CSE211 Homework1

Yakup Talha Yolcu

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1 Problem 1

- a) If it snows tonight, then I will stay at home.
  Converse:If I will stay at home, it snows tonight.
  Contrapositive:If I will not stay at home, it doesn't snow tonight
  Inverse:If it doesn't snow tonight, I will not stay at home

- b) I go to the beach whenever it is a sunny summer day.
  Converse: It is a sunny summer day whenever I go to the beach
  Contrapositive:Neither it is not a sunny summer day nor I go to the beach
  Inverse:I don't go to the beach and It is not a sunny summer day

- c)If I stay up late, then I sleep until noon
  Converse:If I sleep until noon then I stay up late
  Contrapositive:If I don't sleep until noon then I don't stay up late
  Inverse:If I don't stay up late then I don't sleep until noon

2 Problem 2

- a) $(p \bigoplus \neg q)$

p	q	$\neg$ q	p $\bigoplus \neg$q
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

- (b) $(p \Leftrightarrow q) \bigoplus (\neg p \Leftrightarrow \neg r)$

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p	q	r	$\rightharpoonup r$	$p \Leftrightarrow q$	$\rightharpoonup p \Leftrightarrow \rightharpoonup r$	$(p \Leftrightarrow q) \bigoplus (\rightharpoonup p \Leftrightarrow \rightharpoonup r)$
T	T	T	F	T	F	T
T	T	F	T	T	T	F
T	F	F	T	F	T	T
T	F	T	F	F	F	F
F	T	T	F	F	T	T
F	T	F	T	F	F	F
F	F	F	T	T	F	T
F	F	T	F	T	T	F

- (c) $(p \bigoplus q) \Rightarrow (p \bigoplus \rightharpoonup q)$

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p	q	$\rightharpoonup q$	$p \bigoplus q$	$p \bigoplus \rightharpoonup q$	$(p \bigoplus q) \Rightarrow (p \bigoplus \rightharpoonup q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

3 Problem 3

- P(x): "x can speak English."
- Q(x): "x knows Python."
- H(x): "x is happy."

(a) There is a student at the university who can speak English and who knows Python.
(Solution) $\exists x(P(x) \wedge Q(x))$

(b) There is a student at the university who can speak English but who doesn't know Python.
(Solution) $\exists x(P(x) \wedge \rightharpoonup Q(x))$

(c) Every student at the university either can speak English or knows Python.
(Solution) $\forall x(P(x) \vee Q(x))$

(d) No student at the university can speak English or knows Python.
(Solution) $\exists x(P(x) \vee Q(x))$

(e) If there is a student at the university who can speak English and know Python, then she/he is happy.
(Solution) $\exists x((P(x) \wedge Q(x)) \Rightarrow H(x))$

(f) At least two students are happy.
(Solution) $\exists x1 \exists x2(P(x1), P(x2))$

(g) $\neg\forall x(Q(x) \wedge P(x))$
(Solution) "There is not a student who knows Python and there is not a student can speak English"

4 Problem 4

Prove that $3 + 3 \cdot 5 + 3 \cdot 5^2 + \ldots + 3 \cdot 5^n = \frac{3(5^{n+1}-1)}{4}$ whenever n is a nonnegative integer.
Solution:

In Mathematical Induction we try for n=1
1st step:For n=1 Equation becomes: $3 + 3.5 = \frac{3(5^{1+1}-1)}{4}$ so 18=18 n=1 satisfies this equation

2nd step:We accept that equation satisfies for n=k
$3 + 3 \cdot 5 + 3 \cdot 5^2 + \ldots + 3 \cdot 5^k = \frac{3(5^{k+1}-1)}{4}$

3rd step:We will try for n=k+1 to prove the equation

$3 + 3 \cdot 5 + 3 \cdot 5^2 + \ldots + 3 \cdot 5^k + 3 \cdot 5^{k+1} = \frac{3(5^{k+1+1}-1)}{4}$

If we subtitue 2nd step equation from 3rd step equation we will get:

$3 \cdot 5^{k+1} = \frac{3(5^{k+1+1}-1)}{4} - \frac{3(5^{k+1}-1)}{4}$ If we multiply by 4 and divide by 3 two side of the equation we will get:

