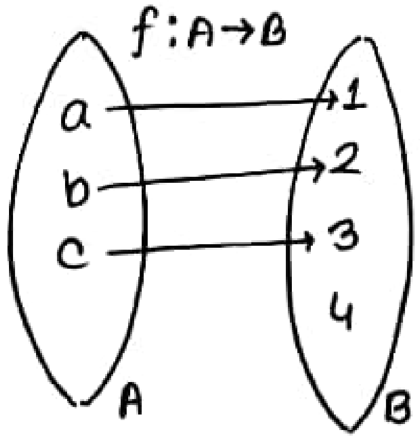
Calculas: Calculas is the mathematical study of continious changes.

Function: Let, A and B two non empty set. If each element of the set A is assaigned by some manner or other, to a unique element of set B, then this assaignment is called function. If we let 'f' denote those assaignments, then we writes $f: A \rightarrow B$ which is read as "f is a function of A into B".

The set A is called the domain of 'f' and the set B is called the co-domain of 'f'.

Again, if $a \in A$, then the element of B which is assaigned to 'a' is called the image of 'a' is denoted by $f(a)$. The set of all elements of images is called the range of 'f'.

Example: Let, $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$ define a function 'f' of A into B by the correspondence $f(a) = 1$, $f(b) = 2$, $f(c) = 3$. Using these function definition, the function $f: A \rightarrow B$ represented by the following diagram.



Here, domain, $A = \{a, b, c\}$

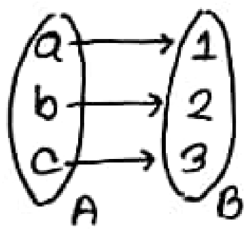
co domain, $B = \{1, 2, 3, 4\}$

range $= \{1, 2, 3\}$

$= \{f(a), f(b), f(c)\}$

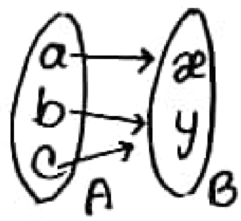
<u>One-One function</u>: Let, $f: A \to B$ be a function. Then 'f' is called one-one if each element of A has distinct images.

<u>example</u>:



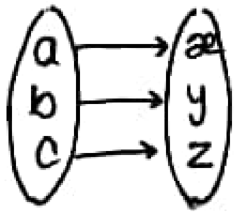
<u>Onto function</u>: Let, $f: A \to B$ be a function. Then 'f' is called onto if every element of B appears as the image of at least one element of A. In these case $f(A) = B$.

<u>example</u>:



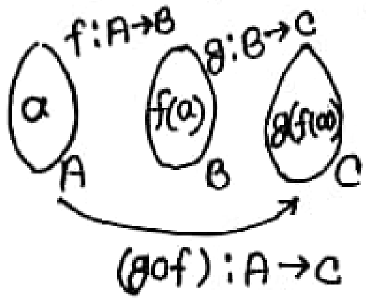
<u>Inverse function</u>: Let, $f: A \to B$ be a function. If 'f' is one-one and onto, then there exists a function f^{-1} from B to A which is called the inverse function. In these case we write $f^{-1}: B \to A$

<u>example</u>:



<u>Composition or product function</u>: Let, 'f' be a function of $A \to B$ and let, 'g' be a function of $B \to C$. Then, we define a function $(g \circ f): A \to C$ by $(g \circ f)(a) = g(f(a))$ where $a \in A$ implies $g(f(a)) \in C$.

We consider the following diagram:



<u>example</u>: $f(x) = x^2$, $g(x) = x+1$

$\therefore fog = f(g(x))$
$= f(x+1)$
$= (x+1)^2$

$\therefore gof = g(f(x))$
$= g(x^2)$
$= x^2 + 1$

<u>Step/Box/Greatest integer function</u>: A function $[x]$ is said to be greatest integer function, when $[x]$ is the greatest integer less than or equal to x that is not exceeding x.

