Algorithm A

$T(n) = 5T(n/2) + \Theta(n)$

$a = 5 \quad b = 2 \quad f(n) = n$

$n^{\log_b a} = n^{\log_2 5} = n^{2.3219}$

$n < n^{2.3219}$

So, $T(n) = \Theta(n^{\log_2 5}) = \Theta(n^{2.3219})$

Algorithm B

$T(n) = 2T(n-1) + 1$

$= 2(2T(n-2)+1)+1 = 2^2 T(n-2) + 2 + 1$

$= 2^k T(n-k) + \sum_{i=0}^{k-1} 2^i$

$k = n-1 \Rightarrow 2^{n-1} T(1) + \sum_{i=0}^{n-2} 2^i$

$\Rightarrow \sum_{i=0}^{n-2} 2^i = 2^{n-1} - 1$

So $T(n) = \Theta(2^n)$

Algorithm C

$a = 9, \quad b = 3, \quad f(n) = n^2$

$n^{\log_b a} = n^{\log_3 9} = n^2$

So the grow rate is $C < A < B$

C is the best

$T(n) = \Theta(n^2 \log n)$

2)

a) $T(n) = 2T(n/3) + 1$

$a = 2, b = 3, f(n) = 1 = n^0$

$n^{\log_3 2} = n^{0.6309}$

$T(n) = \Theta(n^{\log_3 2})$

b) $T(n) = 5T(n/4) + n$

$a = 5, b = 4, f(n) = n = n^1$

$n^{\log_4 5} = n^{1.1609}$

$T(n) = \Theta(n^{\log_4 5})$

c) $T(n) = 7T(n/7) + n$

$a = 7, b = 7, f(n) = n$

$n^{\log_7 7} = n^1$ $T(n) = \Theta(n \log n)$

d) $T(n) = 9T(n/3) + n^2$

$a = 9, b = 3, f(n) = n^2$

$n^{\log_3 9} = n^2$

$T(n) = \Theta(n^2 \log n)$

e) $T(n) = 8T(n/2) + n^3$

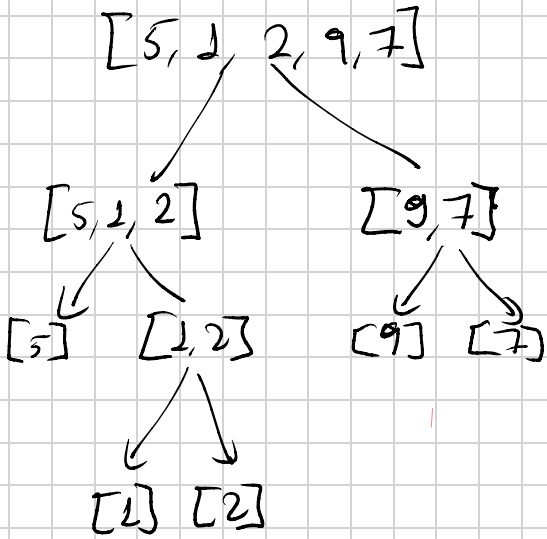
$a = 8, b = 2, f(n) = n^3$

$n^{\log_2 8} = n^3$

$T(n) = \Theta(n^3 \log n)$

3)

Merge Sort



Quick Sort