

GEBZE TECHNICAL UNIVERSITY
DEPARTMENT OF COMPUTER ENGINEERING
CSE222/505 – Spring 2021
Homework 2 Report

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Part 1

I wrote new code for these algorithms. I did not want to use my previous homework codes.

1. Searching a product

Time complexity is just $\Theta(n)$ because for loop will be executed n times because of the branch products's length. In println and get methods, we have constant time.

Time complexity = $\Theta(n)$

```
public void search(String model_name, String color_name) {
    for(int i=0; i<branch_products.length; i++) {
        System.out.println(branch_products.get(i));
    }
}
```

2. Add/remove product.

i) Add product

Time complexity for the worst case is $\Theta(n)$ (amortized) for reallocation + $\Theta(1)$ for get_size method.

Time complexity for the best case is $\Theta(1)$ because of get_size method.

So, at the end our time complexity is $O(n)$.

```
public void add_product(Product a1) {
    if(get_size()==branch_products.length) {
        reallocate();
    }
    branch_products[get_size()+1]=a1;
    size++;
}
```

ii) Remove product

To remove a product, we need to find this product by giving color and model names.

After finding it, we need to decrement the stock.

We have 2 for loops and 2 if's for each. For the worst case we will have $\Theta(n*m)$ complexity n for -> model name, m for -> color name.

For the best case we have $\Theta(1)$ complexity. So our complexity is $O(n*m)$

Getters are considered as constant time.

```
public void remove_product(String color, String model, int stock) {
    for(int i=0; i<branch_products.length; i++) {
        if(model.equals(branch_products.get(i).get_name())) {
            for(int j=0; j<branch_products.get(i).colors.length; j++) {
                if(color.equals(branch_products.get(i).colors.get(j).get_name())) {
                    (branch_products.get(i).colors.get(j).set_stock(get_stock()-stock))
                }
            }
        }
    }
}
```

3. Querying the products that need to be supplied.

Admin can query by giving a branch to the system and learn whether product need to be supplied.

Time complexity is constant time because we have getter function and we have comparison.

$$T(n) = \Theta(1)$$

```
public boolean query(Branch b1){  
    return !(b1.get_stock()==false);  
}
```

Part 2

- a) It's meaningless to say : "The running time of an algorithm A is at least $O(n^2)$ " because when we are using Big Oh notation we are indicating that our algorithm's running time is less than or equal to n^2
- b) We know $\Theta(f(n)+g(n)) = O(f(n)+g(n)) = \Omega(f(n)+g(n))$

$$\max(f(n), g(n)) \geq g(n) \text{ if } \max(f(n), g(n)) = f(n)$$

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Let's say that $\max(f(n), g(n)) = f(n)$ and let $T(f(n)) = \Theta(n)$

So, there must be $\Theta(n) \geq g(n)$

$g(n)$ could be $\Theta(n)$ or $\Theta(1)$

Let's say that $g(n) = \Theta(n) \rightarrow$ then $O(f(n)+g(n)) = O(n+n)$

$n+n$'s time complexity is $\Theta(n)$. $\max(f(n), g(n))$ was $f(n)$ and it was $\Theta(n)$.

It is appropriate.

Let's say that $g(n) = \Theta(1)$. Then $O(f(n)+g(n)) = O(n+1)$.

$O(n+1) = \max(f(n), g(n)) \Rightarrow O(n+1) = \Theta(n)$. yes it is proved.

If we had chose the $g(n)$ as maximum, we would get the same result.

- c)
 - 1) $2^{n+1} = \Theta(2^n)$
 $2^{n+1} = 2^n * 2$, so we have lower order term as 2. We can remove it.
 $2^n = \Theta(2^n)$ we can say this because Θ checks order of magnitude.
It is true
 - 2) $2^{2n} = \Theta(2^n)$
 $2^{2n} = (2^2)^n = 4^n$ so we can't say that. It is wrong
 - 3) Let $f(n) = O(n^2)$ and $g(n) = \Theta(n^2)$. Prove or disprove that: $f(n) * g(n) = \Theta(n^4)$.

$O(n^2)$ means that $f(n)$'s complexity is greater than or equal to n^2 .

$\Theta(n^2)$ means that $g(n)$'s complexity is exactly n^2 .

Let's check $O(n^2) * \Theta(n^2) = \Theta(n^4)$

We can say $O(n^2) = O(n^3)$. So $O(n^3) * \Theta(n^2) = O(n^5)$.

$O(n^5) = \Theta(n^4)$ for just some cases, not always. So we can't say this statement is absolutely correct.

Part 3

$n^{1.01}$, $n \log^2 n$, $2^n n^{1/2}$, $\log^3 n$, $n 2^n$, 3^n , 2^{n+1} , $5^{\log n}$, $\log n$

We know exponential functions are the largest ones.

We need to start from 2^n , $n 2^n$, 3^n , 2^{n+1} , $5^{\log n}$.

$5^{\log n}$ will be the least because $\log n$ is less than n .

2^n and 2^{n+1} will be equal because of constant 2.

Let's say $\lim (3^n) / (n 2^n)$, n goes infinity, we will get 0. It means that $(n 2^n)$ is larger.