Mathematically, $f(x) = [x] = n$ whenever $n \le x \le n+1; n \in R$

<u>example</u>: $x = 3$, $[3] = 3$
$x = 2.3$, $[2.3] = 2$
$x = -1.5$, $[-1.5] = -2$

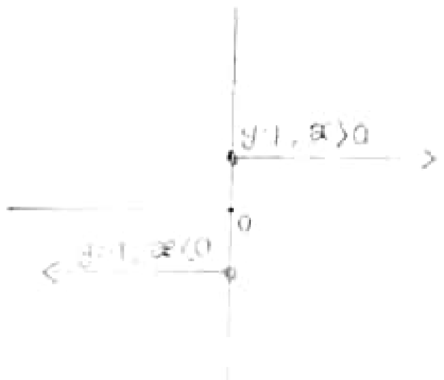
Graph of function:

example: $y = f(x) = \begin{cases} -1, & \text{when } x<0 \\ 0, & \text{when } x=0 \\ 1, & \text{when } x>0 \end{cases}$

$$\text{Domain} = (-\infty, 0) \cup \{0\} \cup (0, \infty)$$
$$= (-\infty, \infty)$$
$$= \mathbb{R}$$

$$\text{Range} = \{-1, 0, 1\}$$

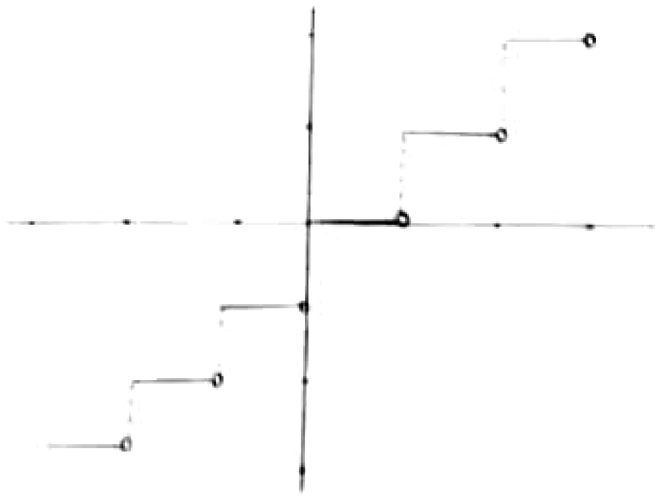
The graph of the given function is as follows:



example: $y = [x]$ where $[x]$ is the greatest integer not exceeding x.

$$y = [x] = \begin{cases} -2 & \text{where } -2 \le x < -1 \\ -1 & \text{where } -1 \le x < 0 \\ 0 & \text{where } 0 \le x < 1 \\ 1 & \text{where } 1 \le x < 2 \\ 2 & \text{where } 2 \le x < 3 \end{cases}$$

The graph of the function:



$$\text{Domain} = \mathbb{R}$$

$$\text{Range} = \{\ldots -2, -1, 0, 1, 2, \ldots\}$$

Page : 8/VI $\quad y = x - [x]$

For, $x = 0$, $y = 0 - [0] = 0$

For, $0 \leq x < 1$, $y = x - [0] = x$

For, $1 \leq x < 2$, $y = x - [1] = x - 1$

For, $2 \leq x < 3$, $y = x - [2] = x - 2$

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For, $-1 \leq x < 0$, $y = x + 1$

$-2 \leq x < -1$, $y = x + 2$

- - - - - - - - - - - -

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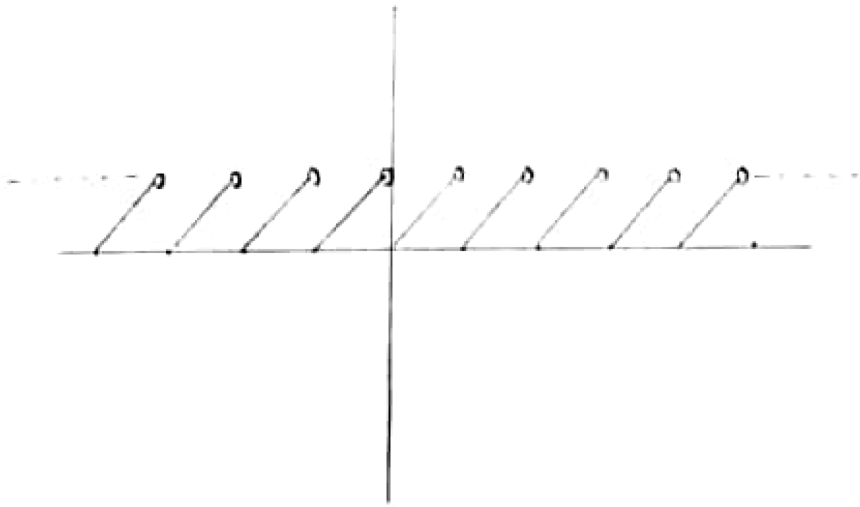
Domain = $\mathbb{R}$

