6) а) $\begin{vmatrix} -6 & 2 & -5 \\ -5 & 1 & -5 \\ -8 & 8 & -3 \end{vmatrix}$ — 1 ЗнЗ

Решение:

$$\det \begin{vmatrix} -\lambda-6 & 2 & -5 \\ -5 & -\lambda+1 & -5 \\ -8 & 8 & -\lambda-3 \end{vmatrix} = (-\lambda-6)(-\lambda+1)(-\lambda-3) + 2(-5)(-8) + (-5)(-5)(8) - (-8)(-\lambda+1)(-5) - 8\cdot(-5)\cdot(-\lambda-6) - (-\lambda-3)\cdot(-5)\cdot 2 =$$

$$= (\lambda^2 - \lambda + 6\lambda - 6)(-\lambda-3) + 80 + 200 - (8\lambda - 8)(-5) - (40\lambda + 240) - (10\lambda + 30) = -\lambda^3 - 3\lambda^2 + \lambda^2 + 3\lambda - 6\lambda^2 - 18\lambda + 6\lambda + 18 + 80 + 200 + 40\lambda - 40 - 40\lambda - 240 - 10\lambda - 30 =$$

$$= \underline{-\lambda^3 - 8\lambda^2 - 19\lambda - 12}$$

$$-\lambda^3 - 8\lambda^2 - 19\lambda - 12 = 0$$

$$-8 - 12 = -1 - 19$$

$$-20 = -20 \Rightarrow \lambda_1 = (-1)$$

Делители 12: ±4 и ±3, подбором вычисляются $\lambda_2 = (-4)$, $\lambda_3 = (-3)$

$\lambda_1 = (-1)$

$$\begin{vmatrix} -5 & 2 & -5 \\ -5 & 2 & -5 \\ -8 & 8 & -2 \end{vmatrix} \Rightarrow \begin{vmatrix} -5 & 2 & -5 \\ -4 & 4 & -1 \end{vmatrix} \xrightarrow{[2]\times 5} \begin{vmatrix} -5 & 2 & -5 \\ -20 & 20 & -5 \end{vmatrix} \xrightarrow{[2]-[1]\times 4} \begin{vmatrix} -5 & 2 & -5 \\ 0 & 12 & 15 \end{vmatrix}$$

$$x_3 = x_3$$

$$12x_2 + 15x_3 = 0$$

$$x_2 = -\frac{5}{4}x_3$$

$$-5x_1 - \frac{5}{2}x_3 - 5x_3 = 0 \Rightarrow x_3 \times \begin{pmatrix} -\frac{3}{2} \\ -\frac{5}{4} \\ 1 \end{pmatrix}, \text{ если } x_3 = 1, \text{ то } V_1 = \begin{pmatrix} -\frac{3}{2} \\ -\frac{5}{4} \\ 1 \end{pmatrix}$$

$$x_1 = -\frac{3}{2}x_3$$

$\lambda_2 = -4$

$$\begin{vmatrix} -(-4)-6 & 2 & -5 \\ -5 & -(-4)+1 & -5 \\ -8 & 8 & -(-4)+3 \end{vmatrix} \longrightarrow \begin{vmatrix} -2 & 2 & -5 \\ -5 & 5 & -5 \\ -8 & 8 & 7 \end{vmatrix} \xrightarrow{[2]\,|:5 \;\leftrightarrow\; 1} \begin{vmatrix} -1 & 1 & -1 \\ -2 & 2 & -5 \\ -8 & 8 & 7 \end{vmatrix} \xrightarrow{[2]-[1]\times 2} \longrightarrow$$

$$\longrightarrow \begin{vmatrix} -1 & 1 & -1 \\ 0 & 0 & -3 \\ -8 & 8 & 7 \end{vmatrix} \xrightarrow{[3]-[1]\times 8} \begin{vmatrix} -1 & 1 & -1 \\ 0 & 0 & -3 \\ 0 & 0 & 9 \end{vmatrix} \xrightarrow{[3]+[2]\times 3} \begin{vmatrix} -1 & 1 & -1 \\ 0 & 0 & -3 \\ & & \end{vmatrix} \Rightarrow$$

$$\Rightarrow -x_1 + x_2 - x_3 = 0 \qquad \Rightarrow x_1 = x_2,\ x_2 = x_2$$
$$x_3 = 0$$

$$\begin{pmatrix} x_2 \\ x_2 \\ 0 \end{pmatrix} = x_2 \underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}}_{\text{ФНР}},\ \text{пусть } x_2 = 1,\ V_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$x_3 = (-3)$

$$\begin{vmatrix} -(-3)-6 & 2 & -5 \\ -5 & -(-3)+1 & -5 \\ -8 & 8 & -(-3)+3 \end{vmatrix} \longrightarrow \begin{vmatrix} -3 & 2 & -5 \\ -5 & 4 & -5 \\ -8 & 8 & 6 \end{vmatrix} \xrightarrow{\substack{[2]\times 3 \\ [3]\times 3}} \begin{vmatrix} -3 & 2 & -5 \\ -15 & 12 & -15 \\ -24 & 24 & 0 \end{vmatrix} \xrightarrow{[2]-[1]\times 5} \longrightarrow$$

$$\longrightarrow \begin{vmatrix} -3 & 2 & -5 \\ 0 & 2 & 10 \\ -24 & 24 & 0 \end{vmatrix} \xrightarrow{[3]-[1]\times 8} \begin{vmatrix} -3 & 2 & -5 \\ 0 & 1 & 5 \\ 0 & 8 & 40 \end{vmatrix} \xrightarrow{[3]-[2]\times 8} \begin{vmatrix} -3 & 2 & -5 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{vmatrix}$$

$$x_3 = x_3$$
$$x_2 = -5x_3$$
$$-3x_1 - 10x_3 - 5x_3 = 0$$
$$x_1 = -5x_3$$

$$\Rightarrow \begin{pmatrix} -5x_3 \\ -5x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} -5 \\ -5 \\ 1 \end{pmatrix} \Rightarrow \text{если } x_3 = 1,\ \text{то}$$

$$V_3 = \begin{pmatrix} -5 \\ -5 \\ 1 \end{pmatrix}$$