CSE 211: Discrete Mathematics **(Due: 07/12/20)**

Homework #2

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Problem 1: Relations (15 points)

Draw the Hasse diagram for the "greater than or equal to" relation on {0, 1, 2, 3, 4, 5}.

(Solution)

Relation is :
{ (0,0) , (1,0) , (1,1) , (2,0) , (2,1) , (2,2) , (3,0) , (3,1) , (3,2) , (3,3) , (4,0) , (4,1) , (4,2) , (4,3) , (4,4) , (5,0) , (5,1) , (5,2) , (5,2) , (5,3) , (5,4) , (5,5) }

Hasse diagram for ({0, 1, 2, 3, 4, 5}, $\leq$) is:

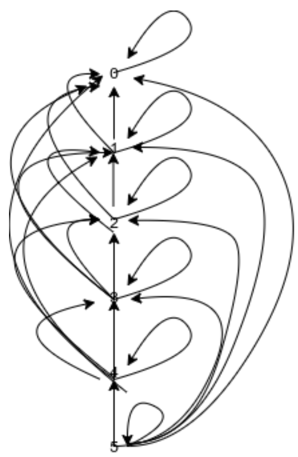

Figure 1: Original Graph

Since we know that a poset MUST provide reflexivity, we also do not need the reflexive relations in A. Hence A can be updated as: A = { (1,0) , (2,0) , (2,1) , (3,0) , (3,1) , (3,2) , (4,0) , (4,1) , (4,2) , (4,3) , (5,0) , (5,1) , (5,2) , (5,2) , (5,3) , (5,4) } In the next step, remove the self-loops in Figure 2:

Remove the transivite edges and the hasse diagram is obtained in Figure 3:

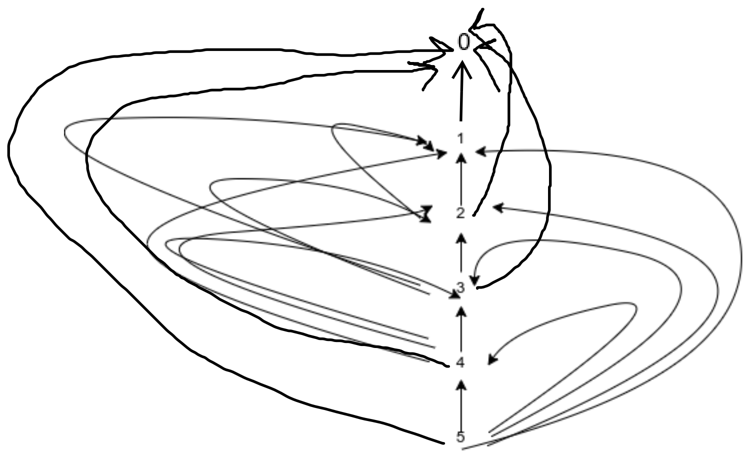

Figure 2: The graph without self-loops

Figure 3: The hasse diagram of ($\{\ 0\ ,\ 1\ ,\ 2\ ,\ 3\ ,\ 4\ ,\ 5\}$, $\leq$)

Problem 2: Relations (15 points)

Answer these questions for the poset ({{1}, {2}, {4}, {1, 2}, {1, 4}, {2, 4}, {3, 4}, {1, 3, 4}, {2, 3, 4}}, $\subseteq$).

Before the solution , we can draw a Hasse diagram for the POSET to determine maximal and minimal elements etc...

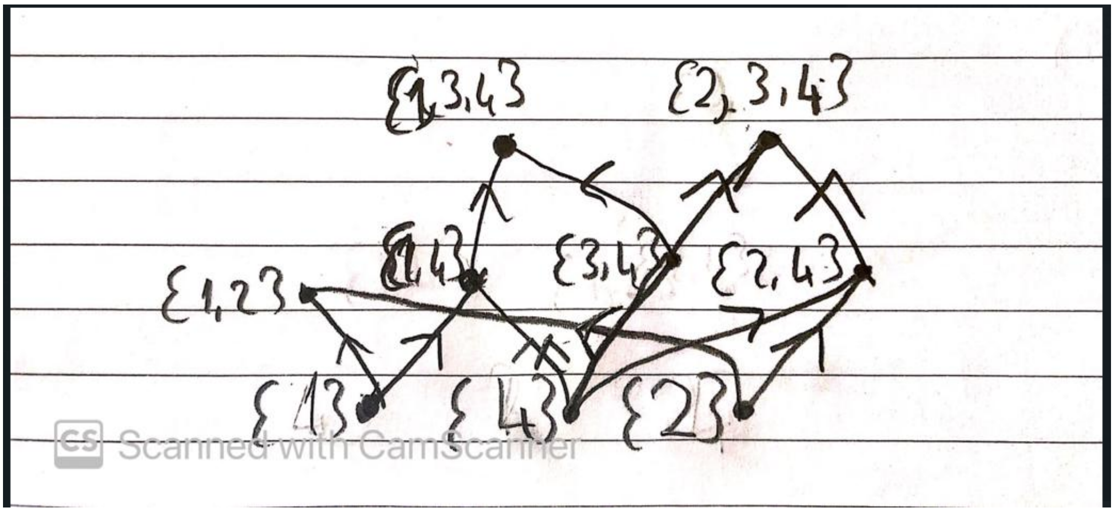

Figure 4: Hasse Diagram

(a) Find the maximal elements.
(Solution) Maximal elements are : {1,3,4} and {2,3,4} They are at the top of the diagram.

(b) Find the minimal elements.
(Solution) Minimal elements are : {1} , {2} and {4} They are at the bottom of the diagram.

(c) Is there a greatest element?
(Solution) There is no greatest element because there is more than one maximal element.

(d) Find all upper bounds of {{2} , {4}}.
(Solution) Upper bounds of {{2} , {4}} are {2,4} and {2,3,4}.

(e) Find the least upper bound of {{2}, {4}}, if it exists.
(Solution) Least upper bound of {{2}, {4}} exists and its {2,4}.

(f) Find all lower bounds of {{1, 3, 4}, {2, 3, 4}}.
(Solution) Lower bounds of $\{1, 3, 4\}$ are : {1} , {4} , {1,4} and {3,4}
Lower bounds of $\{2, 3, 4\}$ are : {2} , {4} , {3,4} , {2,4}

(h) Find the greatest lower bound of {{1, 3, 4}, {2, 3, 4}}, if it exists.
(Solution) Greatest lower bound of $\{1, 3, 4\}$ does not exists.
Greatest lower bound of $\{2, 3, 4\}$ does not exists.

Problem 3: Relations (70 points)

Remember that a relation R on a set A can have the properties reflexive, symmetric, anti-symmetric and transitive.

- **Reflexive:** R is reflexive if (a, a) $\in R$, $\forall$ a $\in$ A.
- **Symmetric:** R is symmetric if (b, a) $\in$ R whenever (a, b) $\in$ R, $\forall$ a, b $\in$ A.
- **Anti-symmetric:** R is antisymmetric if $\forall$ a, b $\in$ A, (a, b) $\in$ R and (b, a) $\in$ R implies that a = b.
- **Transitive:** R is transitive if $\forall$ a, b, c $\in$ A, (a, b) $\in$ R and (b, c) $\in$ R implies that (a, c) $\in$ R.