Range = $[0, 1)$

The graph of the function :



Page : 44 / VIII $f(x) = \frac{|x|}{x}$

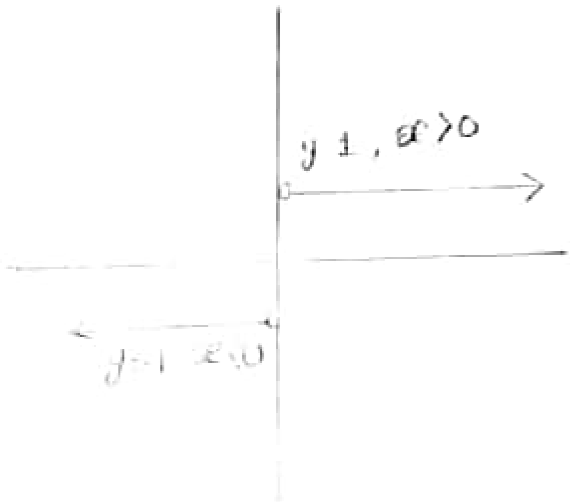
Domain = $\mathbb{R} - 0$

Range = $\mathbb{R} - 0$

For, $x < 0$, $y = -1$

For, $x > 0$, $y = 1$

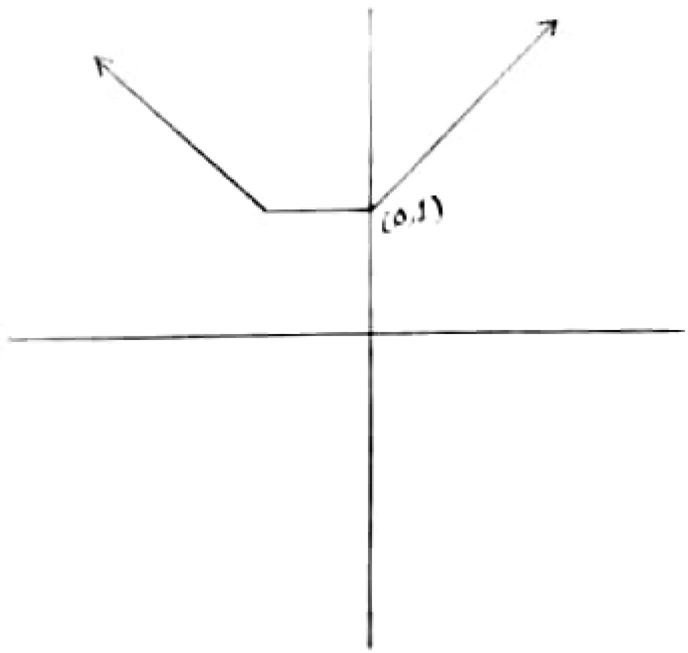
The graph of the function :



$y = |x+1| + |x|$

$$= \begin{cases} -(x+1) - x, & x < -1 \\ (x+1) - x, & -1 \leq x < 0 \\ (x+1) + x, & x \geq 0 \end{cases}$$

$$= \begin{cases} -2x - 1, & x < -1 \\ 1, & -1 \leq x < 0 \\ 2x + 1 & x \geq 0 \end{cases}$$



Domain = $\mathbb{R}$

Range, = $[1, \infty)$

Page : 31

4/(i) $f(x) = \sqrt{x-1} + \sqrt{5-x}$

Since, $f(x)$ is real, the values of x must be such that both $\sqrt{x-1}$ and $\sqrt{5-x}$ are real quantities, which requires that, $(x-1) \geq 0$ and $(5-x) \geq 0$.

So, domain of definition of $f(x)$ is $1 \leq x \leq 5$ or $[1, 5]$

4/(iv) $f(x) = \log(x^2 - 5x + 6)$

$f(x)$ is defined for all real values of x that make $x^2 - 5x + 6 > 0$ or $(x-2)(x-3) > 0$. The inequality holds for all real values of x, except those that lie between 2 and 3 including $x = 2$ and $x = 3$.

So, domain of definition of $f(x)$ is all real value of x, except $2 \leq x \leq 3$.

