

$$2) a) \begin{cases} x_1 - 5x_2 - 19x_3 = 10 \\ 5x_1 + 9x_2 + x_3 = 2 \end{cases}$$

$$\left( \begin{array}{ccc|c} 1 & -5 & -19 & 10 \\ 5 & 9 & 1 & 2 \end{array} \right) \xrightarrow{[2]-[1] \times 5} \left( \begin{array}{ccc|c} 1 & -5 & -19 & 10 \\ 0 & 24 & 96 & -48 \end{array} \right) \xrightarrow{[2]/24}$$

$$\rightarrow \left( \begin{array}{ccc|c} 1 & -5 & -19 & 10 \\ 0 & 1 & 4 & -2 \end{array} \right) \Rightarrow \begin{array}{l} \text{базисные } x_1 \text{ и } x_2 \\ \text{свободный: } x_3 \end{array}$$

Выразим базисные через свободные.

$$x_2 + 4x_3 = -2$$

$$\boxed{x_2 = -2 - 4x_3}$$

$$x_1 + 6 + 12x_3 - 19x_3 = 10$$

$$\boxed{x_1 = 4 + 7x_3}$$

$$\begin{pmatrix} 4 + 7x_3 \\ -2 - 4x_3 \\ x_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}}_{\text{ФМР}} + x_3 \begin{pmatrix} 7 \\ -4 \\ 1 \end{pmatrix}$$

Размерность пространства:  $3 - 2 = 1$

$$b) \begin{cases} x_1 + 2x_2 + x_3 - 2x_4 = 9 \\ 3x_1 - 4x_2 - 7x_3 - 6x_4 = -3 \end{cases}$$

13.3

N2

$$\left( \begin{array}{cccc|c} 1 & 2 & 1 & -2 & 9 \\ 3 & -4 & -7 & -6 & -3 \end{array} \right) \xrightarrow{[2] - [1] \times 3} \left( \begin{array}{cccc|c} 1 & 2 & 1 & -2 & 9 \\ 0 & -10 & -10 & 0 & -30 \end{array} \right) \xrightarrow{[2] / 10}$$

$$\rightarrow \left( \begin{array}{cccc|c} 1 & 2 & 1 & -2 & 9 \\ 0 & 1 & 1 & 0 & 3 \end{array} \right) \quad \begin{array}{l} \text{базисные: } x_1 \text{ и } x_2 \\ \text{свободные: } x_3 \text{ и } x_4 \end{array}$$

Выразим базисные через свободные:

$$x_2 + x_3 = 3 \Rightarrow x_2 = 3 - x_3$$

$$x_1 + 2(3 - x_3) + x_3 - 2x_4 = 9$$

$$x_1 + 6 - 2x_3 + x_3 - 2x_4 = 9$$

$$x_1 = 3 + x_3 + 2x_4$$

$$\begin{pmatrix} 3 + x_3 + 2x_4 \\ 3 - x_3 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

ФМР

Размерность пространства:  $n - r = 4 - 2 = 2$

$$2) \begin{cases} x_1 - x_2 + x_3 = -1 \\ 5x_1 - 6x_2 - 2x_3 - 2x_4 = 1 \end{cases}$$

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N2

~~$$\begin{pmatrix} 1 & -1 & 1 & 0 & -1 \\ 5 & -6 & -2 & -2 & 1 \end{pmatrix}$$~~

$$\xrightarrow{[2] - [1] \times 5} \left( \begin{array}{cccc|c} 1 & -1 & 1 & 0 & -1 \\ 0 & -1 & -7 & -2 & 6 \end{array} \right)$$

БАЗИСНЫЕ  $x_1$  и  $x_2$

свободные  $x_3$  и  $x_4$

ВЫРАЗИМ базисные через свободные

$$-x_2 - 7x_3 - 2x_4 = 6$$

$$\boxed{x_2 = -6 - 7x_3 - 2x_4}$$

$$x_1 + 6 + 7x_3 + 2x_4 + x_3 = -1$$

$$\boxed{x_1 = -7 + 8x_3 + 2x_4}$$

$$\begin{pmatrix} -7 + 8x_3 + 2x_4 \\ -6 - 7x_3 - 2x_4 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -7 \\ -6 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 8 \\ -7 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 2 \\ -2 \\ 0 \\ 1 \end{pmatrix}$$

