# Lab 5: T-tests & Power Analysis

*September 27, 2018*

The goal of this lab is to practice some of the concepts that we have been studying in lecture over the last two weeks, including testing for normality, conducting two-sample tests, and understanding p-values. The emphasis is on understanding Type I and Type II error, statistical power, and the elements of study design that affect power.

After this lab, you should be able to:

- Conduct two-sample and paired t-tests
- Appropriately report statistics and p-values
- Conduct power analysis for t-tests.

As always, work through the lab, running the example code by typing it into R or R Studio (do not copy and paste from the pdf). At the end of the lab, there are four problems to answer. Please answer the problems using R Markdown to show your code and any requested graphs. *Submit your answers and your R-code to the class Sakai site under the Assignments folder before midnight on either Mon., Oct. 1 (Section 03) or Wed., Oct., 3 (Sections 01 and 02).*

## More functions in R

In this lab, we introduce a few new R commands.

- `array()` - An array is a vector that can have one, two or more dimensions. A two dimensional array is the same as a matrix. Within array, `dim` gives the maximum indices in each dimension.
- `ceiling()` - rounds up to the nearest integer
- `floor()` - rounds down to the nearest integer
- `dim()` - determines the dimensions of a data frame or array
- `lines()` - adds lines to an existing plot
- `points()` - adds points to an existing plot
- `abline()` - adds one or more straight lines through a plot; the line can be specified by providing the y-intercept and slope
- `legend()` - adds a legend to a plot, by giving the x, y coordinates for the legend position and supplying the legend content (see graphing details `?legend`)

Here are some examples using the above functions. First, let's fill an array with 5 rows and 10 columns with integers from 1 to 50. `dimnames` defines the row and column names. Here the column names are not specified (`NULL`), but could be by providing a vector of names.

```
array(1:50, dim = c(5,10), dimnames = list(c("a", "b", "c","d","e"), NULL))
```

The second array has 3 rows, 4 columns, and 2 matrices.

```
array(1:24, dim=c(3,4,2))
```

We can also create an array by combining two existing matrices, `a` and `b`. The `dim` function provides the dimensions of the array. Arrays can also be indexed, just like vectors, data frames and matrices.

```
a <- matrix(8, nrow = 2, ncol = 3)
b <- matrix(9, nrow = 2, ncol = 3)
```

```
my.array <- array(c(a,b), c(2,3,2))
dim(my.array)

my.array[,,2]
my.array[1,,]
my.array[,2,]
```

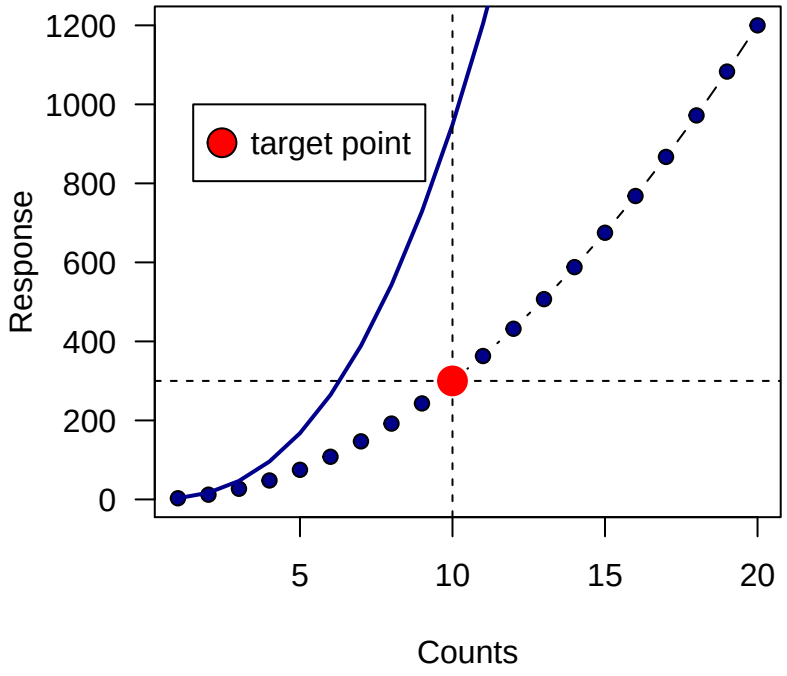
What do these functions do to the vectors of data?

```
ceiling(c(4.2, 5.5, 6.2))
floor(c(4.2, 5.5, 6.2))
```