Limit of a function: If the variable x taking the values greater or less than 'a' in such a way that x is very close to 'a' and hence the values of $f(x)$ approches to a definite number 'L' (say). Then 'L' is called the limit of the function $f(x)$ which we denote by $\lim_{x \to a} f(x) = $ 'L'

Example: Let, $\lim_{x \to 2} \frac{x^2-4}{x-2}$

when, $x \to 1.9$, $f(x) = 3.9$

$x \to 1.99$, $f(x) = 3.99$

Thus, $x \to 2^-$, $f(x) = 4^-$

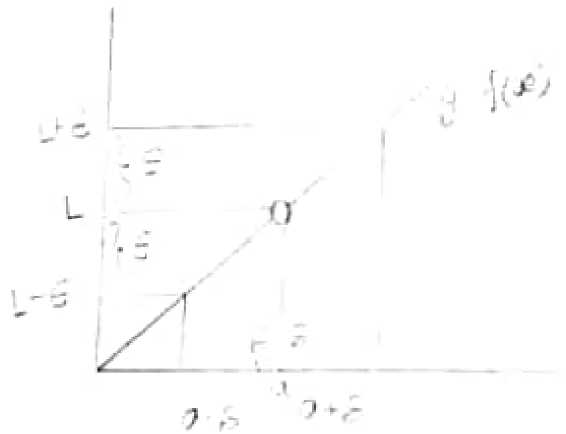
Again when, $x \to 2.1$, $f(x) = 4.1$

$x \to 2.01$, $f(x) = 4.01$

Thus, $x \longrightarrow 2^+$, $f(x) \longrightarrow 4^+$

In this case, we can write, $\lim_{x \to 2} f(x) = 4$

Graphically,



$L - \epsilon < f(x) < L + \epsilon$

$\Rightarrow -\epsilon < f(x) - L < \epsilon$

$\Rightarrow |f(x) - 2| < \epsilon, \ \epsilon > 0$

$a - \delta < x < a + \delta$

$\Rightarrow |x - a| < \delta, \ \delta > 0$

Cauchy's definition: A function $f(x)$ is said to have a limit 'L' at $x \to a$ if for every $\epsilon > 0$, there exists $\delta > 0$ (δ depends on δ) such that, $|f(x) - L| < \epsilon$ whenever $|x-a| < \delta$. In this case we write,

$$\lim_{x \to a} f(x) = L$$

Example: $\lim_{x \to 1} (3x+4) = ?$

Let, $f(x) = 3x+4$

Now, we see that,

for, $x = 0.9$, $f(x) = 6.7$

$x = 0.99$, $f(x) = 6.97$

again when, $x = 1.1$, $f(x) = 7.3$

$x = 1.01$, $f(x) = 7.03$

We observe that, when x comes closer and closer to 1, then $f(x) = 3x+4$ comes closer and closer to 7.

In this case, if we take,

$$|f(x) - 7| < \epsilon, \ \epsilon > 0$$

$$\Rightarrow |3x+4-7| < \epsilon$$

$$\Rightarrow |3x-3| < \epsilon$$

$$\Rightarrow 3|x-1| < \epsilon$$

$$\Rightarrow |x-1| < \epsilon/3 = \delta \text{ (say)}$$

Hence, we see that,

when $|f(x)-7|<\epsilon$ whenever $|x-1|<\delta$

$$\therefore \lim_{x\to 1} f(x)=7$$

note: $x \to a\ [x\to a^- \text{ or } x\to a^+]$

$$\lim_{x\to a} f(x) \to \lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$$

Left hand limit: Let, $f(x)$ be any function and $x\to a$.

Then $\lim_{x\to a^-} f(x)$ is called the left hand limit.

Right hand limit: Let, $f(x)$ be any function and $x\to a$.

Then $\lim_{x\to a^-} f(x)$ is called the Right hand limit.

Existance of a limit: A function $f(x)$ has a limit L at $x\to a$ if L.H.L = R.H.L = L. That is,

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$$

Problem: If $f(x)=\begin{cases} 4 & \text{when } x>5 \\ 0 & \text{when } x=5 \\ -4 & \text{when } x<5 \end{cases}$

Does the limit of the function exist at $x\to 5$?

Now, L.H.L : $\lim_{x\to 5^-} f(x) = -4$

R.H.L : $\lim_{x\to 5^+} f(x) = 4$

Since L.H.L $\neq$ R.H.L , $\lim_{x\to 5}$ does n't exist.