ФНР

Размерность пространства:  $4 - 2 = 2$

$$2d) \begin{cases} 2x_1 - 6x_2 + 3x_3 + 7x_4 + x_5 = -1 \\ x_1 - x_2 + 3x_4 + 3x_5 = -8 \\ 5x_1 - 9x_2 + 5x_3 + 22x_4 + 12x_5 = -7 \end{cases}$$

$\Delta_3 \neq 0$   
N2

$$\left( \begin{array}{ccccc|c} 2 & -6 & 3 & 7 & 1 & -1 \\ 1 & -1 & 0 & 3 & 3 & -8 \\ 5 & -9 & 5 & 22 & 12 & -7 \end{array} \right) \xrightarrow{[2] \leftrightarrow [1]} \left( \begin{array}{ccccc|c} 1 & -1 & 0 & 3 & 3 & -8 \\ 2 & -6 & 3 & 7 & 1 & -1 \\ 5 & -9 & 5 & 22 & 12 & -7 \end{array} \right) \xrightarrow{[2] - [1] \times 2}$$

$$\rightarrow \left( \begin{array}{ccccc|c} 1 & -1 & 0 & 3 & 3 & -8 \\ 0 & -4 & 3 & 1 & -5 & 15 \\ 5 & -9 & 5 & 22 & 12 & -7 \end{array} \right) \xrightarrow{[3] - [1] \times 5} \left( \begin{array}{ccccc|c} 1 & -1 & 0 & 3 & 3 & -8 \\ 0 & -4 & 3 & 1 & -5 & 15 \\ 0 & -4 & 5 & 7 & -3 & 33 \end{array} \right) \xrightarrow{[3] - [2]}$$

$$\rightarrow \left( \begin{array}{ccccc|c} 1 & -1 & 0 & 3 & 3 & -8 \\ 0 & -4 & 3 & 1 & -5 & 15 \\ 0 & 0 & 2 & 6 & 2 & 18 \end{array} \right)$$

базисные  $x_1, x_2, x_3$   
свободные  $x_3, x_4$   
Размерность нр-ва:  $5 - 3 = 2$

Выразим базисные через свободные:

$$2x_3 + 6x_4 + 2x_5 = 18$$

$$x_3 = 9 - 3x_4 - x_5$$

$$-4x_2 + 3(9 - 3x_4 - x_5) + x_4 - 5x_5 = 15$$

$$-4x_2 + 27 - 9x_4 - 3x_5 + x_4 - 5x_5 = 15$$

$$-4x_2 + 12 - 8x_4 - 8x_5 = 0$$

$$x_2 = 3 - 2x_4 - 2x_5$$

$$x_1 - 3 + 2x_4 + 2x_5 + 3x_4 + 3x_5 = -8$$

$$x_1 = -5 - 5x_4 - 5x_5$$

$$\begin{pmatrix} -5 - 5x_4 - 5x_5 \\ 3 - 2x_4 - 2x_5 \\ 9 - 3x_4 - x_5 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \\ 9 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -5 \\ -2 \\ -3 \\ 1 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} -5 \\ -2 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