We will make a quick (meaningless) plot using the power law function (not to be confused with statistical power) to employ `lines()`, `abline()` and `points()`. Run each line one at a time to check out how they change the plot. Play around with the graphing parameters, so that you are comfortable creating your own attractive graphs. In particular, try to get a sense of what the following parameters do: `lty`, `col`, `lwd`, `cex`, `cex.asix`, `las`, `ylim`, `xlim`, `ylab`, `xlab`, `bg`, `type`, `pch`.

```
x <- c(1:20)
a <- 3
b <- 2
b1 <- 2.5

plot(x, y=a*x^b, xlab = "Counts", ylab = "Response", las=1, pch=21,
    bg="darkblue", type = "b")
 lines(x, y=a*x^b1, lwd=2, col="darkblue")
  abline(h=300, lty=2)
   abline(v=10, lty=2)
  points(10, 300, pch=21, cex=2, col="red", bg ="red")
legend(1.5, 1000, c("target point"), pt.bg="red", pch=21, pt.cex=2)
```



## Understanding Power Analysis

Statistical tests are typically used to decide between two hypotheses: the null hypothesis, $H_0$, and the alternative hypothesis, $H_a$. We generally want statistical tests to provide us with evidence that we can reject the null and conclude that the impact or factor that we are studying has an effect. Of course, we are really interested in the strength of the effect, not just whether there is a difference between the null and the alternative hypothesis. With any statistical test there is always the possibility that we will find a difference between groups when one does not actually exist: called *Type I* error and denoted by $\alpha$. Likewise, it is possible that the test fails to detect a difference that really exists: called *Type II* error and denoted by $\beta$.

Statistical *power* is the ability to detect a statistically significant difference when the null hypothesis is false. It is the ability to detect a difference when a difference exists. The power of an experiment or study depends on the sample size, the effect size, and the $\alpha$ level. An effect size is a quantitative measure of the strength of a phenomenon. Generally speaking, as your sample size increases, so does the power of your test. This should intuitively make sense as a larger sample means that you have collected more information – which makes it easier to correctly reject the null hypothesis when you should.

Power analyses are often used prior to a study or experiment to determine an appropriate sample size. In this case, if we know the power of our test (or our desired power), we can find any of the quantities – sample size, effect size, and $\alpha$ – given that we know the other two quantities.

Knowing the desired power and $\alpha$ is pretty easy, but the effect size might be more challenging. A common measure of effect size is Cohen's $d$, which can be used when comparing two means, such as when you do a t-test. It is calculated by taking the difference between the two groups (e.g., mean of the treatment group minus the mean of the control group) and dividing it by the pooled standard deviation: $d = \frac{|\mu_1 - \mu_2|}{\sigma_p}$, where $\sigma_p = \sqrt{(\sigma_1^2 + \sigma_2^2)/2}$. Thus, Cohen's $d$ simply scales the difference in means by the number of standard deviations. With a $d$ of 1, we know that the two groups' means differ by one standard deviation; a $d$ of 0.5 tells us that the two groups' means differ by half a standard deviation, and so on. Cohen (1977, 1988) justifies three levels of effect sizes (small, medium, and large) for different types of statistical tests (see table below). Following this logic, if two groups' means differ by 0.2 standard deviations or less, the difference is trivial, even if it is statistically significant. Note that $d$ can be larger than 1 if there is a really big difference between two means.

| Statistic | Effect Size Index | Small | Medium | Large |
|---|---|---|---|---|
| t-test on means | $d$ | 0.20 | 0.50 | 0.80 |
| t-test on correlations | $r$ | 0.1 | 0.3 | 0.5 |
| F-test ANOVA | $f$ | 0.10 | 0.25 | 0.40 |
| F-test regression | $f^2$ | 0.02 | 0.15 | 0.35 |
| Chi-square test | $w$ | 0.10 | 0.30 | 0.50 |

Because effect size can only be calculated after you collect data, an estimate of effect size is often used for power analysis. It is typical to use an estimate from a prior study or to use a value of 0.5 as it indicates a moderate to large difference.

