CSE 211: Discrete Mathematics **(Due: 17/01/21)**

Homework #4

Instructor: Dr. Zafeirakis Zafeirakopoulos *Name:* Yakup Talha Yolcu *Student Id:* 1801042609
Assistant: Gizem Süngü *1801042609*

Problem 1 (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.
(Solution)
If we replace $a_n = -2^{n+1}$ we will get :
-2^{n+1}= 3 (-2^n) + 2^n
$-2^{n+1} = -2^{n+1}$ we can't have a solution by this equation

(b) Find the solution with $a_0 = 1$.
(Solution)
if we replace a_n = x
characteristic equation = x-3 roots is x=3
my particular guess is A2^n if I replace a_n = A2^n
A2^n=3 A$2^n - 1$ + 2^n
We have A=-2
so $a_n = \alpha 3^n$ - 22^n
If we try $a_0 = 1$ we will get:
1=α-2 so α=3
a_n = 3 3^n - 2 2^n

Problem 2 (35 points)

Solve the recurrence relation f(n) = 4f(n-1) - 4f(n-2) + n^2 for f(0) = 2 and f(1) = 5.
(Solution)
a_n = 4 a_{n-1} - 4 a_{n-2} + n^2 for a_0 = 2 and a_1 = 5
Characteristic equation: x^2 - 4x + 4 = 0
Roots are 2 and 2 so homogenius $a_n = \alpha 2^n + \beta n 2^n$
Particular guess is : $a_n = A_2 n^2 + A_1 n + A_0$
So $n^2 = A_2 n^2 + A_1 n + A_0$ - 4 $(A_2(n-1)^2 + A_1(n-1) + A_0)$ + 4 $(A_2(n-2)^2 + A_1(n-2) + A_0)$
$n^2 = A_2 n^2 + A_1 n + A_0$ - $4A_2 n^2$ + $8A_2 n$ - $4A_2$ - 4 $A_1 n$ + $4A_1$ - $4A_0$ + $4A_2 n^2$ - 16 $A_2 n$ + $16A_2$ + 4 $A_1 n$ - $8A_1$ + $4A_0$
If we compare the n^2 's coefficients 1=A_2 - $4A_2$ + $4A_2$ so A_2=1
If we compare n's coefficients 0 = A_1+8A_2-4A_1-16A_2+4A_1
0=A_1+8-4A_1-16+4A_1
8=A_1
If we compare the constants coefficients: 0=A_0-4A_2-4A_0+16A_2-8A_1+4A_0
0=A_0-4-4A_0+16-64+4A_0 52=A_0
so our guess is become : $a_n = n^2$ + 8n + 52
$a_n = \alpha 2^n + \beta n 2^n + n^2$ + 8n + 52
α= -50 and β = 22 so a_n = (-50)2^n + $22n2^n$ + n^2 + 8n + 52

Problem 3 (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.
(Solution) characteristic equation is: x^2 - 2x + 2 = 0
$\Delta = b^2$ - 4ac=-4¡0
roots are : 1 - i and 1 + i so we will have :
$a_n = \alpha(1-i)^n + \beta(1+i)^n$

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.
(Solution) $a_n = \alpha(1-i)^n + \beta(1+i)^n$
if we try for a_0=α + β = 1
if we try for a_1=$\alpha(1-i)$ + $\beta(1+i)$ = 2
if we replace α=1-β 1-β (1-i) + β(1+i)=2
2βi=1+i multiply both sides with i
-2β = i - 1 so β = (1-i)/2
α=1-(1-i)/2 = (1 + i)/2
so $a_n = \frac{1+i}{2}(1-i)^n + \frac{1-i}{2}(1+i)^n$