← ФМР



$$3) a) \begin{pmatrix} 12 & -4 & 19 \\ 2 & 3 & -4 \\ 4 & -6 & 11 \end{pmatrix} \sim \begin{pmatrix} 2 & 3 & -4 \\ 4 & -6 & 11 \\ 12 & -4 & 19 \end{pmatrix} \xrightarrow{[3]-[1] \times 6} \begin{pmatrix} 2 & 3 & -4 \\ 4 & 6 & 11 \\ 0 & -22 & 43 \end{pmatrix} \xrightarrow{[2]-[1] \times 2} \begin{pmatrix} 2 & 3 & -4 \\ 0 & 0 & 19 \\ 0 & -22 & 43 \end{pmatrix} \quad \boxed{\Delta 33} \\ \text{N3}$$

$$\rightarrow \begin{pmatrix} 2 & 3 & -4 \\ 0 & 0 & 19 \\ 0 & -22 & 43 \end{pmatrix} \sim \begin{pmatrix} 2 & 3 & -4 \\ 0 & -22 & 43 \\ 0 & 0 & 19 \end{pmatrix} \quad r(A) = 3$$

$$3) b) \begin{pmatrix} 3 & 13 & 2 & -5 \\ 1 & 10 & 2 & -4 \\ 13 & -6 & -6 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 10 & 2 & -4 \\ 3 & 13 & 2 & -5 \\ 13 & -6 & -6 & 4 \end{pmatrix} \xrightarrow{[3]-[1] \times 3} \begin{pmatrix} 1 & 10 & 2 & -4 \\ 3 & 13 & 2 & -5 \\ 0 & -136 & -42 & 56 \end{pmatrix} \rightarrow$$

$$\xrightarrow{[2]-[1] \times 3} \begin{pmatrix} 1 & 10 & 2 & -4 \\ 0 & -17 & -4 & 3 \\ 0 & -68 & -21 & 28 \end{pmatrix} \xrightarrow{[3]-[2] \times 4} \begin{pmatrix} 1 & 10 & 2 & -4 \\ 0 & -17 & -4 & 3 \\ 0 & 0 & -13 & 16 \end{pmatrix} \quad r(A) = 3$$

$$3) c) \begin{pmatrix} 6 & -6 & -1 & 1 \\ 11 & -11 & 0 & 2 \\ 4 & -4 & 3 & 1 \\ 9 & -9 & -7 & 1 \end{pmatrix} \xrightarrow{[1]-[3]} \begin{pmatrix} 2 & 2 & -4 & 0 \\ 11 & -11 & 0 & 2 \\ 4 & -4 & 3 & 1 \\ 9 & -9 & -7 & 1 \end{pmatrix} \xrightarrow{[4]/2} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 11 & -11 & 0 & 2 \\ 4 & -4 & 3 & 1 \\ 9 & -9 & -7 & 1 \end{pmatrix} \rightarrow$$

$$\xrightarrow{[2]-[1] \times 11} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -22 & 22 & 2 \\ 4 & -4 & 3 & 1 \\ 9 & -9 & -7 & 1 \end{pmatrix} \xrightarrow{[3]-[1] \times 4} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -22 & 22 & 2 \\ 0 & -8 & -5 & 1 \\ 9 & -9 & -7 & 1 \end{pmatrix} \xrightarrow{[4]-[1] \times 9}$$

$$\rightarrow \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -11 & 11 & 1 \\ 0 & -8 & -5 & 1 \\ 0 & -18 & 11 & 1 \end{pmatrix} \xrightarrow{\begin{matrix} [7] \times 4 \\ [8] \times 11 \end{matrix}} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -11 & 11 & 1 \\ 0 & -88 & -55 & 11 \\ 0 & -198 & 121 & 11 \end{pmatrix} \xrightarrow{[3]-[2] \times 8} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -11 & 11 & 1 \\ 0 & 0 & -143 & 3 \\ 0 & -198 & 121 & 11 \end{pmatrix} \xrightarrow{[4]-[2] \times 18}$$

$$\rightarrow \left( \begin{array}{cccc|c} 4 & -1 & 2 & -3 & -1 \\ 0 & -1 & 2 & -3 & 1 \\ 0 & 0 & 6 & -26 & 1 \\ 0 & 0 & 0 & 2730 & 57 \\ 0 & 0 & 0 & 0 & 1695 \end{array} \right) \begin{array}{c} 10 \\ 10 \\ 111 \\ 12033 \\ -4753 \end{array}$$

