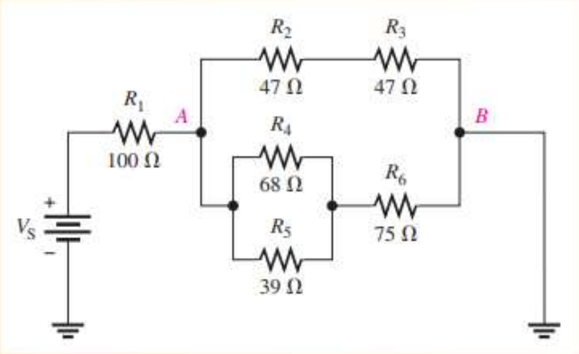


FIGURE 17

Total Current

Once you know the total resistance and the source voltage, you can apply Ohm's law to find the total current in a circuit. Total current is the source voltage divided by the total resistance.

$$I_T = \frac{V_S}{R_T}$$

For example, assuming that the source voltage is 10 V, the total current in the circuit of Example 7 (Figure 17) is

$$I_T = \frac{V_S}{R_T} = \frac{10\text{ V}}{148.4\ \Omega} = 67.4\text{ mA}$$

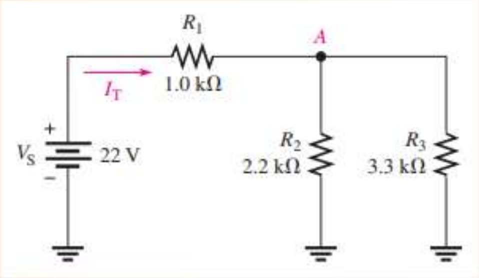
Branch Current

Using the current-divider formula, Kirchhoff's current law, Ohm's law, or combinations of these, you can find the current in any branch of a series-parallel circuit.

In some cases, it may take repeated application of the formula to find a given current.

Branch Current

EXAMPLE 8 Find the current through R_2 and the current through R_3 in Figure 19.



▲ FIGURE 19

Solution First, identify the series and parallel relationship. Next, determine how much current is into node A. This is the total circuit current. To find I_T, you must know R_T.

$$R_T = R_1 + \frac{R_2R_3}{R_2 + R_3} = 1.0\,\text{k}\Omega + \frac{(2.2\,\text{k}\Omega)(3.3\,\text{k}\Omega)}{2.2\,\text{k}\Omega + 3.3\,\text{k}\Omega} = 1.0\,\text{k}\Omega + 1.32\,\text{k}\Omega = 2.32\,\text{k}\Omega$$

$$I_T = \frac{V_S}{R_T} = \frac{22\,\text{V}}{2.32\,\text{k}\Omega} = 9.48\,\text{mA}$$

Use the current-divider rule for two branches to find the current through R_2.

$$I_2 = \left(\frac{R_3}{R_2 + R_3}\right)I_T = \left(\frac{3.3\,\text{k}\Omega}{5.5\,\text{k}\Omega}\right)9.48\,\text{mA} = \mathbf{5.69\,mA}$$

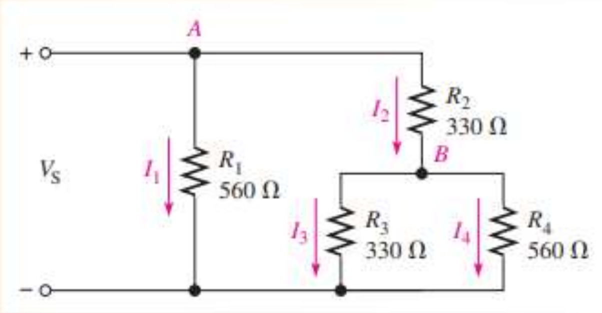
Now you can use Kirchhoff's current law to find the current through R_3.

$$I_T = I_2 + I_3$$
$$I_3 = I_T - I_2 = 9.48\,\text{mA} - 5.69\,\text{mA} = \mathbf{3.79\,mA}$$

Branch Current

EXAMPLE 9 Determine the current through R_4 in Figure 20 if $V_S = 5\text{ V}$.

▶ **FIGURE 20**



Solution First, find the current (I_2) into node B. Once you know this current, use the current-divider formula to find I_4, the current through R_4.

