CSCI 6751 V1 | Artificial Intelligence

Midterm Examination

Oct 14, 2025

Total 50 points **Time: 50 minutes**

GOOD LUCK ***Group 1***

Student Name & ID _____

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/25	/25	/50

Question 1. (25 points)

You are working with a multivariate linear regression model with the hypothesis function (linear model): $y = \theta_0 + \theta_1 x_1 + \theta_2 x_2$

Given:

A single data point: $(x_1, x_2, y) = (1, 3, 8)$

Current model parameters: $\theta = [\theta_0, \theta_1, \theta_2] = [1, 0.5, 1]$

Learning rate for Gradient Descent: $\eta = 0.05$

L2 Regularization parameter: $\lambda = 0.5$

i) Using the single data point, perform one iteration of Gradient Descent.
   a) Calculate the prediction y.
   b) Calculate the prediction error.
   c) Calculate the updated parameter vector θ_new after one GD step.

ii) Using the initial parameters $\theta = [1, 0.5, 1]$, calculate the L2 regularized cost J_Ridge. (Assume we do not regularize θ_0).

Solution:

i)

a) Prediction $\hat{y}=\theta_0+\theta_1x_1+\theta_2x_2= 1+0.5\cdot1+1\cdot3=1+0.5+3=4.5$.

b) $e=\hat{y}-y=4.5-8=-3.5$.

c) MSE Error $J = (1/n) * \Sigma(y_i-\hat{y}_i)^2$

$$\frac{\partial J}{\partial \theta_j} = \frac{2}{n}(\hat{y} - y) \cdot \frac{\partial \hat{y}}{\partial \theta_j}$$

So

$$\frac{\partial J}{\partial \theta_0} = 2(-3.5)(1) = -7.0$$
$$\frac{\partial J}{\partial \theta_1} = 2(-3.5)(x_1) = 2(-3.5)(1) = -7.0$$
$$\frac{\partial J}{\partial \theta_2} = 2(-3.5)(x_2) = 2(-3.5)(3) = -21.0$$

$$\theta_j^{\text{new}} = \theta_j - \eta \frac{\partial J}{\partial \theta_j}$$

So

1. $\theta_0^{\text{new}} = 1 - 0.05(-7.0) = 1 + 0.35 = 1.35$
2. $\theta_1^{\text{new}} = 0.5 - 0.05(-7.0) = 0.5 + 0.35 = 0.85$
3. $\theta_2^{\text{new}} = 1 - 0.05(-21.0) = 1 + 1.05 = 2.05$

ii)

$$J_{\text{Ridge}} = \frac{1}{n}\sum(y_i - \hat{y}_i)^2 + \lambda(\theta_1^2 + \theta_2^2)$$

Given:

$$y = 8, \quad \hat{y} = 4.5, \quad \lambda = 0.5, \quad \theta_1 = 0.5, \quad \theta_2 = 1$$

Step 1: Squared error

$$(y - \hat{y})^2 = (8 - 4.5)^2 = (-3.5)^2 = 12.25$$

Step 2: Regularization term

$$\lambda(\theta_1^2 + \theta_2^2) = 0.5(0.5^2 + 1^2) = 0.5(0.25 + 1) = 0.5(1.25) = 0.625$$

Step 3: Total Ridge cost

$$J_{\text{Ridge}} = 12.25 + 0.625 = \boxed{12.875}$$

Question 2. (25 points)

System: Smart Coffee Maker Strength Control

Input Variables:

Coffee Bean Freshness (days since roast):

Fresh: trapmf([0, 0, 3, 5]) Medium: trapmf([3, 5, 10, 14]) Old: trapmf([10, 14, 21, 21])

Water Quality (ppm minerals):

Soft: trapmf([0, 0, 50, 100]) Medium: trapmf([50, 100, 150, 200]) Hard: trapmf([150, 200, 300, 300])

Output Variable: Brew Strength:

Mild: trapmf([0, 0, 3, 4]) Balanced: trapmf([3, 4, 6, 7]) Strong: trapmf([6, 7, 10, 10])

Rules:

IF Beans are Fresh AND Water is Soft THEN Strength is Mild

IF Beans are Medium AND Water is Medium THEN Strength is Balanced

IF Beans are Old AND Water is Hard THEN Strength is Strong

Current Input:

Bean Freshness = 6 days, Water Quality = 120 ppm

i) Calculate the firing strength of each rule

Solution:

Step 1 Membership values

- Bean Freshness = 6 → μ(Fresh)=0, μ(Medium)=1, μ(Old)=0
- Water Quality = 120 → μ(Soft)=0, μ(Medium)=1, μ(Hard)=0

Step 2: Firing strengths

- Rule 1 (Fresh & Soft → Mild): min(0,0) = 0
- Rule 2 (Medium & Medium → Balanced): min(1,1) = 1
- Rule 3 (Old & Hard → Strong): min(0,0) = 0