CSE 321 HW1 Yakup Talha Yolcu 1801042609

1) a) $(2^n+n^3) \in O(4^n)$ $\quad 2^n+n^3 \leq c.4^n$ $\quad n \geq n_0$ let $c=1$, $n_0=2$ $n=2$

$(2^2+2^3)=12 \leq 1.4^2 \Rightarrow 12 \leq 16$ true ✓ Big-O means less than or equal to.

b) $\sqrt{10n^2+7n+3} \in \Omega(n) \Rightarrow 10n^2+7n+3 \geq c.n$ $\quad n \geq n_0$ let $c=1$ $n_0=1$ $n=1$

$\sqrt{10.1+7.1+3} \geq 1.1 \Rightarrow \sqrt{20} \geq 1$ true ✓ Big-Omega means greater than or equal to

c) $n^2+n \in o(n^2) \Rightarrow n^2+n < c.n^2$ $\quad n \geq n_0$ Little-o means less than.

$1^2+1 < 1.1^2$ $\quad 2<1$ ✗ False. let $c=1$ $n=1$ $n_0=1$

d) $3\log_2^2 n \in \Theta(\log_2 n^2) \Rightarrow c_1.\log_2 n^2 \leq 3\log_2{}^2 n \leq c_2.\log_2 n^2$ $\quad n \geq n_0$

Theta means that equals. let $c_1=1$ $\quad c_2=2$ $\quad n=n_0=16$

$1.\log_2 16^2 \leq 3\log_2^2 16 \leq 2\log_2 16^2 \Rightarrow 8 \leq 3\log_2(\log_2 16) \leq 16$

$8 \leq 6 \leq 16$ ✗ false

e) $(n^3+1)^6 \in O(n^3)$ $\quad (n^3+1)^6 \leq c.n^3$ $\quad n \geq n_0$ Big-O means less than or equal to

$(1^3+1)^6 \leq 1.1^3 \Rightarrow 32 \leq 1$ false ✗ let $c=1$ $n=1$ $n_0=1$

2) a) $2n\log(n+2)^2+(n+2)^2\log\frac{n}{2} \Rightarrow c_1.g(n) \leq 2n\log(n+2)^2+(n+2)^2\log\frac{n}{2} \leq c_2.g(n)$

$n \geq n_0$ $\quad c_1, c_2$ constant

So $1.g(n) \leq 2.2.\log(2+2)^2+(2+2)^2\log(\frac{2}{2}) \leq 4.g(n)$ $\quad$ Let $c_1=1$ $c_2=4$ $n=2$

$g(2) \leq 4\log 16+16\log 1 \leq 4.g(2) \Rightarrow g(2) \leq 16\log 2 \leq 4.g(2)$

$\boxed{g(n) \text{ is } n^2\log n}$ $\quad$ Let $g(n)=n^2\log n$

$=g(2)=4\log 2$ $\quad$ it's true ✓

b) $10^{-3}n^4+3n^3+1 \Rightarrow c_1.g(n) \leq 10^{-3}n^4+3n^3+1 \leq c_2.g(n)$ $\quad$ Let $g(n)=n^4$

$10^{-2}.n^4 \leq 10^{-3}.n^4+3n^3+1 \leq 1.n^4$ $\quad n \geq n_0$ $\quad c_1=10^{-2}$ $\quad c_2=1$

$100 \leq 10+3000+1 \leq 10^4$ $\quad$ let $n=10$

$100 \leq 3011 \leq 10000$ ✓

$g(n)=n^4$

$10^{-2}.g(10) \leq 10^{-3}.10^4+3.10^3+1 \leq g(10)$

$\frac{g(10)}{100} \leq 3011 \leq g(10)$ $\quad$ Let $g(n)=n^4$ $\quad g(10)=10000$

3) a) $\log n, n^{\log n}, n^{1.5}$ First compare $\log n$ and $n^{1.5}$ $\lim_{n\to\infty} \frac{\log n}{n^{1.5}} = \frac{1/n}{1.5 n^{0.5}} = \frac{\text{Number}}{\text{Infinity}}$ (L'Hospital) $\Rightarrow \sim 0$

Result of limit is close to 0, this means that $\log n < n^{1.5}$

Compare $n^{1.5}$ and $n^{\log n}$ $\lim_{n\to\infty} \frac{n^{1.5}}{n^{\log n}} = n^{(1.5-\log n)} = n^{1.5-\infty} = n^{-\infty} = \frac{1}{n^{\infty}} \approx 0$

So $n^{1.5} < n^{\log n}$ So growth order is $\log n < n^{1.5} < n^{\log n}$

b) $n!, 2^n, n^2$ Stirling for $n! \approx \sqrt{2\pi n}\cdot\left(\frac{n}{e}\right)^n$ for large n

First compare n^2 and 2^n $\lim_{n\to\infty} \frac{n^2}{2^n}$ (Lopital) $\Rightarrow \lim_{n\to\infty} \frac{2n}{2^n \ln 2}$ (Lopital) $\Rightarrow \frac{2}{2^n \ln 2 \ln 2} = \frac{\text{Number}}{\infty} \sim 0$

Result of limit close to 0.

