Project Proposal: Social Choice and Preference Aggregation

Modal Ranking: A Uniquely Robust Voting Rule

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Main Topic/Claim

The paper discusses voting rules that output a correct ranking of alternatives by quality from a large range of noisy input rankings. The paper suggests a very simple voting rule – select the most frequent ranking as the output. The paper shows some theoretical desired properties of this algorithm.

Main Results

The paper results summarize three theorems that have been proven. Before going over the theorems, let's define the notations:

- A - set of alternatives, $|A| = m$.
- $L(A)$ - set of rankings over A.
- $D(L(A))$ - set of distributions over A.
- $\pi = \{L_1, \ldots, L_n\}$ - a collection of votes.
- SWF - randomized social welfare function $f : \pi \to D(L(A))$.

Using these notations, we can define 3 families of $SWFs$:

- **PM-c Rules**: all SWF that for every π with a complete acyclic pairwise-majority graph whose vertices are ordered w.r.t $\sigma \in L(A)$, we get $f(\pi) = \sigma$ w.p. 1. Pairwise-majority stands for pairwise more votes that prefer one over the other.
- **PD-c Rules**: all SWF that for every profile π with complete acyclic position-dominance graph whose vertices are ordered w.r.t $\sigma \in L(A)$, we get $f(\pi) = \sigma$ w.p. 1. The position-dominance graph has an edge between two vertices when one has strictly more votes than the other for every position $k \in \{1, 2, ..., m-1\}$.
- **GSRs**: generalized scoring rules, basically contains all voting rules we learned in class and more. We won't elaborate on this definition, but we will say that ranking is based on score and they are characterized by social choice axioms.

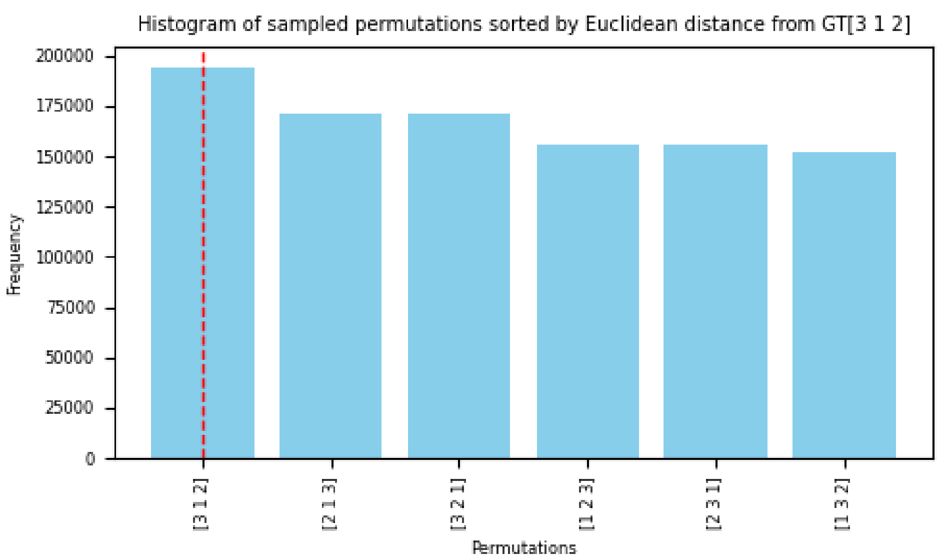
Also, a d-monotonic noise function gets a larger probability for closer samples to the ground truth, with respect to some distance function d. A SWF is monotone robust if it's robust (defined) to every d-monotonic noise function. So, after these definitions, we can show the main results of the paper:

- **Theorem 1**: Let r be a (possibly) randomized GSR without holes. Then, r is monotone-robust with respect to all distance metrics if and only if r coincides with the modal ranking rule on every profile with no ties (i.e., r outputs the most frequent ranking with probability 1 on every profile where it is unique).

- **Theorem 2**: Under uniformly random tie-breaking, all positional scoring rules (including plurality and Borda count), the Kemeny rule, single transferable vote (STV), Copeland's method, Bucklin's rule, the maximin rule, Slater's rule, and the ranked pairs method are GSRs without holes.
- **Theorem 3**: For $m \geq 3$, no $PM - c$ or $PD - c$ rule is monotone-robust to all distance metrics.

These results show that the modal ranking function is the only SWF that's monotone-robust to all distance metrics within a large group of $SWFs$ – The first shows it's the only in the $GSRs$, the second shows that many interesting $SWFs$ are $GSRs$ and the last shows that for $PM-c$ and $PD-c$ the robustness doesn't hold. Furthermore, based on theorem 2, all the scoring rules we learned in class are $GSRs$ without holes.

