Contents

1 All Macros

```
//#pragma GCC optimize("Ofast")
//#pragma GCC optimization ("O3")
//#pragma comment(linker, "/stack
    :200000000")
//#pragma GCC optimize("unroll-loops")
//#pragma GCC target("sse,sse2,sse3,
    ssse3,sse4,popcnt,abm,mmx,avx,tune=
    native")

#include <ext/pb_ds/assoc_container.hpp
    >
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
   //find_by_order(k) --> returns
       iterator to the kth largest
       element counting from 0
   //order_of_key(val) --> returns the
       number of items in a set that
       are strictly smaller than our
       item
template <typename DT>
using ordered_set = tree <DT, null_type
    , less<DT>, rb_tree_tag,
    tree_order_statistics_node_update>;

/*--- DEBUG TEMPLATE STARTS HERE ---*/
#ifdef SFT
void show(int x) {cerr << x;}
void show(long long x) {cerr << x;}
void show(double x) {cerr << x;}
void show(char x) {cerr << '\'' << x <<
    '\'';}
void show(const string &x) {cerr << '\"
    ' << x << '\"';}
void show(bool x) {cerr << (x ? "true"
    : "false");}

template<typename T, typename V>
void show(pair<T, V> x) { cerr << '{';
    show(x.first); cerr << ", "; show(x
    .second); cerr << '}'; }
template<typename T>
void show(T x) {int f = 0; cerr << "{";
     for (auto &i: x) cerr << (f++ ? ",
     " : ""), show(i); cerr << "}";}

void debug_out(string s) {
   cerr << '\n';
}
template <typename T, typename... V>
void debug_out(string s, T t, V... v) {
   s.erase(remove(s.begin(), s.end(), '
       '), s.end());
   cerr << "       "; // 8 spaces
   cerr << s.substr(0, s.find(','));
   s = s.substr(s.find(',') + 1);
   cerr << " = ";
   show(t);
   cerr << endl;
   if(sizeof...(v)) debug_out(s, v...);
}

#define debug(x...) cerr << "LINE: " <<
    __LINE__ << endl; debug_out(#x, x)
    ; cerr << endl;
#else
#define debug(x...)
#endif
```

```
const int RANDOM = chrono::
    high_resolution_clock::now().
    time_since_epoch().count();
unsigned hash_f(unsigned x) {
  x = ((x >> 16) ^ x) * 0x45d9f3b;
  x = ((x >> 16) ^ x) * 0x45d9f3b;
  return x = (x >> 16) ^ x;
}

unsigned hash_combine(unsigned a,
    unsigned b) { return a * 31 + b; }
struct chash {
  int operator()(int x) const { return
      hash_f(x); }
};
typedef gp_hash_table<int, int, chash>
    gp;
gp table;
```

2 Data Structure

2.1 Segment Tree

```
const int N = 1000006;

using DT = LL;
using LT = LL;
constexpr DT I = 0;
constexpr LT None = 0;
DT val[4 * N];
LT lazy[4 * N];
int L, R;

void pull(int s, int e, int node) {
  val[node] = val[node << 1] + val[node
      << 1 | 1];
}
void apply(const LT &U, int s, int e,
    int node) {
  val[node] += (e - s + 1) * U;
  lazy[node] += U;
}
void reset(int node) { lazy[node] =
    None; }
DT merge(const DT &a, const DT &b) {
    return a + b; }
DT get(int s, int e, int node) { return
     val[node]; }
void push(int s, int e, int node) {
  if (s == e) return;
  apply(lazy[node], s, s + e >> 1, node
      << 1);
  apply(lazy[node], s + e + 2 >> 1, e,
      node << 1 | 1);
  reset(node);
}
void build(int s, int e, vector<DT> &v,
     int node = 1) {
  int m = s + e >> 1;
  if (s == e) {
    val[node] = v[s];
    return;
  }
  build(s, m, v, node * 2);
  build(m + 1, e, v, node * 2 + 1);
  pull(s, e, node);
}
void update(int S, int E, LT uval, int
    s = L, int e = R, int node = 1) {
  if (S > E) return;
  if (S == s and E == e) {
    apply(uval, s, e, node);
    return;
  }
  push(s, e, node);
  int m = s + e >> 1;
  update(S, min(m, E), uval, s, m, node
      * 2);
  update(max(S, m + 1), E, uval, m + 1,
      e, node * 2 + 1);
  pull(s, e, node);
}
DT query(int S, int E, int s = L, int e
    = R, int node = 1) {
  if (S > E) return I;
  if (s == S and e == E) return get(s,
      e, node);
  push(s, e, node);
  int m = s + e >> 1;
  DT L = query(S, min(m, E), s, m, node
      * 2);
  DT R = query(max(S, m + 1), E, m + 1,
      e, node * 2 + 1);
  return merge(L, R);
}
void init(int _L, int _R, vector<DT> &v
    ) {
  L = _L, R = _R;
  build(L, R, v);
}
```

2.2 Persistent Segment Tree

```
struct Node {
  Node *l, *r;
  int sum;

  Node(int val) : l(nullptr), r(nullptr
      ), sum(val) {}
  Node(Node* l, Node* r) : l(l), r(r),
      sum(0) {
    if (l) sum += l->sum;
    if (r) sum += r->sum;
  }
};

int a[MAXN];
Node* root[MAXN];

Node* Build(int bg, int ed) {
  if (bg == ed) return new Node(a[bg]);
  int mid = (bg + ed) / 2;
  return new Node(Build(bg, mid), Build
      (mid + 1, ed));
}

int Query(Node* v, int bg, int ed, int
    l, int r) {
  if (l > ed || r < bg) return 0;
  if (l <= bg && ed <= r) return v->sum
      ;
  int mid = (bg + ed) / 2;
  return Query(v->l, bg, mid, l, r) +
      Query(v->r, mid + 1, ed, l, r);
}

Node* Update(Node* v, int bg, int ed,
    int pos, int new_val) {
  if (bg == ed) return new Node(v->sum
      + new_val);
  int mid = (bg + ed) / 2;
  if (pos <= mid)
```

```
    return new Node(Update(v->l, bg, mid
        , pos, new_val), v->r);
  else
    return new Node(v->l, Update(v->r,
        mid + 1, ed, pos, new_val));
}
```

2.3 Implicit Segment Tree

```
struct node {
  int val;
  node *lft, *rt;
  node() {}
  node(int val = 0) : val(val), lft(
      NULL), rt(NULL) {}
};

struct implicit_segtree {
  node *root;
  implicit_segtree() {}
  implicit_segtree(int n) {
    root = new node(n);
  }
  void update(node *now, int L, int R,
      int idx, int val) {
    if (L == R) {
      now -> val += val;
      return;
    }
    int mid = L + (R - L) / 2;
    if (now->lft == NULL) now->lft = new
        node(mid - L + 1);
    if (now->rt == NULL) now->rt = new
        node(R - mid);
    if (idx <= mid) update(now->lft, L,
        mid, idx, val);
    else update(now->rt, mid + 1, R, idx
        , val);
    now->val = (now->lft)->val + (now->
        rt)->val;
  }

  int query(node *now, int L, int R,
      int k) {
    if (L == R) return L;
    int mid = L + (R - L) / 2;
    if (now->lft == NULL) now->lft = new
        node(mid - L + 1);
    if (now->rt == NULL) now->rt = new
        node(R - mid);
    if (k <= (now->lft)->val) return
        query(now->lft, L, mid, k);
    else return query(now->rt, mid + 1,
        R, k - (now->lft)->val);
  }
};
```

2.4 Centroid Decomposition

```
// problem: Xenia and Tree
// Centroid Tree Implementation
#define lg(n) (31 - __builtin_clz(n))

const int N = 1e5 + 1;
const int K = lg(N) + 1;

int n;
vector<int> adj[N];
int anc[N][K], lvl[N], par[N], sz[N],
    vis[N];
int minD[N];
namespace CD {
  // Call dfs at the very outset
  void dfs(int u = 1, int p = 0) {
    sz[u] = 1;
    for (auto v : adj[u])
      if (v != p) {
        dfs_size(v, u);
        sz[u] += sz[v];
      }
  }

  int findCentroid(int u, int p) {
    int total = sz[u];
    for (auto v : adj[u])
      if (v != p and not vis[v] and 2 *
          sz[v] > total) {
        sz[u] = total - sz[v];
        sz[v] = total;
        return findCentroid(v, u);
      }
    return u;
  }

  int query(int u) {
    int ans = 1e6;
    for (int i = u; i; i = par[i])
      ans = min(ans, minD[i] + lca::dist
          (i, u));
    return ans;
  }

  void update(int u) {
    for (int i = u; i; i = par[i])
      minD[i] = min(minD[i], lca::dist(i
          , u));
  }

  int decompose(int u, int p) {
    u = findCentroid(u, p);
    vis[u] = 1;
    for (auto v : adj[u])
      if (not vis[v])
        par[decompose(v, u)] = u;
    return u;
  }
}

int main() {
  cin.tie(NULL)->sync_with_stdio(false)
      ;
  fill(all(minD), 1e9);

  int m;
  cin >> n >> m;
  for (int u, v, i = 1; i < n; i++) {
    cin >> u >> v;
    adj[u].push_back(v);
    adj[v].push_back(u);
  }

  lca::init();
  CD::dfs();
  CD::decompose(1, 0);

  CD::update(1);
  int t, v;
  while (m--) {
    cin >> t >> v;
    if (t == 1) CD::update(v);
    else cout << CD::query(v) << '\n';
  }

  return 0;
}
```

2.5 DSU With Rollbacks

```
struct Rollback_DSU {
  int n;
  vector<int> par, sz;
  vector<pair<int, int>> op;
  Rollback_DSU(int n) : par(n), sz(n,
      1) {
    iota(par.begin(), par.end(), 0);
    op.reserve(n);
  }
  int Anc(int node) {
    for (; node != par[node]; node = par
        [node])
      ; // no path compression
    return node;
  }
  void Unite(int x, int y) {
    if (sz[x = Anc(x)] < sz[y = Anc(y)])
        swap(x, y);
    op.emplace_back(x, y);
    par[y] = x;
    sz[x] += sz[y];
  }
  void Undo(int t) {
    for (; op.size() > t; op.pop_back())
        {
      par[op.back().second] = op.back().
          second;
      sz[op.back().first] -= sz[op.back
          ().second];
    }
  }
};
```

2.6 BIT-2D

```
const int N = 1008;
int bit[N][N], n, m;
int a[N][N], q;
void update(int x, int y, int val) {
  for (; x < N; x += -x & x)
    for (int j = y; j < N; j += -j & j)
        bit[x][j] += val;
}
int get(int x, int y) {
  int ans = 0;
  for (; x; x -= x & -x)
    for (int j = y; j; j -= j & -j) ans
        += bit[x][j];
  return ans;
}
int get(int x1, int y1, int x2, int y2)
    {
  return get(x2, y2) - get(x1 - 1, y2)
      - get(x2, y1 - 1) + get(x1 - 1,
      y1 - 1);
}
```

2.7 Merge Sort Tree

```
vector<LL> Tree[4 * MAXN];
LL arr[MAXN];

vector<LL> merge(vector<LL> v1, vector<
    LL> v2) {
  LL i = 0, j = 0;
  vector<LL> ret;
  while (i < v1.size() || j < v2.size()
      ) {
    if (i == v1.size()) {
      ret.push_back(v2[j]);
      j++;
    } else if (j == v2.size()) {
      ret.push_back(v1[i]);
      i++;
    } else {
      if (v1[i] < v2[j]) {
        ret.push_back(v1[i]);
        i++;
      } else {
        ret.push_back(v2[j]);
        j++;
      }
    }
  }

  return ret;
}

void Build(LL node, LL bg, LL ed) {
  if (bg == ed) {
    Tree[node].push_back(arr[bg]);
    return;
  }

  LL leftNode = 2 * node, rightNode = 2
      * node + 1;
  LL mid = (bg + ed) / 2;

  Build(leftNode, bg, mid);
  Build(rightNode, mid + 1, ed);

  Tree[node] = merge(Tree[leftNode],
       Tree[rightNode]);
}

LL query(LL node, LL bg, LL ed, LL l,
     LL r, LL k) {
  if (ed < l || bg > r) return 0;

  if (l <= bg && ed <= r)
    return upper_bound(Tree[node].begin
         (), Tree[node].end(), k) -
           Tree[node].begin();

  LL leftNode = 2 * node, rightNode = 2
      * node + 1;
  LL mid = (bg + ed) / 2;

  return query(leftNode, bg, mid, l, r,
      k) +
       query(rightNode, mid + 1, ed, l
           , r, k);
}
```