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Distinction between $\lim_{x\to a} f(x)$ and $f(a)$:

The statement $\lim_{x\to a} f(x)$ is a statement about the value of $f(x)$ when x has any value near to a. But $f(a)$ stands for the value of $f(x)$ when x is exactly equal to a, obtained either by the definition of the function at a or else by substitution of a for x in the expression $f(x)$, when it exists.

Page 70: ex-3

$$\lim_{x\to 4} (2x-2) = 6$$

Let us choose, $\epsilon = 0.01$

Then, $|(2x-2)-6| < 0.01$ if $|2x-8| < 0.01$ i.e, if $|x-4| < 0.005$,
i.e, $\delta = 0.005$. Similarly, if $\epsilon = 0.001$, $\delta = 0.0005$ and so on.
Thus, δ depends upon ϵ, i.e, the nearer $(2x-2)$ is to 6, the nearer x is to 4. We have

$|(2x-2)-6| < 0.01$ if $0 < |x-4| < 0.005$

$|(2x-2)-6| < 0.001$ if $0 < |x-4| < 0.0005$,

and generally, $|(2x-2)-6| < \epsilon$ if $0 < |x-4| < \frac{1}{2}\epsilon$.

Hence, 6 is the limit of $2x-2$ as $x \to 4$.

Problem 10: $f(x) = \lim\limits_{x \to \infty} \dfrac{1}{1+x^{2n}}$

Show that, $f(x) = 1, \frac{1}{2}, 0$. $|x| <, =, > 1$

For, $|x| < 1 \Rightarrow -1 < x < 1$

$$\therefore f(x) = \lim_{n \to \infty} \frac{1}{1+x^{2n}} = 1$$

For, $|x| = 1 \Rightarrow x = \pm 1$

$$\therefore f(x) = \lim_{n \to \infty} \frac{1}{1+(\pm 1)^{2n}} = \frac{1}{2}$$

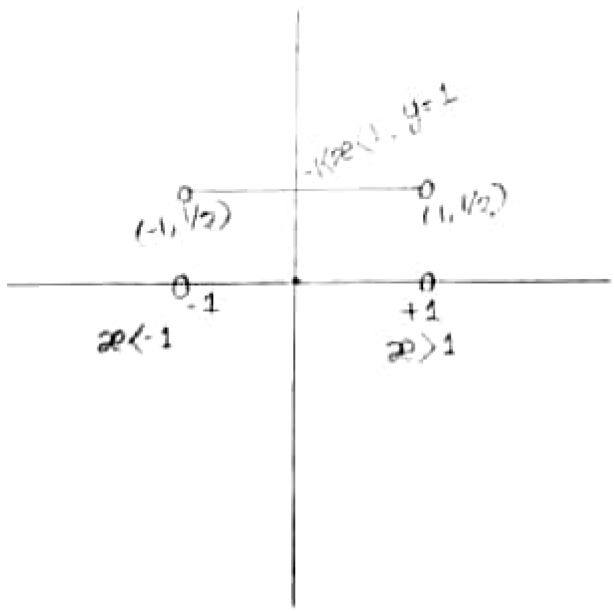
For, $|x| > 1 \Rightarrow -1 > x > 1 \Rightarrow x > 1, x < -1$

$$\therefore f(x) = \lim_{x \to \infty} \frac{1}{1+x^{2n}} = 0$$

when, $-1 < x < 1$, $y = 1$

when, $x = \pm 1$, $y = \frac{1}{2}$

when, $x < -1$, $x > 1$, $y = 0$



Continuity of a function:

A function $f(x)$ is said to be contineous at $x=a$ if $\lim_{x\to a} f(x) = f(a)$ in finite.

Cauchy's definition (δ-ϵ) of continuity:

A function $f(x)$ is said to be continious at $x=a$ if for every $\epsilon>0$, there exists $\delta>0$ (δ depends on ϵ) such that $|f(x)-f(a)|<\epsilon$ whenever $|x-a|<\delta$.

In this case, we write

1. $\lim_{x\to a} f(x) = f(a) =$ finite

$\Rightarrow \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) =$ Functional Value $[f(a)]$

$\Rightarrow$ L.H.L $=$ R.H.L $=$ FV

2. $\lim_{x\to a} f(x) \neq f(a)$

$\Rightarrow f(x)$ is dis continious at $x=a$

Classification of discontinuity:

1. **Ordinary discontinuity:** A function $f(x)$ is said to have an ordinary discontinuity at $x=a$ if $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$

example: $\lim_{x\to 0} (2+e^{1/x})$ has an ordinary discontinuity at $x=0$.