Helpful Figure



In the implementation part, we defined 2 general distributions that for some parameters, distance metric, number of candidates, and ground truth, define a d-monotone distribution of rankings. Here, we can a histogram of sampled data from Distribution 1:

$$\mathbb{P}_\epsilon(r_i) = \frac{1}{Dist(GT, r_i) + \varepsilon} \cdot C$$

with

$$\epsilon = 1, m = 3, GT = [3, 1, 2]$$

, and the distance metric is the Euclidean distance (looking at rankings as vectors). As we can see, the closer (by Euclidean Distance Metric) to the GT, the higher frequency of a ranking.

Cited paper

The most relevant paper to this paper is "When do noisy votes reveal the truth?" by Procaccia, A. D.; and Shah, N. 2013. Our paper is a continuation of the related paper, that uses the definitions that were introduced in the previous paper such as monotone-robust, PM-c and PD-c and more. In the previous paper, the authors fixed the family of voting rules to be PM-c or PD-c rules, and asked which distance metrics induce noise models for which all the rules in these families are robust. They explore this direction and find out that even the most prominent voting rules such as plurality are not accurate in the limit for any noise models that are monotonic with respect to these distances. That led to a future work question - Which voting rules are monotone-robust even with respect to such distance functions? In our paper, instead of fixing the family of rules, the authors fix the family of distances to be all possible distance metrics d, and characterize the "family" of voting rules that are monotone-robust with respect to any d. They also find the algorithm which is robust to all distance metrics.

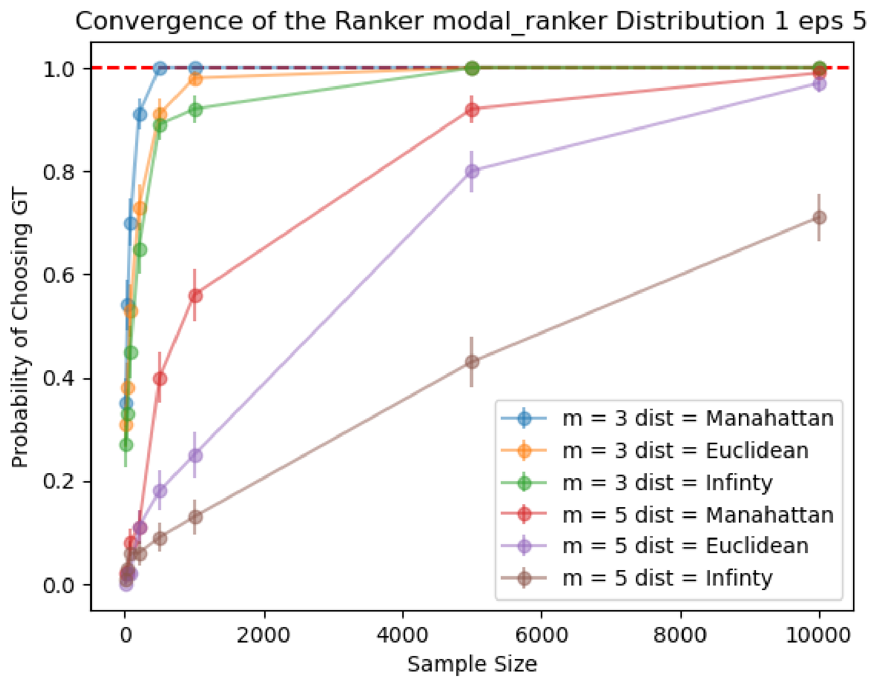
Paper that cited this paper

One major paper that cited our paper is "Beyond majority: Label ranking ensembles based on voting rules" by Werbin-Ofir, H.; Dery, L. and Shmueli, E., published in "Expert Systems With Applications", 2019. This paper focuses on label ranking ensembles (ML task) and the use of voting rules as aggregation techniques to improve the performance of simple label ranking models. Our paper is cited due to the use of the Modal Ranking rule in the experiments and analysis made in this paper.

Part 2 – Implementation

In our work we will try to use simulations to support the proven theorems in the paper. We will use different d-monotone noise distributions and distance metrics defined in the jupyter notebook, and examine the modal-ranking algorithm and other rankings learned through the course. In the implementation part, we:

- Define 2 D-monotone distribution families, and define some distributions for different Ground Truths, number of candidates, and other parameters. We sample a dataset of a million records of rankings for each distribution.
- Show and compare the convergence of the Modal Ranking method to other GSRs (defined score-based methods based on Borda and Plurality scores) on different datasets. Create many convergence plots for different datasets with different distance metrics, number of candidates, and more.



For example, here we can see the convergence of the Modal-Ranking based on different sample sizes. The probability of choosing the right ranking is approximated by some iterations of sampling a dataset and running the algorithm.

- Inference of different ranking methods on a real dataset: we processed and analyzed a dataset of ranking of video games by different critics, and ran the Modal Ranking and other rankings on the inputs, comparing the chosen ranking over games with some "Ground Truth" we found.

The outline here is written in highlights and discussed in more detail in the notebook.