2.8 MO with Update

```
const int N = 1e5 + 5, sz = 2700, bs =
    25;
int arr[N], freq[2 * N], cnt[2 * N], id
    [N], ans[N];
struct query {
  int l, r, t, L, R;
  query(int l = 1, int r = 0, int t =
      1, int id = -1)
      : l(l), r(r), t(t), L(l / sz), R(r
          / sz) {}
  bool operator<(const query &rhs)
      const {
    return (L < rhs.L) or (L == rhs.L
        and R < rhs.R) or
          (L == rhs.L and R == rhs.R
              and t < rhs.t);
  }
} Q[N];
struct update {
  int idx, val, last;
} Up[N];
int qi = 0, ui = 0;
int l = 1, r = 0, t = 0;

void add(int idx) {
  --cnt[freq[arr[idx]]];
  freq[arr[idx]]++;
  cnt[freq[arr[idx]]]++;
}
void remove(int idx) {
  --cnt[freq[arr[idx]]];
  freq[arr[idx]]--;
  cnt[freq[arr[idx]]]++;
}
void apply(int t) {
  const bool f = l <= Up[t].idx and Up[
      t].idx <= r;
  if (f) remove(Up[t].idx);
  arr[Up[t].idx] = Up[t].val;
  if (f) add(Up[t].idx);
}
void undo(int t) {
  const bool f = l <= Up[t].idx and Up[
      t].idx <= r;
  if (f) remove(Up[t].idx);
  arr[Up[t].idx] = Up[t].last;
  if (f) add(Up[t].idx);
}
int mex() {
  for (int i = 1; i <= N; i++)
    if (!cnt[i]) return i;
  assert(0);
}
int main() {
  int n, q;
  cin >> n >> q;
  int counter = 0;
  map<int, int> M;
  for (int i = 1; i <= n; i++) {
    cin >> arr[i];
    if (!M[arr[i]]) M[arr[i]] = ++
        counter;
    arr[i] = M[arr[i]];
  }
  iota(id, id + N, 0);
  while (q--) {
    int tp, x, y;
    cin >> tp >> x >> y;
    if (tp == 1)
      Q[++qi] = query(x, y, ui);
    else {
      if (!M[y]) M[y] = ++counter;
      y = M[y];
      Up[++ui] = {x, y, arr[x]};
      arr[x] = y;
    }
  }
  t = ui;
  cnt[0] = 3 * n;
  sort(id + 1, id + qi + 1, [&](int x,
      int y) { return Q[x] < Q[y]; });
  for (int i = 1; i <= qi; i++) {
    int x = id[i];
    while (Q[x].t > t) apply(++t);
    while (Q[x].t < t) undo(t--);
    while (Q[x].l < l) add(--l);
    while (Q[x].r > r) add(++r);
    while (Q[x].l > l) remove(l++);
    while (Q[x].r < r) remove(r--);
    ans[x] = mex();
  }
  for (int i = 1; i <= qi; i++) cout <<
       ans[i] << '\n';
}
```

2.9 SparseTable (Rectangle Query)

```
#include <bits/stdc++.h>
using namespace std;

const int MAXN = 505;
const int LOGN = 9;

// O(n^2 (logn)^2
// Supports Rectangular Query
int A[MAXN][MAXN];
int M[MAXN][MAXN][LOGN][LOGN];

void Build2DSparse(int N) {
  for (int i = 1; i <= N; i++) {
    for (int j = 1; j <= N; j++) {
      M[i][j][0][0] = A[i][j];
    }
    for (int q = 1; (1 << q) <= N; q++)
        {
      int add = 1 << (q - 1);
      for (int j = 1; j + add <= N; j++)
          {
        M[i][j][0][q] = max(M[i][j][0][q
             - 1], M[i][j + add][0][q -
            1]);
      }
    }
  }

  for (int p = 1; (1 << p) <= N; p++) {
    int add = 1 << (p - 1);
    for (int i = 1; i + add <= N; i++) {
      for (int q = 0; (1 << q) <= N; q
          ++) {
        for (int j = 1; j <= N; j++) {
          M[i][j][p][q] = max(M[i][j][p
               - 1][q], M[i + add][j][p -
              1][q]);
        }
      }
    }
  }
}

// returns max of all A[i][j], where x1
    <=i<=x2 and y1<=j<=y2
int Query(int x1, int y1, int x2, int
    y2) {
  int kX = log2(x2 - x1 + 1);
  int kY = log2(y2 - y1 + 1);
  int addX = 1 << kX;
  int addY = 1 << kY;
  int ret1 = max(M[x1][y1][kX][kY], M[x1][y2 - addY + 1][kX][kY]);
  int ret2 = max(M[x2 - addX + 1][y1][kX][kY],
                 M[x2 - addX + 1][y2 - addY + 1][kX][kY]);
  return max(ret1, ret2);
}
```

2.10 Sparse Table

```
// tested by: https://judge.yosupo.jp/problem/staticrmq

// 0-based indexing, query finds in range [first, last]
#define lg(x) (31 - __builtin_clz(x))

const int N = 1e5 + 1;
const int K = lg(N);

int a[N], tr[N][K + 1];

namespace sparse_table {
  int f(int p1, int p2) { return min(p1, p2); }

  void build() {
    for(int i = 0; i < n; i++)
      tr[i][0] = a[i];
    for(int j = 1; j <= K; j++) {
      for(int i = 0; i + (1<<j) <= n; i++)
        tr[i][j] = f(tr[i][j - 1], tr[i + (1<<(j - 1))][j - 1]);
    }
  }

  int query(int l, int r) {
    int d = lg(r - l + 1);
    return f(table[l][d], table[r - (1<<d) + 1][d]);
  }
}
```

3 DP

3.1 Convex Hull Trick

```
struct line {
  ll m, c;
  line() {}
  line(ll m, ll c) : m(m), c(c) {}
};
struct convex_hull_trick {
  vector<line> lines;
  int ptr = 0;
  convex_hull_trick() {}
  bool bad(line a, line b, line c) {
    return 1.0 * (c.c - a.c) * (a.m - b.m) < 1.0 * (b.c - a.c) * (a.m - c.m);
  }
  void add(line L) {
    int sz = lines.size();
    while (sz >= 2 && bad(lines[sz - 2], lines[sz - 1], L)) {
      lines.pop_back();
      sz--;
    }
    lines.pb(L);
  }
  ll get(int idx, int x) { return (1ll * lines[idx].m * x + lines[idx].c); }
  ll query(int x) {
    if (lines.empty()) return 0;
    if (ptr >= lines.size()) ptr = lines.size() - 1;
    while (ptr < lines.size() - 1 && get(ptr, x) > get(ptr + 1, x)) ptr++;
    return get(ptr, x);
  }
};
ll sum[MAX];
ll dp[MAX];
int arr[MAX];
int main() {
  fastio;
  int t;
  cin >> t;
  while (t--) {
    int n, a, b, c;
    cin >> n >> a >> b >> c;
    for (int i = 1; i <= n; i++) cin >> sum[i];
    for (int i = 1; i <= n; i++) dp[i] = 0, sum[i] += sum[i - 1];
    convex_hull_trick cht;
    cht.add(line(0, 0));
    for (int pos = 1; pos <= n; pos++) {
      dp[pos] = cht.query(sum[pos]) - 1ll * a * sqr(sum[pos]) - c;
      cht.add(line(2ll * a * sum[pos], dp[pos] - a * sqr(sum[pos])));
    }
    ll ans = (-1ll * dp[n]);
    ans += (1ll * sum[n] * b);
    cout << ans << "\n";
  }
}
```

3.2 Dynamic CHT

```
#include <bits/stdc++.h>

using namespace std;

typedef long long ll;

const ll IS_QUERY = -(1LL << 62);

struct line {
  ll m, b;
  mutable function <const line*()> succ;

  bool operator < (const line &rhs) const {
    if (rhs.b != IS_QUERY) return m < rhs.m;
    const line *s = succ();
    if (!s) return 0;
    ll x = rhs.m;
    return b - s -> b < (s -> m - m) * x;
  }
};

struct HullDynamic : public multiset <line> {
  bool bad (iterator y) {
    auto z = next(y);
    if (y == begin()) {
      if (z == end()) return 0;
      return y -> m == z -> m && y -> b <= z -> b;
    }
    auto x = prev(y);
    if (z == end()) return y -> m == x -> m && y -> b <= x -> b;
    return 1.0 * (x -> b - y -> b) * (z -> m - y -> m) >= 1.0 * (y -> b - z -> b) * (y -> m - x -> m);
  }

  void insert_line (ll m, ll b) {
    auto y = insert({m, b});
    y -> succ = [=] {return next(y) == end() ? 0 : &*next(y);};
    if (bad(y)) {erase(y); return;}
    while (next(y) != end() && bad(next(y))) erase(next(y));
    while (y != begin() && bad(prev(y))) erase(prev(y));
  }

  ll eval (ll x) {
    auto l = *lower_bound((line) {x, IS_QUERY});
    return l.m * x + l.b;
  }
};

int main() {
  HullDynamic hull;
  hull.insert_line(1, 1);
  hull.insert_line(-1, 1);
  cout << hull.eval(69) << endl;
  cout << hull.eval(420) << endl;
  return 0;
}
```

3.3 Li Chao Tree

```
struct line {
  LL m, c;
  line(LL m = 0, LL c = 0) : m(m), c(c) {}
};
LL calc(line L, LL x) { return 1LL * L.m * x + L.c; }
struct node {
  LL m, c;
  line L;
  node *lft, *rt;
  node(LL m = 0, LL c = 0, node *lft = NULL, node *rt = NULL)
    : L(line(m, c)), lft(lft), rt(rt) {}
};
struct LiChao {
  node *root;
  LiChao() { root = new node(); }
  void update(node *now, int L, int R, line newline) {
    int mid = L + (R - L) / 2;
    line lo = now->L, hi = newline;
    if (calc(lo, L) > calc(hi, L)) swap(lo, hi);
    if (calc(lo, R) <= calc(hi, R)) {
      now->L = hi;
      return;
    }
    if (calc(lo, mid) < calc(hi, mid)) {
      now->L = hi;
      if (now->rt == NULL) now->rt = new node();
      update(now->rt, mid + 1, R, lo);
    } else {
      now->L = lo;
      if (now->lft == NULL) now->lft = new node();
      update(now->lft, L, mid, hi);
    }
  }
  LL query(node *now, int L, int R, LL x) {
    if (now == NULL) return -inf;
    int mid = L + (R - L) / 2;
    if (x <= mid)
      return max(calc(now->L, x), query(now->lft, L, mid, x));
    else
      return max(calc(now->L, x), query(now->rt, mid + 1, R, x));
  }
};
```

3.4 SOS DP

```
for(int i = 0; i<(1<<N); ++i)
 F[i] = A[i];
for(int i = 0;i < N; ++i) for(int mask = 0; mask < (1<<N); ++mask){
 if(mask & (1<<i))
  F[mask] += F[mask^(1<<i)];
}
```

4 Geometry

4.1 Point

```
typedef double Tf;
typedef double Ti; /// use long long for exactness
const Tf PI = acos(-1), EPS = 1e-9;
int dcmp(Tf x) { return abs(x) < EPS ? 0 : (x < 0 ? -1 : 1); }

struct Point {
  Ti x, y;
  Point(Ti x = 0, Ti y = 0) : x(x), y(y) {}

  Point operator+(const Point& u) const { return Point(x + u.x, y + u.y); }
  Point operator-(const Point& u) const { return Point(x - u.x, y - u.y); }
  Point operator*(const LL u) const { return Point(x * u, y * u); }
  Point operator*(const Tf u) const { return Point(x * u, y * u); }
  Point operator/(const Tf u) const { return Point(x / u, y / u); }

  bool operator==(const Point& u) const {
    return dcmp(x - u.x) == 0 && dcmp(y - u.y) == 0;
  }
  bool operator!=(const Point& u) const { return !(*this == u); }
  bool operator<(const Point& u) const {
    return dcmp(x - u.x) < 0 || (dcmp(x - u.x) == 0 && dcmp(y - u.y) < 0);
  }
};
Ti dot(Point a, Point b) { return a.x * b.x + a.y * b.y; }
Ti cross(Point a, Point b) { return a.x * b.y - a.y * b.x; }
Tf length(Point a) { return sqrt(dot(a, a)); }
Ti sqLength(Point a) { return dot(a, a); }
Tf distance(Point a, Point b) { return length(a - b); }
Tf angle(Point u) { return atan2(u.y, u.x); }

// returns angle between oa, ob in (-PI, PI]
Tf angleBetween(Point a, Point b) {
  Tf ans = angle(b) - angle(a);
  return ans <= -PI ? ans + 2 * PI : (ans > PI ? ans - 2 * PI : ans);
}
// Rotate a ccw by rad radians, Tf Ti same
Point rotate(Point a, Tf rad) {
  return Point(a.x * cos(rad) - a.y * sin(rad),
               a.x * sin(rad) + a.y * cos(rad));
}
// rotate a ccw by angle th with cos(th) = co && sin(th) = si, tf ti same
Point rotatePrecise(Point a, Tf co, Tf si) {
  return Point(a.x * co - a.y * si, a.y * co + a.x * si);
}
Point rotate90(Point a) { return Point(-a.y, a.x); }
// scales vector a by s such that length of a becomes s, Tf Ti same
Point scale(Point a, Tf s) { return a / length(a) * s; }
// returns an unit vector perpendicular to vector a, Tf Ti same
Point normal(Point a) {
  Tf l = length(a);
  return Point(-a.y / l, a.x / l);
}
// returns 1 if c is left of ab, 0 if on ab && -1 if right of ab
int orient(Point a, Point b, Point c) {
    return dcmp(cross(b - a, c - a));
  }
/// Use as sort(v.begin(), v.end(), polarComp(O, dir))
/// Polar comparator around O starting at direction dir
struct polarComp {
  Point O, dir;
  polarComp(Point O = Point(0, 0), Point dir = Point(1, 0)) : O(O), dir(dir) {}
  bool half(Point p) {
    return dcmp(cross(dir, p)) < 0 || (dcmp(cross(dir, p)) == 0 && dcmp(dot(dir, p)) > 0);
  }
  bool operator()(Point p, Point q) {
    return make_tuple(half(p), 0) < make_tuple(half(q), cross(p, q));
  }
};
struct Segment {
  Point a, b;
  Segment(Point aa, Point bb) : a(aa), b(bb) {}
};
typedef Segment Line;
struct Circle {
  Point o;
  Tf r;
  Circle(Point o = Point(0, 0), Tf r = 0) : o(o), r(r) {}
  // returns true if point p is in || on the circle
  bool contains(Point p) { return dcmp(sqLength(p - o) - r * r) <= 0; }
  // returns a point on the circle rad radians away from +X CCW
  Point point(Tf rad) {
    static_assert(is_same<Tf, Ti>::value);
    return Point(o.x + cos(rad) * r, o.y + sin(rad) * r);
  }
  // area of a circular sector with central angle rad
  Tf area(Tf rad = PI + PI) { return rad * r * r / 2; }
  // area of the circular sector cut by a chord with central angle alpha
  Tf sector(Tf alpha) { return r * r * 0.5 * (alpha - sin(alpha)); }
};
```