The R library, `pwr`, performs power analysis for proportions, t-tests, one-way ANOVA, correlation, and linear models. Download the `pwr` package and install it in your R library for use in this lab.

## T-tests & Power Analysis

Today we will focus on power analysis of t-tests, employing the functions: `pwr.t.test()` or `pwr.t2n.test()`. The first function is used if both samples have the same sample size, the second is used if they have different sample sizes. Type `?pwr` to see the other available functions for power calculations.

As an example, let's find the power of a one-tailed t-test, with a significance level of 0.05, an effect size $d$

equal to 0.75, and 25 subjects in each group. Is this a small or large effect size?

```
require(pwr)
pwr.t.test(n = 25, d = 0.75, sig.level=.05, alternative = "greater")
```

What is the power of this test? Note that you could specify, `alternative = "two.sided"`, `"less"`, or `"greater"` to indicate a two-tailed, or one-tailed test. A two-tailed test is the default. Change the above code to `"two.sided"` and see how this affects the power.

Note that if you know the power you want, you could find the sample size by changing $n$ to 'NULL':

```
pwr.t.test(n = NULL, d = 0.75, sig.level = 0.05, alternative = "greater", power = 0.9)
```

## Graphing Power, Effect Size, and Sample Size

What is the relationship between power, sample size, and effect size? To visualize it, let's graph sample size and effect size over a range of statistical power. Run the below code line-by-line so that you understand exactly what it is doing.

```
# Set the range of effect sizes, d

d <- seq(from = 0.1, to = 0.9, by = 0.01)
nd <- length(d)

# Set the range of power values, p

p <- seq(from = 0.4, to = 0.9, by = 0.1)
np <- length(p)

# Loop over 6 power values and 81 effect sizes to calculate the
# sample sizes. What does the samsize array contain?

samsize <- array(numeric(nd*np), dim=c(nd,np))

 for (i in 1:np){
  for (j in 1:nd){
    result <- pwr.t.test(n=NULL, d = d[j], sig.level = .05,
                         power = p[i], alternative = "two.sided")

      samsize[j,i] <- ceiling(result$n)
  }
}

# Set up the graph

xrange <- range(d)
yrange <- round(range(samsize))
colors <- rainbow(length(p))

plot(xrange, yrange, type="n", xlab="Effect size (d)",
        ylab="Sample Size (n)", las = 1)

# Add the power curves and make the graph pretty
```