$$\frac{13 \times 3}{3}$$

$$\begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -11 & 11 & 1 \\ 0 & 0 & -143 & 3 \\ 0 & 0 & -77 & -7 \end{pmatrix} \xrightarrow{4/7 \times 13} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -11 & 11 & 1 \\ 0 & 0 & -143 & 3 \\ 0 & 0 & -143 & -13 \end{pmatrix} \xrightarrow{[4] - [3]} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -11 & 11 & 1 \\ 0 & 0 & -143 & 3 \\ 0 & 0 & 0 & -16 \end{pmatrix}$$

$$r(A) = 4$$

$$\begin{aligned}
 a) \begin{vmatrix} 0 & -8 & 5 & 1 \\ 0 & -1 & 6 & 1 \\ -6 & -7 & 5 & 1 \\ 6 & -9 & 0 & 0 \end{vmatrix} &= 6 \cdot (-1)^{(4+1)} \begin{vmatrix} -8 & 5 & 1 \\ -1 & 6 & 1 \\ -7 & 5 & 1 \end{vmatrix} + (-9) \cdot (-1)^{(4+2)} \begin{vmatrix} 0 & 5 & 1 \\ 0 & 6 & 1 \\ -6 & 5 & 1 \end{vmatrix} + \\
 0 \cdot (-1)^{(4+3)} \dots + 0 \cdot (-1)^{(4+4)} \dots &= -6 \left[ -8 \cdot (-1)^{(1+1)} \begin{vmatrix} 6 & 1 \\ 5 & 1 \end{vmatrix} + 5 \cdot (-1)^{(1+2)} \begin{vmatrix} -1 & 1 \\ -7 & 1 \end{vmatrix} + \right. \\
 \left. + 1 \cdot (-1)^{(1+3)} \begin{vmatrix} -1 & 6 \\ -7 & 5 \end{vmatrix} \right] + (-9) \left[ 0 \cdot (-1)^{(1+1)} \dots + 5 \cdot (-1)^{(1+2)} \begin{vmatrix} 0 & 1 \\ -6 & 1 \end{vmatrix} + 1 \cdot (-1)^{(1+3)} \begin{vmatrix} 0 & 6 \\ -6 & 5 \end{vmatrix} \right] = \\
 = (-6) \cdot [ -8 \cdot (6 \cdot 1 - 1 \cdot 5) - 5 \cdot ((-1) \cdot 1 - 1 \cdot (-7)) + 1 \cdot ((-1) \cdot 5 - 6 \cdot (-7)) ] + \\
 + (-9) \cdot [ -5 (0 \cdot 1 - 1 \cdot (-6)) + 1 (0 \cdot 5 - 6 \cdot (-6)) ] = \\
 = (-6) \cdot [ -8 - 30 + 37 ] + (-9) [ -30 + 36 ] = \\
 = (46) + (-54) = \boxed{-8} \quad \boxed{-48} \quad \checkmark
 \end{aligned}$$

