

# COMP 598

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## Question 1

$$((ba)^* + b^*)^*b + \epsilon$$

## Question 2

First, we minimize this DFA. If it can be minimize as one state. Then we check if this state is accepted. If this state is accepted, then it can accept  $\Sigma^*$

## Question 3

### 3.1

It is decidable.

Since  $L$  is a CFL and  $R$  is a regular language. So their intersection is also a CFL. We know that whether a CFL is nil is decidable. So we done!

### 3.2

It is undecidable. To decide whether  $L \cap R = \Sigma^*$ , it is similar as whether  $L = \Sigma^*$ . But we know that this is undecidable. So we done!

## Question 4

It is context free but not regular.

It is not regular. Prove shown below.

$L \cap a^*c^* = \{a^n c^n | n \geq 0\}$ . Clearly,  $\{a^n c^n | n \geq 0\}$  is not regular but  $a^*c^*$  is regular.

So  $L$  is not regular.

It is context free because we can write a grammar to generate it.

Here comes the grammar:

$S \rightarrow aSc \mid B \mid \epsilon$

$B \rightarrow bBc \mid \epsilon$

So we prove it!

## Question 5

### 5.1

It is not decidable because if we pick a input which is not in that set,  $M$  never halts.

### 5.2

We will use the reductions to do this. First, define  $M'$  as a turing machine. Pick arbitrary input  $w \in \Sigma^*$  and run on the  $M$ . If  $M$  accept it, then  $M'$  accept  $\Sigma^*$ . Otherwise pick another input to run on  $M$ .

Since  $EMPTY$  is not CE, so is  $FIN$ .

## Question 6

### 6.1

It is decidable. First, Inversing the DFA. Change the reject and accepting states. By doing this, we will get a DFA for  $\bar{L}$ . By looking at the new DFA, assume the states' number is  $n$ . we can checking all the strings with length between  $n$  to  $2n$ . If exists a string with length between  $n$  to  $2n$  and accepted by this new DFA, which means this DFA has a loop.

Then by pumping lemma,  $\bar{L}$  is infinite. so the original regular language is not cofinite. Otherwise, it is cofinite.

## 6.2

It is undecidable.

Since  $L$  is a CFL, so  $\bar{L}$  is recursive. We know that it is undecidable for a given Turing machine to accept finite or infinite inputs. So it's undecidable.

## 6.3

It is not decidable. We can use Rice's Thm to prove it.

## Question 7

### 7.1

True

### 7.2

True

### 7.3

True

### 7.4

True

### 7.5

False

## Finally

I solemnly swear that I am up to no mischief. I did not consult anyone nor did I use the internet to search for answers to these questions.