$4 \cdot 5^{k+1} = (5^{k+2} - 1) - (5^{k+1} - 1)$
$20 \cdot 5^k = 5^k \cdot 25 - 1 - 5^k \cdot 5 + 1$
$20 \cdot 5^k = 20 \cdot 5^k$
So for n=k+1 equation is satisfied and it means that this equation is true

5 Problem 5

Prove that n^2 - 1 is divisible by 8 whenever n is an odd positive integer
Solution:

Let's say n=2k-1 because of the term that n is an odd positive integer
For Inductıon's 1st step, let's say n=3
$3^2 - 1 \equiv 0 \pmod 8$
$8 \equiv 0 \pmod 8$

For Induction's 2st step accept this equation satisfies for n=2 . k - 1

(2 . k - 1) ^{2} - 1 ≡ 0 (mod 8)
4 . k ^{2} - 4 . k + 1 - 1 ≡ 0 (mod 8)
4 . k ^{2} - 4 . k ≡ 0 (mod 8)

For Induction's 3rd step, we will try for n = 2 . k + 1 (n is still odd positive integer)
(2 .k + 1) ^{2} - 1 ≡ 0 (mod 8)
4 .k ^{2} + 4 .k + 1 - 1 ≡ 0 (mod 8) we can write 4 .k ^{2} + 4.k as 4 .k ^{2} - 4 .k + 8.k
4 . k ^{2} - 4 . k + 8 . k ≡ 0 (mod 8) 4 . k ^{2} - 4 . k was divisible by 8
8 . k ≡ 0 (mod 8) this equation satisfies the main equation
We concluded that n^2 - 1 is divisible by 8 whenever n is an odd positive integer

6 Problem 6

Which of the following sets are equal? Show your work step by step.
a) {t : t is a root of x^2 – 6x + 8 = 0}
b) {y : y is a real number in the closed interval [2, 3]}
c) {4, 2, 5, 4}
d) {4, 5, 7, 2} - {5, 7}
e) {q: q is either the number of sides of a rectangle or the number of digits in any integer between 11 and 99}

Solution: a)roots of the x^2 – 6x + 8 = 0} can be find by factoring
(x-4)*(x-2)=0
(x-4).(x-2)=0
x=4 and x=2 the A set is
A={2, 4}

b)B set has infinite number elements so it can be expressed as:
B={2, ..., 2.5, ...3}

c) C set is C={4, 2, 5, 4}

d) D set is D={4, 5, 7, 2} - {5, 7} this means that
D={4, 2}

e) Number of the sides of a rectangle is 4
Number of digits in any integer between 11 and 99 is 2
But in the expression it says that either 4 or 2
This means that there should be just 1 element in the set
So E={2} or E={4}

We find A set and D set are equal
A=D=$\{2, 4\}$

7 Problem Bonus

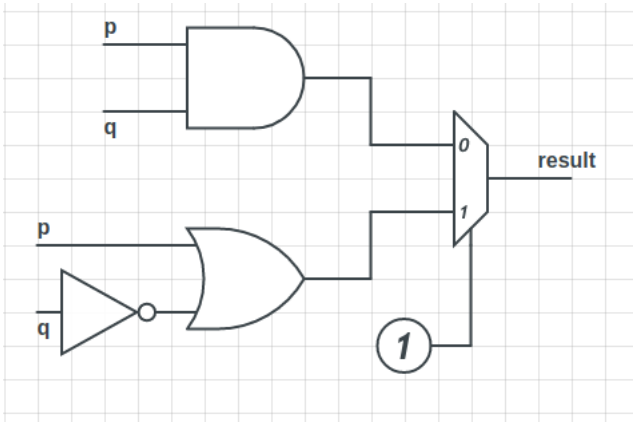

Figure 1: Combinational Circuit

- p : It is sunny.
- q : The flowers are blooming.