2. Removeable discontinuity: A function $f(x)$ is said to ~~be~~ have an removeable discontinuity at $x=a$ if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) \neq f(a)$ or $f(a)$ can not be defined.

ex: If $f(x) = \frac{x^2-a^2}{x-a}$, then $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) \neq f(a)$

3. Infinite discontinuity: A function $f(x)$ is said to have an infinite discontinuity at $x=a$ if one or both of $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ tend to $+\infty$ or $-\infty$. Here, $f(a)$ may or may not exist.

ex: If $f(x) = \frac{x^2}{x-3}$, then $\lim_{x\to 3^-} f(x) \to -\infty$ and $\lim_{x\to 3^+} f(x) \to +\infty$ but $f(3)$ can not be defined.

4. Oscillatory discontinuity:

Let, $f(x) = \sin\frac{1}{x}$, here $\sin 1/x$ oscillates between -1 and $+1$ and at $x=0$, $f(x)$ is discontinuous.

Differentiability

Derivative / Differential co-efficient: Let, $f(x)$ be a function defined on $[a,b]$ and $c \in (a,b)$. Then, $f(x)$ is said to be diffrentiable / derivable at $x=c$ if $\lim_{x\to c} \frac{f(x)-f(c)}{x-c}$ exists and finite. This limit is known as the derivative of $f(x)$ at

$x=c$ and which we denote it by $f'(c)$. That is,

$$f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c} \quad \dots (1)$$

If we put $x=c+h$, then $h\to 0$ (but +ve) as $x\to c$

Therefore, (1) becomes $f'(c) = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$

⊞ Here, $\lim_{x\to c^+} \frac{f(x)-f(c)}{x-c}$ or $\lim_{h\to 0^+} \frac{f(c+h)-f(c)}{h}$ $[x=c+h, h\to 0]$

is called Right hand derivative of $f(x)$ at $x=c$ and which we define by $Rf'(c) = \lim_{h\to 0^+} \frac{f(c+h)-f(c)}{h} = \lim_{x\to c^+} \frac{f(x)-f(c)}{x-c}$

And $\lim_{x\to c^-} \frac{f(x)-f(c)}{x-c}$ or $\lim_{h\to 0^+} \frac{f(c-h)-f(c)}{-h}$ $[x=c-h, h\to 0]$

is called Left hand derivative which we define by $Lf'(c) = \lim_{h\to 0^+} \frac{f(c-h)-f(c)}{-h} = \lim_{x\to c^-} \frac{f(x)-f(c)}{x-c}$

⊞ Now, if $Lf'(c) = Rf'(c) =$ finite, then we say that $f(x)$ is differentiable at $x=c$.

<u>Theorem</u>: Every differentiable function is continous but the converse is not always true.

<u>Proof</u>: Let, $f(x)$ be any differentiable function at $x=a$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

Now, we write,

$$f(a+h)-f(a) = \frac{f(a+h)-f(a)}{h} \cdot h$$

$$\Rightarrow \lim_{h\to 0}\{f(a+h)-f(a)\} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \cdot \lim_{h\to 0} h$$

$$\Rightarrow \lim_{h\to 0} f(a+h)-f(a) = f'(a)\times 0$$

$$\Rightarrow \lim_{h\to 0} f(a+h) = f(a)$$

$$\Rightarrow \lim_{h\to a} f(x) = f(a)$$

$a+h=x$
$h\to 0$
$x\to a$

$\Rightarrow f(x)$ is continious at $x=a$.

For the converse case we consider $f(x)=|x|$ which is obiviously continous at $x=0$.

We find, $Rf'(0) = \lim_{h\to 0^+}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+}\frac{h-0}{h} = 1$

We find, $Lf'(0) = \lim_{h\to 0^+}\frac{f(0-h)-f(0)}{h} = \lim_{h\to 0^+}\frac{-(-h)-0}{-h} = -1$

Since, $Lf'(0) \neq Rf'(0)$

$\Rightarrow f(x)$ is not differntiable.

Page: 188 (17) If $f(x) = \begin{cases} 1+x, & x<0 \\ 1, & 0\le x\le 1 \\ 2x^2+4x+5, & x>1 \end{cases}$

find $f'(x)$ for all values of x for which it ~~exi~~ exists. Does $\lim_{x\to 0} f'(x)$ exists?