Notice that there are two main branches in the circuit. The left-most branch consists of only R_1. The right-most branch has R_2 in series with the parallel combination of R_3 and R_4. The voltage across both of these main branches is the same and equal to 5 V. Calculate the equivalent resistance ($R_{2+3\|4}$) of the right-most main branch and then apply Ohm's law; I_2 is the total current through this main branch. Thus,

$$R_{2+3\|4} = R_2 + \frac{R_3R_4}{R_3 + R_4} = 330\ \Omega + \frac{(330\ \Omega)(560\ \Omega)}{890\ \Omega} = 538\ \Omega$$

$$I_2 = \frac{V_S}{R_{2+3\|4}} = \frac{5\text{ V}}{538\ \Omega} = 9.3\text{ mA}$$

Use the two-resistor current-divider formula to calculate I_4.

$$I_4 = \left(\frac{R_3}{R_3 + R_4}\right)I_2 = \left(\frac{330\ \Omega}{890\ \Omega}\right)9.3\text{ mA} = \mathbf{3.45\text{ mA}}$$

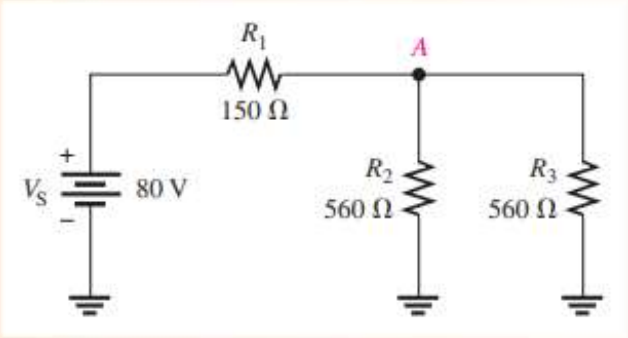
Voltage Drops

To find the voltages across certain parts of a series-parallel circuit, you can use the voltage divider formula, Kirchhoff's voltage law, Ohm's law, or combinations of each

EXAMPLE 10

Determine the voltage drop from node A to ground in Figure 21. Then find the voltage (V_1) across R_1.

▶ FIGURE 21



Voltage Drops

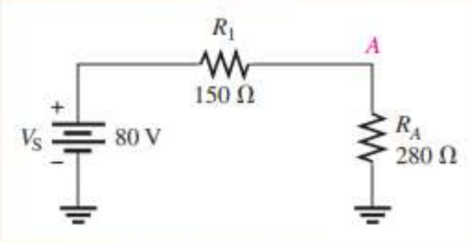
Solution Note that R_2 and R_3 are in parallel in this circuit. Since they are equal in value, their equivalent resistance from node A to ground is

$$R_A = \frac{560\ \Omega}{2} = 280\ \Omega$$

In the equivalent circuit shown in Figure 22, R_1 is in series with R_A. The total circuit resistance as seen from the source is

$$R_T = R_1 + R_A = 150\ \Omega + 280\ \Omega = 430\ \Omega$$

FIGURE 22



Use the voltage-divider formula to find the voltage across the parallel combination of Figure 21 (between node A and ground).

$$V_A = \left(\frac{R_A}{R_T}\right)V_S = \left(\frac{280\ \Omega}{430\ \Omega}\right)80\text{ V} = \mathbf{52.1\ V}$$

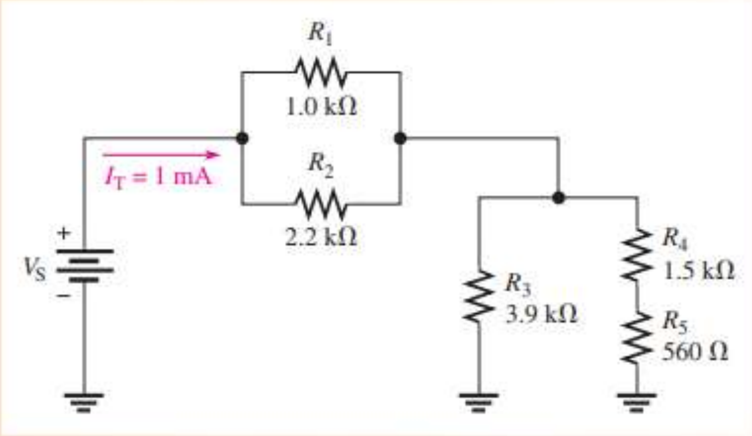
Now use Kirchhoff's voltage law to find V_1.

