EEE3096S: Embedded Systems II

LECTURE 20:

INFORMATION THEORY AND SAMPLING

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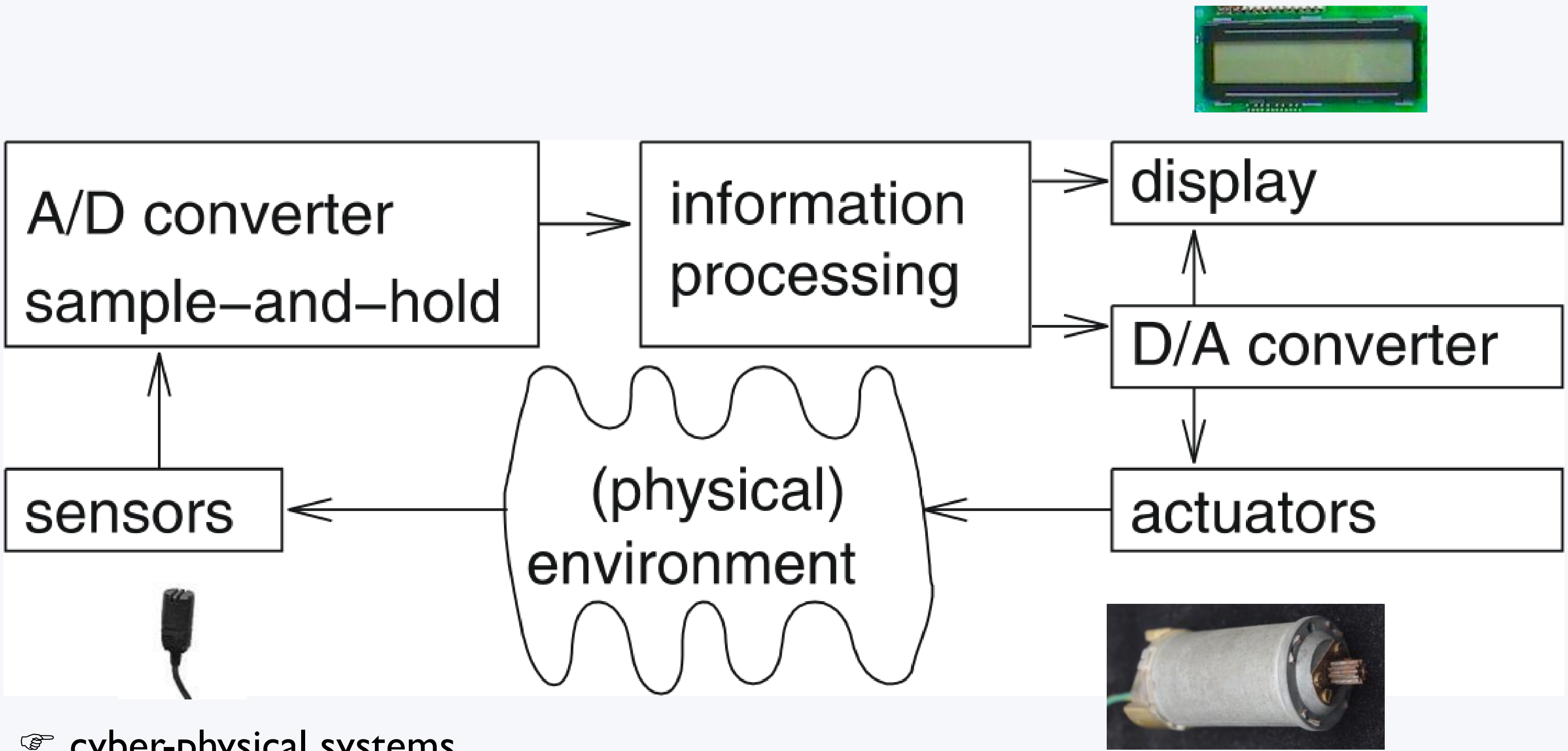
OUTLINE OF LECTURE

- Hardware in the loop
- Signals
- Sample-and-hold circuits (also called 'SAH')
- Sampling (recap)
- Shannon and Nyquist & relevance to ADCs

This presentation contains some slides adapted from course textbook "Embedded System Design Embedded Systems Foundations of Cyber-Physical Systems, and the Internet of Things" by Peter Marwedel

HARDWARE-IN-THE-LOOP (HITL)

Embedded system hardware is frequently used in a loop of sampling and actuating (i.e. ***"hardware in a loop"***):



