

3)a)

$$x = \begin{pmatrix} -7 \\ 2 \\ 0 \end{pmatrix}, e_1 = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}, e_2 = \begin{pmatrix} -5 \\ -5 \\ 0 \end{pmatrix}$$

13 N3  
8

$$\begin{cases} \langle x, e_1 \rangle - \lambda_1 \langle e_1, e_1 \rangle - \lambda_2 \langle e_1, e_2 \rangle = 0 \\ \langle x, e_2 \rangle - \lambda_1 \langle e_1, e_2 \rangle - \lambda_2 \langle e_2, e_2 \rangle = 0 \end{cases}$$

$$\begin{cases} +1 - \lambda_1 \cdot 26 - \lambda_2 (-25) = 0 \\ 25 - \lambda_1 \cdot (-25) - \lambda_2 \cdot (-50) = 0 \end{cases}$$

$$\begin{cases} 2 - \lambda_1 \cdot 52 - \lambda_2 \cdot (-50) = 0 \\ 25 - \lambda_1 \cdot (-25) - \lambda_2 \cdot (-50) = 0 \end{cases}$$

$$27 - \lambda_1 (52 - 25) = 0$$

$$27 - \lambda_1 \cdot 27 = 0 \Rightarrow \lambda_1 = 1$$

$$1 - 1 \cdot 26 - \lambda_2 \cdot (-25) = 0$$

$$-25 + 25 \lambda_2 = 0 \Rightarrow \lambda_2 = 1$$

$$1 \cdot \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} + 1 \cdot \begin{pmatrix} -5 \\ -5 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \\ -3 \end{pmatrix} = \hat{x}$$

$$x - \hat{x} = \begin{pmatrix} -7 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} -4 \\ -1 \\ -3 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} = \tilde{x}$$

$$\langle \tilde{x}, e_1 \rangle = (-3) \cdot 1 + 3 \cdot 4 + 3 \cdot (-3) = 0$$

$$\langle \tilde{x}, e_2 \rangle = (-3) \cdot (-5) + 3 \cdot (-5) + 3 \cdot 0 = 0$$

$$|\tilde{x}| = \sqrt{\langle \tilde{x}, \tilde{x} \rangle} = \sqrt{27} = 3\sqrt{3}$$

6) b)

$$X = \begin{pmatrix} -2 \\ 0 \\ 4 \\ 2 \end{pmatrix}, e_1 = \begin{pmatrix} -1 \\ 2 \\ 0 \\ -1 \end{pmatrix}, e_2 = \begin{pmatrix} 2 \\ 1 \\ 2 \\ 2 \end{pmatrix}, e_3 = \begin{pmatrix} 0 \\ -5 \\ 2 \\ 0 \end{pmatrix}$$

$$\begin{cases} \langle X, e_1 \rangle - \lambda_1 \langle e_1, e_1 \rangle - \lambda_2 \langle e_2, e_1 \rangle - \lambda_3 \langle e_3, e_1 \rangle = 0 \\ \langle X, e_2 \rangle - \lambda_1 \langle e_1, e_2 \rangle - \lambda_2 \langle e_2, e_2 \rangle - \lambda_3 \langle e_3, e_2 \rangle = 0 \\ \langle X, e_3 \rangle - \lambda_1 \langle e_1, e_3 \rangle - \lambda_2 \langle e_2, e_3 \rangle - \lambda_3 \langle e_3, e_3 \rangle = 0 \end{cases}$$

$$\left( \begin{array}{ccc|c} \langle e_1, e_1 \rangle & \langle e_2, e_1 \rangle & \langle e_3, e_1 \rangle & \langle X, e_1 \rangle \\ \langle e_1, e_2 \rangle & \langle e_2, e_2 \rangle & \langle e_3, e_2 \rangle & \langle X, e_2 \rangle \\ \langle e_1, e_3 \rangle & \langle e_2, e_3 \rangle & \langle e_3, e_3 \rangle & \langle X, e_3 \rangle \end{array} \right) \Rightarrow$$

$$\Rightarrow \left( \begin{array}{ccc|c} 6 & -2 & 10 & 6 \\ -2 & 13 & -1 & 8 \\ 10 & -1 & 29 & 8 \end{array} \right) \xrightarrow{[1]/6} \left( \begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{5}{3} & 0 \\ -2 & 13 & -1 & 8 \\ -10 & -1 & -29 & 8 \end{array} \right) \xrightarrow{[2]+[1] \times 2} \left( \begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{5}{3} & 0 \\ 0 & \frac{37}{3} & -\frac{13}{3} & 8 \\ -10 & -1 & -29 & 8 \end{array} \right)$$

$$\xrightarrow{[3]+[1] \times 10} \left( \begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{5}{3} & 0 \\ 0 & \frac{37}{3} & -\frac{13}{3} & 8 \\ 0 & -\frac{13}{3} & \frac{37}{3} & 8 \end{array} \right) \xrightarrow{[2] \times \frac{3}{37}} \left( \begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{5}{3} & 0 \\ 0 & 1 & -\frac{13}{37} & \frac{24}{37} \\ 0 & -\frac{13}{3} & \frac{37}{3} & 8 \end{array} \right)$$

$$\xrightarrow{[3]+[2] \times \frac{13}{3}} \left( \begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{5}{3} & 0 \\ 0 & 1 & -\frac{13}{37} & \frac{24}{37} \\ 0 & 0 & \frac{66}{37} & \frac{8}{37} \end{array} \right) \xrightarrow{[3] \times \frac{37}{66}} \left( \begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{5}{3} & 0 \\ 0 & 1 & -\frac{13}{37} & \frac{24}{37} \\ 0 & 0 & 1 & \frac{2}{37} \end{array} \right)$$

$$\xrightarrow{[1]+[3] \times \frac{66}{37}} \left( \begin{array}{ccc|c} 1 & 0 & -\frac{66}{37} & \frac{8}{37} \\ 0 & 1 & -\frac{13}{37} & \frac{24}{37} \\ 0 & 0 & 1 & \frac{2}{37} \end{array} \right) \xrightarrow{[1]+[3] \times \frac{66}{37}} \left( \begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -\frac{13}{37} & \frac{24}{37} \\ 0 & 0 & 1 & \frac{2}{37} \end{array} \right)$$

$$\Rightarrow \left( \begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \Rightarrow \begin{cases} \lambda_1 = 2 \\ \lambda_2 = 1 \\ \lambda_3 = 1 \end{cases}$$



$$\lambda_1 e_1 + \lambda_2 e_2 + \lambda_3 e_3 = \begin{pmatrix} -2+2+0 \\ 4+1-5 \\ 0+2+2 \\ -2+2+0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \\ 0 \end{pmatrix} = \hat{x}$$

$$x - \hat{x} = \begin{pmatrix} -2 \\ 0 \\ 4 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 0 \\ 2 \end{pmatrix} = \tilde{x}$$

$$|\tilde{x}| = \sqrt{\langle \tilde{x} | \tilde{x} \rangle} = \sqrt{8} = 2\sqrt{2}$$

$$\langle \tilde{x} | e_1 \rangle = (-2)(-1) + 0 \cdot 2 + 0 \cdot 0 + 2(-1) = 0$$