$$V_S = V_1 + V_A$$
$$V_1 = V_S - V_A = 80\text{ V} - 52.1\text{ V} = \mathbf{27.9\ V}$$

Voltage Drops

EXAMPLE 11

Determine the voltage drop across each resistor in the circuit of Figure 23.



▲ FIGURE 23

Voltage Drops

Solution The source voltage is not given, but you know the total current from the figure. Since R_1 and R_2 are in parallel, they each have the same voltage. The current through R_1 is

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right)I_T = \left(\frac{2.2\text{ k}\Omega}{3.2\text{ k}\Omega}\right)1\text{ mA} = 688\ \mu\text{A}$$

The voltages across R_1 and R_2 are

$$V_1 = I_1R_1 = (688\ \mu\text{A})(1.0\text{ k}\Omega) = \mathbf{688\ mV}$$
$$V_2 = V_1 = \mathbf{688\ mV}$$

The series combination of R_4 and R_5 form the branch resistance, R_{4+5}. Apply the current-divider formula to determine the current through R_3.

$$I_3 = \left(\frac{R_{4+5}}{R_3 + R_{4+5}}\right)I_T = \left(\frac{2.06\text{ k}\Omega}{5.96\text{ k}\Omega}\right)1\text{ mA} = 346\ \mu\text{A}$$

The voltage across R_3 is

$$V_3 = I_3R_3 = (346\ \mu\text{A})(3.9\text{ k}\Omega) = \mathbf{1.35\ V}$$

The currents through R_4 and R_5 are the same because these resistors are in series.

$$I_4 = I_5 = I_T - I_3 = 1\text{ mA} - 346\ \mu\text{A} = 654\ \mu\text{A}$$

Calculate the voltages across R_4 and R_5 as follows:

$$V_4 = I_4R_4 = (654\ \mu\text{A})(1.5\text{ k}\Omega) = \mathbf{981\ mV}$$
$$V_5 = I_5R_5 = (654\ \mu\text{A})(560\ \Omega) = \mathbf{366\ mV}$$

Voltage Drops

EXAMPLE 12 Determine the voltage drop across each resistor in Figure 24.

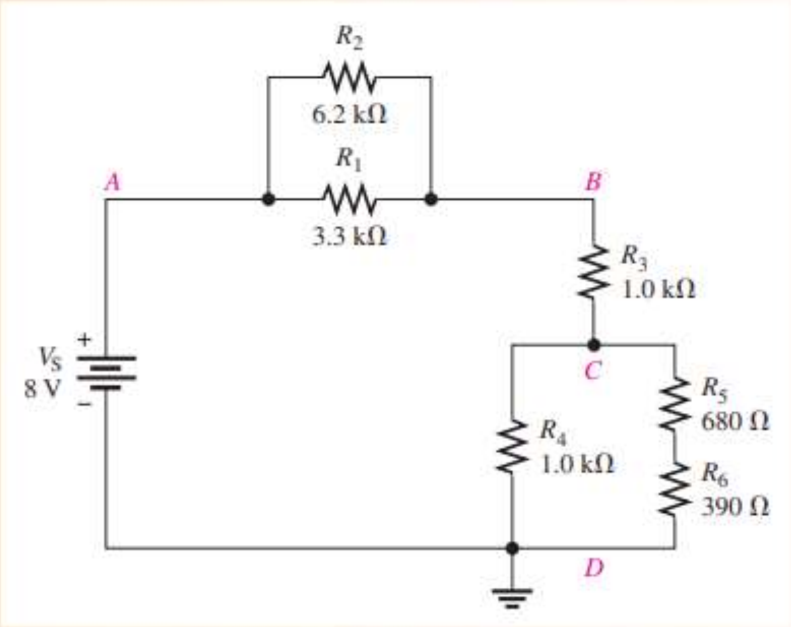


FIGURE 24