Up to now we have $3^n > n 2^n > 2^{n+1} = 2^n > 5^{\log_2 n}$

Remained: $n^{1.01}$, $n \log^2 n$, $n^{1/2}$, $\log^3 n$ and $\log n$

We know $n^{1.01} > n \log^2 n > \log^3 n > \log n$

If we have $\lim (n \text{ goes infinity}) (n^{1/2}) / \log^3 n = (\text{By L'Hopital}) (1/(2 * n^{1/2})) / (1/n(\ln 2)) * 3$ close the infinity. We will get $\ln 2$ at the end. So $(n^{1/2})$ is larger.

The growth order is ->

$3^n > n 2^n > 2^{n+1} = 2^n > 5^{\log_2 n} > n^{1.01} > n \log^2 n > \sqrt{n} > \log^3 n > \log n$

Part 4

1) Minimum valued item

```
1  ArrayList<int> x;  
2  set min=x.get(0);  
3  for (i in iterable x) {  
4      if i<min do  
5          i=min  
6      end  
7  }  
8  return min
```

Time complexity is -> in for loop we have if statement. In if statement we have comparison and it is constant time. We have $\Theta(1)$. Loop is executed n times. So we have $\Theta(n) * \Theta(1) = \Theta(n)$. Its complexity is $\Theta(n)$.

2) Median item

- For set $j=0, j<n, j++$
 - -Determine minimum and maximum of the ArrayList // $\Theta(n)$
- -While
 - -For set $i=0, i<n, i++$ // $\Theta(n^2)$
 - If $\text{min} < \text{ArrayList}[i]$ and there is no such element between these two values // $\Theta(n)$
Min=ArrayList[i]
 - If $\text{max} > \text{ArrayList}[i]$ and there is no such element between these two values // $\Theta(n)$
Max=ArrayList[i]
 - If max equals min, break
 - If absolute value of max-min less than or equal to 1, break

//For best case, while will be executed one time, and complexity will be $\Theta(n^2)$

//For the worst case, while will be executed $n/2$ times, So complexity will be $\Theta(n^2 * n/2)$ which is $\Theta(n^3)$

//Total complexity will be $O(n^3)$

- Return $(\text{min} + \text{max}) / 2$

3) Find two elements whose sum is equal to given value.

- Set boolean flag=false // $\Theta(1)$
- For $i=0, i<n, i++$ // $\Theta(1)$ for best case, $\Theta(n^2)$ for worst case (in total)
 - For $j=0, j<n, j++$ // $\Theta(1)$ for best case, $\Theta(n)$ for worst case
 - If (the value that in the i th index) + (the value that in the j th index) equals the given value // $\Theta(1)$
 - Flag=true
 - Break
 - If flag equals true // $\Theta(1)$
 - Break
- We have i and j now
- **//At the end, we have $O(n^2)$**

4) Assume there are two ordered array list of n elements. Merge these two lists to get a single list in increasing order.

- For $i=0, i < n+m, i++$ // $\Theta(n+m)$
 - If $i < n$ // $\Theta(1)$
 - Set list's ith index value as the first list's ith index value
 - Else $\Theta(1)$
 - Set list's ith index value as the second list's i-n'th index value
- Sort the merged list // $O(n \log n)$ considered as merge sort
- **So our time complexity is $O(n \log n)$ at the end**

Part 5

1)

```
int p_1 (int array[]): {  
    return array[0] * array[2] // Complexity is  $\Theta(1)$   
    //Total time complexity is  $\Theta(1)$   
    //Total space complexity is  $S(1)$   
}
```

2)

```
int p_2 (int array[], int n): {  
    Int sum = 0 //  $\Theta(1)$   
    for (int i = 0; i < n; i=i+5) //  $\Theta(n)$   
        sum += array[i] * array[i] //  $\Theta(1)$   
    return sum //  $\Theta(1)$   
  
    //Total space complexity is  $S(1)$   
    //Total time complexity is  $\Theta(1) + \Theta(n) + \Theta(1) = \Theta(n)$   
}
```

3)

```
void p_3 (int array[], int n): {  
    for (int i = 0; i < n; i++) //  $\Theta(n) * O(\log n) = O(n * \log n)$   
        for (int j = 1; j < i; j=j*2) //  $O(\log n)$   
            printf("%d", array[i]*array[j]) //  $\Theta(1)$   
    }  
  
    //Total space complexity is  $S(1)$   
    //Total time complexity is  $\Theta(1) * O(n * \log n) = O(n * \log n)$ 
```

4)

```
void p_4 (int array[], int n): {  
    if (p_2(array, n)) > 1000) //Space complexity = S(1) T(n)=  $\Theta(n)$   
        p_3(array, n) // Space complexity = S(1) T(n)= $O(n*\log n)$   
    else printf(“%d”, p_1(array) * p_2(array, n)) // Space complexity= S(1)  
    //T(n)=  $\Theta(n)$  +  $\Theta(1)$  =  $\Theta(n)$   
  
    //Best case S(n)= Worst case S(n)=S(1)+S(1)  
  
    //Best case T(n)=  $\Theta(n)$ +  $\Theta(n)$ =  $\Theta(n)$   
    //Worst case T(n)  $\Theta(n)$ +  $O(n*\log n)$ =  $O(n*\log n)$   
    Total time complexity =  $O(n*\log n)$   
    Total space complexity = S(1)  
  
}
```

//I thought that space complexity does not depend on the input size, so all space complexities are 1 as constant