$$\begin{aligned}
 b) \begin{vmatrix} -4 & 4 & -6 & 3 \\ 5 & 4 & -6 & 6 \\ 4 & -1 & 3 & 1 \\ -9 & 9 & -9 & -9 \end{vmatrix} &= -4 \cdot (-1)^{(1+1)} \begin{vmatrix} 4 & -6 & 6 \\ -1 & 3 & 1 \\ 9 & -9 & -9 \end{vmatrix} + 4 \cdot (-1)^{(1+2)} \begin{vmatrix} 5 & -6 & 6 \\ 4 & 3 & 1 \\ -9 & -9 & -9 \end{vmatrix} + \\
 4 \cdot (-1)^{(1+3)} \begin{vmatrix} 5 & 4 & 6 \\ 4 & -1 & 1 \\ -9 & 9 & -9 \end{vmatrix} + 3 \cdot (-1)^{(1+4)} \begin{vmatrix} 5 & 4 & -6 \\ 4 & -1 & 3 \\ -9 & 9 & -9 \end{vmatrix} = \\
 = (-4) \cdot \left[ 4 \cdot (-1)^{(1+1)} \begin{vmatrix} 3 & 1 \\ -9 & -9 \end{vmatrix} + (-6) \cdot (-1)^{(1+2)} \begin{vmatrix} -1 & 1 \\ 9 & -9 \end{vmatrix} + 6 \cdot (-1)^{(1+3)} \begin{vmatrix} -1 & 3 \\ 9 & -9 \end{vmatrix} \right] \oplus \\
 \oplus (-4) \left[ 5 \cdot (-1)^{(1+1)} \begin{vmatrix} 3 & 1 \\ -9 & -9 \end{vmatrix} + (-6) \cdot (-1)^{(1+2)} \begin{vmatrix} 4 & 1 \\ -9 & -9 \end{vmatrix} + 6 \cdot (-1)^{(1+3)} \begin{vmatrix} 4 & 3 \\ -9 & -9 \end{vmatrix} \right] \oplus \\
 \oplus (-6) \left[ 5 \cdot (-1)^{(1+1)} \begin{vmatrix} -1 & 1 \\ 9 & -9 \end{vmatrix} + 4 \cdot (-1)^{(1+2)} \begin{vmatrix} 4 & 1 \\ -9 & -9 \end{vmatrix} + 6 \cdot (-1)^{(1+3)} \begin{vmatrix} 4 & -1 \\ -9 & 9 \end{vmatrix} \right] \oplus \\
 \oplus -3 \left[ 5 \cdot (-1)^{(1+1)} \begin{vmatrix} -1 & 3 \\ 9 & -9 \end{vmatrix} + 4 \cdot (-1)^{(1+2)} \begin{vmatrix} 4 & 3 \\ -9 & -9 \end{vmatrix} + (-6) \cdot (-1)^{(1+3)} \begin{vmatrix} 4 & -1 \\ -9 & 9 \end{vmatrix} \right] =
 \end{aligned}$$



$$\begin{aligned}
 &= (-4) [4 \cdot (3 \cdot (-9) - 1 \cdot (-9)) + 6((-1) \cdot (-9) - 1 \cdot 9) + ((-1) \cdot (-9) - 3 \cdot 9)] + \\
 &+ (-4) [5(3 \cdot (-9) - 1 \cdot (-9)) + 6(4 \cdot (-9) - 1 \cdot (-9)) + 6(4 \cdot (-9) - 3 \cdot (-9))] + \\
 &+ (-6) [5((-1) \cdot (-9) - 1 \cdot 9) + (-4)(4 \cdot (-9) - 1 \cdot (-9)) + 6(4 \cdot 9 - (-1) \cdot (-9))] + \\
 &+ (-3) [5(-1) \cdot (-9) - 3 \cdot 9] + (-4)(4 \cdot (-9) - 3 \cdot (-9)) + (-6)(4 \cdot 9 - (-1) \cdot (-9))] =
 \end{aligned}$$

13(8)  
N5

$$\begin{aligned}
 &= (-4) [4((-27 - (-9)) + 6(9 - 9) + 6(9 - 27)] + \\
 &+ (-4) [5(-27 - (-9)) + 6(-36 - (-9)) + 6(-36 - (-27))] + \\
 &+ (-6) [5(9 - 9) + (-4)(-36 - (-27)) + (-6)(36 - 9)] + \\
 &+ (-3) [5(9 - 27) + (-4)(-36 - (-27)) + (-6)(36 - 9)] = \\
 &= -16(-18) - 24(0) - 24(-18) + -20(-18) - 24(-27) - 24(-9) + \\
 &(-30) \cdot 6 + 24(-27) + 36 \cdot 27 - 15(-18) + 12(-9) + 18 \cdot 27 = \\
 &= 288 - 0 + 432 + 360 + 648 + 216 + 0 - 648 - 972 + 270 + \\
 &108 + 486 = \boxed{972}
 \end{aligned}$$