4.2 Linear

```
// **** LINE LINE INTERSECTION START ****
// returns true if point p is on segment s
bool onSegment(Point p, Segment s) {
  return dcmp(cross(s.a - p, s.b - p)) == 0 && dcmp(dot(s.a - p, s.b - p)) <= 0;
}
// returns true if segment p && q touch or intersect
bool segmentsIntersect(Segment p, Segment q) {
  if (onSegment(p.a, q) || onSegment(p.b, q)) return true;
  if (onSegment(q.a, p) || onSegment(q.b, p)) return true;

  Ti c1 = cross(p.b - p.a, q.a - p.a);
  Ti c2 = cross(p.b - p.a, q.b - p.a);
  Ti c3 = cross(q.b - q.a, p.a - q.a);
  Ti c4 = cross(q.b - q.a, p.b - q.a);
  return dcmp(c1) * dcmp(c2) < 0 && dcmp(c3) * dcmp(c4) < 0;
}
bool linesParallel(Line p, Line q) {
  return dcmp(cross(p.b - p.a, q.b - q.a)) == 0;
}
// lines are represented as a ray from a point: (point, vector)
// returns false if two lines (p, v) && (q, w) are parallel or collinear
// true otherwise, intersection point is stored at o via reference, Tf Ti Same
bool lineLineIntersection(Point p, Point v, Point q, Point w, Point& o) {
  if (dcmp(cross(v, w)) == 0) return false;
  Point u = p - q;
  o = p + v * (cross(w, u) / cross(v, w));
  return true;
}
// returns false if two lines p && q are parallel or collinear
// true otherwise, intersection point is stored at o via reference
bool lineLineIntersection(Line p, Line q, Point& o) {
  return lineLineIntersection(p.a, p.b - p.a, q.a, q.b - q.a, o);
}
// returns the distance from point a to line l
// **** LINE LINE INTERSECTION FINISH ****
Tf distancePointLine(Point p, Line l) {
  return abs(cross(l.b - l.a, p - l.a) / length(l.b - l.a));
}
// returns the shortest distance from point a to segment s
Tf distancePointSegment(Point p, Segment s) {
  if (s.a == s.b) return length(p - s.a);
  Point v1 = s.b - s.a, v2 = p - s.a, v3 = p - s.b;
  if (dcmp(dot(v1, v2)) < 0)
    return length(v2);
  else if (dcmp(dot(v1, v3)) > 0)
    return length(v3);
  else
    return abs(cross(v1, v2) / length(v1));
}
// returns the shortest distance from segment p to segment q
Tf distanceSegmentSegment(Segment p, Segment q) {
  if (segmentsIntersect(p, q)) return 0;
  Tf ans = distancePointSegment(p.a, q);
  ans = min(ans, distancePointSegment(p.b, q));
  ans = min(ans, distancePointSegment(q.a, p));
  ans = min(ans, distancePointSegment(q.b, p));
  return ans;
}
// returns the projection of point p on line l, Tf Ti Same
Point projectPointLine(Point p, Line l) {
  Point v = l.b - l.a;
  return l.a + v * ((Tf)dot(v, p - l.a) / dot(v, v));
}
```

4.3 Circular

```
// Extremely inaccurate for finding near touches
// compute intersection of line l with circle c
// The intersections are given in order of the ray (l.a, l.b), Tf Ti same
vector<Point> circleLineIntersection(Circle c, Line l) {
  vector<Point> ret;
  Point b = l.b - l.a, a = l.a - c.o;
  Tf A = dot(b, b), B = dot(a, b);
  Tf C = dot(a, a) - c.r * c.r, D = B * B - A * C;
  if (D < -EPS) return ret;
  ret.push_back(l.a + b * (-B - sqrt(D + EPS)) / A);
  if (D > EPS) ret.push_back(l.a + b * (-B + sqrt(D)) / A);
  return ret;
}
// signed area of intersection of circle(c.o, c.r) &&
// triangle(c.o, s.a, s.b) [cross(a-o, b-o)/2]
Tf circleTriangleIntersectionArea(Circle c, Segment s) {
  using Linear::distancePointSegment;
  Tf OA = length(c.o - s.a);
  Tf OB = length(c.o - s.b);
  // sector
  if (dcmp(distancePointSegment(c.o, s) - c.r) >= 0)
    return angleBetween(s.a - c.o, s.b - c.o) * (c.r * c.r) / 2.0;
  // triangle
  if (dcmp(OA - c.r) <= 0 && dcmp(OB - c.r) <= 0)
    return cross(c.o - s.b, s.a - s.b) / 2.0;
  // three part: (A, a) (a, b) (b, B)
  vector<Point> Sect = circleLineIntersection(c, s);
  return circleTriangleIntersectionArea(c, Segment(s.a, Sect[0])) +
    circleTriangleIntersectionArea(c, Segment(Sect[0], Sect[1])) +
    circleTriangleIntersectionArea(c, Segment(Sect[1], s.b));
}
// area of intersecion of circle(c.o, c.r) && simple polyson(p[])
Tf circlePolyIntersectionArea(Circle c, Polygon p) {
  Tf res = 0;
  int n = p.size();
  for (int i = 0; i < n; ++i)
    res += circleTriangleIntersectionArea(c, Segment(p[i], p[(i + 1) % n]));
  return abs(res);
}
// locates circle c2 relative to c1
// interior           (d < R - r)         ----> -2
// interior tangents (d = R - r)         ----> -1
// concentric       (d = 0)
// secants           (R - r < d < R + r) ----> 0
// exterior tangents (d = R + r)         ----> 1
// exterior          (d > R + r)         ----> 2
int circleCirclePosition(Circle c1, Circle c2) {
  Tf d = length(c1.o - c2.o);
  int in = dcmp(d - abs(c1.r - c2.r)), ex = dcmp(d - (c1.r + c2.r));
  return in < 0 ? -2 : in == 0 ? -1 : ex == 0 ? 1 : ex > 0 ? 2 : 0;
}
// compute the intersection points between two circles c1 && c2, Tf Ti same
vector<Point> circleCircleIntersection(Circle c1, Circle c2) {
  vector<Point> ret;
  Tf d = length(c1.o - c2.o);
  if (dcmp(d) == 0) return ret;
  if (dcmp(c1.r + c2.r - d) < 0) return ret;
  if (dcmp(abs(c1.r - c2.r) - d) > 0) return ret;

  Point v = c2.o - c1.o;
  Tf co = (c1.r * c1.r + sqLength(v) - c2.r * c2.r) / (2 * c1.r * length(v));
  Tf si = sqrt(abs(1.0 - co * co));
  Point p1 = scale(rotatePrecise(v, co, -si), c1.r) + c1.o;
  Point p2 = scale(rotatePrecise(v, co, si), c1.r) + c1.o;

  ret.push_back(p1);
  if (p1 != p2) ret.push_back(p2);
  return ret;
}
// intersection area between two circles c1, c2
Tf circleCircleIntersectionArea(Circle c1, Circle c2) {
  Point AB = c2.o - c1.o;
  Tf d = length(AB);
  if (d >= c1.r + c2.r) return 0;
  if (d + c1.r <= c2.r) return PI * c1.r * c1.r;
  if (d + c2.r <= c1.r) return PI * c2.r * c2.r;

  Tf alpha1 = acos((c1.r * c1.r + d * d - c2.r * c2.r) / (2.0 * c1.r * d));
  Tf alpha2 = acos((c2.r * c2.r + d * d - c1.r * c1.r) / (2.0 * c2.r * d));
  return c1.sector(2 * alpha1) + c2.sector(2 * alpha2);
}
// returns tangents from a point p to circle c, Tf Ti same
vector<Point> pointCircleTangents(Point p, Circle c) {
  vector<Point> ret;
  Point u = c.o - p;
  Tf d = length(u);
  if (d < c.r)
    ;
  else if (dcmp(d - c.r) == 0) {
    ret = {rotate(u, PI / 2)};
  } else {
    Tf ang = asin(c.r / d);
    ret = {rotate(u, -ang), rotate(u, ang)};
  }
  return ret;
}
// returns the points on tangents that touches the circle, Tf Ti Same
vector<Point> pointCircleTangencyPoints(Point p, Circle c) {
  Point u = p - c.o;
  Tf d = length(u);
  if (d < c.r)
    return {};
  else if (dcmp(d - c.r) == 0)
    return {c.o + u};
  else {
    Tf ang = acos(c.r / d);
    u = u / length(u) * c.r;
    return {c.o + rotate(u, -ang), c.o + rotate(u, ang)};
  }
}
// for two circles c1 && c2, returns two list of points a && b
// such that a[i] is on c1 && b[i] is c2 && for every i
// Line(a[i], b[i]) is a tangent to both circles
// CAUTION: a[i] = b[i] in case they touch | -1 for c1 = c2
int circleCircleTangencyPoints(Circle c1, Circle c2, vector<Point> &a,
                               vector<Point> &b) {
  a.clear(), b.clear();
  int cnt = 0;
  if (dcmp(c1.r - c2.r) < 0) {
    swap(c1, c2);
    swap(a, b);
  }
  Tf d2 = sqLength(c1.o - c2.o);
  Tf rdif = c1.r - c2.r, rsum = c1.r + c2.r;
  if (dcmp(d2 - rdif * rdif) < 0)
    return 0;
  if (dcmp(d2) == 0 && dcmp(c1.r - c2.r) == 0) return -1;

  Tf base = angle(c2.o - c1.o);
  if (dcmp(d2 - rdif * rdif) == 0) {
    a.push_back(c1.point(base));
    b.push_back(c2.point(base));
    cnt++;
    return cnt;
  }
  Tf ang = acos((c1.r - c2.r) / sqrt(d2));
  a.push_back(c1.point(base + ang));
  b.push_back(c2.point(base + ang));
  cnt++;
  a.push_back(c1.point(base - ang));
  b.push_back(c2.point(base - ang));
  cnt++;

  if (dcmp(d2 - rsum * rsum) == 0) {
    a.push_back(c1.point(base));
    b.push_back(c2.point(PI + base));
    cnt++;
  } else if (dcmp(d2 - rsum * rsum) > 0) {
    Tf ang = acos((c1.r + c2.r) / sqrt(d2));
    a.push_back(c1.point(base + ang));
    b.push_back(c2.point(PI + base + ang));
    cnt++;
    a.push_back(c1.point(base - ang));
    b.push_back(c2.point(PI + base - ang));
    cnt++;
  }
  return cnt;
}
```

