Задача № 1

a) ~~$(y^3)' = y = \frac{1+x-x^2}{1+x+x^2}$~~

~~$y' = \frac{1-2x}{1+2x}$~~

ДЗ2 №1

$$y = \frac{1+x-x^2}{1-x+x^2}$$

$$y' = \left(\frac{1+x-x^2}{1-x+x^2}\right)' = \frac{(1+x-x^2)'\cdot(1-x+x^2)-(1+x-x^2)(1-x+x^2)'}{(1-x+x^2)^2} =$$

$$= \frac{(1-2x)(1-x+x^2)-(1+x-x^2)(-1+2x)}{(1-x-x^2)^2} =$$

$$= \frac{(1-x+x^2-2x+2x^2-2x^3)-(-1+2x-x+2x^2+x^2-2x^3)}{(1-x-x^2)^2}$$

~~$(2-7)(4+3)$~~ ~~$8+6-42-21$~~

$$= \frac{1-3x+\cancel{3x^2}-\cancel{2x^3}+1-x-\cancel{3x^2}+\cancel{2x^3}}{1-x^2-x^4} =$$

$$= \frac{2-4x}{1-x^2-x^4}$$ — Вроде так, насчет знаменателя не уверен. наверное лучше оставит $(1-x-x^2)^2$

b) $y = \frac{1}{x} + \frac{1}{\sqrt{x}} + \frac{1}{\sqrt[3]{x}}$

$$y' = \left(\frac{1}{x}\right)' + \left(\frac{1}{\sqrt{x}}\right)' + \left(\frac{1}{\sqrt[3]{x}}\right)' = (x^{-1})' + \left(x^{-\frac{1}{2}}\right)' + \left(x^{-\frac{2}{3}}\right)' =$$

$$= -x^{-2} + \left(-\frac{1}{2}x^{-\frac{3}{2}}\right) + \left(-\frac{2}{3}x^{-\frac{5}{3}}\right) =$$

$$= -\frac{1}{x^2} + \left(-\frac{1}{2}\cdot\frac{1}{\sqrt[3]{x^2}}\right) + \left(-\frac{2}{3}\cdot\frac{1}{\sqrt[3]{x^5}}\right) =$$

$$= -\frac{1}{x^2} - \frac{1}{2\sqrt[3]{x^2}} - \frac{2}{3\sqrt[3]{x^5}}$$

d) $y = \sqrt[3]{\dfrac{1+x^3}{1-x^3}} = \left((1+x^3)(1-x^3)^{-1}\right)^{\frac{1}{3}}$ ДЗ 2

$$y' = \frac{1}{3}\left((1+x^3)(1-x^3)^{-1}\right)^{-\frac{2}{3}} \cdot \left((1+x^3)(1-x^3)^{-1}\right)' =$$

$$= \frac{1}{3}\left(\frac{(1+x^3)}{1-x^3}\right)^{-\frac{2}{3}} \cdot \left((1+x^3)' \cdot (1-x^3)^{-1} + (1+x^3)\left(\left((1-x^3)^{-1}\right)'\right)\right) =$$

$$= \frac{1}{3}\left(\frac{1+x^3}{1-x^3}\right)^{-\frac{2}{3}} \cdot \left(3x^2(1-x^3)^{-1} + (1+x^3)\left(-1(1-x^3)^{-2}\right)\right) =$$

$$= \frac{1}{3}\left(\frac{1+x^3}{1-x^3}\right)^{-\frac{2}{3}} \left(\frac{3x^2}{1-x^3} + \frac{1+x^3}{(-1)(1-x^3)^2}\right)$$

не уверен.

e) $y = \underset{\text{внеш}}{\operatorname{arctg}} \underset{\text{внутр}}{\frac{x^2}{a}}$

$$y' = \left(\operatorname{arctg}\frac{x^2}{a}\right)' = \frac{1}{1+\left(\frac{x^2}{a}\right)^2} \cdot \left(\frac{x^2}{a}\right)' = \frac{1}{1+\left(\frac{x^2}{a}\right)^2} \cdot \cancel{\left(\frac{x^2}{a}\right)} \frac{2x}{a} =$$

$$= \frac{2x}{\left(1+\left(\frac{x^2}{a}\right)^2\right) \cdot a}.$$

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F) $\cancel{y = \arcsin \frac{1-x^2}{1+x^2} = \arcsin (1-x^2)\cdot(1+x^2)^{-1}}$

$$\cancel{y' = \left(\arcsin (1-x^2)\cdot(1+x^2)^{-1}\right)' =}$$

$$\cancel{= \frac{1}{\sqrt{1-\left((1-x^2)\cdot(1+x^2)^{-1}\right)^2}} \cdot \left((1-x^2)\cdot(1+x^2)^{-1}\right)' =}$$

$$\cancel{= \frac{1}{\sqrt{1}}}$$

$$\cancel{= \frac{1}{\sqrt{1 - \frac{1-x^2}{1+x^2}}} \cdot \left(\frac{-2x}{1+x^2}\right)}$$