$$\begin{aligned}
 c) \begin{vmatrix} -2 & -3 & -6 & -9 \\ -9 & 5 & 4 & -5 \\ -7 & -3 & 4 & 3 \\ -8 & -5 & -2 & -7 \end{vmatrix} &= (-2)(-1) \begin{vmatrix} 5 & 4 & -5 \\ -3 & 4 & 3 \\ -5 & -2 & -7 \end{vmatrix} + (-3)(-1) \begin{vmatrix} -9 & 4 & -5 \\ -7 & 4 & 3 \\ -8 & -2 & 7 \end{vmatrix} + \\
 &+ (-6)(-1) \begin{vmatrix} -9 & 5 & -5 \\ -7 & -3 & 3 \\ -8 & -5 & -7 \end{vmatrix} + (-9)(-1) \begin{vmatrix} -9 & 5 & 4 \\ -7 & -3 & 4 \\ -8 & -5 & -2 \end{vmatrix} =
 \end{aligned}$$

$$\begin{aligned}
 &= (-2) \left[ 5 \begin{vmatrix} 4 & 3 \\ -2 & -7 \end{vmatrix} + 4 \begin{vmatrix} -3 & 3 \\ -5 & -7 \end{vmatrix} + (-5) \begin{vmatrix} -3 & 4 \\ -5 & -2 \end{vmatrix} \right] + \\
 &+ 3 \left[ (-9) \begin{vmatrix} 4 & 3 \\ -2 & -7 \end{vmatrix} + 4 \begin{vmatrix} -7 & 3 \\ -8 & -7 \end{vmatrix} + (-5) \begin{vmatrix} -7 & 4 \\ -8 & -2 \end{vmatrix} \right] + \\
 &+ (-6) \left[ (-9) \begin{vmatrix} -3 & 3 \\ -5 & -7 \end{vmatrix} + 5 \begin{vmatrix} -7 & 3 \\ -8 & -7 \end{vmatrix} + (-5) \begin{vmatrix} -7 & -3 \\ -8 & -5 \end{vmatrix} \right] + \\
 &+ 9 \left[ (-9) \begin{vmatrix} -3 & 4 \\ -5 & -2 \end{vmatrix} + 5 \begin{vmatrix} -7 & 4 \\ -8 & -2 \end{vmatrix} + 4 \begin{vmatrix} -7 & -3 \\ -8 & -5 \end{vmatrix} \right] =
 \end{aligned}$$

A.2.



$$\begin{aligned}
 &= (-10)(4(-7) - 3(-2)) + 8(21 - (-15)) + 10(6 - (-20)) + \\
 &\quad + (-27)(-28 - (-6)) + (-12)(49 - (-24)) + (-15)(14 - (-32)) + \\
 &\quad + 54(21 - (-15)) + 30(49 - (-24)) + 30(35 - 24) + \\
 &\quad + (-81)(6 - (-20)) + (-45)(14 - (4 \cdot (-8))) + 36(35 - 24) =
 \end{aligned}$$

$$\begin{array}{r}
 13.5 \\
 \hline
 25
 \end{array}$$

$$\begin{aligned}
 &= 220 + 288 + 260 + 594 - 300 - 690 + \\
 &\quad + 324 + 2190 + 330 - 2106 + 2070 - 396 = -564
 \end{aligned}$$

$$\begin{aligned}
 &= 220 + 288 + 260 + 594 - 876 - 690 + \\
 &\quad + 1944 + 2190 + 330 - 2106 - 2070 + 396 =
 \end{aligned}$$

$$\boxed{480}$$

$$\sqrt{1.3}$$

а)

$$A = \begin{vmatrix} -6 & 2 & -5 \\ -5 & 1 & -5 \\ -8 & 8 & -3 \end{vmatrix}$$

 $\begin{vmatrix} 1 & 3 & 3 \\ & 1 & 6 \end{vmatrix}$ 

$$A - \lambda E = \begin{pmatrix} -6-\lambda & 2 & -5 \\ -5 & 1-\lambda & -5 \\ -8 & 8 & -3-\lambda \end{pmatrix}$$