4.4 Convex

```
/// minkowski sum of two polygons in O(n)
Polygon minkowskiSum(Polygon A, Polygon B) {
  int n = A.size(), m = B.size();
  rotate(A.begin(), min_element(A.begin(), A.end()), A.end());
  rotate(B.begin(), min_element(B.begin(), B.end()), B.end());

  A.push_back(A[0]);
  B.push_back(B[0]);
  for (int i = 0; i < n; i++) A[i] = A[i + 1] - A[i];
  for (int i = 0; i < m; i++) B[i] = B[i + 1] - B[i];

  Polygon C(n + m + 1);
  C[0] = A.back() + B.back();
  merge(A.begin(), A.end() - 1, B.begin(), B.end() - 1, C.begin() + 1,
        polarComp(Point(0, 0), Point(0, -1)));
  for (int i = 1; i < C.size(); i++) C[i] = C[i] + C[i - 1];
  C.pop_back();
  return C;
}
// finds the rectangle with minimum area enclosing a convex polygon and
// the rectangle with minimum perimeter enclosing a convex polygon
// Tf Ti Same
pair<Tf, Tf>
    rotatingCalipersBoundingBox(const Polygon &p) {
  using Linear::distancePointLine;
  int n = p.size();
  int l = 1, r = 1, j = 1;
  Tf area = 1e100;
  Tf perimeter = 1e100;
  for (int i = 0; i < n; i++) {
    Point v = (p[(i + 1) % n] - p[i]) / length(p[(i + 1) % n] - p[i]);
    while (dcmp(dot(v, p[r % n] - p[i]) - dot(v, p[(r + 1) % n] - p[i])) < 0)
      r++;
    while (j < r || dcmp(cross(v, p[j % n] - p[i]) -
                         cross(v, p[(j + 1) % n] - p[i])) < 0)
      j++;
    while (l < j ||
           dcmp(dot(v, p[l % n] - p[i])
                - dot(v, p[(l + 1) % n] - p[i])) > 0)
      l++;
    Tf w = dot(v, p[r % n] - p[i]) - dot(v, p[l % n] - p[i]);
    Tf h = distancePointLine(p[j % n], Line(p[i], p[(i + 1) % n]));
    area = min(area, w * h);
    perimeter = min(perimeter, 2 * w + 2 * h);
  }
  return make_pair(area, perimeter);
}
// returns the left side of polygon u after cutting it by ray a->b
Polygon cutPolygon(Polygon u, Point a, Point b) {
  using Linear::lineLineIntersection;
  using Linear::onSegment;

  Polygon ret;
  int n = u.size();
  for (int i = 0; i < n; i++) {
    Point c = u[i], d = u[(i + 1) % n];
    if (dcmp(cross(b - a, c - a)) >= 0)
      ret.push_back(c);
    if (dcmp(cross(b - a, d - c)) != 0) {
      Point t;
      lineLineIntersection(a, b - a, c, d - c, t);
      if (onSegment(t, Segment(c, d)))
        ret.push_back(t);
    }
  }
  return ret;
}
// returns true if point p is in or on triangle abc
bool pointInTriangle(Point a, Point b, Point c, Point p) {
  return dcmp(cross(b - a, p - a)) >= 0
      && dcmp(cross(c - b, p - b)) >= 0 &&
         dcmp(cross(a - c, p - c)) >= 0;
}
// pt must be in ccw order with no three collinear points
// returns inside = -1, on = 0, outside = 1
int pointInConvexPolygon(const Polygon &pt, Point p) {
  int n = pt.size();
  assert(n >= 3);

  int lo = 1, hi = n - 1;
  while (hi - lo > 1) {
    int mid = (lo + hi) / 2;
    if (dcmp(cross(pt[mid] - pt[0], p -
        pt[0])) > 0)
      lo = mid;
    else
      hi = mid;
  }

  bool in = pointInTriangle(pt[0], pt[
      lo], pt[hi], p);
  if (!in) return 1;

  if (dcmp(cross(pt[lo] - pt[lo - 1], p
       - pt[lo - 1])) == 0) return 0;
  if (dcmp(cross(pt[hi] - pt[lo], p -
      pt[lo])) == 0) return 0;
  if (dcmp(cross(pt[hi] - pt[(hi + 1) %
       n], p - pt[(hi + 1) % n])) == 0)
    return 0;
  return -1;
}
// Extreme Point for a direction is the
     farthest point in that direction
// u is the direction for extremeness
int extremePoint(const Polygon &poly,
    Point u) {
  int n = (int)poly.size();
  int a = 0, b = n;
  while (b - a > 1) {
    int c = (a + b) / 2;
    if (dcmp(dot(poly[c] - poly[(c + 1)
        % n], u)) >= 0 &&
        dcmp(dot(poly[c] - poly[(c - 1 +
            n) % n], u)) >= 0) {
      return c;
    }

    bool a_up = dcmp(dot(poly[(a + 1) %
        n] - poly[a], u)) >= 0;
    bool c_up = dcmp(dot(poly[(c + 1) %
        n] - poly[c], u)) >= 0;
    bool a_above_c = dcmp(dot(poly[a] -
        poly[c], u)) > 0;

    if (a_up && !c_up)
      b = c;
    else if (!a_up && c_up)
      a = c;
    else if (a_up && c_up) {
      if (a_above_c)
        b = c;
      else
        a = c;
    } else {
      if (!a_above_c)
        b = c;
      else
        a = c;
    }
  }

  if (dcmp(dot(poly[a] - poly[(a + 1) %
      n], u)) > 0 &&
      dcmp(dot(poly[a] - poly[(a - 1 + n
          ) % n], u)) > 0)
    return a;
```

```
  return b % n;
}
// For a convex polygon p and a line l,
     returns a list of segments
// of p that touch or intersect line l.
// the i’th segment is considered (p[i
     ], p[(i + 1) modulo |p|])
// #1 If a segment is collinear with
     the line, only that is returned
// #2 Else if l goes through i’th point
     , the i’th segment is added
// Complexity: O(lg |p|)
vector<int> lineConvexPolyIntersection(
    const Polygon &p, Line l) {
  assert((int)p.size() >= 3);
  assert(l.a != l.b);

  int n = p.size();
  vector<int> ret;

  Point v = l.b - l.a;
  int lf = extremePoint(p, rotate90(v))
      ;
  int rt = extremePoint(p, rotate90(v)
      * Ti(-1));
  int olf = orient(l.a, l.b, p[lf]);
  int ort = orient(l.a, l.b, p[rt]);

  if (!olf || !ort) {
    int idx = (!olf ? lf : rt);
    if (orient(l.a, l.b, p[(idx - 1 + n)
        % n]) == 0)
      ret.push_back((idx - 1 + n) % n);
    else
      ret.push_back(idx);
    return ret;
  }
  if (olf == ort) return ret;

  for (int i = 0; i < 2; ++i) {
    int lo = i ? rt : lf;
    int hi = i ? lf : rt;
    int olo = i ? ort : olf;

    while (true) {
      int gap = (hi - lo + n) % n;
      if (gap < 2) break;

      int mid = (lo + gap / 2) % n;
      int omid = orient(l.a, l.b, p[mid
          ]);
      if (!omid) {
        lo = mid;
        break;
      }
      if (omid == olo)
        lo = mid;
      else
        hi = mid;
    }
    ret.push_back(lo);
  }
  return ret;
}
// Calculate [ACW, CW] tangent pair
     from an external point
constexpr int CW = -1, ACW = 1;
bool isGood(Point u, Point v, Point Q,
    int dir) {
  return orient(Q, u, v) != -dir;
```

```
}
Point better(Point u, Point v, Point Q,
     int dir) {
  return orient(Q, u, v) == dir ? u : v
      ;
}
Point pointPolyTangent(const Polygon &
    pt, Point Q, int dir, int lo, int
    hi) {
  while (hi - lo > 1) {
    int mid = (lo + hi) / 2;
    bool pvs = isGood(pt[mid], pt[mid -
        1], Q, dir);
    bool nxt = isGood(pt[mid], pt[mid +
        1], Q, dir);

    if (pvs && nxt) return pt[mid];
    if (!(pvs || nxt)) {
      Point p1 = pointPolyTangent(pt, Q,
          dir, mid + 1, hi);
      Point p2 = pointPolyTangent(pt, Q,
          dir, lo, mid - 1);
      return better(p1, p2, Q, dir);
    }

    if (!pvs) {
      if (orient(Q, pt[mid], pt[lo]) ==
          dir)
        hi = mid - 1;
      else if (better(pt[lo], pt[hi], Q,
          dir) == pt[lo])
        hi = mid - 1;
      else
        lo = mid + 1;
    }
    if (!nxt) {
      if (orient(Q, pt[mid], pt[lo]) ==
          dir)
        lo = mid + 1;
      else if (better(pt[lo], pt[hi], Q,
          dir) == pt[lo])
        hi = mid - 1;
      else
        lo = mid + 1;
    }
  }

  Point ret = pt[lo];
  for (int i = lo + 1; i <= hi; i++)
      ret = better(ret, pt[i], Q, dir);
  return ret;
}
// [ACW, CW] Tangent
pair<Point, Point> pointPolyTangents(
    const Polygon &pt, Point Q) {
  int n = pt.size();
  Point acw_tan = pointPolyTangent(pt,
      Q, ACW, 0, n - 1);
  Point cw_tan = pointPolyTangent(pt, Q
      , CW, 0, n - 1);
  return make_pair(acw_tan, cw_tan);
}
```

4.5 Polygon

```
typedef vector<Point> Polygon;
// removes redundant colinear points
// polygon can’t be all colinear points
Polygon RemoveCollinear(const Polygon &
    poly) {
  Polygon ret;
  int n = poly.size();
  for (int i = 0; i < n; i++) {
    Point a = poly[i];
    Point b = poly[(i + 1) % n];
    Point c = poly[(i + 2) % n];
    if (dcmp(cross(b - a, c - b)) != 0
         && (ret.empty() || b != ret.
         back()))
      ret.push_back(b);
  }
  return ret;
}
// returns the signed area of polygon p
      of n vertices
Tf signedPolygonArea(const Polygon &p)
     {
  Tf ret = 0;
  for (int i = 0; i < (int)p.size() -
       1; i++)
    ret += cross(p[i] - p[0], p[i + 1] -
          p[0]);
  return ret / 2;
}
// given a polygon p of n vertices,
     generates the convex hull in in CCW
// Tested on https://acm.timus.ru/
     problem.aspx?space=1&num=1185
// Caution: when all points are
     colinear AND removeRedundant ==
     false
// output will be contain duplicate
     points (from upper hull) at back
Polygon convexHull(Polygon p, bool
     removeRedundant) {
  int check = removeRedundant ? 0 : -1;
  sort(p.begin(), p.end());
  p.erase(unique(p.begin(), p.end()), p
       .end());

  int n = p.size();
  Polygon ch(n + n);
  int m = 0; // preparing lower hull
  for (int i = 0; i < n; i++) {
    while (m > 1 &&
           dcmp(cross(ch[m - 1] - ch[m -
                2], p[i] - ch[m - 1]))
               <= check)
      m--;
    ch[m++] = p[i];
  }
  int k = m; // preparing upper hull
  for (int i = n - 2; i >= 0; i--) {
    while (m > k &&
           dcmp(cross(ch[m - 1] - ch[m -
                2], p[i] - ch[m - 2]))
               <= check)
      m--;
    ch[m++] = p[i];
  }
  if (n > 1) m--;
  ch.resize(m);
  return ch;
}
// returns inside = -1, on = 0, outside
      = 1
int pointInPolygon(const Polygon &p,
     Point o) {
  using Linear::onSegment;
  int wn = 0, n = p.size();
  for (int i = 0; i < n; i++) {
    int j = (i + 1) % n;
    if (onSegment(o, Segment(p[i], p[j])
         ) || o == p[i]) return 0;
    int k = dcmp(cross(p[j] - p[i], o -
         p[i]));
    int d1 = dcmp(p[i].y - o.y);
    int d2 = dcmp(p[j].y - o.y);
    if (k > 0 && d1 <= 0 && d2 > 0) wn
         ++;
    if (k < 0 && d2 <= 0 && d1 > 0) wn
         --;
  }
  return wn ? -1 : 1;
}
// Given a simple polygon p, and a line
      l, returns (x, y)
// x = longest segment of l in p, y =
      total length of l in p.
pair<Tf, Tf> linePolygonIntersection(
     Line l, const Polygon &p) {
  using Linear::lineLineIntersection;
  int n = p.size();
  vector<pair<Tf, int>> ev;
  for (int i = 0; i < n; ++i) {
    Point a = p[i], b = p[(i + 1) % n],
         z = p[(i - 1 + n) % n];
    int ora = orient(l.a, l.b, a), orb =
          orient(l.a, l.b, b),
         orz = orient(l.a, l.b, z);
    if (!ora) {
      Tf d = dot(a - l.a, l.b - l.a);
      if (orz && orb) {
        if (orz != orb) ev.emplace_back(
             d, 0);
        // else // Point Touch
      } else if (orz)
        ev.emplace_back(d, orz);
      else if (orb)
        ev.emplace_back(d, orb);
    } else if (ora == -orb) {
      Point ins;
      lineLineIntersection(l, Line(a, b)
           , ins);
      ev.emplace_back(dot(ins - l.a, l.b
           - l.a), 0);
    }
  }
  sort(ev.begin(), ev.end());

  Tf ans = 0, len = 0, last = 0, tot =
       0;
  bool active = false;
  int sign = 0;
  for (auto &qq : ev) {
    int tp = qq.second;
    Tf d = qq.first; /// current Segment
          is (last, d)
    if (sign) {     /// On Border
      len += d - last;
      tot += d - last;
      ans = max(ans, len);
      if (tp != sign) active = !active;
      sign = 0;
    } else {
      if (active) { /// Strictly Inside
        len += d - last;
        tot += d - last;
        ans = max(ans, len);
      }
      if (tp == 0)
        active = !active;
      else
        sign = tp;
    }
    last = d;
    if (!active) len = 0;
  }
  ans /= length(l.b - l.a);
  tot /= length(l.b - l.a);
  return {ans, tot};
}
```

5 Graph

5.1 LCA, ETT, VT

```
#define lg(n) (31 - __builtin_clz(n))

const int N = 1e5 + 1;
const int K = lg(N) + 1;

vector<int> adj[N];
int anc[N][K], lvl[N];

namespace lca {
  void init(int u = 1, int p = 0, int d
        = 0) {
    lvl[u] = d;
    anc[u][0] = p;
    for (int i = 1; i < K; i++)
      anc[u][i] = anc[anc[u][i - 1]][i -
           1];
    for (auto v : adj[u])
      if (v != p)
        init(v, u, d + 1);
  }

  int getAnc(int u, int k) {
    for (int i = 0; u and i < K; i++)
      if ((k >> i) & 1)
        u = anc[u][i];
    return u;
  }

  int lca(int u, int v) {
    if (lvl[u] < lvl[v]) swap(u, v);
    u = getAnc(u, lvl[u] - lvl[v]);
    if (u == v) return u;
    for (int i = K - 1; ~i; i--)
      if (anc[u][i] != anc[v][i])
        u = anc[u][i], v = anc[v][i];
    return anc[u][0];
  }

  int dist(int u, int v) {
    return lvl[u] + lvl[v] - 2 * lvl[lca
         (u, v)];
  }
}

struct euler_tour {
  int time = 0;
  tree &T;
  int n;
  vector<int> start, finish, level, par
      ;
  euler_tour(tree &T, int root = 0)
      : T(T), n(T.n), start(n), finish(n
           ), level(n), par(n) {
    time = 0;
    call(root);
  }
  void call(int node, int p = -1) {
    if (p != -1) level[node] = level[p] + 1;
    start[node] = time++;
    for (int e : T[node])
      if (e != p) call(e, node);
    par[node] = p;
    finish[node] = time++;
  }
  bool isAncestor(int node, int par) {
    return start[par] <= start[node] and finish[par] >= finish[node];
  }
  int subtreeSize(int node) { return finish[node] - start[node] + 1 >> 1; }
};

tree virtual_tree(vector<int> &nodes, lca_table &table, euler_tour &tour) {
  sort(nodes.begin(), nodes.end(), [&](int x, int y) { return tour.start[x] < tour.start[y]; });
  int n = nodes.size();
  for (int i = 0; i + 1 < n; i++)
    nodes.push_back(table.lca(nodes[i], nodes[i + 1]));
  sort(nodes.begin(), nodes.end());
  nodes.erase(unique(nodes.begin(), nodes.end()), nodes.end());
  sort(nodes.begin(), nodes.end(), [&](int x, int y) { return tour.start[x] < tour.start[y]; });
  n = nodes.size();
  stack<int> st;
  st.push(0);
  tree ans(n);
  for (int i = 1; i < n; i++) {
    while (!tour.isAncestor(nodes[i], nodes[st.top()])) st.pop();
    ans.addEdge(st.top(), i);
    st.push(i);
  }
  return ans;
}

set<int> getCenters(tree &T) {
  int n = T.n;
  vector<int> deg(n), q;
  set<int> s;
  for (int i = 0; i < n; i++) {
    deg[i] = T[i].size();
    if (deg[i] == 1) q.push_back(i);
    s.insert(i);
  }
  for (vector<int> t; s.size() > 2; q = t) {
    for (auto x : q) {
      for (auto e : T[x])
        if (--deg[e] == 1) t.push_back(e);
      s.erase(x);
    }
  }
  return s;
}
```