☞ cyber-physical systems

MANY EXAMPLES OF SUCH LOOPS

- Heating
- Lights
- Engine control
- Power supply
- …
- Robots

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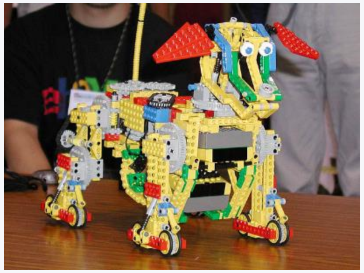
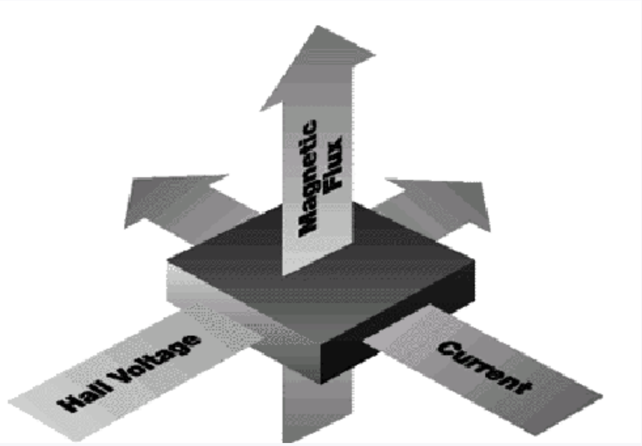


EXAMPLES OF COMMON SENSORS USED IN THE LOOP

- Rain sensors for wiper control
  (“Sensors multiply like rabbits“ [ITT automotive])
- Pressure sensors
- Proximity sensors
- Engine control sensors
- Hall effect sensors



SIGNALS AND SENSING

- Sensors detect aspects in the environment, possibly signals, movement or characteristics within different media (e.g. vibration, pressure)
- Sensors generate signals that computers can use
- Definition: a signal *s* is a mapping from the time domain D_T to a value domain D_V