So $n^2 < 2^n$

Compare 2^n with $n!$ $\lim_{n\to\infty} \frac{n!}{2^n}$ (Stirling) $= \lim_{n\to\infty} \frac{\sqrt{2\pi}\, n \cdot n^{2n}}{e^n \cdot 2^n}$

$\lim_{n\to\infty} \frac{n^{\frac{2n+1}{2}}\cdot\sqrt{2\pi}}{2^n \cdot e^n} \Rightarrow \left(\frac{n}{2e}\right)^n \cdot \sqrt{2\pi n} \sim \infty$

$n! > 2^n$ So growth order is $n^2 < 2^n < n!$

c) $n\log n, \sqrt{n}$ $\lim_{n\to\infty} \frac{\sqrt{n}}{n\log n}$ (Lopital) $\Rightarrow \lim_{n\to\infty} \frac{\frac{1}{2} n^{-1/2}}{\log n + \frac{1}{\ln 10}} \Rightarrow \frac{1}{2\left(\log n + \frac{1}{\ln 10}\right)\cdot\sqrt{n}} \Rightarrow \frac{\text{Number}}{\infty} \sim 0$

Growth order is $\sqrt{n} < n\log n$

$(n\log n)' = \log n + n\cdot\frac{1}{n \ln 10}$

d) $n\cdot 2^n, 3^n$ $\lim_{n\to\infty} \frac{3^n}{n\cdot 2^n} = \lim_{n\to\infty} \frac{\left(\frac{3}{2}\right)^n}{n}$ (Lopital) $= \lim_{n\to\infty} \frac{\ln 3/2 \cdot \left(\frac{3}{2}\right)^n}{1} \Rightarrow \infty$

Growth order is $3^n > n\cdot 2^n$

e) $\sqrt{n+10}, n^3$ $\lim_{n\to\infty} \frac{\sqrt{n+10}}{n^3}$ (Lop) $= \lim_{n\to\infty} \frac{\frac{1}{2}(n+10)^{-\frac{1}{2}}}{3n^2} = \lim_{n\to\infty} \frac{1}{6\cdot n^2\cdot\sqrt{n+10}} \Rightarrow \frac{\text{Number}}{\infty} \approx 0$

Growth order is $\sqrt{n+10} < n^3$

4) a) Basic operation: Independent from worst case, basic operation is checking equality of elements - comparison of pairs. It is most used operation and it is the operation that contributing most to the total running time.

b) At worst case we should have reach return true statement or we should return false at the last iteration of the for loops. According to them

Sum expression is $= \sum_{i=0}^{n-2}\sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} (n-1) - (i+1) + 1 = \sum_{i=0}^{n-2} (n-i-1) = (n-1)+(n-2)+ \ldots n-(n-2)-1$

$= (n-1)(n) \ldots 1$

$= \frac{(n-1)\cdot(n)}{2} = \frac{n^2-n}{2}$ times needed.

c) To calculate the time complexity firsty we should identify basic operation. We determined it. Then we should find how many times this basic operation is executed. We found it. Then we converted summation to closed formula. Then we should eliminate low-order terms and constant factors => $\frac{n^2-n}{2}$ => n^2

$f(n) \in \Theta(g(n))$ iff $c_1 \cdot g(n) \le f(n) \le c_2 \cdot g(n)$ $n \ge n_0$

$c_1 \cdot n^2 \le \frac{n^2-n}{2} \le c_2 \cdot n^2 \Rightarrow$ $n=4$ $c_1 = 1/10$ $c_2=10$

$\frac{1}{10} \cdot 4^2 \le \frac{4^2-4}{2} \le 10 \cdot 4^2$

$\frac{16}{10} \le 6 \le 160$ ✓ it is true So $\frac{n^2-n}{2}$ is $\Theta(n^2)$

5) a) Basic operation is multiplication. But we also have addition and assignment.

b) $\left(\sum_{i=0}^{n-1}\left(\sum_{j=0}^{n-1}\left(\sum_{k=0}^{n-1} 1\right)\right)\right) = \left(\sum_{i=0}^{n-1}\left(\sum_{j=0}^{n-1}(n-1-0+1)\right)\right) = \left(\sum_{i=0}^{n-1}(n.(n-1+1))\right) = (n-1+1)(n)(n)$

n^3 Basic operation n^3 times executed

Remove constants and lower order terms => n^3 Time complexity is $\Theta(n^3)$

There is no branching, so best - average - worst cases are the same

6) algorithm (int desired, A[0, ... n-1])

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for (i = 0 to n-1) do
    for (j = i+1 to n-1) do
        if (A[i] * A[j] == desired)
            print (A[i], A[j])
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Time complexity => Basic operation is multiplication.

$\sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} 1 =$ Worst case - average case - best case are the same

$= \sum_{i=0}^{n-1}((n-1)-(i+1)+1) = \sum_{i=0}^{n-1}(n-i-1) = n-1 + \ldots 0$

$= \frac{(n-1).(n)}{2} = \frac{n^2-n}{2}$

Remove constants and lower order terms => n^2

$c_1 n^2 \le \frac{n^2-n}{2} \le c_2 n^2$ $n=4$ $c_1 = 1/10$ $c_2 = 10$

$\frac{16}{10} \le \frac{12}{2} \le 160$ $1{,}6 \le 6 \le 160$ So $\frac{n^2-n}{2}$ is $\Theta(n^2)$

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