5.2 Euler Tour on Edge

```
// for simplicity, G[idx] contains the adjacency list of a node
// while G(e) is a reference to the e-th edge.
const int N = 2e5 + 5;
int in[N], out[N], fwd[N], bck[N];
int t = 0;
void dfs(graph &G, int node, int par) {
  out[node] = t;
  for (int e : G[node]) {
    int v = G(e).to(node);
    if (v == par) continue;
    fwd[e] = t++;
    dfs(G, v, node);
    bck[e] = t++;
  }
  in[node] = t - 1;
}
void init(graph &G, int node) {
  t = 0;
  dfs(G, node, node);
}
```

5.3 HLD

```
const int N = 1e6 + 7;
template <typename DT>
struct Segtree {
  // write lazy segtree here
};
Segtree<int> tree(N);
vector<int> adj[N];
int depth[N], par[N], pos[N];
int head[N], heavy[N], cnt;

int dfs(int u, int p) {
  int SZ = 1, mxsz = 0, heavyc;
  depth[u] = depth[p] + 1;

  for (auto v : adj[u]) {
    if (v == p) continue;
    par[v] = u;
    int subsz = dfs(v, u);
    if (subsz > mxsz) heavy[u] = v, mxsz = subsz;
    SZ += subsz;
  }
  return SZ;
}
void decompose(int u, int h) {
  head[u] = h, pos[u] = ++cnt;
  if (heavy[u] != -1) decompose(heavy[u], h);

  for (int v : adj[u]) {
    if (v == par[u]) continue;
    if (v != heavy[u]) decompose(v, v);
  }
}
int query(int a, int b) {
  int ret = 0;
  for (; head[a] != head[b]; b = par[head[b]]) {
    if (depth[head[a]] > depth[head[b]]) swap(a, b);
    ret += tree.query(1, 0, cnt, pos[head[b]], pos[b]);
  }

  if (depth[a] > depth[b]) swap(a, b);
  ret += tree.query(1, 0, cnt, pos[a], pos[b]);
  return ret;
}
```

5.4 SCC

```
typedef long long LL;
const LL N = 1e6 + 7;

bool vis[N];
int root[N];
vector<int> adj[N], adjr[N], adj_scc[N];
vector<int> order, component, roots;
// tp = 0 ,finding topo order, tp = 1 , reverse edge traversal

void dfs(int u, int tp = 0) {
  vis[u] = true;
  if (tp) component.push_back(u);
  auto& ad = (tp ? adjr : adj);
  for (int v : ad[u])
    if (!vis[v]) dfs(v, tp);
  if (!tp) order.push_back(u);
}

int main() {
  for (int i = 1; i <= n; i++) {
    if (!vis[i]) dfs(i);
  }
  memset(vis, 0, sizeof vis);
  reverse(order.begin(), order.end());
  for (int i : order) {
    if (!vis[i]) {
      dfs(i, 1);
      int rt = component[0];
      for (auto j : component)
        root[j] = rt;
      roots.push_back(rt);
      component.clear();
    }
  }

  for (int u = 1; u <= n; u++)
    for (auto v : adj[u]) {
      int rtu = root[u],
          rtv = root[v];
      if (rtu != rtv)
        adj_scc[rtu].push_back(rtv);
    }
}
```

5.5 LCA in O(1)

```
/* LCA in O(1)
 * depth calculates weighted distance
 * level calculates distance by number of edges
 * Preprocessing in NlongN */
LL depth[N];
int level[N];

int st[N], en[N], LOG[N], par[N];
int a[N], id[N], table[L][N];

vector<PII> adj[N];
int n, root, Time, cur;

void init(int nodes, int root_) {
  n = nodes, root = root_, LOG[0] = LOG[1] = 0;
  for (int i = 2; i <= n; i++) LOG[i] = LOG[i >> 1] + 1;
  for (int i = 0; i <= n; i++) adj[i].clear();
}

void addEdge(int u, int v, int w) {
  adj[u].push_back(PII(v, w));
  adj[v].push_back(PII(u, w));
}

int lca(int u, int v) {
  if (en[u] > en[v]) swap(u, v);
  if (st[v] <= st[u] && en[u] <= en[v]) return v;

  int l = LOG[id[v] - id[u] + 1];
  int p1 = id[u], p2 = id[v] - (1 << l) + 1;
  int d1 = level[table[l][p1]], d2 = level[table[l][p2]];

  if (d1 < d2)
    return par[table[l][p1]];
  else
    return par[table[l][p2]];
}

LL dist(int u, int v) {
  int l = lca(u, v);
  return (depth[u] + depth[v] - (depth[l] * 2));
}

/* Euler tour */
void dfs(int u, int p) {
  st[u] = ++Time, par[u] = p;

  for (auto [v, w] : adj[u]) {
    if (v == p) continue;
    depth[v] = depth[u] + w;
    level[v] = level[u] + 1;
    dfs(v, u);
  }

  en[u] = ++Time;
  a[++cur] = u, id[u] = cur;
}

/* RMQ */
void pre() {
  cur = Time = 0, dfs(root, root);
  for (int i = 1; i <= n; i++) table[0][i] = a[i];

  for (int l = 0; l < L - 1; l++) {
    for (int i = 1; i <= n; i++) {
      table[l + 1][i] = table[l][i];

      bool C1 = (1 << l) + i <= n;
      bool C2 = level[table[l][i + (1 << l)]] < level[table[l][i]];

      if (C1 && C2) table[l + 1][i] = table[l][i + (1 << l)];
    }
  }
}
```

5.6 Dinic Max Flow

```
/// flow with demand(lower bound) only for DAG
// create new src and sink
// add_edge(new src, u, sum(in_demand[u]))
// add_edge(u, new sink, sum(out_demand[u]))
// add_edge(old sink, old src, inf)
//  if (sum of lower bound == flow) then demand satisfied
// flow in every edge i = demand[i] + e.flow

using Ti = long long;
const Ti INF = 1LL << 60;
struct edge {
  int v, u;
  Ti cap, flow = 0;
  edge(int v, int u, Ti cap) : v(v), u(u), cap(cap) {}
};
const int N = 1e5 + 50;
vector<edge> edges;
vector<int> adj[N];
int m = 0, n;
int level[N], ptr[N];
queue<int> q;
bool bfs(int s, int t) {
  for (q.push(s), level[s] = 0; !q.empty(); q.pop()) {
    for (int id : adj[q.front()]) {
      auto &ed = edges[id];
      if (ed.cap - ed.flow > 0 and level[ed.u] == -1)
        level[ed.u] = level[ed.v] + 1, q.push(ed.u);
    }
  }
  return level[t] != -1;
}
Ti dfs(int v, Ti pushed, int t) {
  if (pushed == 0) return 0;
  if (v == t) return pushed;
  for (int &cid = ptr[v]; cid < adj[v].size(); cid++) {
    int id = adj[v][cid];
    auto &ed = edges[id];
    if (level[v] + 1 != level[ed.u] || ed.cap - ed.flow < 1) continue;
    Ti tr = dfs(ed.u, min(pushed, ed.cap - ed.flow), t);
    if (tr == 0) continue;
    ed.flow += tr;
    edges[id ^ 1].flow -= tr;
    return tr;
  }
  return 0;
}
void init(int nodes) {
  m = 0, n = nodes;
  for (int i = 0; i < n; i++) level[i] = -1, ptr[i] = 0, adj[i].clear();
}
void addEdge(int v, int u, Ti cap) {
  edges.emplace_back(v, u, cap), adj[v].push_back(m++);
  edges.emplace_back(u, v, 0), adj[u].push_back(m++);
}
Ti maxFlow(int s, int t) {
  Ti f = 0;
  for (auto &ed : edges) ed.flow = 0;
  for (; bfs(s, t); memset(level, -1, n * 4)) {
    for (memset(ptr, 0, n * 4); Ti pushed = dfs(s, INF, t); f += pushed)
      ;
  }
  return f;
}
```

5.7 Min Cost Max Flow

```
mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
const LL inf = 1e9;
struct edge {
  int v, rev;
  LL cap, cost, flow;
  edge() {}
  edge(int v, int rev, LL cap, LL cost)
    : v(v), rev(rev), cap(cap), cost(cost), flow(0) {}
};
struct mcmf {
  int src, sink, n;
  vector<int> par, idx, Q;
  vector<bool> inq;
  vector<LL> dis;
  vector<vector<edge>> g;
  mcmf() {}
  mcmf(int src, int sink, int n)
    : src(src),
      sink(sink),
      n(n),
      par(n),
      idx(n),
      inq(n),
      dis(n),
      g(n),
      Q(10000005) {} // use Q(n) if not using random
  void add_edge(int u, int v, LL cap, LL cost, bool directed = true) {
    edge _u = edge(v, g[v].size(), cap, cost);
    edge _v = edge(u, g[u].size(), 0, -cost);
    g[u].pb(_u);
    g[v].pb(_v);
    if (!directed) add_edge(v, u, cap, cost, true);
  }
  bool spfa() {
    for (int i = 0; i < n; i++) {
      dis[i] = inf, inq[i] = false;
    }
    int f = 0, l = 0;
    dis[src] = 0, par[src] = -1, Q[l++] = src, inq[src] = true;
    while (f < l) {
      int u = Q[f++];
      for (int i = 0; i < g[u].size(); i++) {
        edge &e = g[u][i];
        if (e.cap <= e.flow) continue;
        if (dis[e.v] > dis[u] + e.cost) {
          dis[e.v] = dis[u] + e.cost;
          par[e.v] = u, idx[e.v] = i;
          if (!inq[e.v]) inq[e.v] = true
               , Q[l++] = e.v;
          // if (!inq[e.v]) {
          //   inq[e.v] = true;
          //   if (f && rnd() & 7) Q[--f]
               = e.v;
          //   else Q[l++] = e.v;
          // }
        }
      }
      inq[u] = false;
    }
    return (dis[sink] != inf);
  }
  pair<LL, LL> solve() {
    LL mincost = 0, maxflow = 0;
    while (spfa()) {
      LL bottleneck = inf;
      for (int u = par[sink], v = idx[
          sink]; u != -1; v = idx[u], u
          = par[u]) {
        edge &e = g[u][v];
        bottleneck = min(bottleneck, e.
            cap - e.flow);
      }
      for (int u = par[sink], v = idx[
          sink]; u != -1; v = idx[u], u
          = par[u]) {
        edge &e = g[u][v];
        e.flow += bottleneck;
        g[e.v][e.rev].flow -= bottleneck
            ;
      }
      mincost += bottleneck * dis[sink],
           maxflow += bottleneck;
    }
    return make_pair(mincost, maxflow);
  }
};
// want to minimize cost and don't care
    about flow
// add edge from sink to dummy sink (
    cap = inf, cost = 0)
// add edge from source to sink (cap =
    inf, cost = 0)
// run mcmf, cost returned is the
    minimum cost
```

5.8 Bridge Tree

```
vector<vector<int>> components;
vector<int> depth, low;
stack<int> st;
vector<int> id;
vector<edge> bridges;
graph tree;
void find_bridges(int node, graph &G,
    int par = -1, int d = 0) {
  low[node] = depth[node] = d;
  st.push(node);
  for (int id : G[node]) {
    int to = G(id).to(node);
    if (par != to) {
      if (depth[to] == -1) {
        find_bridges(to, G, node, d + 1)
            ;
        if (low[to] > depth[node]) {
          bridges.emplace_back(node, to)
              ;
          components.push_back({});
          for (int x = -1; x != to; x =
               st.top(), st.pop())
            components.back().push_back(
                st.top());
        }
      }
      low[node] = min(low[node], low[to
          ]);
    }
  }
  if (par == -1) {
    components.push_back({});
    while (!st.empty()) components.back
        ().push_back(st.top()), st.pop
        ();
  }
}
graph &create_tree() {
  for (auto &comp : components) {
    int idx = tree.addNode();
    for (auto &e : comp) id[e] = idx;
  }
  for (auto &[l, r] : bridges) tree.
      addEdge(id[l], id[r]);
  return tree;
}
void init(graph &G) {
  int n = G.n;
  depth.assign(n, -1), id.assign(n, -1)
      , low.resize(n);
  for (int i = 0; i < n; i++)
    if (depth[i] == -1) find_bridges(i,
        G);
}
```