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F) $y = \arcsin \frac{1-x^2}{1+x^2}$

$$y' = \left(\arcsin \frac{1-x^2}{1+x^2}\right)' = \frac{1}{\sqrt{1-\frac{1-x^2}{1+x^2}}} \cdot \left(\frac{1-x^2}{1+x^2}\right)' =$$

$$= \frac{1}{\sqrt{1-\frac{1-x^2}{1+x^2}}} \cdot \left(\frac{(1-x^2)'(1+x^2) - (1-x^2)(1+x^2)'}{(1+x^2)^2}\right) =$$

$$= \frac{1}{\sqrt{1-\frac{1-x^2}{1+x^2}}} \cdot \frac{-2x(1+x^2) - (1-x^2)\cdot 2x}{1+x^4} =$$

$$= \frac{1}{\sqrt{1-\frac{1-x^2}{1+x^2}}} \cdot \frac{2x(-1-\cancel{x^2}-1+\cancel{x^2})}{1+x^4} = \frac{1}{\sqrt{1-\frac{1-x^2}{1+x^2}}} \cdot \frac{-4x}{1+x^4}$$

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d) $y = \sin|\sin(\sin x)| =$

$= \sin\sqrt{(\sin(\sin x))^2}$

$y' = \left(\sin\sqrt{(\sin(\sin x))^2}\right)' = \cos\cdot\sqrt{(\sin(\sin x))^2}\cdot\left(\sqrt{(\sin(\sin x))^2}\right)' =$

$= \cos|\sin(\sin x)|\cdot\frac{1}{2\sqrt{(\sin(\sin x))^2}}\cdot\left(\left(\sin(\sin x)\right)^2\right)' =$

$= \cos|\sin(\sin x)|\cdot\frac{1}{2|\sin(\sin x)|}\cdot 2\cdot\sin(\sin x)\cdot(\sin(\sin x))' =$

$= \cos|\sin(\sin x)|\cdot\frac{1}{2|\sin(\sin x)}\cdot 2\cdot\sin(\sin x)\cdot\cos(\sin x)\cdot(\sin x)' =$

$= \cos|\cancel{\sin(\sin x)}|\cdot\frac{1}{\cancel{2}|\cancel{\sin(\sin x)}}\cdot\cancel{2}\cdot\sin(\sin x)\cdot\cos(\sin x)\cdot(\cos x =$

$= \frac{\cos|\sin(\sin x)|\cdot\sin(\sin x)\cdot\cos(\sin x)\cdot\cos x}{|\sin(\sin x)}$

h) $y=(x-a_1)^{\alpha_1}\cdot(x-a_2)^{\alpha_2}\dots(x-a_n)^{\alpha_n}$ 13 2.1

$$y'=\left((x-a_1)^{\alpha_1}\cdot(x-a_2)^{\alpha_2}\dots(x-a_n)^{\alpha_n}\right)'=$$

$$=\left((x-a_1)^{\alpha_1}\right)'\cdot(x-a_2)^{\alpha_2}\dots(x-a_n)^{\alpha_n}+(x-a_1)^{\alpha_1}\cdot\left((x-a_2)^{\alpha_2}\right)'\dots\cdot(x-a_n)^{\alpha_n}+$$

$$+(x-a_1)^{\alpha_1}\cdot(x-a_2)^{\alpha_2}\dots\cdot\left((x-a_n)^{\alpha_n}\right)'=$$

$$=\alpha_1(x-a_1)^{\alpha_1-1}\cdot\overbrace{(x-a_1)'}^{=1}\cdot(x-a_2)^{\alpha_2}\cdot\dots(x-a_n)^{\alpha_n}+$$

$$+(x-a_1)^{\alpha_1}\cdot\alpha_2(x-a_2)^{\alpha_2-1}\cdot\overbrace{(x-a_2)'}^{=1}\cdot\dots(x-a_n)^{\alpha_n}+$$

$$+(x-a_1)^{\alpha_1}\cdot(x-a_2)^{\alpha_2}\cdot\dots\cdot\alpha_n(x-a_n)^{\alpha_n-1}\cdot\overbrace{(x-a_n)'}^{=1}=$$

$$=\alpha_1(x-a_1)^{\alpha_1-1}\cdot(x-a_2)^{\alpha_2}\cdot\dots\cdot(x-a_n)^{\alpha_n}+$$

$$+(x-a_1)^{\alpha_1}\cdot\alpha_2(x-a_2)^{\alpha_2-1}\cdot\dots\cdot(x-a_n)^{\alpha_n}+$$

$$+(x-a_1)^{\alpha_1}\cdot(x-a_2)^{\alpha_2}\cdot\dots\cdot\alpha_n(x-a_n)^{\alpha_n-1}$$

