

HW 3

Answers to part A (theoretical part)

1. The formula $Posterior \propto Prior \cdot Likelihood$ represents Bayes' theorem for state estimation. Let's explain each of its components:

Posterior-Our updated belief about the system's state x_t after incorporating both the motion model prediction and the current measurements. This is what we want to compute.

Prior-Our predicted belief about state x_t based only on the motion model (before seeing the current measurement). This comes from the previous time step.

Likelihood-The probability of observing our actual measurement z_t , assuming the true state is x_t . This tells us how likely each possible state is, given what we actually observed.

Both Kalman and particle filters implement the discussed formula, but in different ways.

In **Kalman Filter** the posterior is an updated Gaussian distribution (mean and covariance), representing our belief about the system state. The Bayesian update is computed using closed form linear equations under Gaussian assumptions.

In **Particle Filter** the posterior is represented by updated particle weights. We multiply each particle's prior weight by how well that particle's location matches our target: $w[i] = w[i] \times likelihood[i]$. For example, if a particle lands where the image patch looks very similar to our tracked object, it gets high likelihood. This is simulation based since we don't assume Gaussian distributions.

2.

- a. We use histograms because they capture what colors appear in a patch without caring where those colors are located. This makes them good for tracking objects that might move or change shape slightly.

Pros:

- Position independent-Only cares about colour frequency, not location.
- Quick to calculate for real-time tracking.
- Handles some illumination changes.

Cons:

- Can't tell where colors appear in the patch.
- Different objects with similar colors get mixed up.

- b. SSD requires that every pixel stays exactly the same between frames and that the object doesn't move even slightly. In real videos, this never happens-objects move around, lighting changes, and even small shadows

make SSD give wrong results. SSD is very sensitive to any changes, while histograms are more flexible and can handle these normal variations that happen during tracking.

- c. We can compare patches using **SSIM**-an image quality metric that measures how similar two patches are by comparing their brightness, contrast, and structure.

Pros:

- Considers where colors appear in the patch.
- Works better when lighting changes.
- Can tell apart objects with similar colors but different patterns.

Cons:

- Takes more time to calculate than histograms.
- Gets confused by image noise.

3. The standard particle filter will fail when objects change scale significantly because the state vector $[x, y, w, h, vx, vy]$ has fixed width and height parameters with no mechanism to adapt object size.

Performance degrades under viewpoint changes due to appearance variations. When a person rotates while dancing, the color histogram changes substantially as different body parts become visible or occluded, reducing tracking accuracy.

4. A simple approach is to monitor the particle weights over time. When the average particle weight drops below a user specified threshold and stays low for several consecutive frames, this indicates the template no longer matches the object's appearance and needs updating.

To determine the new template, we would use the best performing particle from recent frames-specifically, the particle with the highest weight over the last 2-3 frames. Rather than completely replacing the old template, we would blend them together (70% old template+30% new patch) to avoid sudden changes. This gradual update allows the template to adapt to appearance changes while maintaining tracking stability.