5.9 Tree Isomorphism

```
mp["01"] = 1;
ind = 1;
int dfs(int u, int p) {
  int cnt = 0;
  vector<int> vs;
  for (auto v : g1[u]) {
    if (v != p) {
      int got = dfs(v, u);
      vs.pb(got);
      cnt++;
    }
  }
  if (!cnt) return 1;

  sort(vs.begin(), vs.end());
  string s = "0";
  for (auto i : vs) s += to_string(i);
  vs.clear();
  s.pb('1');
  if (mp.find(s) == mp.end()) mp[s] =
      ++ind;
  int ret = mp[s];
  return ret;
}
```

6 Math

6.1 Linear Sieve

```
const int N = 1e7;
vector<int> primes;
int spf[N + 5], phi[N + 5], NOD[N + 5],
      cnt[N + 5], POW[N + 5];
bool prime[N + 5];
int SOD[N + 5];
void init() {
  fill(prime + 2, prime + N + 1, 1);
  SOD[1] = NOD[1] = phi[1] = spf[1] =
      1;
  for (LL i = 2; i <= N; i++) {
    if (prime[i]) {
      primes.push_back(i), spf[i] = i;
      phi[i] = i - 1;
      NOD[i] = 2, cnt[i] = 1;
      SOD[i] = i + 1, POW[i] = i;
    }
    for (auto p : primes) {
      if (p * i > N or p > spf[i]) break
          ;
      prime[p * i] = false, spf[p * i] =
           p;
      if (i % p == 0) {
        phi[p * i] = p * phi[i];
        NOD[p * i] = NOD[i] / (cnt[i] +
            1) * (cnt[i] + 2),
                cnt[p * i] = cnt[i] + 1;
        SOD[p * i] = SOD[i] / SOD[POW[i
            ]] * (SOD[POW[i]] + p * POW[
            i]),
                POW[p * i] = p * POW[i];
        break;
      } else {
        phi[p * i] = phi[p] * phi[i];
        NOD[p * i] = NOD[p] * NOD[i],
            cnt[p * i] = 1;
        SOD[p * i] = SOD[p] * SOD[i],
            POW[p * i] = p;
      }
    }
  }
}
```

6.2 Pollard Rho

```
LL mul(LL a, LL b, LL mod) {
  return (__int128)a * b % mod;
  // LL ans = a * b - mod * (LL) (1.L /
       mod * a * b);
  // return ans + mod * (ans < 0) - mod
       * (ans >= (LL) mod);
}
LL bigmod(LL num, LL pow, LL mod) {
  LL ans = 1;
  for (; pow > 0; pow >>= 1, num = mul(
      num, num, mod))
    if (pow & 1) ans = mul(ans, num, mod
        );
  return ans;
}
bool is_prime(LL n) {
  if (n < 2 or n % 6 % 4 != 1) return (
      n | 1) == 3;
  LL a[] = {2, 325, 9375, 28178,
      450775, 9780504, 1795265022};
  LL s = __builtin_ctzll(n - 1), d = n
      >> s;
  for (LL x : a) {
    LL p = bigmod(x % n, d, n), i = s;
    for (; p != 1 and p != n - 1 and x %
         n and i--; p = mul(p, p, n))
      ;
    if (p != n - 1 and i != s) return
        false;
  }
  return true;
}
LL get_factor(LL n) {
  auto f = [&](LL x) { return mul(x, x,
      n) + 1; };
  LL x = 0, y = 0, t = 0, prod = 2, i =
      2, q;
  for (; t++ % 40 or gcd(prod, n) == 1;
      x = f(x), y = f(f(y))) {
    (x == y) ? x = i++, y = f(x) : 0;
    prod = (q = mul(prod, max(x, y) -
        min(x, y), n)) ? q : prod;
  }
  return gcd(prod, n);
}
map<LL, int> factorize(LL n) {
  map<LL, int> res;
  if (n < 2) return res;
  LL small_primes[] = {2, 3, 5, 7, 11,
      13, 17, 19, 23, 29, 31, 37, 41,
                      43, 47, 53, 59,
                          61, 67, 71,
                          73, 79, 83,
                          89, 97};
  for (LL p : small_primes)
    for (; n % p == 0; n /= p, res[p]++)
      ;

  auto _factor = [&](LL n, auto &
      _factor) {
    if (n == 1) return;
    if (is_prime(n))
      res[n]++;
    else {
      LL x = get_factor(n);
      _factor(x, _factor);
      _factor(n / x, _factor);
    }
  };
  _factor(n, _factor);
  return res;
}
```

6.3 Chinese Remainder Theorem

```
// given a, b will find solutions for
// ax + by = 1
tuple<LL, LL, LL> EGCD(LL a, LL b) {
  if (b == 0)
    return {1, 0, a};
  else {
    auto [x, y, g] = EGCD(b, a % b);
    return {y, x - a / b * y, g};
  }
}
// given modulo equations, will apply
    CRT
PLL CRT(vector<PLL> &v) {
  LL V = 0, M = 1;
  for (auto &[v, m] : v) { // value %
      mod
    auto [x, y, g] = EGCD(M, m);
    if ((v - V) % g != 0) return {-1,
        0};
    V += x * (v - V) / g % (m / g) * M,
        M *= m / g;
    V = (V % M + M) % M;
  }
  return make_pair(V, M);
}
```

6.4 Mobius Function

```
const int N = 1e6 + 5;
int mob[N];
void mobius() {
  memset(mob, -1, sizeof mob);
  mob[1] = 1;
  for (int i = 2; i < N; i++)
    if (mob[i]) {
      for (int j = i + i; j < N; j += i)
        mob[j] -= mob[i];
    }
}
```

6.5 FFT

```
using CD = complex<double>;
typedef long long LL;
const double PI = acos(-1.0L);

int N;
vector<int> perm;
vector<CD> wp[2];
void precalculate(int n) {
  assert((n & (n - 1)) == 0), N = n;
  perm = vector<int>(N, 0);
  for (int k = 1; k < N; k <<= 1) {
    for (int i = 0; i < k; i++) {
      perm[i] <<= 1;
      perm[i + k] = 1 + perm[i];
    }
  }
  wp[0] = wp[1] = vector<CD>(N);
  for (int i = 0; i < N; i++) {
    wp[0][i] = CD(cos(2 * PI * i / N),
        sin(2 * PI * i / N));
    wp[1][i] = CD(cos(2 * PI * i / N), -
        sin(2 * PI * i / N));
  }
}
void fft(vector<CD> &v, bool invert =
    false) {
  if (v.size() != perm.size())
    precalculate(v.size());
  for (int i = 0; i < N; i++)
    if (i < perm[i]) swap(v[i], v[perm[i
        ]]);
  for (int len = 2; len <= N; len *= 2)
      {
    for (int i = 0, d = N / len; i < N;
        i += len) {
      for (int j = 0, idx = 0; j < len /
          2; j++, idx += d) {
        CD x = v[i + j];
        CD y = wp[invert][idx] * v[i + j
            + len / 2];
        v[i + j] = x + y;
        v[i + j + len / 2] = x - y;
      }
    }
  }
  if (invert) {
    for (int i = 0; i < N; i++) v[i] /=
        N;
  }
}
void pairfft(vector<CD> &a, vector<CD>
    &b, bool invert = false) {
  int N = a.size();
  vector<CD> p(N);
  for (int i = 0; i < N; i++) p[i] = a[
      i] + b[i] * CD(0, 1);
  fft(p, invert);
  p.push_back(p[0]);
  for (int i = 0; i < N; i++) {
    if (invert) {
      a[i] = CD(p[i].real(), 0);
      b[i] = CD(p[i].imag(), 0);
    } else {
      a[i] = (p[i] + conj(p[N - i])) *
          CD(0.5, 0);
      b[i] = (p[i] - conj(p[N - i])) *
          CD(0, -0.5);
    }
  }
}
vector<LL> multiply(const vector<LL> &a
    , const vector<LL> &b) {
  int n = 1;
  while (n < a.size() + b.size()) n <<=
      1;
  vector<CD> fa(a.begin(), a.end()), fb
      (b.begin(), b.end());
  fa.resize(n);
  fb.resize(n);
  //      fft(fa); fft(fb);
  pairfft(fa, fb);
  for (int i = 0; i < n; i++) fa[i] =
      fa[i] * fb[i];
  fft(fa, true);
  vector<LL> ans(n);
  for (int i = 0; i < n; i++) ans[i] =
      round(fa[i].real());
  return ans;
}
const int M = 1e9 + 7, B = sqrt(M) + 1;
vector<LL> anyMod(const vector<LL> &a,
    const vector<LL> &b) {
  int n = 1;
  while (n < a.size() + b.size()) n <<=
      1;
  vector<CD> al(n), ar(n), bl(n), br(n)
      ;
  for (int i = 0; i < a.size(); i++) al
      [i] = a[i] % M / B, ar[i] = a[i]
      % M % B;
  for (int i = 0; i < b.size(); i++) bl
      [i] = b[i] % M / B, br[i] = b[i]
      % M % B;
  pairfft(al, ar);
  pairfft(bl, br);
  //      fft(al); fft(ar); fft(bl);
      fft(br);
  for (int i = 0; i < n; i++) {
    CD ll = (al[i] * bl[i]), lr = (al[i]
        * br[i]);
    CD rl = (ar[i] * bl[i]), rr = (ar[i]
        * br[i]);
    al[i] = ll;
    ar[i] = lr;
    bl[i] = rl;
    br[i] = rr;
  }
  pairfft(al, ar, true);
  pairfft(bl, br, true);
  //      fft(al, true); fft(ar, true)
      ; fft(bl, true); fft(br, true);
  vector<LL> ans(n);
  for (int i = 0; i < n; i++) {
    LL right = round(br[i].real()), left
        = round(al[i].real());
    ;
    LL mid = round(round(bl[i].real()) +
        round(ar[i].real()));
    ans[i] = ((left % M) * B * B + (mid % M) * B + right) % M;
  }
  return ans;
}
```

6.6 NTT

```
const LL N = 1 << 18;
const LL MOD = 786433;

vector<LL> P[N];
LL rev[N], w[N | 1], a[N], b[N], inv_n, g;
LL Pow(LL b, LL p) {
  LL ret = 1;
  while (p) {
    if (p & 1) ret = (ret * b) % MOD;
    b = (b * b) % MOD;
    p >>= 1;
  }
  return ret;
}
LL primitive_root(LL p) {
  vector<LL> factor;
  LL phi = p - 1, n = phi;
  for (LL i = 2; i * i <= n; i++) {
    if (n % i) continue;
    factor.emplace_back(i);
    while (n % i == 0) n /= i;
  }
  if (n > 1) factor.emplace_back(n);
  for (LL res = 2; res <= p; res++) {
    bool ok = true;
    for (LL i = 0; i < factor.size() && ok; i++)
      ok &= Pow(res, phi / factor[i]) != 1;
    if (ok) return res;
  }
  return -1;
}
void prepare(LL n) {
  LL sz = abs(31 - __builtin_clz(n));
  LL r = Pow(g, (MOD - 1) / n);
  inv_n = Pow(n, MOD - 2);
  w[0] = w[n] = 1;
  for (LL i = 1; i < n; i++) w[i] = (w[i - 1] * r) % MOD;
  for (LL i = 1; i < n; i++)
    rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (sz - 1));
}
void NTT(LL *a, LL n, LL dir = 0) {
  for (LL i = 1; i < n - 1; i++)
    if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (LL m = 2; m <= n; m <<= 1) {
    for (LL i = 0; i < n; i += m) {
      for (LL j = 0; j < (m >> 1); j++) {
        LL &u = a[i + j], &v = a[i + j + (m >> 1)];
        LL t = v * w[dir ? n - n / m * j : n / m * j] % MOD;
        v = u - t < 0 ? u - t + MOD : u - t;
        u = u + t >= MOD ? u + t - MOD : u + t;
      }
    }
  }
  if (dir)
    for (LL i = 0; i < n; i++) a[i] = (inv_n * a[i]) % MOD;
}
vector<LL> mul(vector<LL> p, vector<LL> q) {
  LL n = p.size(), m = q.size();
  LL t = n + m - 1, sz = 1;
  while (sz < t) sz <<= 1;
  prepare(sz);

  for (LL i = 0; i < n; i++) a[i] = p[i];
  for (LL i = 0; i < m; i++) b[i] = q[i];
  for (LL i = n; i < sz; i++) a[i] = 0;
  for (LL i = m; i < sz; i++) b[i] = 0;

  NTT(a, sz);
  NTT(b, sz);
  for (LL i = 0; i < sz; i++) a[i] = (a[i] * b[i]) % MOD;
  NTT(a, sz, 1);

  vector<LL> c(a, a + sz);
  while (c.size() && c.back() == 0) c.pop_back();
  return c;
}
```

