影像處理 02 數位影像基礎

教師:許閔傑、蕭兆翔

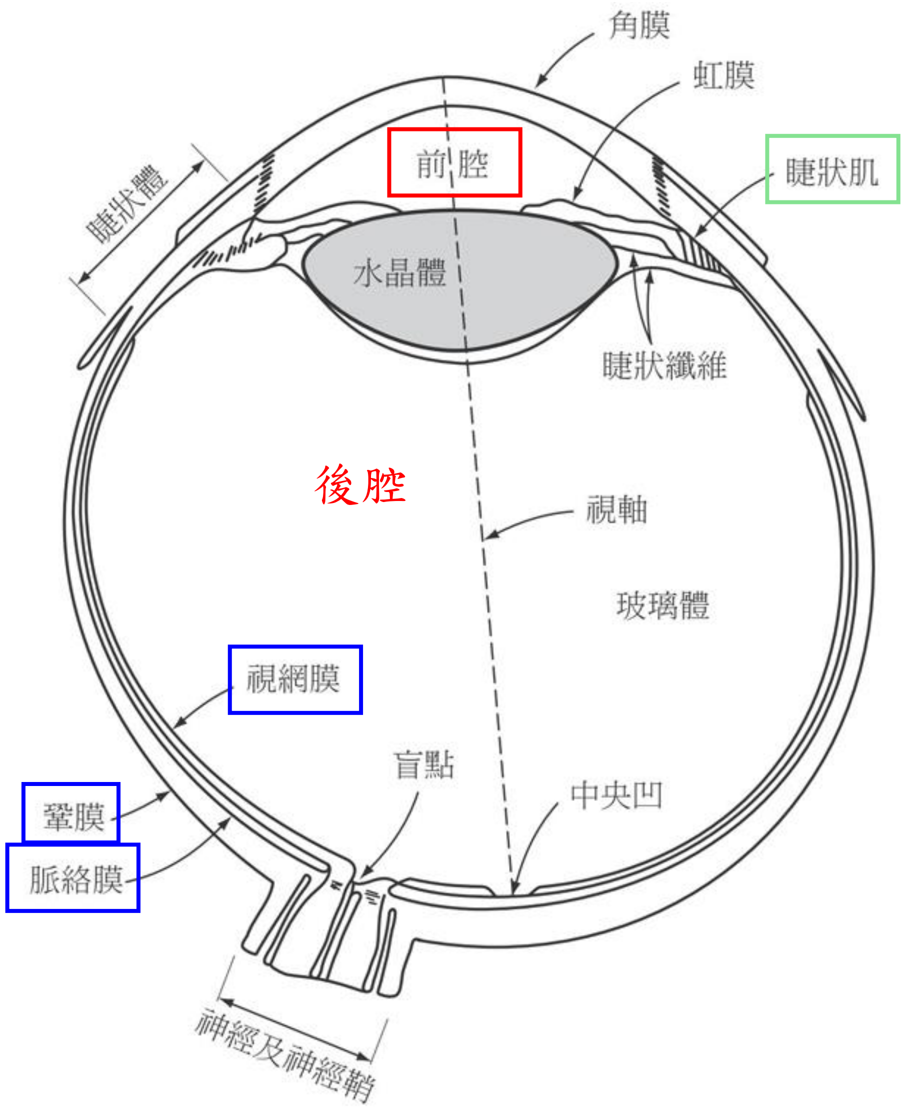
助教:莊媞涵

大綱

- 視覺感知
- 影像感應與擷取
- 影像取樣與量化
- 內插法(Interpolation)
- 空間轉換

2.1 視覺感知的要素

圖 2.1
人類眼睛橫剖面
的簡化圖形



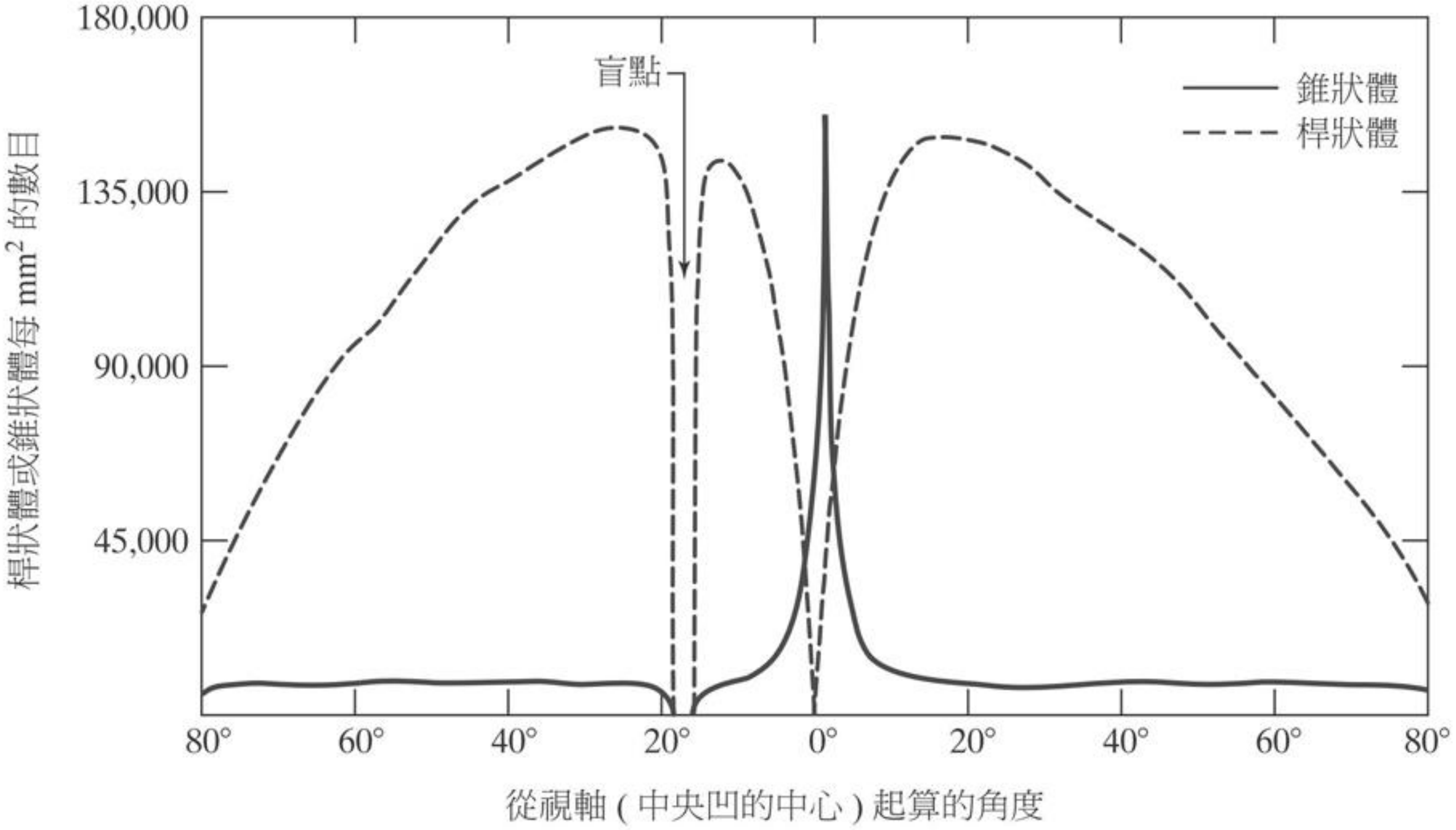


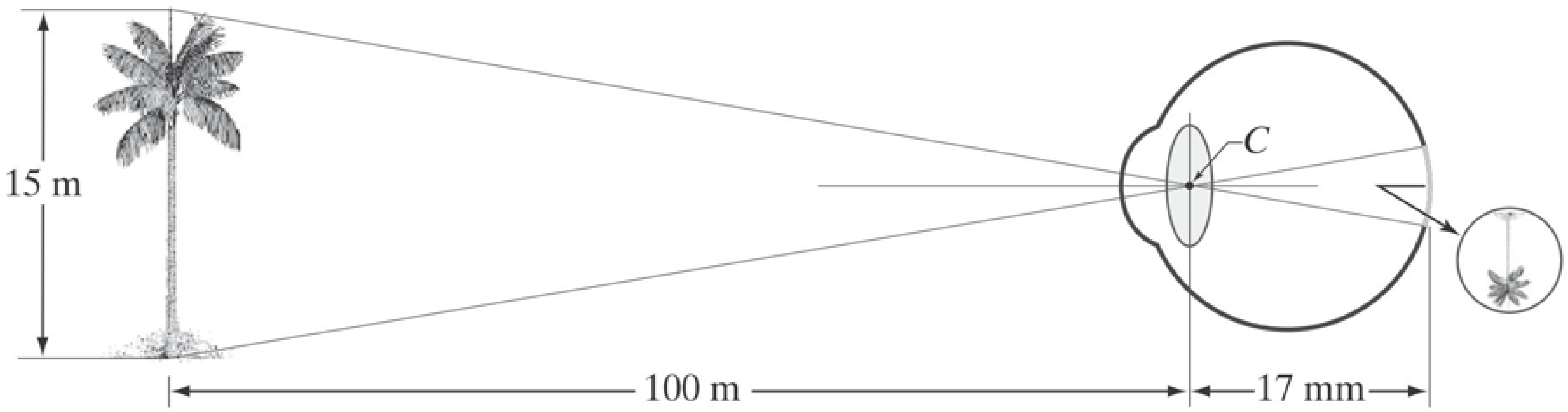
圖 2.2
視網膜上桿狀體和錐狀體的分布

錐細胞(Cone):
負責亮光視覺

桿細胞(Rod):
負責昏暗視覺

眼睛中的細胞

圖 2.3
注視一棵棕櫚樹時眼睛的圖形表示。C 點是水晶體的聚焦中心。



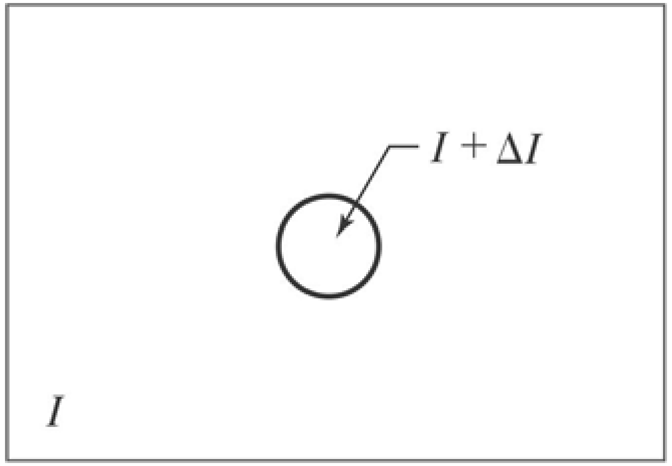