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p	q	$\neg$q	p $\wedge$ q	p $\vee \neg$ q
T	T	F	T	T
T	F	T	F	T
F	T	F	F	F
F	F	T	F	T

Circuit can be expressed as (p $\wedge$ q) Multiplexer (p $\vee \neg$ q)
Multiplexer selection is 1
These 3 are connected to OR gate
Circuit becomes

Multiplexer is 1 so result will be p $\vee \neg$ q
It means It is sunny or The flowers are not blooming

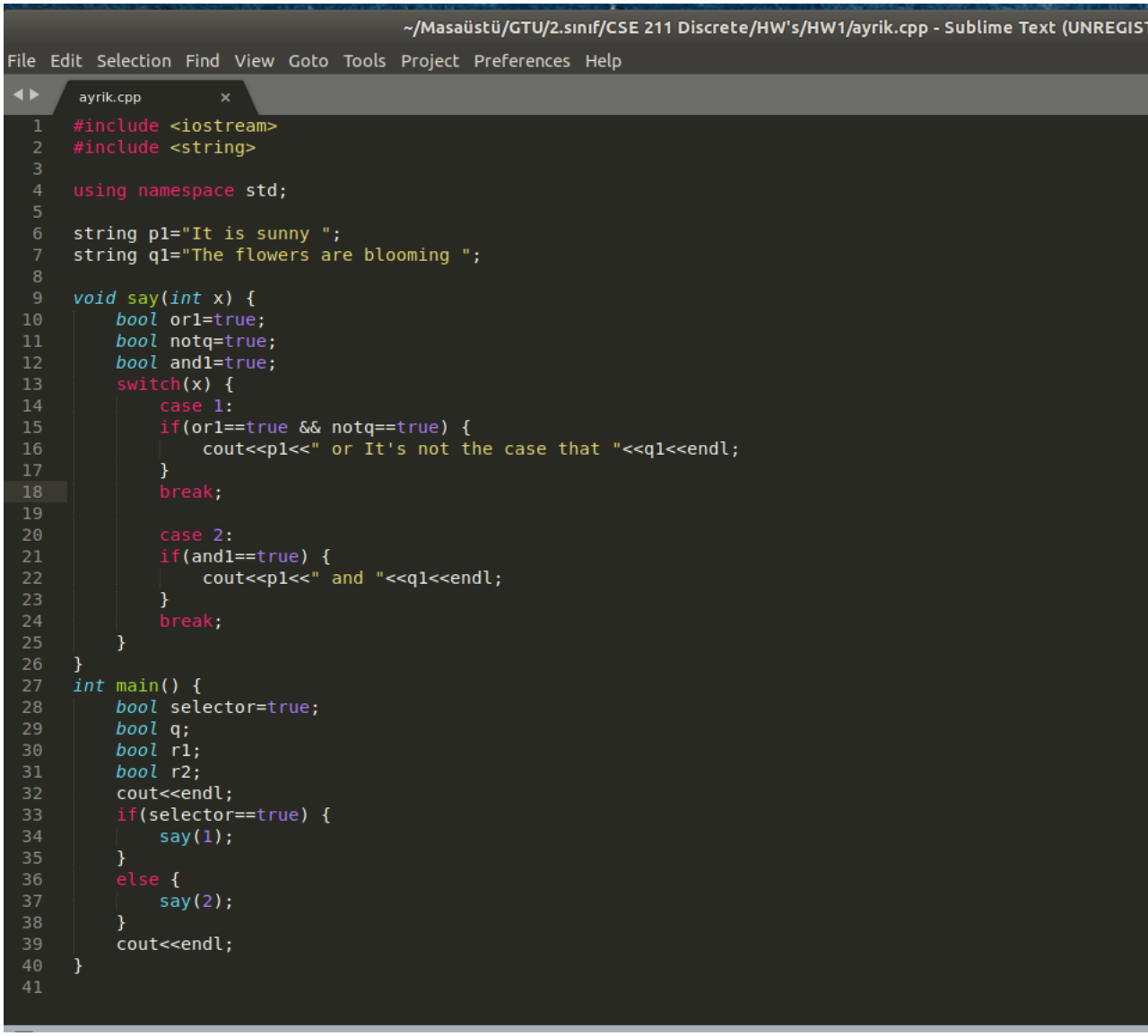

Figure 2: Code