7 String

7.1 Aho Corasick

```
const int sg = 26, N = 1e3 + 9;
struct aho_corasick {
  struct node {
    node *link, *out, *par;
    bool leaf;
    LL val;
    int cnt, last, len;
    char p_ch;
    array<node*, sg> to;
    node(node* par = NULL, char p_ch = '$', int len = 0)
      : par(par), p_ch(p_ch), len(len) {
      val = leaf = cnt = last = 0;
      link = out = NULL;
    }
  };
  vector<node> trie;
  node* root;
  aho_corasick() {
    trie.reserve(N), trie.emplace_back();
    root = &trie[0];
    root->link = root->out = root;
  }
  inline int f(char c) { return c - 'a'; }
  inline node* add_node(node* par = NULL, char p_ch = '$', int len = 0) {
    trie.emplace_back(par, p_ch, len);
    return &trie.back();
  }
  void add_str(const string& s, LL val = 1) {
    node* now = root;
    for (char c : s) {
      int i = f(c);
      if (!now->to[i]) now->to[i] = add_node(now, c, now->len + 1);
      now = now->to[i];
    }
    now->leaf = true, now->val++;
  }
  void push_links() {
    queue<node*> q;
    for (q.push(root); q.empty(); q.pop()) {
      node *cur = q.front(), *link = cur->link;
      cur->out = link->leaf ? link : link->out;
      int idx = 0;
      for (auto& next : cur->to) {
        if (next != NULL) {
          next->link = cur != root ? link->to[idx++] : root;
          q.push(next);
        } else
          next = link->to[idx++];
      }
    }
    cur->val += link->val;
  }
};
```

7.2 Double hash

```
// define +, -, * for (PLL, LL) and (PLL, PLL), % for (PLL, PLL);
PLL base(1949313259, 1997293877);
PLL mod(2091573227, 2117566807);

PLL power(PLL a, LL p) {
  PLL ans = PLL(1, 1);
  for(; p; p >>= 1, a = a * a % mod) {
    if(p & 1) ans = ans * a % mod;
  }
  return ans;
}

PLL inverse(PLL a) { return power(a, (mod.ff - 1) * (mod.ss - 1) - 1); }
PLL inv_base = inverse(base);
PLL val;
vector<PLL> P;

void hash_init(int n) {
  P.resize(n + 1);
  P[0] = PLL(1, 1);
  for (int i = 1; i <= n; i++) P[i] = (P[i - 1] * base) % mod;
}
PLL append(PLL cur, char c) { return (cur * base + c) % mod; }
/// prepends c to string with size k
PLL prepend(PLL cur, int k, char c) {
  return (P[k] * c + cur) % mod; }
/// replaces the i-th (0-indexed) character from right from a to b;
PLL replace(PLL cur, int i, char a, char b) {
  cur = (cur + P[i] * (b - a)) % mod;
  return (cur + mod) % mod;
}
```

```
/// Erases c from the back of the
    string
PLL pop_back(PLL hash, char c) {
  return (((hash - c) * inv_base) % mod
     + mod) % mod;
}
/// Erases c from front of the string
    with size len
PLL pop_front(PLL hash, int len, char c
    ) {
  return ((hash - P[len - 1] * c) % mod
     + mod) % mod;
}
/// concatenates two strings where
    length of the right is k
PLL concat(PLL left, PLL right, int k)
    { return (left * P[k] + right) %
    mod; }
/// Calculates hash of string with size
     len repeated cnt times
/// This is O(log n). For O(1), pre-
    calculate inverses
PLL repeat(PLL hash, int len, LL cnt) {
  PLL mul = (P[len * cnt] - 1) *
      inverse(P[len] - 1);
  mul = (mul % mod + mod) % mod;
  PLL ret = (hash * mul) % mod;
  if (P[len].ff == 1) ret.ff = hash.ff
      * cnt;
  if (P[len].ss == 1) ret.ss = hash.ss
      * cnt;
  return ret;
}
LL get(PLL hash) { return ((hash.ff <<
    32) ^ hash.ss); }
struct hashlist {
  int len;
  vector<PLL> H, R;
  hashlist() {}
  hashlist(string& s) {
    len = (int)s.size();
    hash_init(len);
    H.resize(len + 1, PLL(0, 0)), R.
        resize(len + 2, PLL(0, 0));
    for (int i = 1; i <= len; i++) H[i]
        = append(H[i - 1], s[i - 1]);
    for (int i = len; i >= 1; i--) R[i]
        = append(R[i + 1], s[i - 1]);
  }
  /// 1-indexed
  PLL range_hash(int l, int r) {
    return ((H[r] - H[l - 1] * P[r - l +
        1]) % mod + mod) % mod;
  }
  PLL reverse_hash(int l, int r) {
    return ((R[l] - R[r + 1] * P[r - l +
        1]) % mod + mod) % mod;
  }
  PLL concat_range_hash(int l1, int r1,
      int l2, int r2) {
    return concat(range_hash(l1, r1),
        range_hash(l2, r2), r2 - l2 +
        1);
  }
  PLL concat_reverse_hash(int l1, int
      r1, int l2, int r2) {
    return concat(reverse_hash(l2, r2),
        reverse_hash(l1, r1), r1 - l1 +
        1);
  }
};
```

7.3 KMP

```
vector<int> prefix_function(string s) {
  int n = (int)s.length();
  vector<int> pi(n);
  for (int i = 1; i < n; i++) {
    int j = pi[i-1];
    while (j > 0 && s[i] != s[j])
      j = pi[j-1];
    if (s[i] == s[j])
      j++;
    pi[i] = j;
  }
  return pi;
}
```

7.4 Manacher's

```
vector<int> d1(n);
// d[i] = number of palindromes taking
    s[i] as center
for (int i = 0, l = 0, r = -1; i < n; i
    ++) {
  int k = (i > r) ? 1 : min(d1[l + r -
      i], r - i + 1);
  while (0 <= i - k && i + k < n && s[i
      - k] == s[i + k]) k++;
  d1[i] = k--;
  if (i + k > r) l = i - k, r = i + k;
}
vector<int> d2(n);
// d[i] = number of palindromes taking
    s[i-1] and s[i] as center
for (int i = 0, l = 0, r = -1; i < n; i
    ++) {
  int k = (i > r) ? 0 : min(d2[l + r -
      i + 1], r - i + 1);
  while (0 <= i - k - 1 && i + k < n &&
      s[i - k - 1] == s[i + k]) k++;
  d2[i] = k--;
  if (i + k > r) l = i - k - 1, r = i +
      k;
}
```

7.5 String Match FFT

```
//find occurrences of t in s where '?'s
     are automatically matched with any
     character
//res[i + m - 1] = sum_j=0 to m - 1_{s[
    i + j] * t[j] * (s[i + j] - t[j])
vector<int> string_matching(string &s,
    string &t) {
  int n = s.size(), m = t.size();
  vector<int> s1(n), s2(n), s3(n);
  for(int i = 0; i < n; i++) s1[i] = s[
      i] == '?' ? 0 : s[i] - 'a' + 1;
      //assign any non zero number for
      non '?'s
  for(int i = 0; i < n; i++) s2[i] = s1
      [i] * s1[i];
  for(int i = 0; i < n; i++) s3[i] = s1
      [i] * s2[i];
  vector<int> t1(m), t2(m), t3(m);
  for(int i = 0; i < m; i++) t1[i] = t[
      i] == '?' ? 0 : t[i] - 'a' + 1;
  for(int i = 0; i < m; i++) t2[i] = t1
      [i] * t1[i];
  for(int i = 0; i < m; i++) t3[i] = t1
      [i] * t2[i];
  reverse(t1.begin(), t1.end());
  reverse(t2.begin(), t2.end());
  reverse(t3.begin(), t3.end());
  vector<int> s1t3 = multiply(s1, t3);
  vector<int> s2t2 = multiply(s2, t2);
  vector<int> s3t1 = multiply(s3, t1);
  vector<int> res(n);
  for(int i = 0; i < n; i++) res[i] =
      s1t3[i] - s2t2[i] * 2 + s3t1[i];
  vector<int> oc;
  for(int i = m - 1; i < n; i++) if(res
      [i] == 0) oc.push_back(i - m + 1)
      ;
  return oc;
}
```

7.6 Suffix Array

```
#include <bits/stdc++.h>

using namespace std;

/*
  O(|S| + |alphabet|) Suffix Array
  LIM := max{s[i]} + 2
*/

void inducedSort (const vector <int> &
    vec, int val_range, vector <int> &
    SA, const vector <int> &sl, const
    vector <int> &lms_idx) {
  vector <int> l(val_range, 0), r(
      val_range, 0);
  for (int c : vec) {
    ++r[c]; if (c + 1 < val_range) ++l[c
        + 1];
  }
  partial_sum(l.begin(), l.end(), l.
      begin());
  partial_sum(r.begin(), r.end(), r.
      begin());
  fill(SA.begin(), SA.end(), -1);
  for (int i = lms_idx.size() - 1; i >=
      0; --i) SA[--r[vec[lms_idx[i]]]]
      = lms_idx[i];
  for (int i : SA) if (i > 0 and sl[i -
      1]) SA[l[vec[i - 1]]++] = i - 1;
  fill(r.begin(), r.end(), 0);
  for (int c : vec) ++r[c];
  partial_sum(r.begin(), r.end(), r.
      begin());
  for (int k = SA.size() - 1, i = SA[k
      ]; k; --k, i = SA[k]) {
    if (i and !sl[i - 1]) SA[--r[vec[i -
        1]]] = i - 1;
  }
}

vector <int> suffixArray (const vector
    <int> &vec, int val_range) {
  const int n = vec.size();
  vector <int> sl(n), SA(n), lms_idx;
  for (int i = n - 2; i >= 0; --i) {
    sl[i] = vec[i] > vec[i + 1] or (vec[
        i] == vec[i + 1] and sl[i + 1])
        ;
    if (sl[i] and !sl[i + 1]) lms_idx.
        emplace_back(i + 1);
  }
  reverse(lms_idx.begin(), lms_idx.end
      ());
  inducedSort(vec, val_range, SA, sl, lms_idx);
  vector <int> new_lms_idx(lms_idx.size()), lms_vec(lms_idx.size());
  for (int i = 0, k = 0; i < n; ++i) {
    if (SA[i] > 0 and !sl[SA[i]] and sl[SA[i] - 1]) new_lms_idx[k++] = SA[i];
  }
  int cur = 0; SA[n - 1] = 0;
  for (int k = 1; k < new_lms_idx.size(); ++k) {
    int i = new_lms_idx[k - 1], j = new_lms_idx[k];
    if (vec[i] ^ vec[j]) {
      SA[j] = ++cur; continue;
    }
    bool flag = 0;
    for (int a = i + 1, b = j + 1; ; ++a, ++b) {
      if (vec[a] ^ vec[b]) {
        flag = 1; break;
      }
      if ((!sl[a] and sl[a - 1]) or (!sl[b] and sl[b - 1])) {
        flag = !(!sl[a] and sl[a - 1] and !sl[b] and sl[b - 1]);
        break;
      }
    }
    SA[j] = flag ? ++cur : cur;
  }
  for (int i = 0; i < lms_idx.size(); ++i) lms_vec[i] = SA[lms_idx[i]];
  if (cur + 1 < lms_idx.size()) {
    auto lms_SA = suffixArray(lms_vec, cur + 1);
    for (int i = 0; i < lms_idx.size(); ++i) new_lms_idx[i] = lms_idx[lms_SA[i]];
  }
  inducedSort(vec, val_range, SA, sl, new_lms_idx); return SA;
}

vector <int> getSuffixArray (const string &s, const int LIM = 128) {
  vector <int> vec(s.size() + 1);
  copy(begin(s), end(s), begin(vec)); vec.back() = '$';
  auto ret = suffixArray(vec, LIM);
  ret.erase(ret.begin()); return ret;
}

// build RMQ on it to get LCP of any two suffix
vector <int> getLCParray (const string &s, const vector <int> &SA) {
  int n = s.size(), k = 0;
  vector <int> lcp(n), rank(n);
  for (int i = 0; i < n; ++i) rank[SA[i]] = i;
  for (int i = 0; i < n; ++i, k ? --k : 0) {
    if (rank[i] == n - 1) {
      k = 0; continue;
    }
    int j = SA[rank[i] + 1];
    while (i + k < n and j + k < n and s[i + k] == s[j + k]) ++k;
    lcp[rank[i]] = k;
  }
  lcp[n - 1] = 0; return lcp;
}
const int MX = 1e6+7, K = 20;
int lg[MX];

void pre()
{
    lg[1] = 0;
    for (int i=2; i<MX; i++)
        lg[i] = lg[i/2]+1;
}

struct RMQ{
    int N;
    VI v[K];
    RMQ(const VI &a) {
        N = a.size();
        v[0] = a;

        for (int k = 0; (1<<(k+1)) <= N; k++) {
            v[k+1].resize(N);
            for (int i = 0; i-1+(1<<(k+1)) < N; i++) {
                v[k+1][i] = min(v[k][i], v[k][i+(1<<k)]);
            }
        }
    }

    int findMin(int i, int j) {
        int k = lg[j-i+1];
        return min(v[k][i], v[k][j+1-(1<<k)]);
    }
};

int main() {
  string s; cin >> s;
  for (const int i : getSuffixArray(s)) printf("%d ", i);
  puts("");
  return 0;
}
```

7.7 Trie

```
template<int sz>
struct Trie {
  Trie() : id(1) {
    memset(endMark, 0, sizeof endMark);
    for_each(all(trie), [](vector<int> &v) { v.assign(sz, 0); });
  }

  void insert(const string &s) {
    int cur = 0;
    for (auto c : s) {
      int val = c - 'a';
      if (not trie[cur][val])
        trie[cur][val] = id++;
      cur = trie[cur][val];
    }
    endMark[cur] = true;
  }

  bool search(const string &s) {
    int cur = 0;
    for (auto c : s) {
      int val = c - 'a';
      if (not trie[cur][val])
        return false;
      cur = trie[cur][val];
    }
    return endMark[cur];
  }
private:
  int id, endMark[100005];
  vector<int> trie[100005];
};
```