$$\frac{15}{100}=\frac{h}{17}$$

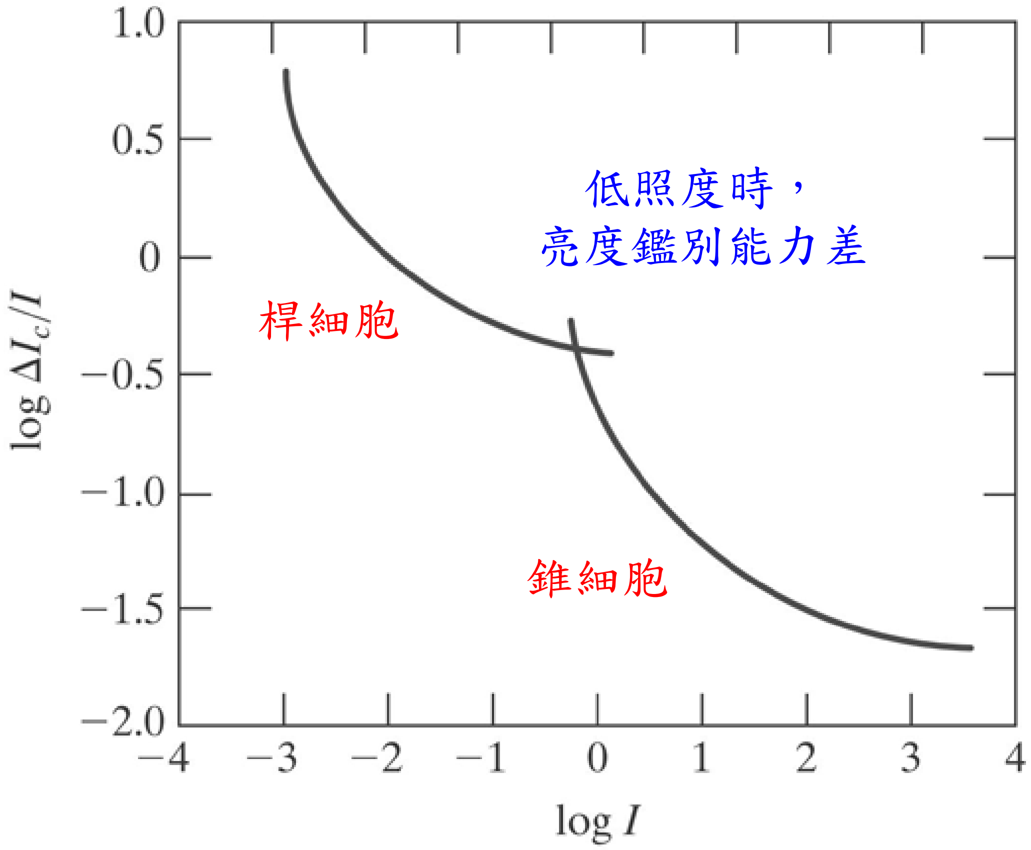
h = 2.25mm

眼睛的亮度鑑別

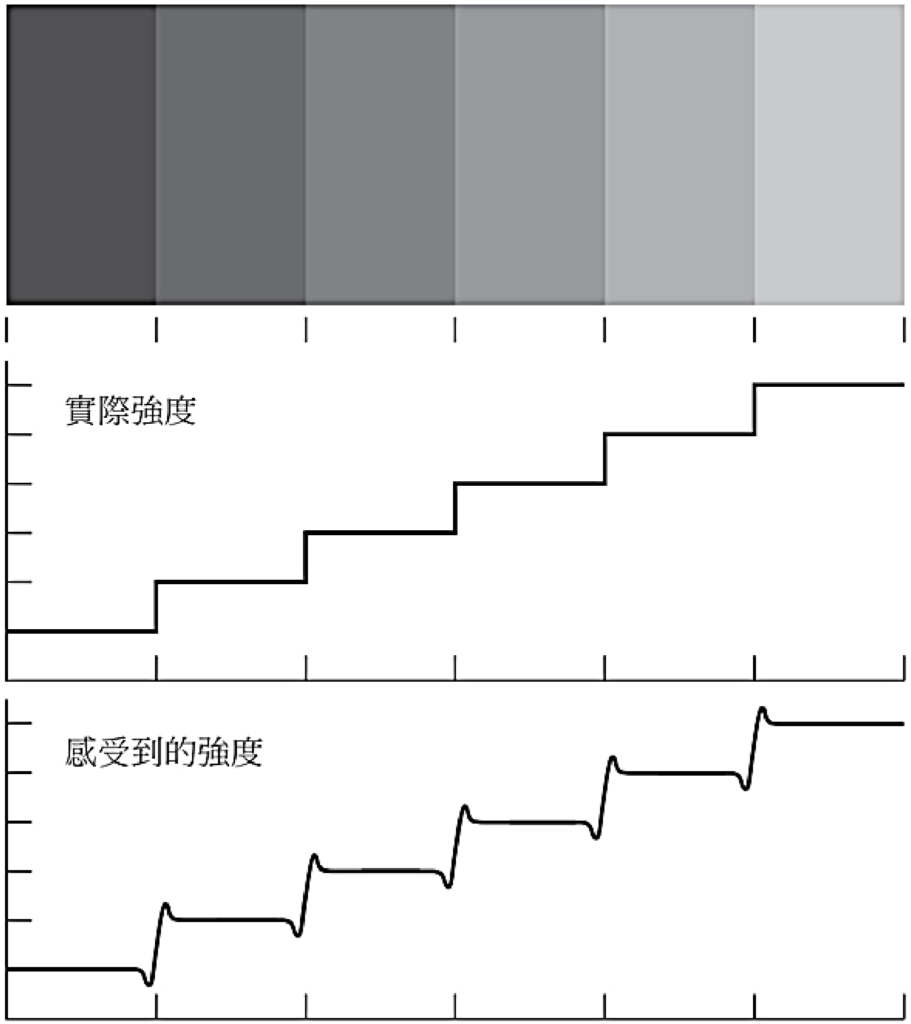
韋伯比(Weber ratio)

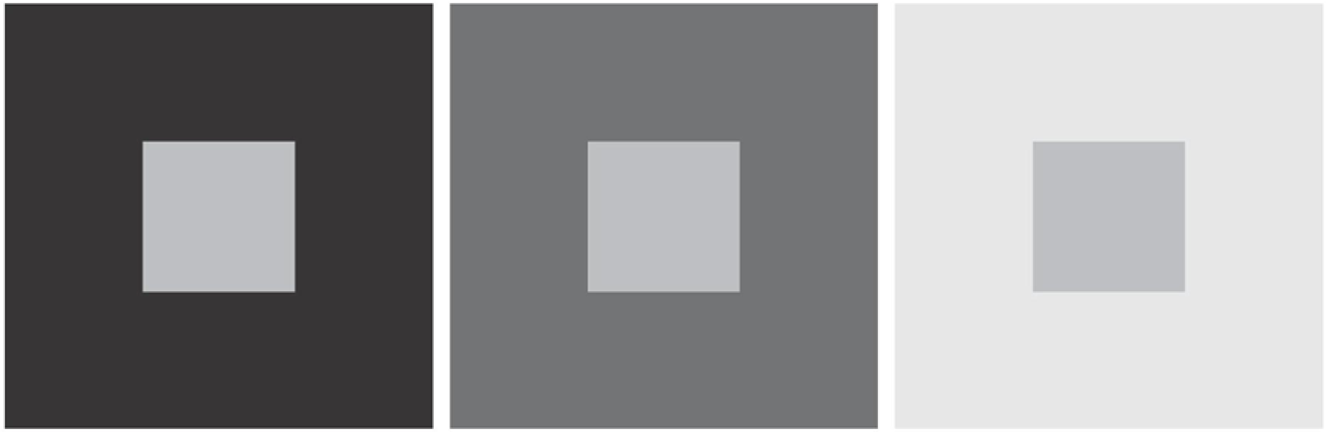
- ➢ $\Delta I_c/\mathrm{I}$



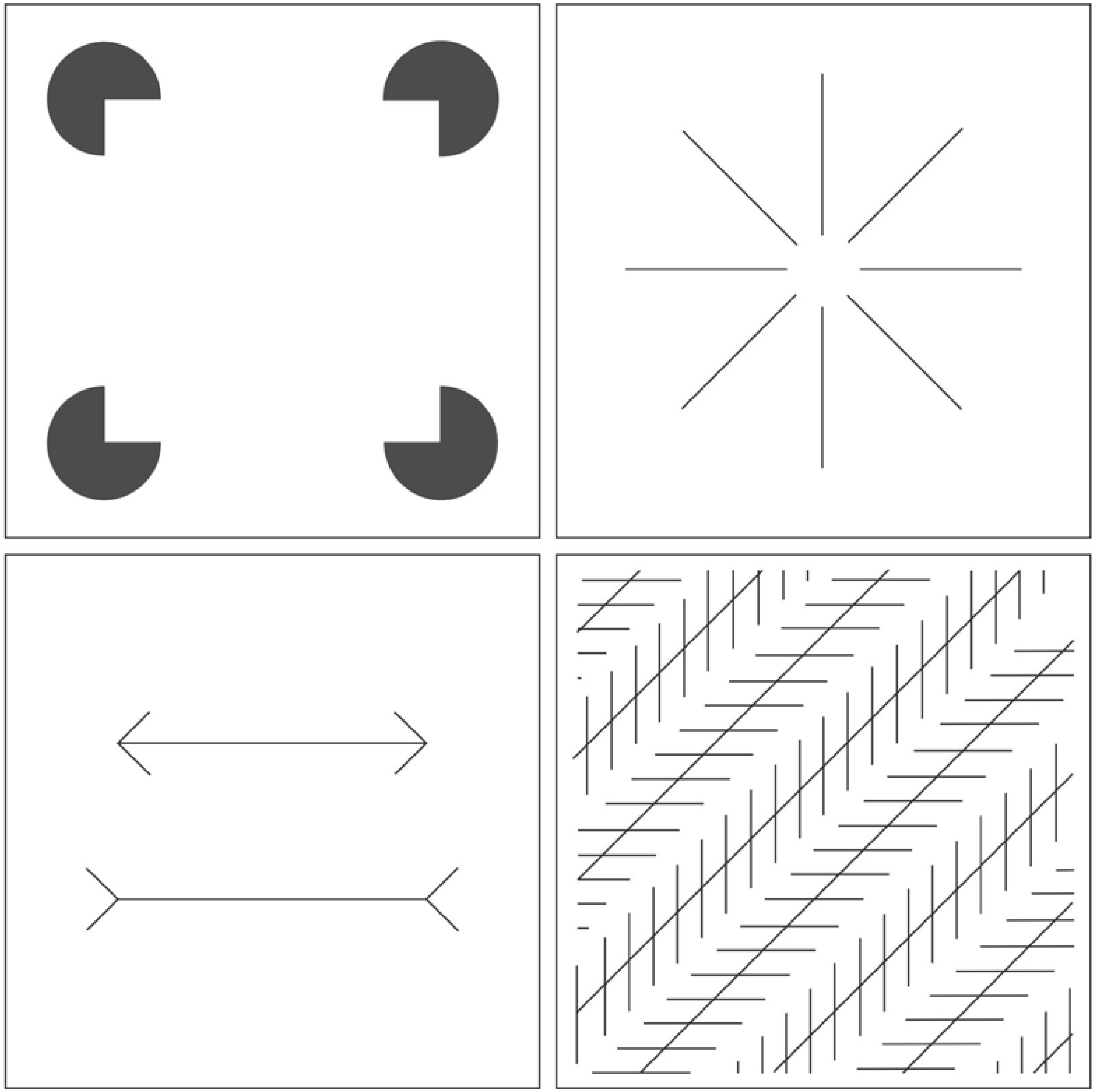


人眼的視覺假象(1/2)

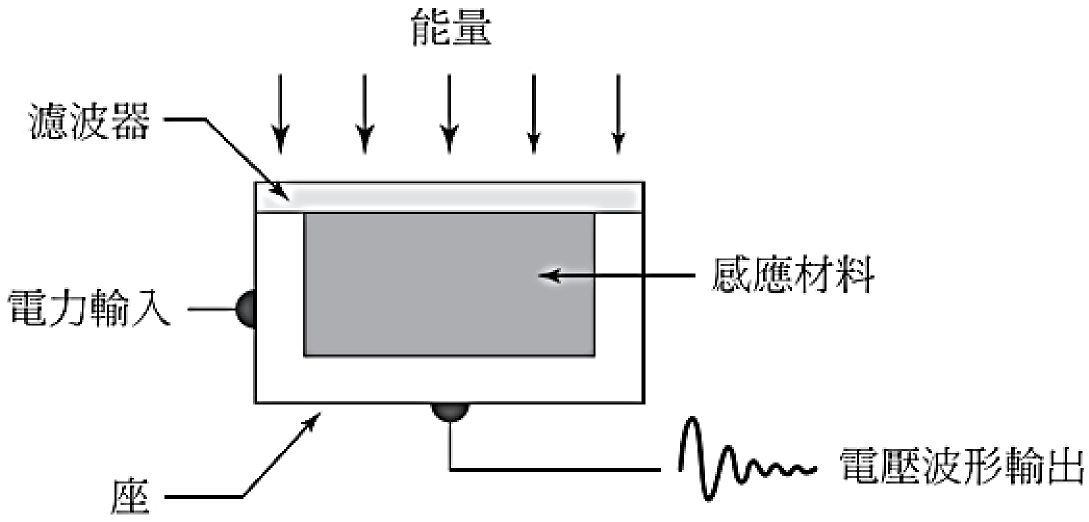




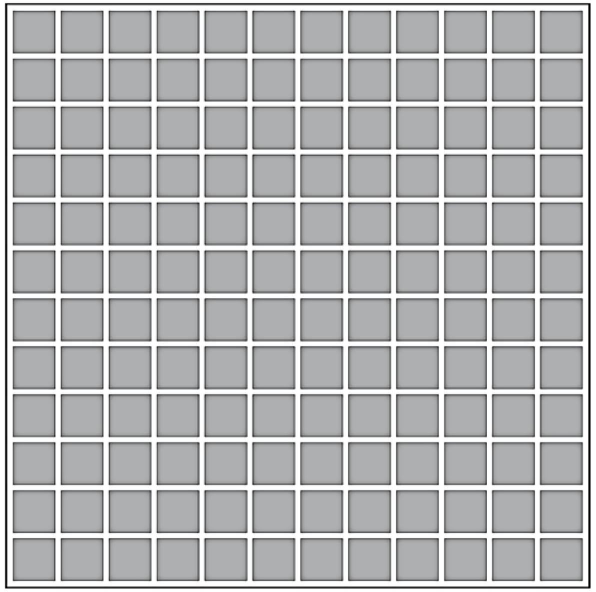
人眼的視覺假象(2/2)