Soln:

For, $x<0$, $f(x)=1+x$, $f'(x)=1$

For, $0\leq x\leq 1$, $f'(x)=0$

For, $x>1$, $f'(x)=4x+4$

Now, test the differentiability,

For, $x=1$ $Lf'(1)=\lim\limits_{h\to0^+}\frac{f(1-h)-f(1)}{-h}=\lim\limits_{h\to0^+}\frac{1-1}{-h}=0$

$Rf'(1)=\lim\limits_{h\to0^+}\frac{f(1+h)-f(1)}{h}=\lim\limits_{h\to0}\frac{2(1+h)+4(1+h)+5-1}{h}=\infty$

$\therefore f'(x)$ does not exists at $x=1$.

Again, for $x=0$, $Lf'(0)=1$ and $Rf'(0)=0$

$\Rightarrow Lf'(0)\neq Rf'(0)$

$\therefore f'(x)$ does not exist at $x=0$

Here, $f'(x)$ exists excluding $x=0,1$

For, $x>0$, $f'(x)=1$

For, $x=0$, $f'(x)$ does not exists

For, $x<0$, $f'(x)=1+x$

$\Rightarrow \lim\limits_{x\to0}f'(x)$ does not exist.

Page : 103 (7) $f(x)=[x]+[-x]$

Let, $x=k$ be an integer

$\therefore [x]=k$, $[-K]=-k$

$$\lim_{x \to k^+} f(x) = \lim_{h \to 0} f(k+h)$$

$$= \lim_{h \to 0} [k+h] + \lim_{h \to 0} [-k-h], \ h \to 0$$

$$= k - (k+1) = -1 \quad \text{but } f(k) = 0$$

So, f has a discontinuity at $x = k$, where k is any integer. If define $f(k) = -1$, then the function becomes continous at $x = k$.

$$Rf'(0) = \lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0^+} \frac{\{3-2(0+h)\}-3}{h}$$

$$= \lim_{h\to 0^+} \frac{-2h}{h} = -2$$

Since, $Lf'(0) \neq Rf'(0)$

$f(x)$ does not exist at $x=0$.

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8/ $f(x) = 1$ when $x<0$

$= 1 + \sin x$ when $0 \le x \le \pi/2$

$= 2 + (x - \frac{\pi}{2})^2$ when $\frac{\pi}{2} \le x$

$$Rf'(\pi/2) = \lim_{h\to 0} \frac{f(\pi/2+h) - f(\pi/2)}{h}$$

$$= \lim_{h\to 0} \frac{2+(\pi/2+h-\pi)^2 - \{2+0\}}{h}$$

$$= \lim_{h\to 0} \frac{2+h^2-2}{h} = 0$$

$$Lf'(\pi/2) = \lim_{h\to 0} \frac{f(\pi/2-h) - f(\pi/2)}{-h}$$

$$= \lim_{h\to 0} \frac{1+\sin(\pi/2-h)-2}{-h}$$

$$= \lim_{h\to 0} \frac{-1+\cos h}{-h} = 0$$

$\therefore f'(x)$ exists at $x = \pi/2$

Again, $Lf'(0) = \lim_{h \to 0} \dfrac{f(0-h) - f(0)}{-h}$

$= \lim_{h \to 0} \dfrac{1 - (1 + \sin 0)}{-h}$

$= 0$

$Rf'(0) = \lim_{h \to 0} \dfrac{f(0+h) - f(0)}{h}$

$= \lim_{h \to 0} \dfrac{1 + \sin h - 1}{h}$

$= 1$

So, $f'(x)$ does not exist at $x = 0$

10(1)/ $f(x) = x \; ; \; 0 < x < 1$

$= 2 - x \; ; \; 1 \leq x \leq 2$

$= x - \frac{1}{2}x^2 \; ; \; x > 2$

consider the point $x = 1$ for continuity,

L.H.L

$\lim_{x \to 1-0} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} (1-h) = 1$

R.H.L

$\lim_{x \to 1+0} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} 2 - (1+h) = 1$

Also, $f(1) = 1$

$\therefore f(x)$ is continuous at $x=1$.