$s : D_T \rightarrow D_V$

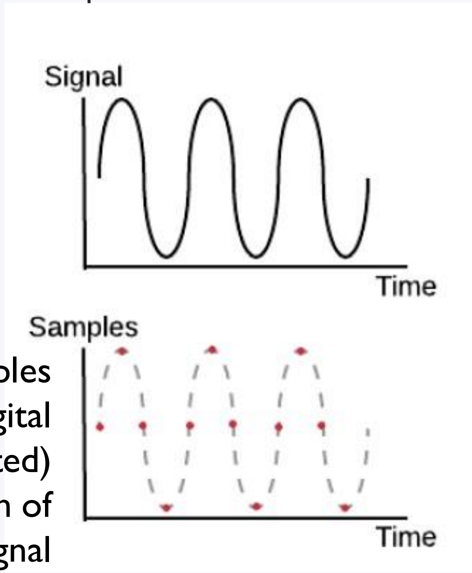
where

D_T : continuous or discrete time domain

D_V : continuous or discrete value domain

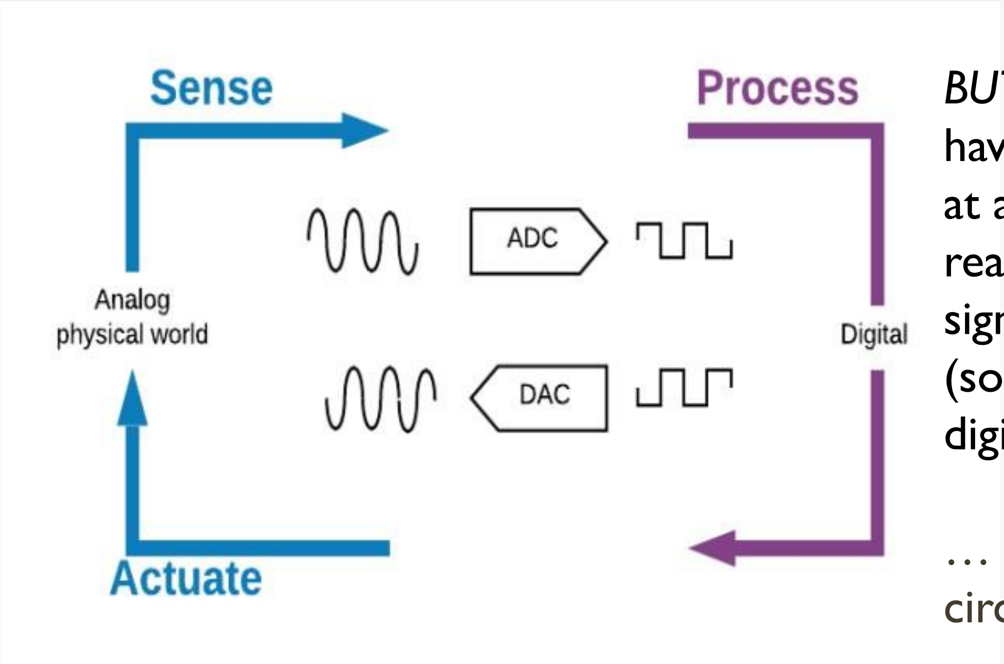
Real sampled signal

Series of Samples providing digital (approximated) representation of signal



REASONS FOR SAMPLING AND DIGITAL SIGNALS

- Digital signals are:
  - Less susceptible to noise



BUT a digital signal are discrete and have a finite representation of a signal at a certain time instance. Whereas real signals are continuous. Thus real signals tends to be approximated (sometimes very accurately) by a digital representation.

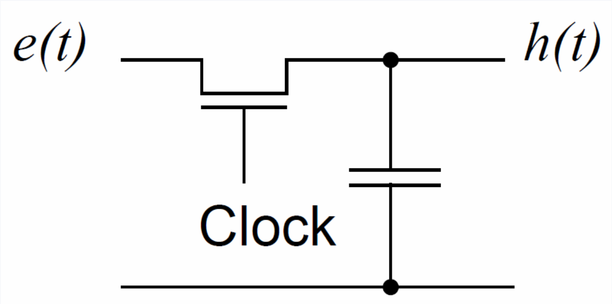
… and this leads us to the S/H circuit…

SAMPLE-AND-HOLD CIRCUITS (ALSO CALLED 'S/H')

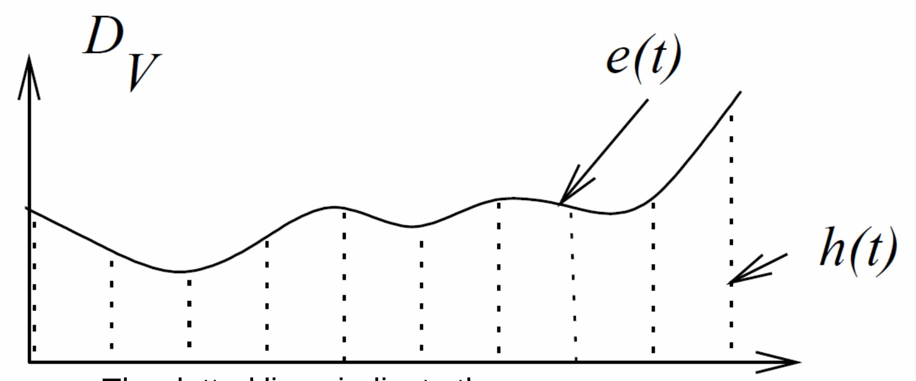
Clocked transistor + capacitor;

Capacitor stores sequence values

A good S/H is an essential part of an ADC because you ideally want to represent the voltage at a certain instant.



Simple S/H implementation



The dotted lines indicate the h(t) value held after a clock

$e(t)$ is a mapping $\mathbb{R} \rightarrow \mathbb{R}$

$h(t)$ is a **sequence** of values or a mapping $\mathbb{Z} \rightarrow \mathbb{R}$

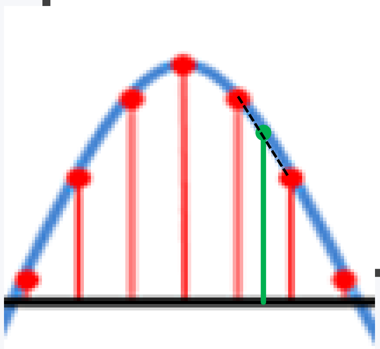
Basically what we are saying here is that you've got a certain countable number of instances that map to a value.

Where Z is of course integers (negative, zero, and positive)

Information Theory & Sampling

(a brief recap)

Simon's Guide To The Essence of Sampling (recap)



SAMPLING

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SAMPLING

Q: Do we lose information due to sampling? …

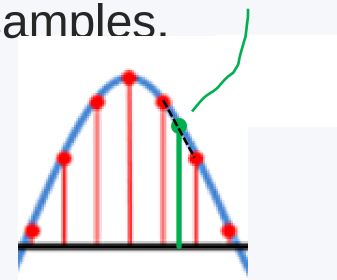
Well, this question might seem to most like asking "Is the pope catholic?"