7.8 Z Algo

```
vector<int> calcz(string s) {
  int n = s.size();
  vector<int> z(n);
  int l = 0, r = 0;
  for (int i = 1; i < n; i++) {
    if (i > r) {
      l = r = i;
      while (r < n && s[r] == s[r - l]) r++;
      z[i] = r - l, r--;
    } else {
      int k = i - l;
      if (z[k] < r - i + 1) z[i] = z[k];
      else {
        l = i;
        while (r < n && s[r] == s[r - l]) r++;
        z[i] = r - l, r--;
      }
    }
  }
  return z;
}
```

8 Extra

8.1 Make File

```
.PHONY: clean

D ?= 0
ifeq ($(D), 1)
 CXXFLAGS=-std=c++2a -g -DMUSHFIQ_DEBUG -DDBG_MACRO_NO_WARNING -Wall -Wextra -Wpedantic -Wshadow -Wformat=2 -Wfloat-equal -Wconversion -Wlogical-op -Wshift-overflow=2 -Wduplicated-cond -Wcast-qual -Wcast-align -Wno-variadic-macros -D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC -fsanitize=address -fsanitize=undefined -fno-sanitize-recover -fstack-protector -fsanitize-address-use-after-scope
else
 CXXFLAGS=-O2 -std=c++2a
endif

all:
 @echo 'Error: No file given.'

clean:
 @echo 'Cleaning executables...'
 @find . ! -name stress.sh -type f -executable -delete
 @echo 'Executables cleaned.'
```

8.2 Stress Tester

```
make $1
make $2
make $3

# $1 is actual code
# $2 is good code
# $3 is generator
for i in ‘seq 1 1000‘; do
  echo 'Test #'$i

  timeout 5s ./$3 $RANDOM > in
  timeout 5s ./$1 < in> out
  timeout 5s ./$2 < in> ans

  diff ans out || break
done
```

8.3 Sublime Build

```
{
  "shell_cmd": "make $file_base_name &&
       timeout 5s ./$file_base_name <
      in > out",
  "working_dir": "$file_path",
  "selector": "source.cpp"
}
```

8.4 Vimrc

```
nmap <F5> :NERDTreeToggle<CR>
" nnoremap <silent> <expr> <F5> g:
    NERDTree.IsOpen() ? "\:
    NERDTreeClose<CR>" : bufexists(
    expand('%')) ? "\:NERDTreeFind<CR>"
     : "\:NERDTree<CR>"

" Auto import & Compile
:autocmd BufNewFile *.cpp 0r ~/template
    .cpp

nnoremap <F4> :!xclip -o -sel clip > ~/
    cp/in <CR><CR>
inoremap <F4> <ESC>:!xclip -o -sel clip
     > ~/cp/in <CR><CR>
nnoremap <F6> :!xclip -sel clip % <CR><
    CR>
inoremap <F6> <ESC>:!xclip -sel clip %
    <CR><CR>

map <F3> :50 vsplit in<CR>:split out<CR
    ><C-w>h

autocmd filetype cpp nnoremap <F9>   :
    wa \| !make %:r && timeout 5s ./%:r
     < ~/cp/in> ~/cp/out<CR>
autocmd filetype cpp inoremap <F9> <ESC
    >:wa \| !make %:r && timeout 5s
    ./%:r < ~/cp/in> ~/cp/out<CR>
autocmd filetype cpp nnoremap <F10>  :
    wa \| !make clean && make %:r D=1
    && ./%:r < ~/cp/in> ~/cp/out<CR>
autocmd filetype cpp inoremap <F10> <
    ESC>:wa \| !make clean && make %:r
    D=1 && ./%:r < ~/cp/in > ~/cp/out<
    CR>

autocmd filetype python nnoremap <F9> :
    wa \| !python % < ~/cp/in> ~/cp/out
    <CR>
autocmd filetype python inoremap <F9> <
    ESC>:wa \| !python % < ~/cp/in> ~/
    cp/out<CR>

" Auto Completion
inoremap ( ()<left>
inoremap <expr> ) strpart(getline('.'),
     col('.')-1, 1) == ")" ? "\<Right>"
     : ")"
inoremap { {}<left>
inoremap <expr> } strpart(getline('.'),
     col('.')-1, 1) == "}" ? "\<Right>"
     : "}"
inoremap [ []<left>
inoremap <expr> ] strpart(getline('.'),
     col('.')-1, 1) == "]" ? "\<Right>"
     : "]"
inoremap <expr> " strpart(getline('.'),
     col('.')-1, 1) == "\"" ? "\<Right>
    " : "\"\"\<left>"
inoremap <expr> ' strpart(getline('.'),
     col('.')-1, 1) == "\'" ? "\<Right
    >" : "\'\'\<left>"

inoremap <expr> <CR> <sid>
    insert_newline()
function s:insert_newline() abort
  let pair = strpart(getline('.'), col(
     '.')-2, 2)
  return stridx('(){}[]', pair) % 2 ==
      0 && strlen(pair) == 2 ? "\<CR>\<
      ESC>\O" : "\<CR>"
endfunction

inoremap <expr> <space> <sid>
    insert_space()
function s:insert_space() abort
  let pair = strpart(getline('.'), col(
     '.')-2, 2)
  return stridx('(){}[]', pair) % 2 ==
      0 && strlen(pair) == 2 ? "\<space
      >\<space>\<left>" : "\<space>"
endfunction

inoremap <expr> <bs> <sid>rm_pair()
function s:rm_pair() abort
 let pair = strpart(getline('.'), col('
     .')-2, 2)
 return stridx('(){}[]''''""', pair) %
     2 == 0 && strlen(pair) == 2 ? "\<
     del>\<bs>" : "\<bs>"
endfunction

set nocompatible             " be
    iMproved, required
filetype on                  " required
filetype plugin on
filetype plugin indent on
syntax on

set splitright splitbelow
set mouse=a
set number
set relativenumber
set tabstop=2
set shiftwidth=2
set expandtab
set softtabstop=2
set smartindent
set smarttab
set autoindent
set cindent
set noerrorbells
set ruler
set guifont=*
set backspace=indent,eol,start
set ignorecase
set incsearch
set nowrap
set hlsearch

" bubt site
" set termguicolors
set foldmethod=indent
set nofoldenable
" set cursorline
set laststatus=2
set showcmd
set wildmenu
" colorscheme torte


if !has('nvim')
  set clipboard=unnamedplus
endif
if !has('nvim')
  set ttymouse=xterm2
endif

nnoremap <S-j> :m .+1<CR>==
nnoremap <S-k> :m .-2<CR>==
vnoremap <S-j> :m '>+1<CR>gv==gv
vnoremap <S-k> :m '<-2<CR>gv==gv

filetype plugin indent on " required
```

9 Equations and Formulas

9.1 Catalan Numbers

$C_n = \frac{1}{n+1}\binom{2n}{n}$ $C_0 = 1, C_1 = 1$ and $C_n = \sum_{k=0}^{n-1} C_k C_{n-1-k}$

The number of ways to completely parenthesize n+1 factors.

The number of triangulations of a convex polygon with n+2 sides (i.e. the number of partitions of polygon into disjoint triangles by using the diagonals).

The number of ways to connect the $2n$ points on a circle to form n disjoint i.e. non-intersecting chords.

The number of rooted full binary trees with n+1 leaves (vertices are not numbered). A rooted binary tree is full if every vertex has either two children or no children.

Number of permutations of $1, \ldots, n$ that avoid the pattern 123 (or any of the other patterns of length 3); that is, the number of permutations with no three-term increasing sub-sequence. For $n = 3$, these permutations are 132, 213, 231, 312 and 321.

9.2 Stirling Numbers First Kind

The Stirling numbers of the first kind count permutations according to their number of cycles (counting fixed points as cycles of length one).

$S(n, k)$ counts the number of permutations of n elements with k disjoint cycles.

$S(n, k) = (n-1) \cdot S(n-1, k) + S(n-1, k-1)$, $where,\ S(0, 0) = 1, S(n, 0) = S(0, n) = 0$

$\sum_{k=0}^{n} S(n, k) = n!$

The unsigned Stirling numbers may also be defined algebraically, as the coefficient of the rising factorial:

$$x^{\bar{n}} = x(x+1)...(x+n-1) = \sum_{k=0}^{n} S(n, k)x^k$$

Lets $[n, k]$ be the stirling number of the first kind, then

$$\left[{}_{n}\overset{n}{-}{}_{k}\right] = \sum_{0 \le i_1 < i_2 < i_k < n} i_1 i_2 .... i_k.$$

9.3 Stirling Numbers Second Kind

Stirling number of the second kind is the number of ways to partition a set of n objects into k non-empty subsets.

$S(n, k) = k \cdot S(n-1, k) + S(n-1, k-1)$, $where\ S(0, 0) = 1, S(n, 0) = S(0, n) = 0$ $S(n, 2) = 2^{n-1} - 1$ $S(n, k) \cdot k! =$ number of ways to color n nodes using colors from 1 to k such that each color is used at least once.

An r-associated Stirling number of the second kind is the number of ways to partition a set of n objects into k subsets, with each subset containing at least r elements. It is denoted by $S_r(n, k)$ and obeys the recurrence relation. $S_r(n+1, k) = kS_r(n, k) + \binom{n}{r-1} S_r(n-r+1, k-1)$

Denote the n objects to partition by the integers $1, 2, \ldots, n$. Define the reduced Stirling numbers of the second kind, denoted $S^d(n, k)$, to be the number of ways to partition the integers $1, 2, \ldots, n$ into k nonempty subsets such that all elements in each subset have pairwise distance at least d. That is, for any integers i and j in a given subset, it is required that $|i - j| \ge d$. It has been shown that these numbers satisfy, $S^d(n, k) = S(n-d+1, k-d+1), n \ge k \ge d$

9.4 Other Combinatorial Identities

$\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$

$\sum_{i=0}^{k} \binom{n+i}{i} = \sum_{i=0}^{k} \binom{n+i}{n} = \binom{n+k+1}{k}$

$n, r \in N, n > r, \sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$

If $P(n) = \sum_{k=0}^{n} \binom{n}{k} \cdot Q(k)$, then,

$$Q(n) = \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} \cdot P(k)$$

If $P(n) = \sum_{k=0}^{n} (-1)^k \binom{n}{k} \cdot Q(k)$, then,

$$Q(n) = \sum_{k=0}^{n} (-1)^k \binom{n}{k} \cdot P(k)$$

9.5 Different Math Formulas

Picks Theorem : $A = i + b/2 - 1$

Deragements : $d(i) = (i-1) \times (d(i-1) + d(i-2))$

$$\frac{n}{ab} - \left\{\frac{b'n}{a}\right\} - \left\{\frac{a'n}{b}\right\} + 1$$

9.6 GCD and LCM

if m is any integer, then $\gcd(a + m \cdot b, b) = \gcd(a, b)$

The gcd is a multiplicative function in the following sense: if a_1 and a_2 are relatively prime, then $\gcd(a_1 \cdot a_2, b) = \gcd(a_1, b) \cdot \gcd(a_2, b)$.

$\gcd(a, \operatorname{lcm}(b, c)) = \operatorname{lcm}(\gcd(a, b), \gcd(a, c))$.

$\operatorname{lcm}(a, \gcd(b, c)) = \gcd(\operatorname{lcm}(a, b), \operatorname{lcm}(a, c))$.

For non-negative integers a and b, where a and b are not both zero, $\gcd(n^a - 1, n^b - 1) = n^{\gcd(a,b)} - 1$

$\gcd(a, b) = \sum_{k|a \text{ and } k|b} \phi(k)$

$\sum_{i=1}^{n} [\gcd(i, n) = k] = \phi\left(\frac{n}{k}\right)$

$\sum_{k=1}^{n} \gcd(k, n) = \sum_{d|n} d \cdot \phi\left(\frac{n}{d}\right)$

$\sum_{k=1}^{n} x^{\gcd(k,n)} = \sum_{d|n} x^d \cdot \phi\left(\frac{n}{d}\right)$

$\sum_{k=1}^{n} \frac{1}{\gcd(k, n)} = \sum_{d|n} \frac{1}{d} \cdot \phi\left(\frac{n}{d}\right) = \frac{1}{n} \sum_{d|n} d \cdot \phi(d)$

$\sum_{k=1}^{n} \frac{k}{\gcd(k, n)} = \frac{n}{2} \cdot \sum_{d|n} \frac{1}{d} \cdot \phi\left(\frac{n}{d}\right) = \frac{n}{2} \cdot \frac{1}{n} \cdot \sum_{d|n} d \cdot \phi(d)$

$\sum_{k=1}^{n} \frac{n}{\gcd(k, n)} = 2 * \sum_{k=1}^{n} \frac{k}{\gcd(k, n)} - 1$, for $n > 1$

$\sum_{i=1}^{n} \sum_{j=1}^{n} [\gcd(i, j) = 1] = \sum_{d=1}^{n} \mu(d) \lfloor \frac{n}{d} \rfloor^2$

$\sum_{i=1}^{n} \sum_{j=1}^{n} \gcd(i, j) = \sum_{d=1}^{n} \phi(d) \lfloor \frac{n}{d} \rfloor^2$

$\sum_{i=1}^{n} \sum_{j=1}^{n} i \cdot j [\gcd(i, j) = 1] = \sum_{i=1}^{n} \phi(i) i^2$

$F(n) = \sum_{i=1}^{n} \sum_{j=1}^{n} \operatorname{lcm}(i, j) = \sum_{l=1}^{n} \left(\frac{\left(1 + \lfloor \frac{n}{l} \rfloor\right) \left(\lfloor \frac{n}{l} \rfloor\right)}{2} \right)^2 \sum_{d|l} \mu(d) l d$