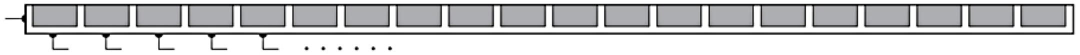
2.2 影像的感應與擷取



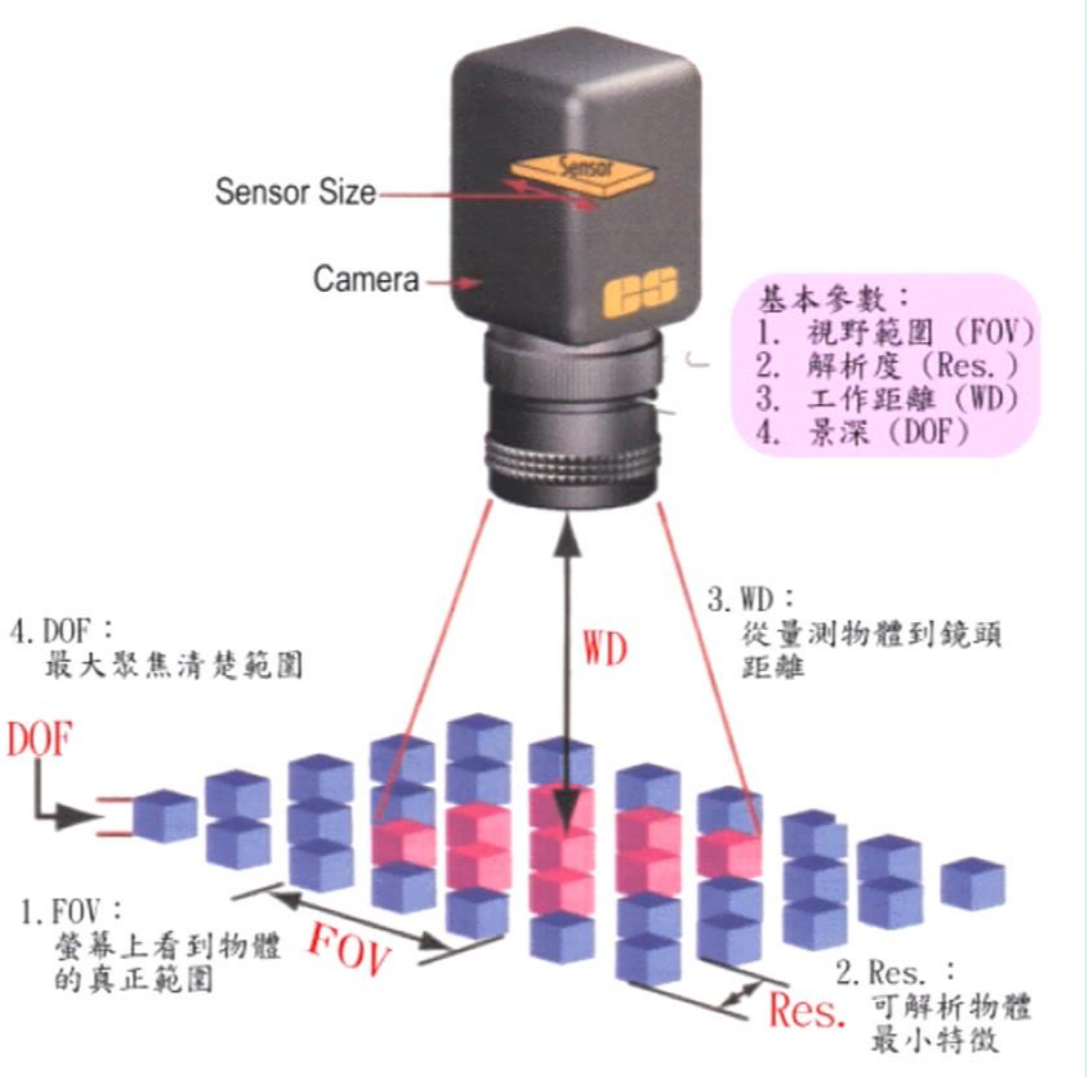
面影像感測器



線影像感測器



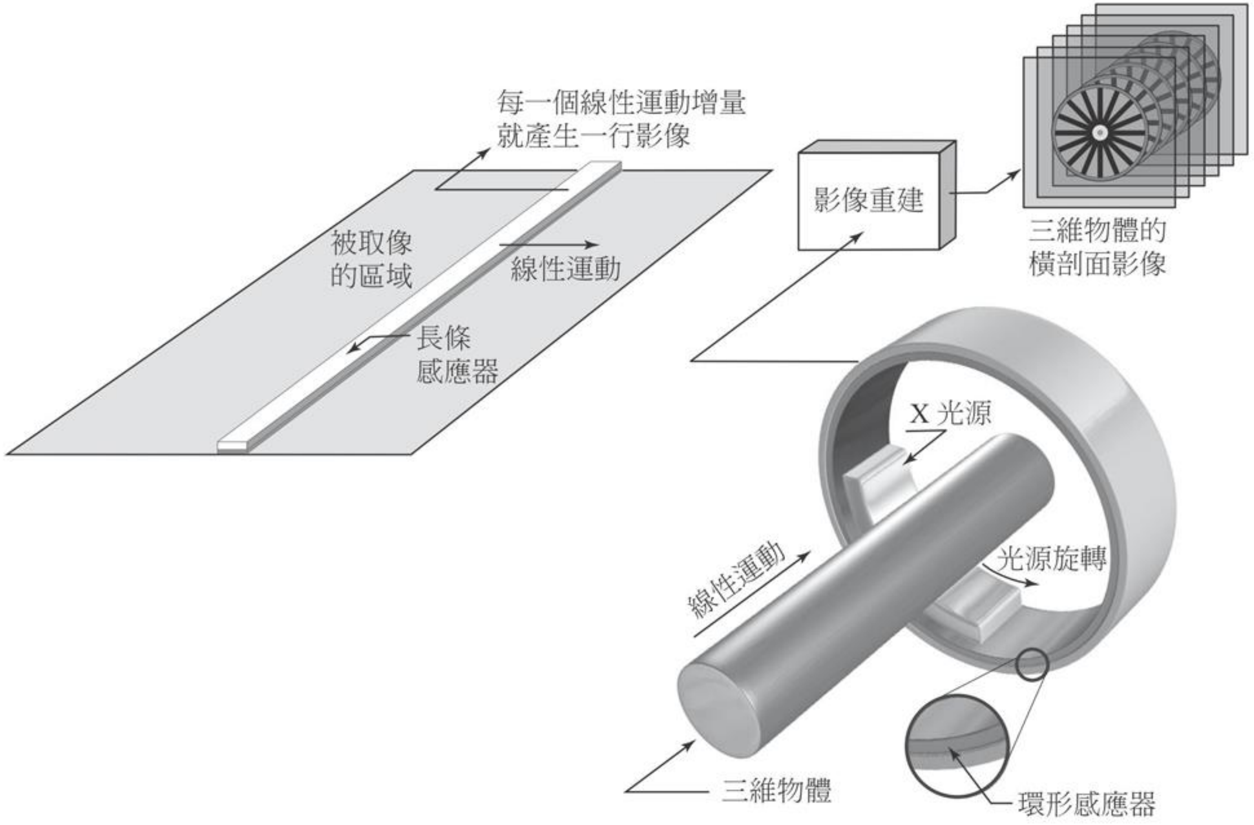
面影像感測器



Sensor Size
Sensor
Camera
基本參數：
1. 視野範圍（FOV）
2. 解析度（Res.）
3. 工作距離（WD）
4. 景深（DOF）
4. DOF：
最大聚焦清楚範圍
WD
3. WD：
從量測物體到鏡頭距離
DOF
1. FOV：
螢幕上看到物體的真正範圍
FOV
Res.
2. Res.：
可解析物體最小特徵