A: In case you are wondering, the answer I'm looking for is: **Yes!**

But we may not need all the information that is lost, what we sample may be sufficient. For example, high logic may be +5V and low logic may be 0V with a +/- 1V tolerance. So if we got a 5.1V that extra 0.1V is not carrying any useful information.

SAMPLING & INTERPOLATION

- Sampling and quantization are important:
  - This translates the signal from the analog world to the digital world
- Sampling happens at particular instances*
- When the goal is to determine the value of the signal at some particular point in time, it may be necessary to use interpolation between actual samples.



** For all intense and purpose we can say 'instance' although the physics implies it is actually an averaged value over a tiny period starting at a particular moment.*

SHANNON'S THEOREM

- Shannon's Theorem gives an upper bound to the capacity of a link, in bits per second (bps), as a function of the available bandwidth and the signal-to-noise ratio of the link.
- The Theorem can be stated as:

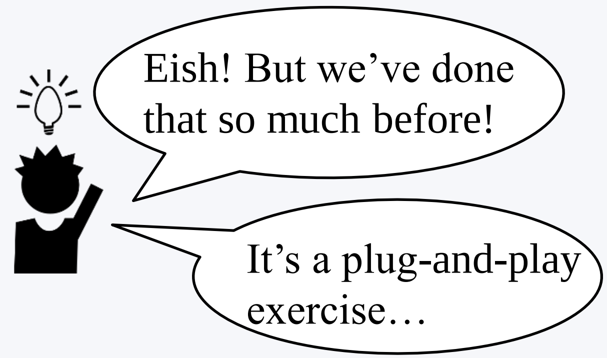
$C = B * \log_2(1+ S/N)$

where C is the achievable channel capacity,
B is the bandwidth of the line,
S is the average signal power and
N is the average noise power.

Example…

EXAMPLE (SHANNON'S THEOREM)

- For a typical telephone line with a signal-to-noise ratio of 30dB and an audio bandwidth of 3kHz, what do we get as the maximum data rate? . . .



Do this quickly on paper or using your calculator… how many kbps is needed

Answer…

Using $C = B * \log_2(1 + S/N)$ where C is the achievable channel capacity, B is the bandwidth of the line, S is the average signal power and N is the average noise power.

S/N bB $= 10\log_{10}(S/N) \rightarrow S/N = 10\text{^}3$

$C = 3000 * \log_2(1001)$

$= 29901$ bps

→ little less than **30 kbps**.

Hint S/N is essentially given already but may need to de-log it

Based on example from http://www.inf.fu-berlin.de/lehre/WS01/19548-U/shannon.html

LIMITATIONS OF SHANNON'S THEOREM

- The theorem gives max channel capacity for an 'archetypal' communication link
- It is a bound on the maximum amount of error-free digital data (i.e. information) that can be transmitted in a specified bandwidth given a certain amount of noise interference

In reality there may be many factors to make it impossible to achieve this 'Shannon optimal' e.g. imperfect sensors, timer drift, etc.

BUT technically you could also get 'super Shannon' rates using compression etc.

Good Nyquist Bad Nyquist

The Embedded Engineers Best Friend & Worst Nemesis

NYQUIST'S THEOREM

- Nyquist's Theorem:
  - "Any signal can be represented by discrete samples if the sampling frequency is <u>at least</u> twice the bandwidth of the signal."
  - i.e. $f_{sampling} > 2 \text{ x } f_{signal}$

    where $f_{sampling}$ is sampling rate and f_{signal} is the maximum frequency of the signal you want to sample.

Further reading: http://whatis.techtarget.com/definition/Nyquist-Theorem , https://en.wikipedia.org/wiki/Nyquist%E2%80%93Shannon_sampling_theorem

NYQUIST'S THEOREM

- A *bandlimited* analog signal that has been sampled can be perfectly reconstructed from an infinite sequence of samples if the sampling rate f_s exceeds $2f_{max}$ samples per second, where f_{max} is the highest frequency in the original signal.

  - If the analog signal does contain frequency components larger than $(1/2)f_s$, then there will be an aliasing error.
  - **Aliasing:** when the digital signal appears to have a different frequency than the original analog signal.