Рассмотрим характеристическое ур-е:  
определятельно 3-ей строки:

$$(-8)(-1) \begin{vmatrix} 2 & -5 \\ 1-\lambda & -5 \end{vmatrix} + 8(-1) \begin{vmatrix} -6-\lambda & -5 \\ -5 & -5 \end{vmatrix} + (-3-\lambda)(-1) \begin{vmatrix} -6-\lambda & 2 \\ -5 & 1-\lambda \end{vmatrix} = 0$$

$$(-8)(-10 - (-5+5\lambda)) + (-8)(30+5\lambda-25) + (-3-\lambda)(-6+5\lambda+\lambda^2 - (-10)) = 0$$

$$(-8)(-5+5\lambda) + (-8)(5+5\lambda) + (-3-\lambda)(4+5\lambda+\lambda^2) = 0$$

$$\cancel{40+40\lambda} - \cancel{40-40\lambda} - 12 - 15\lambda - 3\lambda^2 - 4\lambda - 5\lambda^2 - \lambda^3 = 0$$

$$\cancel{-\lambda^3 - 8\lambda^2 - 19\lambda - 12 = 0}$$

$$(-3-\lambda)(4+5\lambda+\lambda^2) = 0$$

$$-3-\lambda = 0 \quad \lambda^2 + 5\lambda + 4 = 0$$

$$\underline{\lambda_1 = -3} \quad D = 25 - 4 \cdot 1 \cdot 4 = 9 \quad D > 0$$

$$\underline{\lambda_2 = \frac{-5-3}{2} = -4}$$

$$\underline{\lambda_3 = \frac{-5+3}{2} = -1}$$

Собственные значения:  $\lambda_1 = -3, \lambda_2 = -4, \lambda_3 = -1$

Берем  $\lambda_1 = (-3)$

$$\begin{vmatrix} -6-(-3) & 2 & -5 \\ -5 & 1-(-3) & -5 \\ -8 & 8 & -3-(-3) \end{vmatrix} = \begin{vmatrix} -3 & 2 & -5 \\ -5 & 4 & -5 \\ -8 & 8 & 0 \end{vmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

a)

13 u3  
N 6

$$\begin{cases} -3x_1 + 2x_2 - 5x_3 = 0 \\ -5x_1 + 4x_2 - 5x_3 = 0 \\ -8x_1 + 8x_2 = 0 \end{cases}$$

$$\begin{cases} -3x_1 + 2x_2 - 5x_3 = 0 \\ -5x_1 + 4x_2 - 5x_3 = 0 \\ -x_1 + x_2 = 0 \end{cases} \Rightarrow \begin{cases} -x_1 + x_2 = 0 \\ -5x_1 + 4x_2 - 5x_3 = 0 \\ -3x_1 + 2x_2 - 5x_3 = 0 \end{cases}$$

$$\left( \begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ -5 & 4 & -5 & 0 \\ -3 & 2 & -5 & 0 \end{array} \right) \xrightarrow{[3]-[1] \times 3} \left( \begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ -5 & 4 & -5 & 0 \\ 0 & -1 & -5 & 0 \end{array} \right) \xrightarrow{[2]-[1] \times 5} \left( \begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & -1 & -5 & 0 \\ 0 & -1 & -5 & 0 \end{array} \right) \xrightarrow{[3]-[2]}$$

$$\Rightarrow \left( \begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & -1 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow \begin{cases} -x_1 - x_2 = 0 \\ -x_2 - 5x_3 = 0 \\ x_3 = -\frac{x_2}{5} \\ x_2 = -5x_3 \\ x_1 = 5x_3 \end{cases}$$

$$\lambda = (-3)$$

$$\text{Вектор } x = \left( x_1; x_2; -\frac{x_2}{5} \right), \text{ где } x_1, x_2 \in \mathbb{R} \setminus \{0\}$$