線影像感測器

- 刷載具
- 無人機空中成像
- 核磁共振MRI
- PET



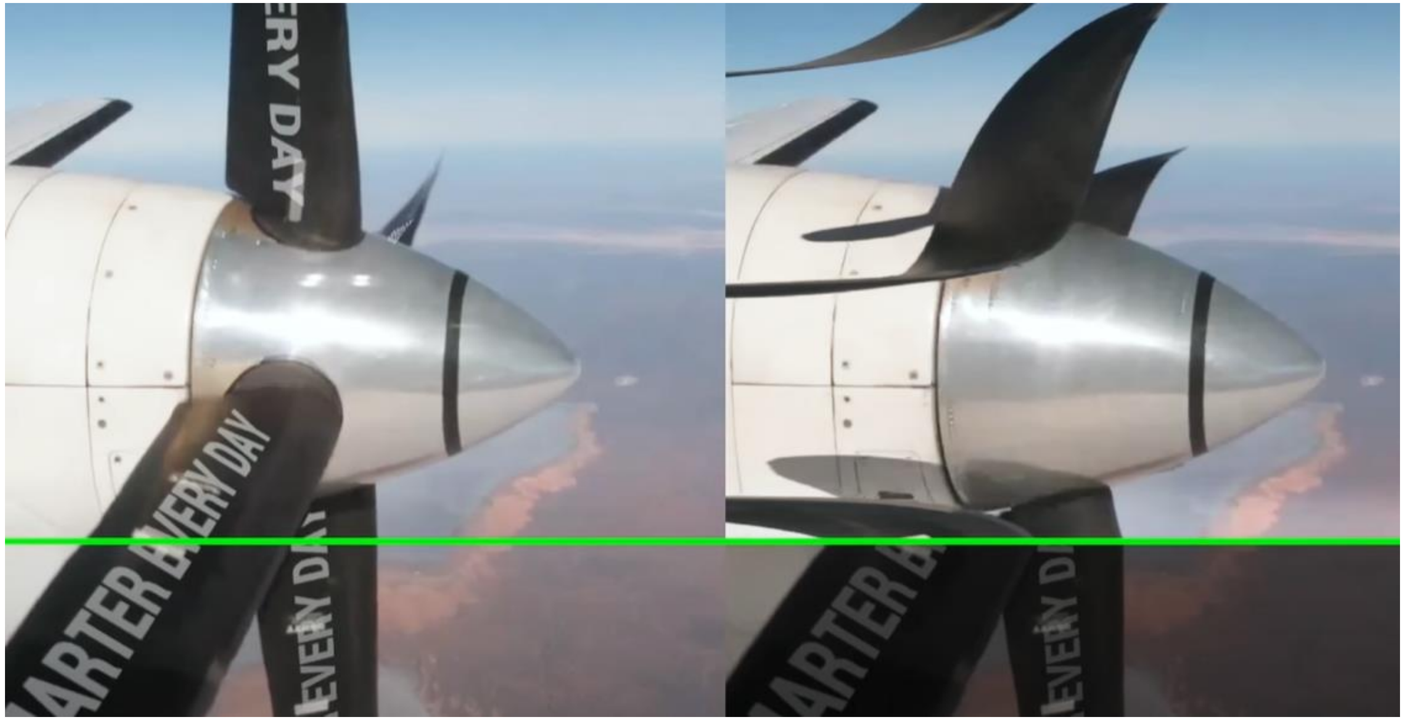
a b

圖 2.12

(a) 用一個長條線型感應器的影像擷取；(b) 用一個圓形長條感應器的影像擷取。

Rolling Shutter Effect

https://www.youtube.com/watch?v=dNVtMmLlnoE&t=33s



2D影像形成模型

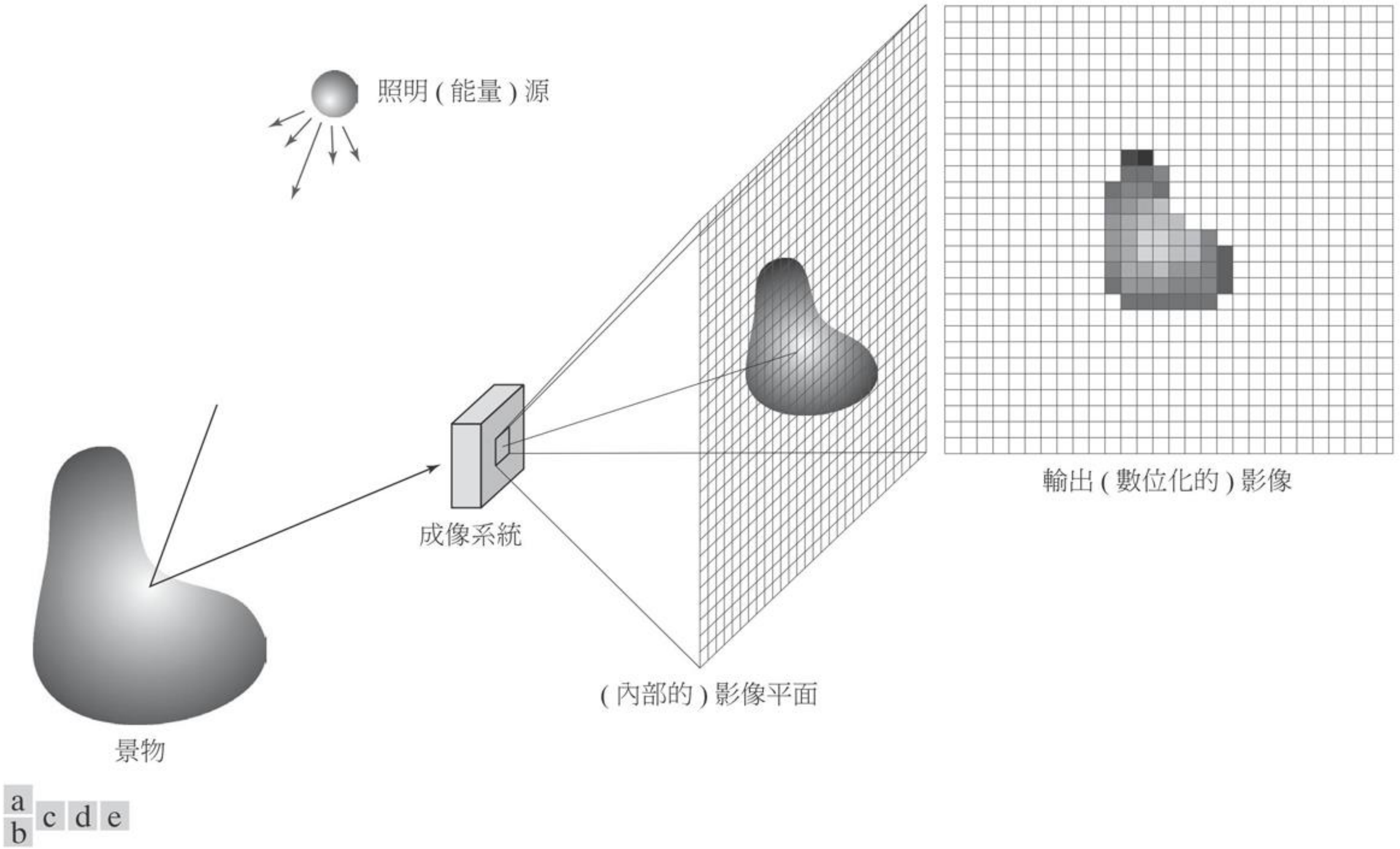


圖 2.13 一個數位影像擷取的例子：(a) 照明（能量）源；(b) 一個景物；(c) 成像系統；(d) 將景物投影到影像平面上；(e) 數位化的影像。

$$0 \le f(x, y) < \infty$$

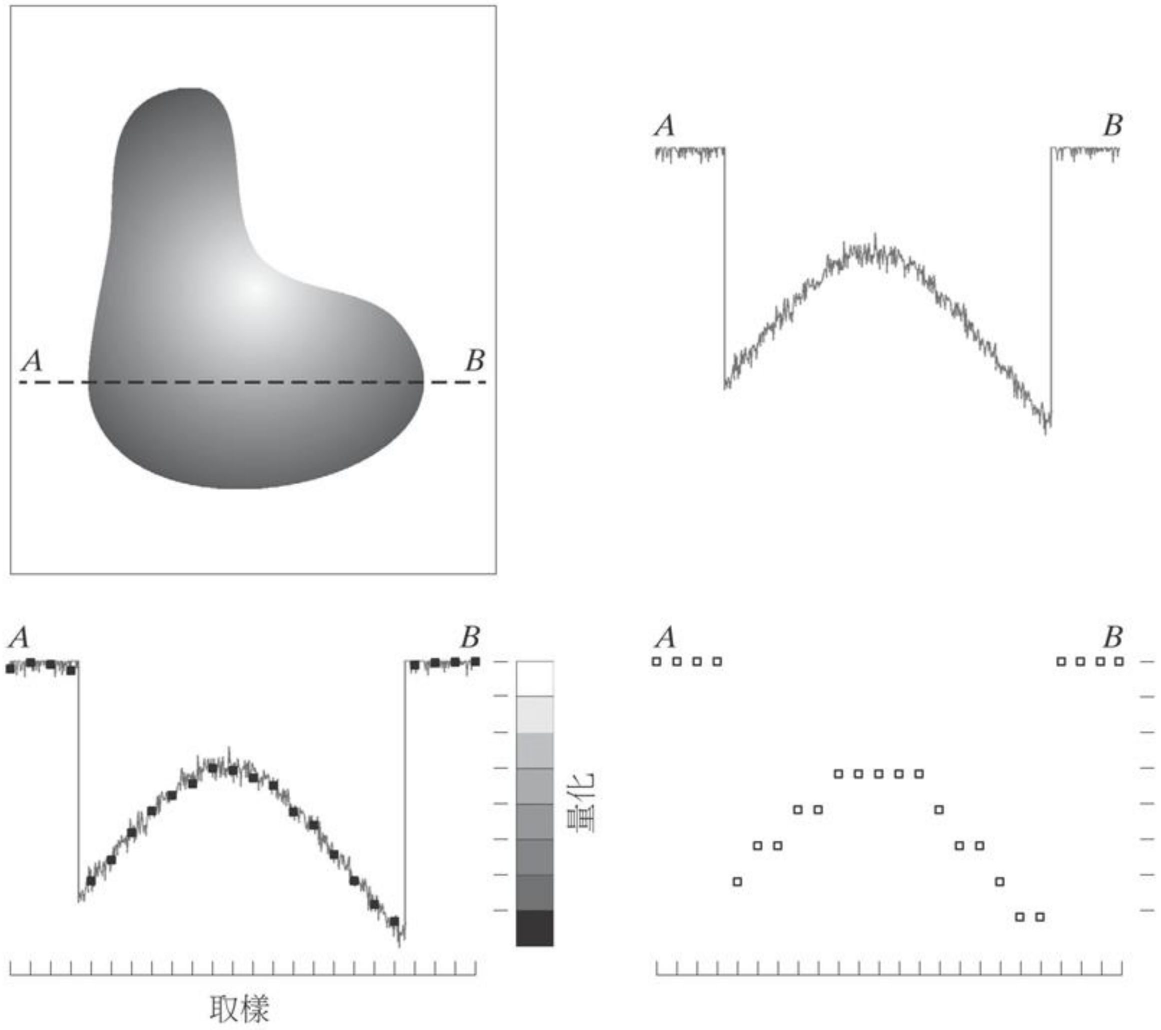
照明量×反射量

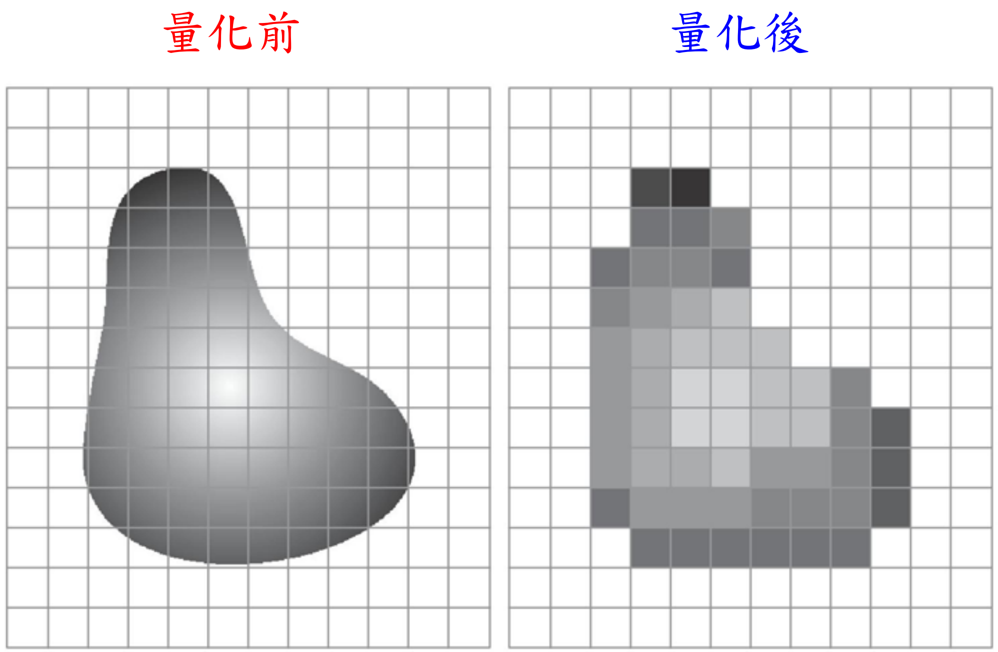
$$\boxed{f(x, y) = i(x, y) r(x, y)}$$