DIFFERENCE BETWEEN: SHANNON AND NYQUIST'S THEOREMS

- **Nyquist's Theorem** deals with the concept of <u>sampling rates</u>
  - This is extremely important for embedded engineers, since the sampling needs to be twice as fast a the speed of the signal changes that you want to detect.
  - The general rule of thumb is that your sampling rate must be at least 2x the frequency of the signal you are sampling.
- **Shannon's Theorem** has to do with <u>throughput</u>
  - No cable or signal is perfectly clean. There is a lot of interference from other sources that cause noise on the line. What Shannon's theorem does is predicts the speed of the data that you can push through the cable before noise becomes too much of a factor. (Looking at this aspect may be useful in the ECE design project)

Summary: Nyquist is all about sampling. Shannon is all about noise. They are related as speed of data impacts both theorems.

SHANNON & NYQUIST RELEVANT TO ADCS

- These formulae are useful tools for planning what ADCs you need and what your system will be capable of in terms of sampling and utilizing channels
  - Shannon will tell you the most you can get out of your channel (excluding ‘super Shannon’)
  - Nyquist will tell you what frequencies you can expect to get out for a certain ADC (assuming the ADC is linear and pretty perfect, we will see later how ADCs may be imperfect which may mean you might not even be able to reliably sample $F_S/2$ frequencies)

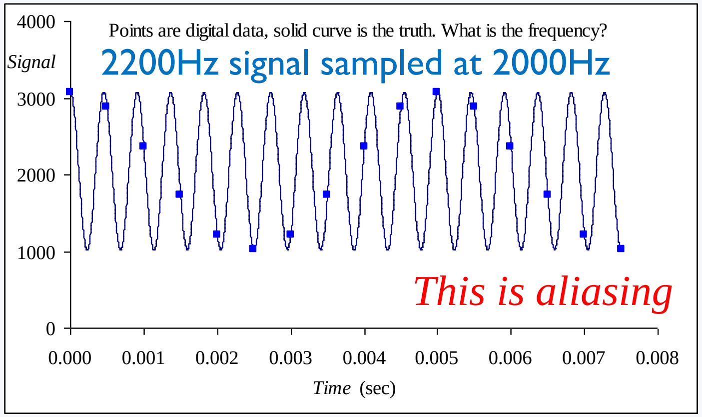
SAMPLING AND ALIASING

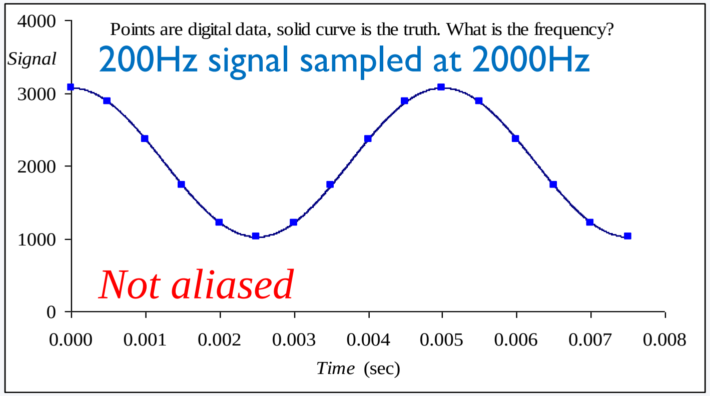
- Aliasing is an effect that causes different signals to become indistinguishable (or aliases of one another) when sampled.
- It also refers to the distortion or artefact that results when the signal reconstructed from samples is different from the original continuous signal.

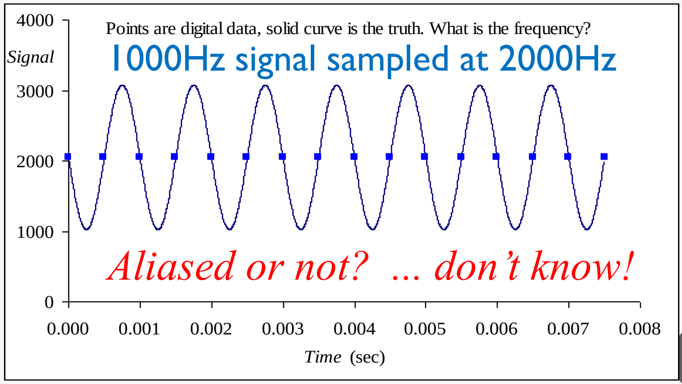
https://en.wikipedia.org/wiki/Aliasing

SAMPLING AND ALIASING

Some examples…







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TOWARDS ADCS…

- You probably suspected already… this is all getting your ready to dive into further aspects of ADCs, as essential part of many embedded products.

Next Episode: