

$$3) a) \begin{pmatrix} 12 & -4 & 19 \\ 2 & 3 & -4 \\ 4 & -6 & 11 \end{pmatrix} \sim \begin{pmatrix} 2 & 3 & -4 \\ 4 & -6 & 11 \\ 12 & -4 & 19 \end{pmatrix} \xrightarrow{[3]-[1] \times 6} \begin{pmatrix} 2 & 3 & -4 \\ 4 & 6 & 11 \\ 0 & -22 & 43 \end{pmatrix} \xrightarrow{[2]-[1] \times 2} \boxed{\Delta 33} \\ \text{N3}$$

$$\rightarrow \begin{pmatrix} 2 & 3 & -4 \\ 0 & 0 & 19 \\ 0 & -22 & 43 \end{pmatrix} \sim \begin{pmatrix} 2 & 3 & -4 \\ 0 & -22 & 43 \\ 0 & 0 & 19 \end{pmatrix} \quad r(A) = 3$$

$$3) b) \begin{pmatrix} 3 & 13 & 2 & -5 \\ 1 & 10 & 2 & -4 \\ 13 & -6 & -6 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 10 & 2 & -4 \\ 3 & 13 & 2 & -5 \\ 13 & -6 & -6 & 4 \end{pmatrix} \xrightarrow{[3]-[1] \times 3} \begin{pmatrix} 1 & 10 & 2 & -4 \\ 3 & 13 & 2 & -5 \\ 0 & -136 & -42 & 56 \end{pmatrix} \rightarrow$$

$$\xrightarrow{[2]-[1] \times 3} \begin{pmatrix} 1 & 10 & 2 & -4 \\ 0 & -17 & -4 & 3 \\ 0 & -68 & -21 & 28 \end{pmatrix} \xrightarrow{[3]-[2] \times 4} \begin{pmatrix} 1 & 10 & 2 & -4 \\ 0 & -17 & -4 & 3 \\ 0 & 0 & -13 & 16 \end{pmatrix} \quad r(A) = 3$$

$$3) c) \begin{pmatrix} 6 & -6 & -1 & 1 \\ 11 & -11 & 0 & 2 \\ 4 & -4 & 3 & 1 \\ 9 & -9 & -7 & 1 \end{pmatrix} \xrightarrow{[1]-[3]} \begin{pmatrix} 2 & 2 & -4 & 0 \\ 11 & -11 & 0 & 2 \\ 4 & -4 & 3 & 1 \\ 9 & -9 & -7 & 1 \end{pmatrix} \xrightarrow{[4]/2} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 11 & -11 & 0 & 2 \\ 4 & -4 & 3 & 1 \\ 9 & -9 & -7 & 1 \end{pmatrix} \rightarrow$$

$$\xrightarrow{[2]-[1] \times 11} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -22 & 22 & 2 \\ 4 & -4 & 3 & 1 \\ 9 & -9 & -7 & 1 \end{pmatrix} \xrightarrow{[3]-[1] \times 4} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -22 & 22 & 2 \\ 0 & -8 & -5 & 1 \\ 9 & -9 & -7 & 1 \end{pmatrix} \xrightarrow{[4]-[1] \times 9}$$

$$\rightarrow \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -11 & 11 & 1 \\ 0 & -8 & -5 & 1 \\ 0 & -18 & 11 & 1 \end{pmatrix} \xrightarrow{\begin{matrix} [7] \times 4 \\ [8] \times 11 \end{matrix}} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -11 & 11 & 1 \\ 0 & -88 & -55 & 11 \\ 0 & -198 & 121 & 11 \end{pmatrix} \xrightarrow{[3]-[2] \times 8} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -11 & 11 & 1 \\ 0 & 0 & -143 & 3 \\ 0 & -198 & 121 & 11 \end{pmatrix} \xrightarrow{[4]-[2] \times 18}$$

$$\rightarrow \left( \begin{array}{cccc|c} 4 & -1 & 2 & -3 & -1 \\ 0 & -1 & 2 & -3 & 1 \\ 0 & 0 & 6 & -26 & 1 \\ 0 & 0 & 0 & 2730 & 57 \\ 0 & 0 & 0 & 0 & 1695 \end{array} \right) \begin{array}{c} 10 \\ 10 \\ 111 \\ 12033 \\ -4753 \end{array}$$

$$\frac{13 \times 3}{3}$$

$$\begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -11 & 11 & 1 \\ 0 & 0 & -143 & 3 \\ 0 & 0 & -77 & -7 \end{pmatrix} \xrightarrow{4/7 \times 13} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -11 & 11 & 1 \\ 0 & 0 & -143 & 3 \\ 0 & 0 & -143 & -13 \end{pmatrix} \xrightarrow{[4] - [3]} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -11 & 11 & 1 \\ 0 & 0 & -143 & 3 \\ 0 & 0 & 0 & -16 \end{pmatrix}$$

$$r(A) = 4$$