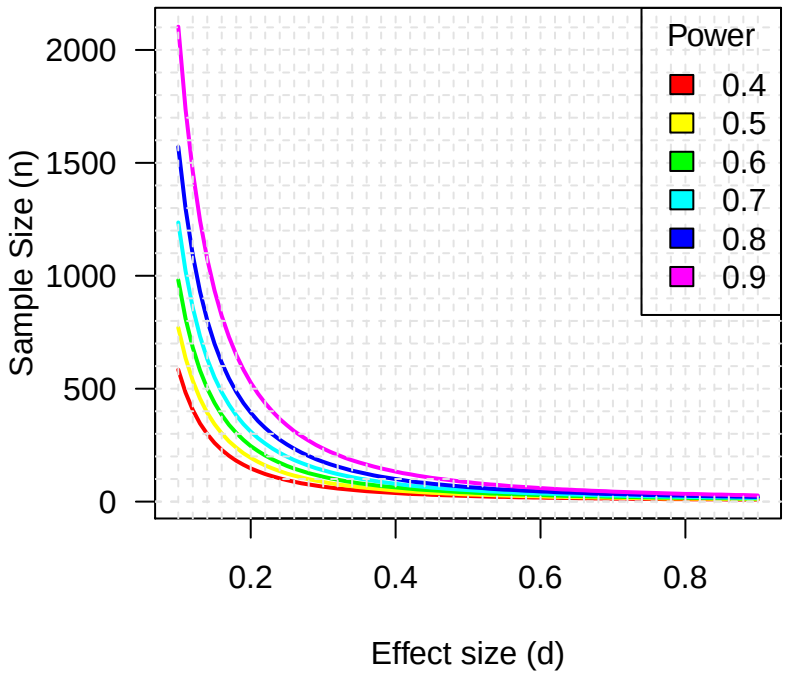



Figure 1: Sample size estimation for two-tailed t-test with sig=0.05

```
for (i in 1:np){
     lines(d, samsize[,i], type="l", lwd=2, col=colors[i])
}

abline(v=0, h=seq(0,yrange[2], by = 100), lty=2, col="grey89")
abline(h=0, v=seq(xrange[1],xrange[2], by = .02), lty=2, col="grey89")
    legend("topright", title="Power", as.character(p),
           fill=colors)
```

# Problems

## Problem 1

Analyze the following data. Write a brief paragraph that includes the following information: (a) your null and alternate hypotheses, (b) the type of test required (one-sample, two-sample, paired), (c) your critical statistic $t_c$, (d) your correctly reported results (e.g., ($t = 2.10$, df $= 32$, $p = 0.22$), and (e) a clear statement of your conclusion from the analysis. Finally, if you made an error in statistical hypothesis testing, what type of error would you have made and why?

A research study was conducted to examine the differences between older and younger adults on perceived life satisfaction. A pilot study was conducted to examine this hypothesis. Ten older adults (over the age of 50) and ten younger adults (between 20 and 30) were given a life satisfaction test (known to have high reliability and validity). Scores on the measure range from 0 to 60 with high scores indicative of high life satisfaction, and low scores indicative of low life satisfaction. The data are presented below. Compute the appropriate t-test.

young: 34, 22, 15, 27, 37, 41, 24, 19, 26, 36

```
old: 41, 24, 46, 39, 21, 33, 43, 40, 50, 41
```

## Problem 2

Using the life satisfaction data, now do the following: (a) calculate the effect size, $d$, for the study, (b) conduct a power analysis and report the statistical power. Write a sentence explaining what the power tells you.

## Problem 3

A researcher hypothesizes that electrical stimulation of the lateral habenula will result in decreased food intake (in grams) in rats. Rats undergo stereotaxic surgery and an electrode is implanted in the right lateral habenula. Following a ten-day recovery period, rats (kept at 80% body weight) are tested for the amount of food consumed during a 10-minute period of time both with and without electrical stimulation. The rats are tested under one of the treatments (with or without electrical stimulation), and then tested under the other treatment after a period of rest. Compute the appropriate t-test for the data provided below, and write a brief descriptive paragraph including all relevant information (as in Problem 1).

```
nostimulation: 18.4, 16.1,  9.2, 32.2, 13.8, 16.1, 27.6, 11.5, 11.5, 18.4,
               22.3, 21.1, 16.4, 29.5, 27.9

stimulation: 27.6, 16.1,  6.9, 25.3, 18.4, 11.5, 32.2, 16.1, 20.7, 23.0,
             24.7, 18.1, 26.5, 23.4, 8.3
```

## Problem 4

For your Master's project you have chosen to study whether exposure of Cardinal eggs to direct sunlight increases the mass of the hatchlings. You plan to locate Cardinal nests and open the vegetation around half of the nests (randomly chosen) to increase exposure to sunlight. A previous study on the nesting of the Eastern Bluebird found the effect size to be 0.4.

What sample size should you aim for to have sufficient power (0.80) to detect a difference? Alter the above power graph to highlight the needs of your project, assuming that the effect size could vary between 0.3 and 0.5 and that your desired statistical power is between 0.60 and 0.80. Remember that your hypothesis is a one-sided test, and so you need to take this into account in your graph. Add a horizontal black line on your power graph that crosses the power curves at the required sample size.

## Problem 5

Change the power curve graph in the lab text so that it plots the range of statistical power with sample sizes from 10 to 100. Make sure that sample size is on the x-axis and effect size is on the y-axis of your graph. What conclusion can you draw about your ability to conduct experiments with 80% power?