Берем  $\lambda_2 = (-4)$

$$\begin{vmatrix} -6-(-4) & 2 & -5 \\ -5 & 1-(-4) & -5 \\ -8 & 8 & -3-(-4) \end{vmatrix} = \begin{vmatrix} -2 & 2 & -5 \\ -5 & 5 & -5 \\ -8 & 8 & 1 \end{vmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{cases} -2x_1 + 2x_2 - 5x_3 = 0 \\ -x_1 + x_2 - x_3 = 0 \\ -8x_1 + 8x_2 + x_3 = 0 \end{cases} \Rightarrow \left( \begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ -2 & 2 & -5 & 0 \\ -8 & 8 & 1 & 0 \end{array} \right) \xrightarrow{[2]-[1] \times 2} \left( \begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & 0 & -3 & 0 \\ -8 & 8 & 1 & 0 \end{array} \right) \xrightarrow{[3]+[1] \times 8}$$

$$\left( \begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 9 & 0 \end{array} \right) \Rightarrow \begin{cases} -x_1 + x_2 - x_3 = 0 \\ -3x_3 = 0 \\ -9x_3 = 0 \end{cases} \quad \begin{cases} -x_1 + x_2 - x_3 = 0 \\ x_3 = 0 \\ x_1 = x_2 \end{cases}$$

$$\text{Вектор } x = (x_1; x_1; 0), \text{ где } x_1 \in \mathbb{R} \setminus \{0\}$$

1.2

Берем  $\lambda_3 = (-1)$

$\Delta_3 \Delta_3$   
6

$$\begin{pmatrix} -6 - (-1) & 2 & -5 \\ -5 & 1 - (-1) & -5 \\ -8 & 8 & -3 - (-1) \end{pmatrix} = \begin{pmatrix} -5 & 2 & -5 \\ -5 & 2 & -5 \\ -8 & 8 & -2 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} -5 & 2 & -5 & | & 0 \\ -5 & 2 & -5 & | & 0 \\ -8 & 8 & -2 & | & 0 \end{pmatrix} \xrightarrow{[3]-[2]} \begin{pmatrix} -5 & 2 & -5 & | & 0 \\ -5 & 2 & -5 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \xrightarrow{2 \times 4} \begin{pmatrix} -4 & 4 & -1 & | & 0 \\ -20 & 8 & -20 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \xrightarrow{[2]-[1] \times 5}$$

$$\rightarrow \begin{pmatrix} -4 & 4 & -1 & | & 0 \\ 0 & -12 & -15 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \rightarrow \begin{aligned} -12x_2 - 15x_3 &= 0 \\ x_2 &= \frac{15x_3}{-12} = -\frac{5x_3}{4} \end{aligned}$$

$$-4x_1 + 5x_3 - x_3 = 0$$

$$x_1 = -x_3$$

$$\text{Вектор } x = \left( x_1; \frac{4x_3}{5}; -x_3 \right) \quad x_3 \in \mathbb{R} \setminus \{0\}.$$

Проверим  $\lambda = (-3)$   $x_1 = 1, x_2 = -5x_3 = \frac{1}{5} \cdot 1$

$$x = \begin{pmatrix} 1 \\ -5 \\ 1 \end{pmatrix}$$

$$\begin{array}{ccc} 5x_1 & -5x_2 & x_3 \\ 5 & -5 & 1 \end{array}$$

$$\begin{pmatrix} -6 & 2 & -5 \\ -5 & 1 & -5 \\ -8 & 8 & -3 \end{pmatrix} \times \begin{pmatrix} 1 \\ -5 \\ 1 \end{pmatrix} = (-3) \begin{pmatrix} 1 \\ -5 \\ 1 \end{pmatrix}$$

$$-6 - 10 - 5 = -21$$

$$-30 - 10 - 5 = -45$$

$$+25 - 5 - 5 = -5$$

$$-40 - 40 - 3 = -83$$

что правильно



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