Consider the point $x=2$

L.H.L

$$\lim_{x\to 2-0} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0} 2-(2-h) = 0$$

R.H.L

$$\lim_{x\to 2+0} f(x) = \lim_{h\to 0} f(2+h) - \frac{1}{2}(2+h)^2 = 0$$

$\therefore f(x)$ is continuous at $x=2$.

For derivative at $x=1$

$$Lf'(1) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0} \frac{(1-h)-1}{-h} = 0$$

$$Rf'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{2(1+h)-1}{h} = -1$$

$\therefore f'(1)$ does not exist.

Again derivative at $x=2$,

$$Lf'(2) = \lim_{h\to 0} \frac{f(2-h)-f(2)}{-h} = \lim_{h\to 0} \frac{2-(2-h)-0}{-h} = -1$$

$$Rf'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{(2+h)-\frac{1}{2}(2+h)^2-0}{h} = -1$$

$\therefore f'(2)$ exists and it's value $=-1$

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+

17/ $f(x) = 1+x \; ; \; x<0$

$= 1 \; ; \; 0 \le x \le 1$

$= 2x^2+4x+5 \; ; \; x>1$

When, $x<0$, $f(x) = 1+x \quad \therefore f'(x) = 1$

When, $x>1$, $f(x) = 2x^2+4x+5 \quad \therefore f'(x) = 4x-4$

$$Lf'(0) = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0} \frac{1-h-1}{-h} = 1$$

$$Rf'(0) = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{1-1}{h} = 0$$

$\therefore f'(0)$ does not exist.

Again, $$Lf'(1) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{x\to 0} \frac{1-1}{-h} = 0$$

$$Rf'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{1(1+h)^2+4(1+h)+5-1}{h} = \infty$$

$\therefore f'(1)$ does not exist.

18/(1) $f(x) = -\frac{x^2}{2} \; ; \; x \le 0$

$= x^n \sin\frac{1}{x} \; ; \; x>0$

When, $n=1$,

$$Lf'(0) = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0} \frac{-h^2/2-0}{-h} = 0$$

$$Rf'(0) = \lim_{h\to 0} \frac{f(0)h-f(0)}{h} = \lim_{h\to 0} \frac{h\sin 1/h-0}{h}$$ which does not exist

$\therefore f'(0)$ does not exist.

When, $n=2$

$Lf'(0) = 0$ (same as $n=1$)

$$Rf'(0) = \lim_{h\to 0} \frac{f(0)+h-f(0)}{h} = \lim_{h\to 0} \frac{h^2 \sin 1/h - 0}{h} = 0$$

$\therefore f'(0)$ does not exist.

18/(ii) $f(x) = [x]$

When, $1 \le x \le 2$, $f(x) = 1$ $\therefore f'(x) = 0$

when, $2 \le x < 3$, $f(x) = 2$ $\therefore f'(x) = 0$ and so on.

$$Lf'(1) = \lim_{h\to 0} \frac{f(1-h)-f(1)}{-h} = \lim_{h\to 0} \frac{1-1}{-h} = 0$$

$$Rf'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{2-1}{h} = \infty$$

$\therefore f'(1)$ does not exist.

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Ex. 4/ We have, $|x| = x$ when $x > 0$
$= 0$ when $x = 0$
$= -x$ when $x < 0$ ... (1)

and, $|x-2| = x-2$. when $x > 2$
$= 0$ when $x = 2$
$= 2-x$ when $x < 2$ ... (2)

From, (1) and (2), $f(x) = 2x + (2-x)$
$= x+2$ when, $0 < x < 2$

$$f'(x) = \lim_{x\to1} \frac{f(x)-f(1)}{x-1}$$

$$= \lim_{x\to1} \frac{(x+2)-3}{x-1}$$

$$= \lim_{x\to1} \frac{x-1}{x-1} = 1 \quad [\because x-1 \neq 0]$$

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12(ii) Here, $f(x) = x^2$; $x > 0$

$= 0$; $x = 0$

$= -x^2$; $x < 0$

$$Lf'(0) = \lim_{x\to0^-} \frac{f(x)-f(0)}{x-0} = \lim_{x\to0^-} \frac{-x^2}{x} = 0$$

$$Rf'(0) = \lim_{x\to0^+} \frac{f(x)-f(0)}{x-0} = \lim_{x\to0^+} \frac{x^2}{x} = 0$$

$\therefore f(x)$ is differentiable at $x = 0$ and $f'(0) = 0$.