ДЗ 2.7

g) $y = \ln(e^x + \sqrt{1+e^{2x}})$

$$y' = \left(\ln\left(e^x + \sqrt{1+e^{2x}}\right)\right)' = \frac{1}{e^x + \sqrt{1+e^{2x}}} \cdot \left(e^x + \sqrt{1+e^{2x}}\right)' =$$

$$= \frac{1}{e^x + \sqrt{1+e^{2x}}} \cdot \left(e^x + \frac{1}{2}(1+e^{2x})^{-\frac{1}{2}} \cdot e^{2x}\right.$$

вот тут не уверен.

ДЗ 2

4.б)

$$\lim_{\substack{x\to+\infty\\ y\to+\infty}} \frac{x^2y}{x^4+y^2}$$

1) $M(x,y) \longrightarrow (+\infty,+\infty)$ по прямой $y=kx$

$$\lim_{\substack{x\to+\infty\\ y\to+\infty}} \frac{x^2y}{x^4+y^2} = \left\{\begin{array}{l} y=kx \\ x\to+\infty, y\to+\infty \end{array}\right\} = \lim_{x\to+\infty} \frac{x^2\cdot kx}{x^4+(kx)^2} =$$

$$= \lim_{x\to+\infty} \frac{k\cdot \cancel{x^3}}{\cancel{x^2}(x^2+k^2)} = \lim_{x\to0} \frac{kx}{x^2+k^2} = \frac{+\infty}{+\infty} = +\infty$$

2) $M(x,y) \to (+\infty,+\infty)$ по параболе $y=ax^2$

$$\lim_{\substack{x\to+\infty\\ y\to+\infty}} \frac{x^2y}{x^4+y^2} = \left|\begin{array}{l} y=ax^2 \\ x\to+\infty, y\to+\infty \end{array}\right\} = \lim_{x\to+\infty} \frac{x^2\cdot ax^2}{x^4+(ax^2)^2} =$$

$$= \lim_{x\to+\infty} \frac{a\cancel{x^4}}{\cancel{x^4}\cdot(1+a^2)} = \frac{a}{1+a^2}$$

Вывод: предел зависит от пути, по которому точка $M(x,y)$ стремится к точке $(+\infty,+\infty)$

$\Rightarrow$ предел $\lim\limits_{\substack{x\to+\infty\\ y\to+\infty}} \frac{x^2y}{x^4+y^2}$ не сущ-ет.

4.а. $$\lim_{\substack{x\to0\\ y\to0}} \frac{x^3y^3}{x^2+y^2}$$

1) $M(x,y) \to$ к $(0,0)$ по прямой $y=kx$

$$\lim_{\substack{x\to0\\ y\to0}} \frac{x^3y^3}{x^2+y^2} = \left\{\begin{array}{l} y=kx \\ x\to0, y\to0 \end{array}\right| = \lim_{x\to0} \frac{x^3\cdot(kx)^3}{x^2+(kx)^2} =$$

$$= \lim_{x\to0} \frac{k^3x^6}{x^2(1+k^2)} = \lim_{x\to0} \frac{k^3x^4}{1+k^2} = \frac{0}{1} = 0$$

2) $M(x,y) \to$ к $(0,0)$ по ~~прямой~~ параболе $y=ax^2$

$$\lim_{\substack{x\to0\\ y\to0}} \frac{x^3y^3}{x^2+y^2} = \left|\begin{array}{l} y=ax^2 \\ x\to0, y\to0 \end{array}\right\} = \lim_{x\to0} \frac{x^3\cdot(ax^2)^3}{x^2+(ax^2)^2} =$$

$$= \lim_{x\to0} \frac{a^3\cdot x^9}{x^2(1+ax^2)} = \frac{a^3x^7}{1+ax^2} = \frac{0}{1} = 0$$

Предел сущ-ет и равен 0.

N 2 ДЗ 2

a) $f(x) = \begin{cases} x^2 & \text{если } x \le x_0 \\ ax+b & \text{если } x > x_0 \end{cases}$

~~Как следует~~

Непрерывность:

$x_0^2 = a \cdot x_0 + b$

$x_0^2 - a \cdot x_0 = b$

$2x_0 dx \qquad 1 \cdot a \cdot dx$

$2x_0 dx = a \cdot dx \implies a = 2x_0 \implies x_0 = a/2$

$b = x_0^2 - 2x_0^2 = -x_0^2$

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b) $F(x) = \begin{cases} f(x) & \text{если } x \le x_0 \\ ax+b, & \text{если } x > x_0 \end{cases}$