CSCI 6751 V1 | Artificial Intelligence

Midterm Examination

Oct 14, 2025

Total 50 points **Time: 50 minutes**

GOOD LUCK ***Group 2***

Student Name & ID _____

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/25	/25	/50

Question 1. (25 points)

You are working with a multivariate linear regression model with the hypothesis function (linear model):
$y = \theta_0 + \theta_1 x_1 + \theta_2 x_2$

Given:

A single data point: $(x_1, x_2, y) = (2, 2, 5)$

Current model parameters: $\theta = [\theta_0, \theta_1, \theta_2] = [0.2, 0.5, 0.1]$

Learning rate for Gradient Descent: $\eta = 0.01$

L2 Regularization parameter: $\lambda = 0.5$

i) Using the single data point, perform one iteration of Gradient Descent.
   a) Calculate the prediction y.
   b) Calculate the prediction error.
   c) Calculate the updated parameter vector θ_new after one GD step.

ii) Using the initial parameters $\theta = [0.2, 0.5, 0.1]$, calculate the L2 regularized cost J_Ridge. (Assume we do not regularize θ_0).

Solution:

Solution:

i)

a) Prediction $\hat{y}=\theta_0+\theta_1x_1+\theta_2x_2= 0.2+0.5\cdot2+0.1\cdot2=0.2+1.0+0.2=1.4$

b) $e=\hat{y}-y=1.4-5=-3.6$.

c) MSE Error $J = (1/n) * \Sigma(y_i-\hat{y}_i)^2$

$$\frac{\partial J}{\partial \theta_j} = \frac{2}{n}(\hat{y} - y) \cdot \frac{\partial \hat{y}}{\partial \theta_j}$$

So

$$\frac{\partial J}{\partial \theta_0} = 2 \cdot (-3.6) = -7.2$$

$$\frac{\partial J}{\partial \theta_1} = 2 \cdot (-3.6) \cdot 2 = -14.4$$

$$\frac{\partial J}{\partial \theta_2} = 2 \cdot (-3.6) \cdot 2 = -14.4$$

$$\theta_j^{\text{new}} = \theta_j - \eta \frac{\partial J}{\partial \theta_j}$$

So

$$\theta_0^{\text{new}} = 0.2 - 0.01 \cdot (-7.2) = 0.2 + 0.072 = 0.272$$

$$\theta_1^{\text{new}} = 0.5 - 0.01 \cdot (-14.4) = 0.5 + 0.144 = 0.644$$

$$\theta_2^{\text{new}} = 0.1 - 0.01 \cdot (-14.4) = 0.1 + 0.144 = 0.244$$

$$\boxed{\theta_{\text{new}} = [0.272, 0.644, 0.244]}$$

ii)

Given:

$y = 5, \hat{y} = 1.4, \lambda = 0.5, \theta_1 = 0.5, \theta_2 = 0.1$

Formula:

$$J_{\text{Ridge}} = \frac{1}{n} \sum (y_i - \hat{y}_i)^2 + \lambda(\theta_1^2 + \theta_2^2)$$

Step 1: Squared error

$$(y - \hat{y})^2 = (5 - 1.4)^2 = (3.6)^2 = 12.96$$

Step 2: Regularization term

$$\lambda(\theta_1^2 + \theta_2^2) = 0.5 \cdot (0.5^2 + 0.1^2) = 0.5 \cdot (0.25 + 0.01) = 0.5 \cdot 0.26 = 0.13$$

Step 3: Total Ridge cost

$$J_{\text{Ridge}} = 12.96 + 0.13 = 13.09$$

$$\boxed{J_{\text{Ridge}} = 13.09}$$

Question 2. (25 points)

System: Smart Blender Speed Control

Input Variables:

Fruit Ripeness (days since picked):

Unripe: trapmf([0,0,2,4]) Ripe: trapmf([2,4,6,8]) Overripe: trapmf([6,8,10,10])

Solid Content (% of solids in mix):

Low: trapmf([0,0,30,50]) Medium: trapmf([30,50,60,70]) High: trapmf([60,70,100,100])

Output Variable: Blender Speed:

Slow: trapmf([0,0,3,5]) Medium: trapmf([3,5,7,9]) Fast: trapmf([7,9,10,10])

Rules: R1: IF Fruit is Unripe AND Solid is High THEN Speed is Fast

R2: IF Fruit is Ripe AND Solid is Medium THEN Speed is Medium

R3: IF Fruit is Overripe AND Solid is Low THEN Speed is Slow

Current Input:

Fruit Ripeness = 5 days; Solid Content = 55%; **i)** Calculate the firing strength.

Solution

Step 1: Membership values

- Fruit Ripeness = 5
  - μ(Unripe) = 0
  - μ(Ripe) = 1
  - μ(Overripe) = 0
- Solid Content = 55%
  - μ(Low) = 0
  - μ(Medium) = 1
  - μ(High) = 0

Step 2: Firing strengths

- Rule 1 (Unripe & High → Fast): min(0, 0) = 0
- Rule 2 (Ripe & Medium → Medium): min(1, 1) = 1
- Rule 3 (Overripe & Low → Slow): min(0, 0) = 0