$$0 \le i(x, y) < \infty$$

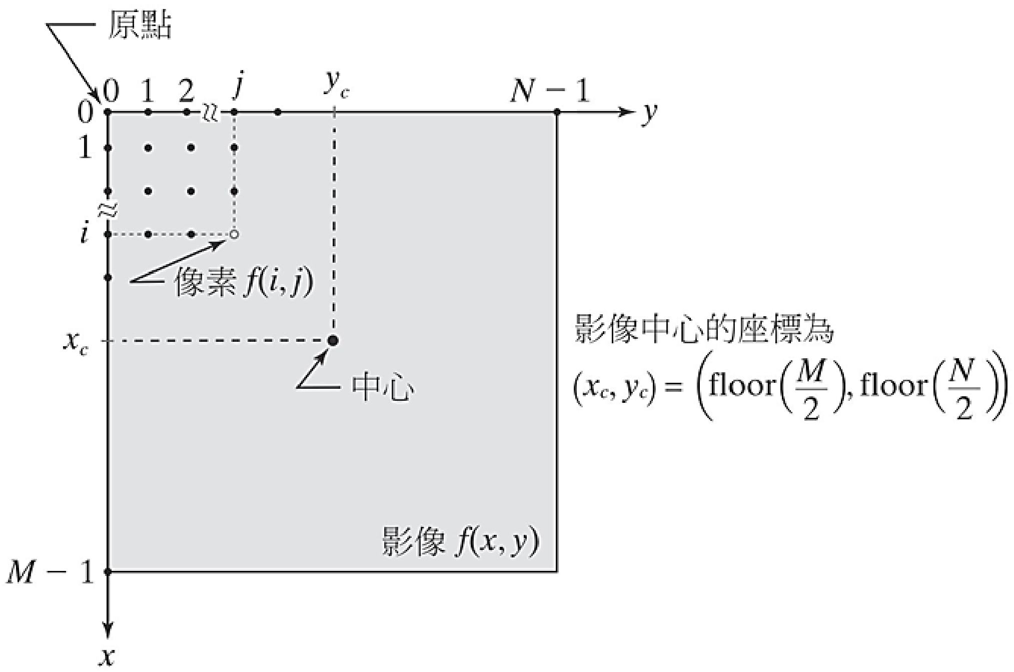
$$0 \le r(x, y) \le 1$$

2.3 影像的取樣和量化

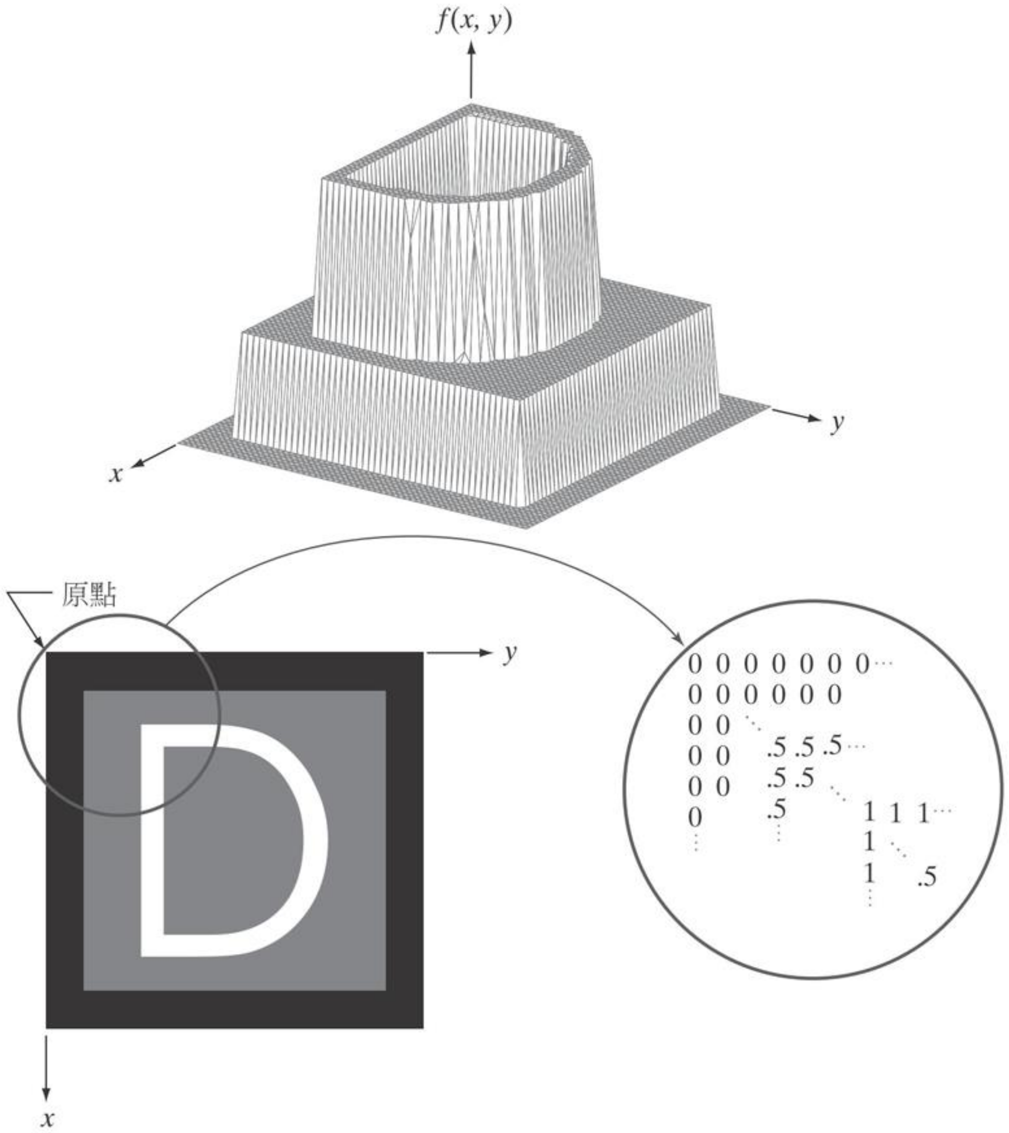




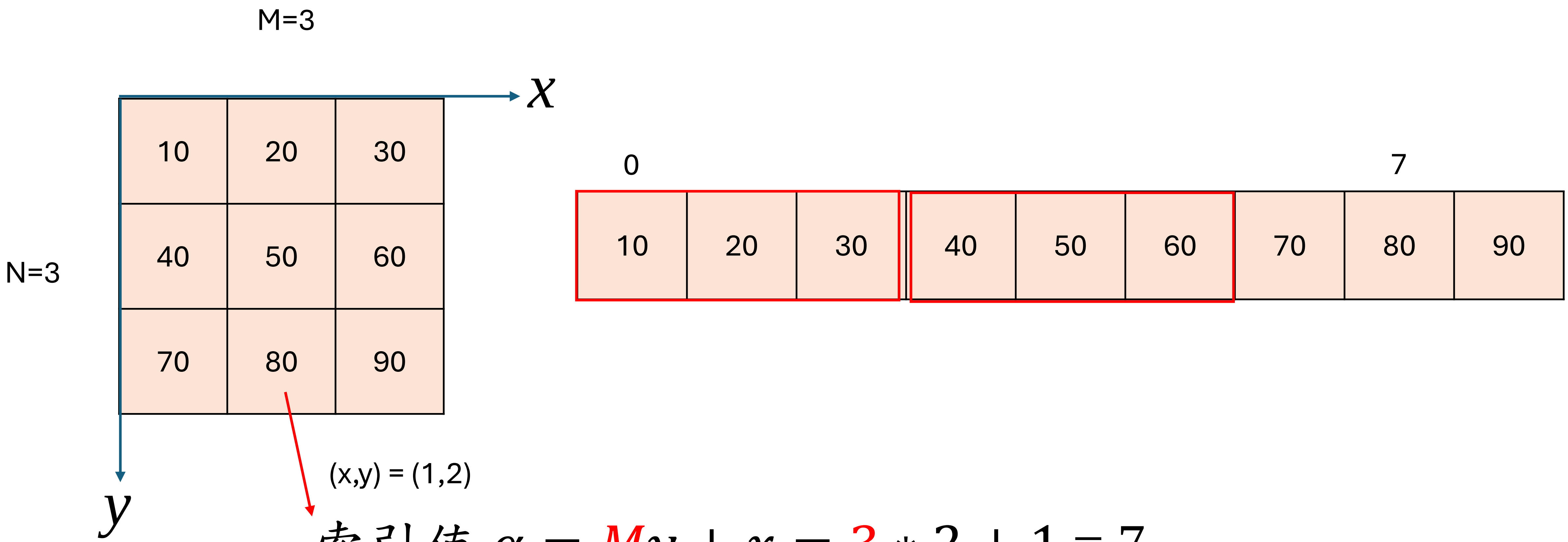
數位影像表達



$$f(x,y)=\begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix} \qquad \textbf{(2-7)}$$



影像索引2D



索引值 $\alpha = {\color{red}M}y + x = {\color{red}3} * 2 + 1 = 7$

影像索引2D

M=3

N=3

10	20	30
40	50	60 (2,1)
70	80	90

x , y

0					5			
10	20	30	40	50	60	70	80	90

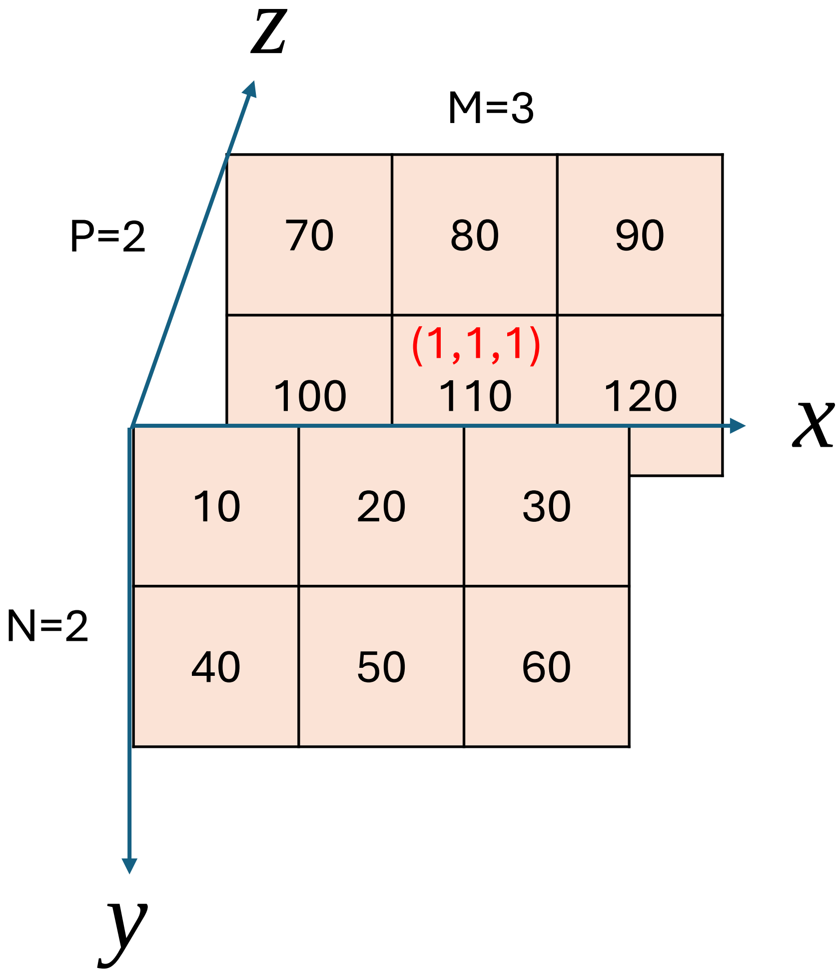
已知索引值 α 求(x,y)

- $x = \alpha \; mod \; M$
- $y = (\alpha - x) \,/\, M$

$x = 5 \; mod \; 3 = 2$

$y = (5 - 2) \,/3 = 1$

影像索引3D

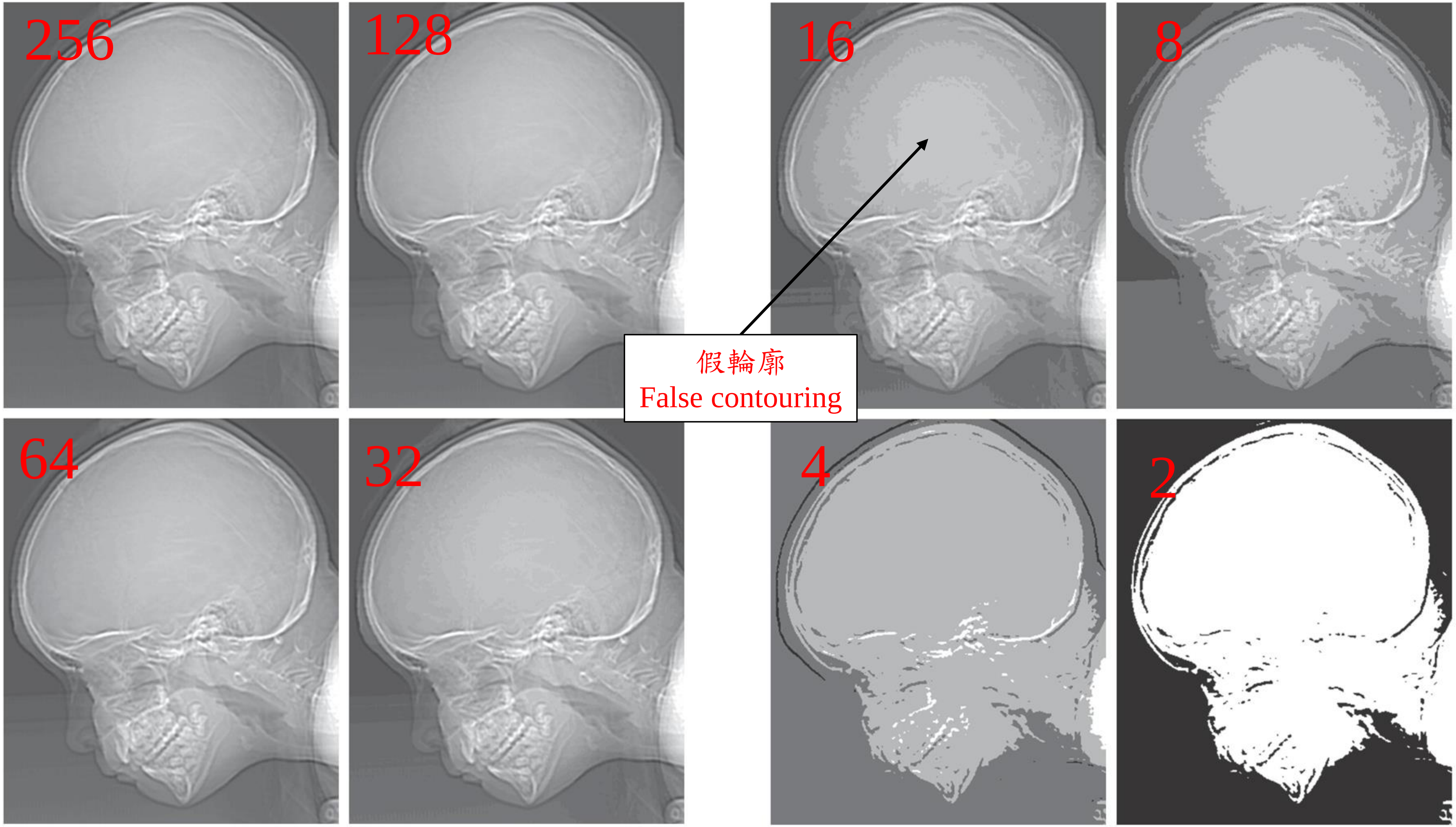


0										10	
10	20	30	40	50	60	70	80	90	100	110	120

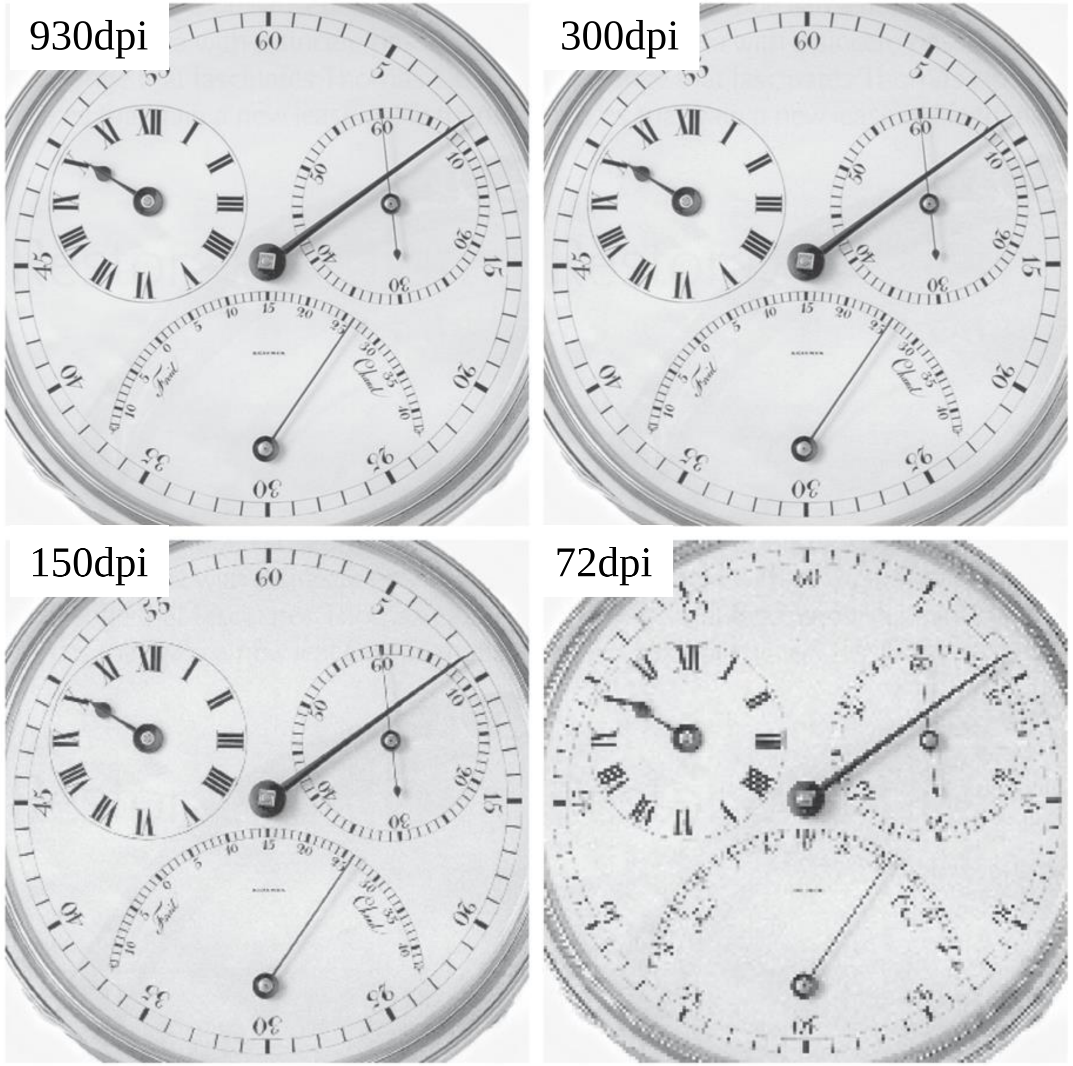
索引值 $S = {\color{red}x} + My + MNz = x + M(y + Nz)$

- $x = S \bmod MN \bmod M$
- $y = S \bmod MN / M$ (取商數)
- $z = S/MN$ (取商數)

強度解析度



影像內插(Interpolation)



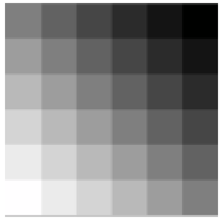
鄰近差補　　雙線性差補　　雙立方差補

a b c

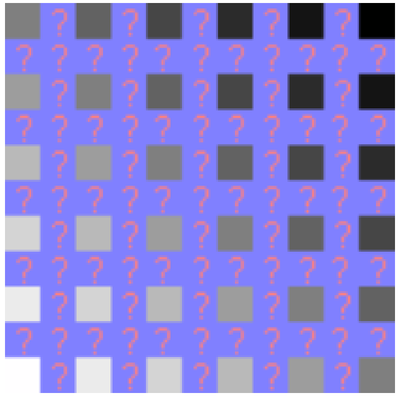
圖 2.25 (a) 縮小到 72 dpi 再用最近鄰插補放大回其原來 930 dpi 大小的影像。此圖與圖 2.21(d) 相同；(b) 縮小再用雙線性插補放大的影像；(c) 與 (b) 同但用雙立方插補。

影像內插(Interpolation)

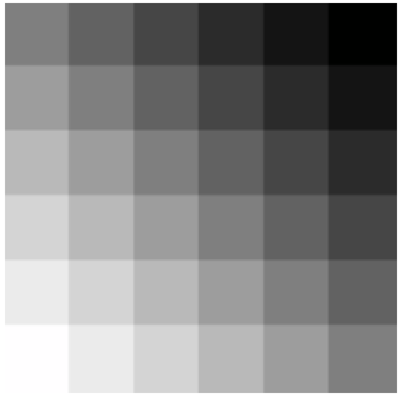
- Common interpolation techniques
  - Nearest Neighbor (NN)
  - Bilinear
  - Bicubic



Original



Enlarged



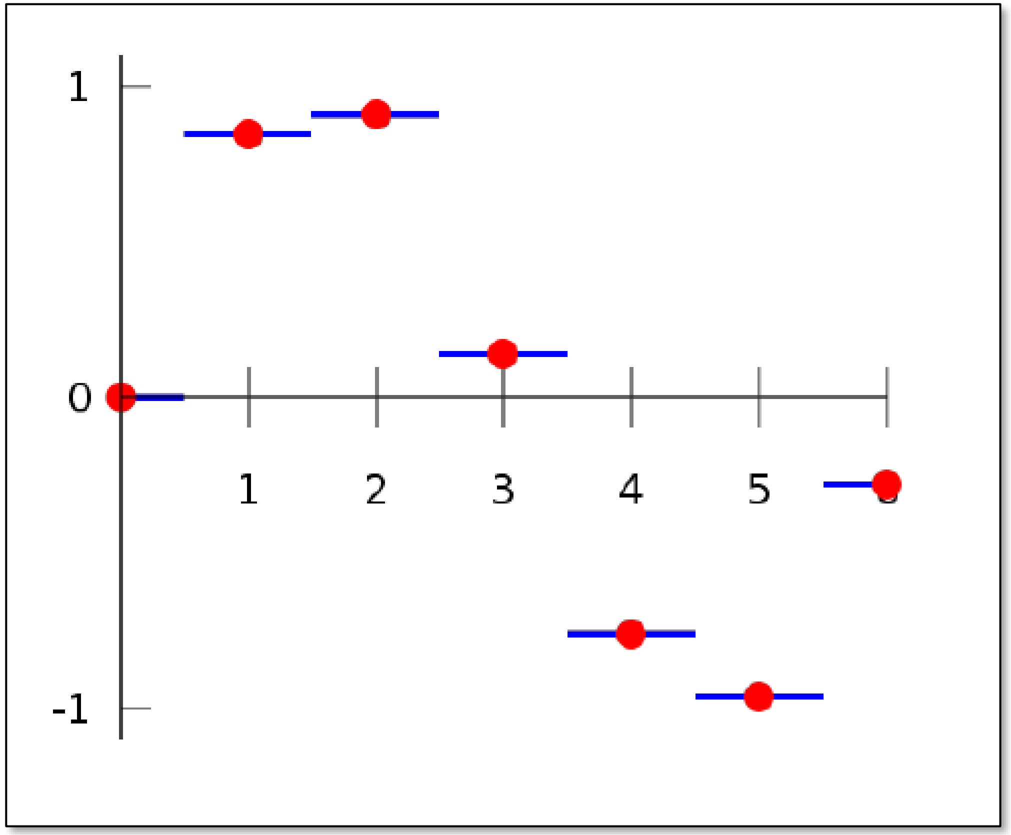
NN

Nearest Neighbor

Bilinear

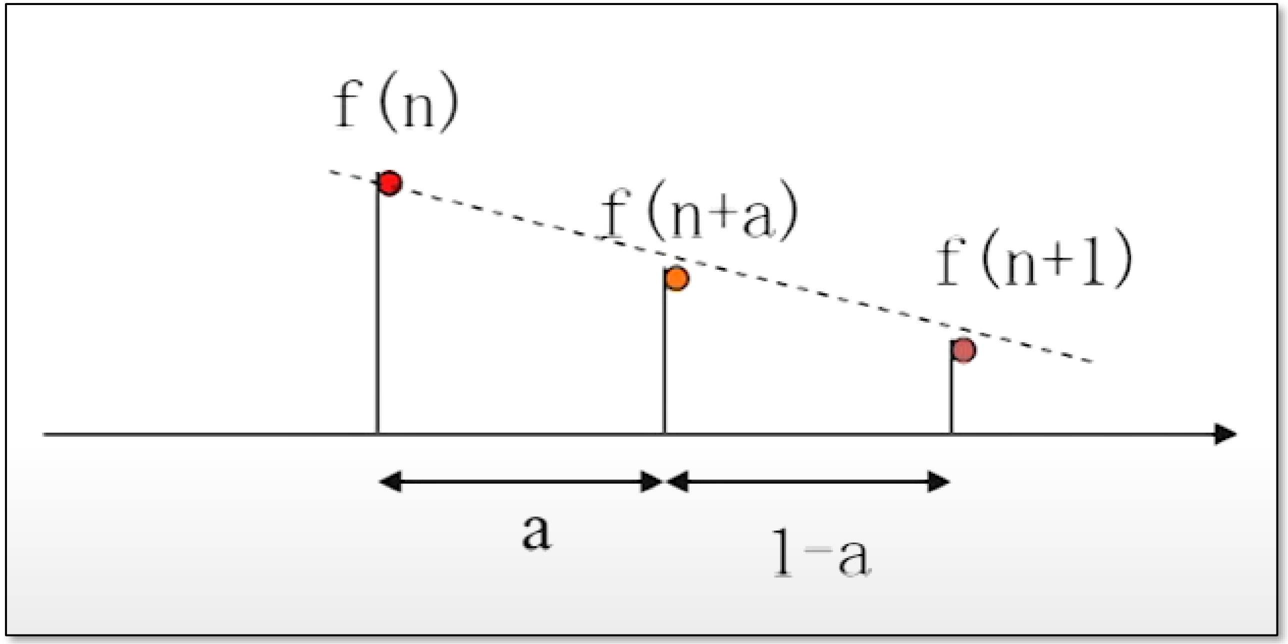
1D-影像內插(Interpolation)

1-D最近鄰內插法



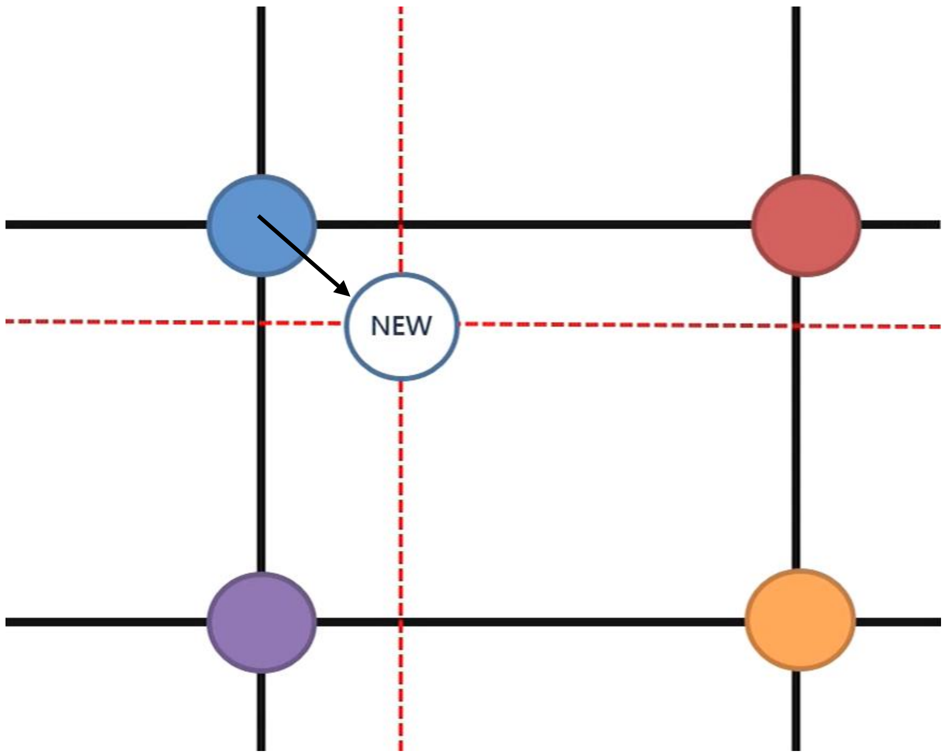
1-D線性內插法

$$f(n+a) = (1-a) \times f(n) + a \times f(n+1)$$



2-D最鄰近內插法 (Nearest Neighbor Interpolation)

2-D最鄰近內插法



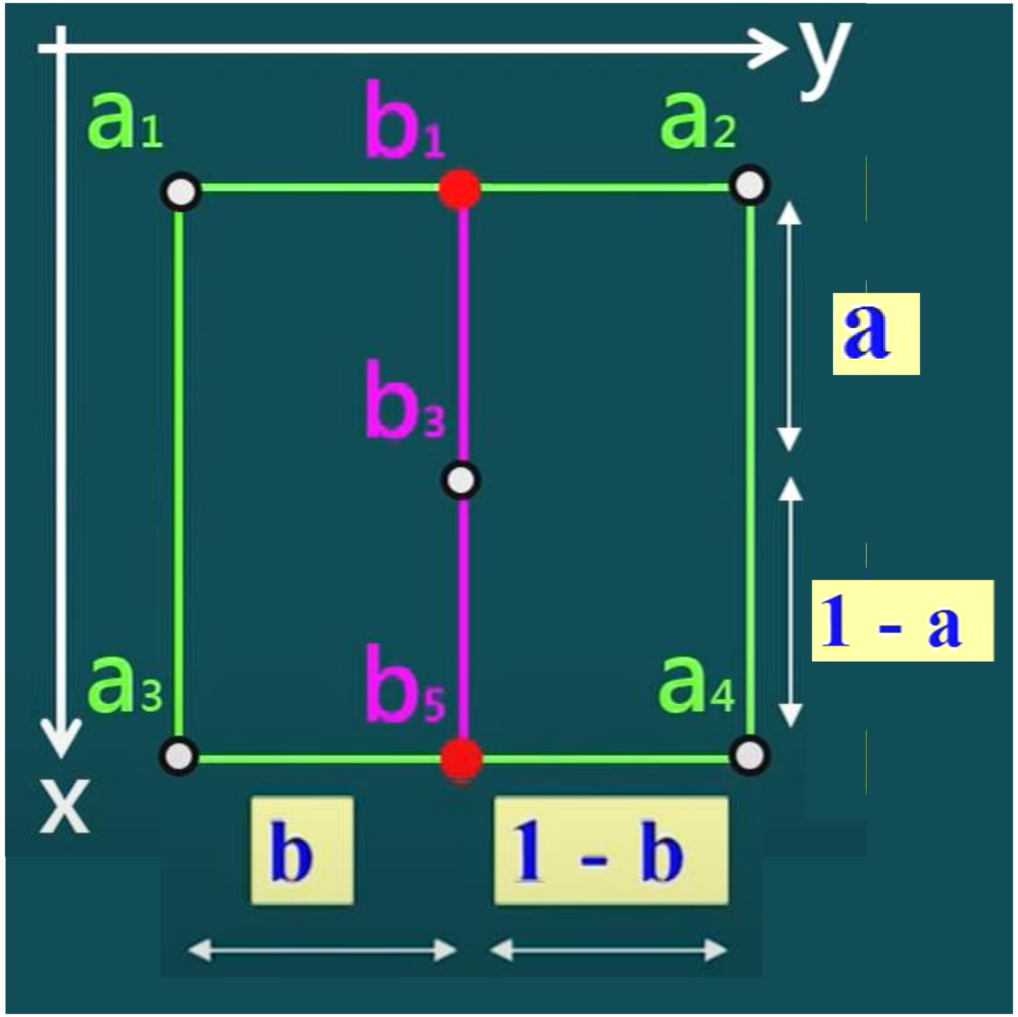
★ 2-D 雙線性內插法(Bilinear Interpolation)

1-D線性內插法

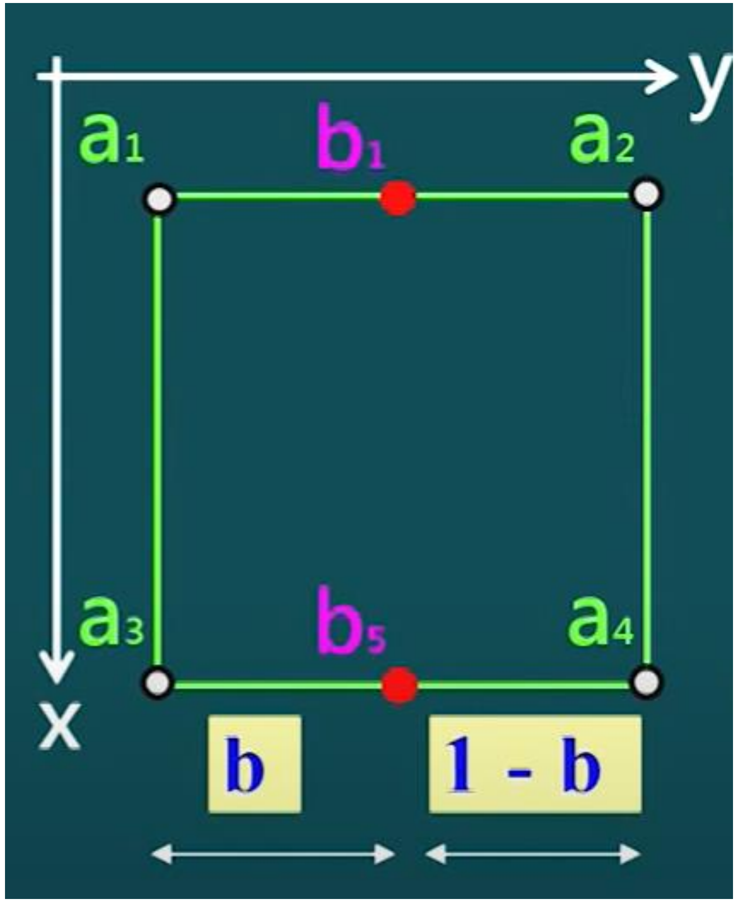
$$f(n+a) = (1-a) \times f(n) + a \times f(n+1)$$

2-D線性內插法

$$b_3 = (1-a) \times [(1-b) \times a_1 + b \times a_2]$$
$$+a \times [(1-b) \times a_3 + b \times a_4]$$



2-D 雙線性內插法 推導(1/3):水平方向內插



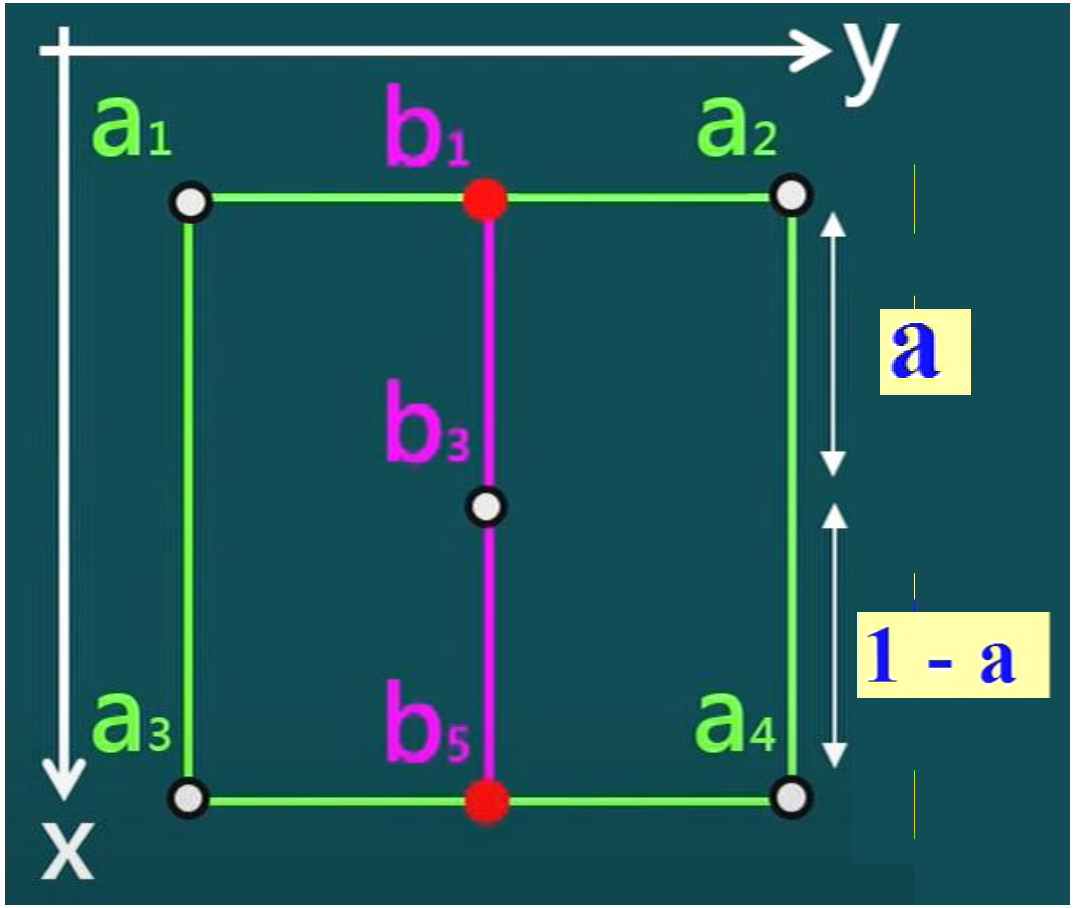
用像素a1、a2 估測出 b1

$$b_1 = (1 - b) \times a_1 + b \times a_2$$

用像素a3、a4 估測出 b5

$$b_5 = (1 - b) \times a_3 + b \times a_4$$

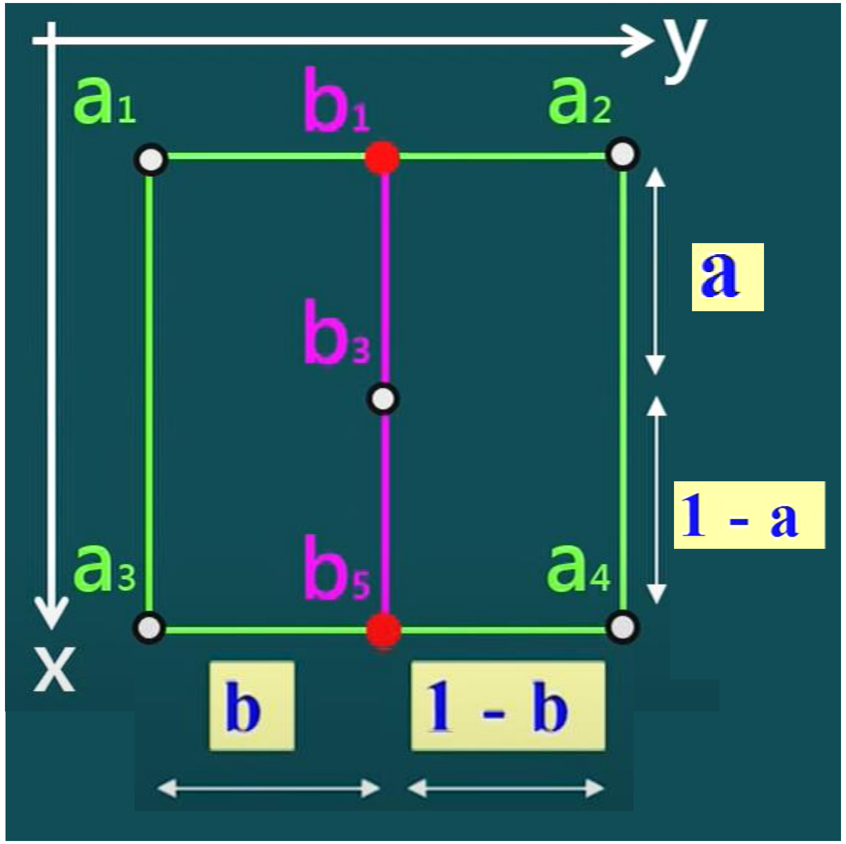
2-D 雙線性內插法 推導(2/3):垂直方向內插



用像素b1、b5 估測出 b3

$$b_3 = (1 - a) \times b_1 + a \times b_5$$

2-D 雙線性內插法 推導(3/3):合併



用像素a1、a2 估測出 b1

$$b_1 = (1 - b) \times a_1 + b \times a_2$$

用像素a3、a4 估測出 b5

$$b_5 = (1 - b) \times a_3 + b \times a_4$$

用像素b1、b5 估測出 b3

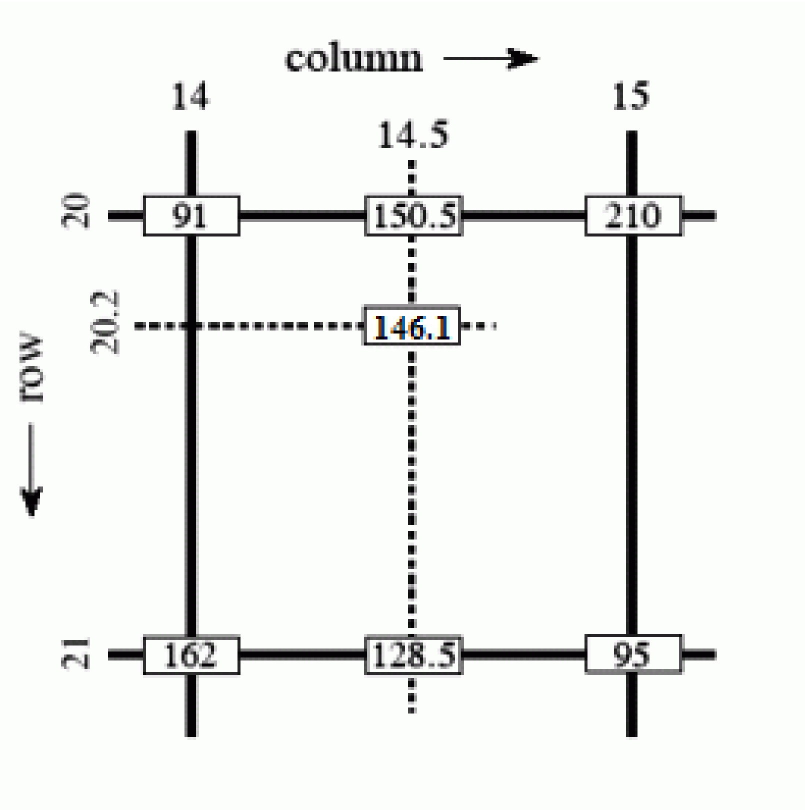
$$b_3 = (1 - a) \times b_1 + a \times b_5$$

通式

$$b_3 = (1 - a) \times [(1 - b) \times a_1 + b \times a_2] + a \times [(1 - b) \times a_3 + b \times a_4]$$

2-D 雙線性內插法 範例Example

https://en.wikipedia.org/wiki/Bilinear_interpolation



$$I_{21,14.5} = \frac{15 - 14.5}{15 - 14} \cdot 91 + \frac{14.5 - 14}{15 - 14} \cdot 210 = 150.5$$

$$I_{21,14.5} = \frac{15 - 14.5}{15 - 14} \cdot 162 + \frac{14.5 - 14}{15 - 14} \cdot 95 = 128.5$$

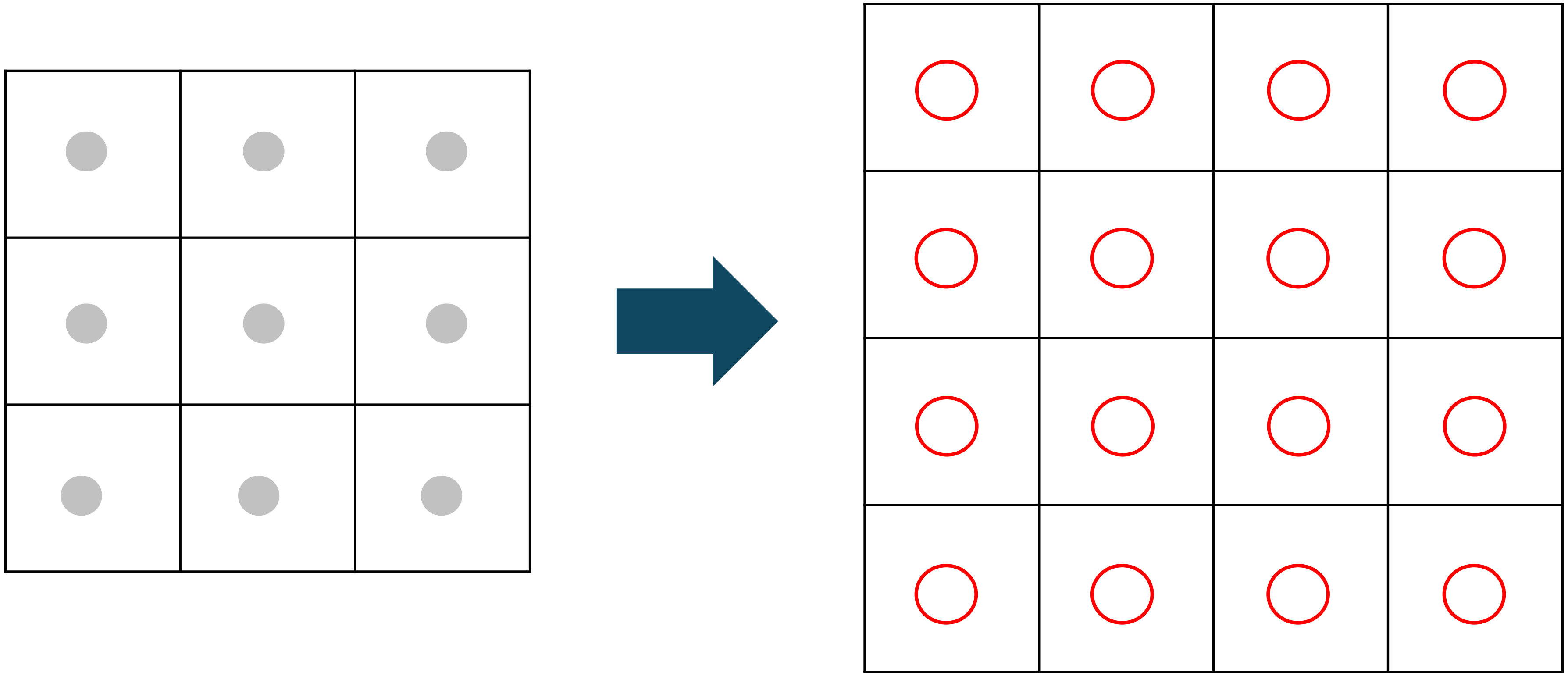
$$I_{20.2,14.5} = \frac{21 - 20.2}{21 - 20} \cdot 150.5 + \frac{20.2 - 20}{21 - 20} \cdot 128.5 = 146.1$$

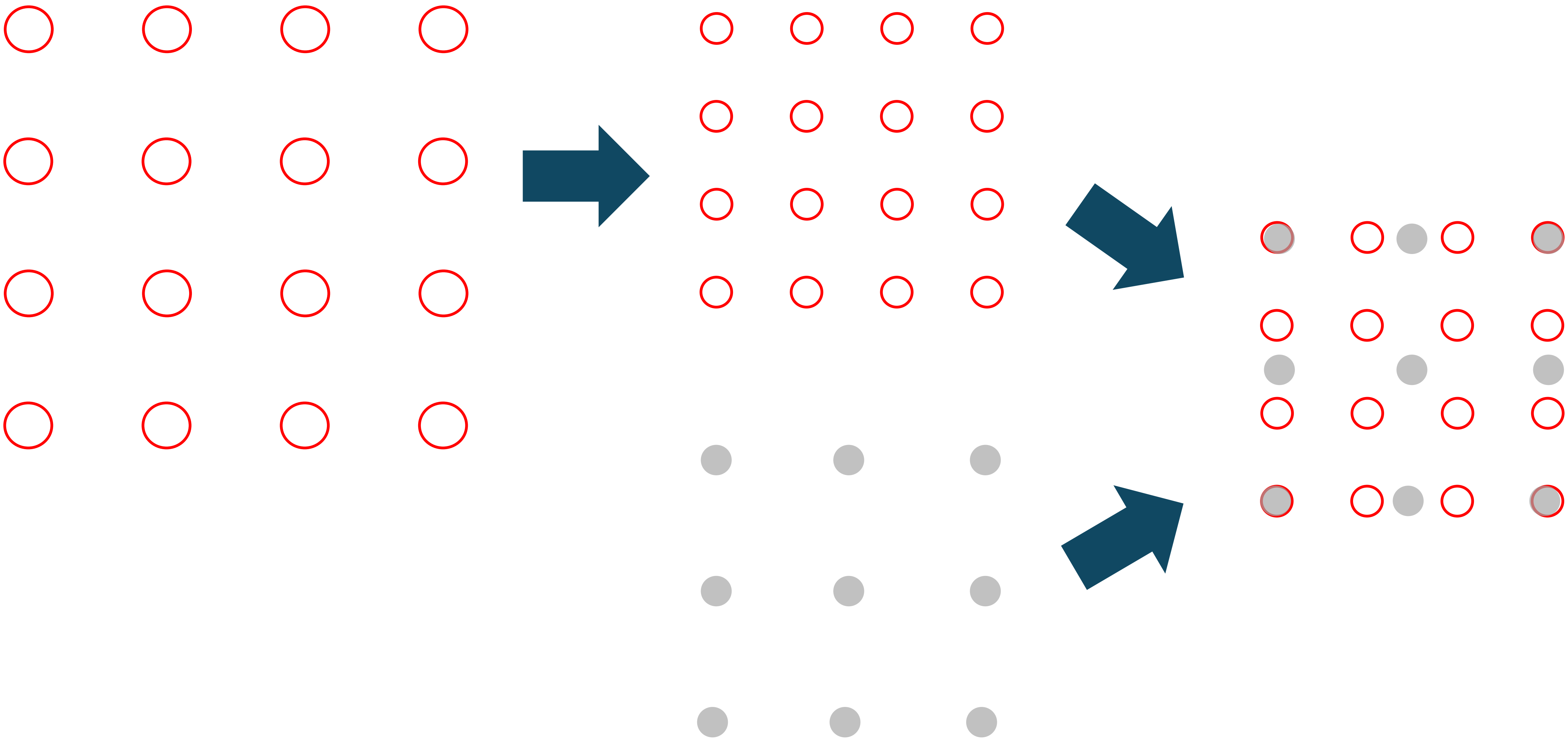
2-D 雙線性內插法 範例Example 2

10	20	30
40	50	60
70	80	90

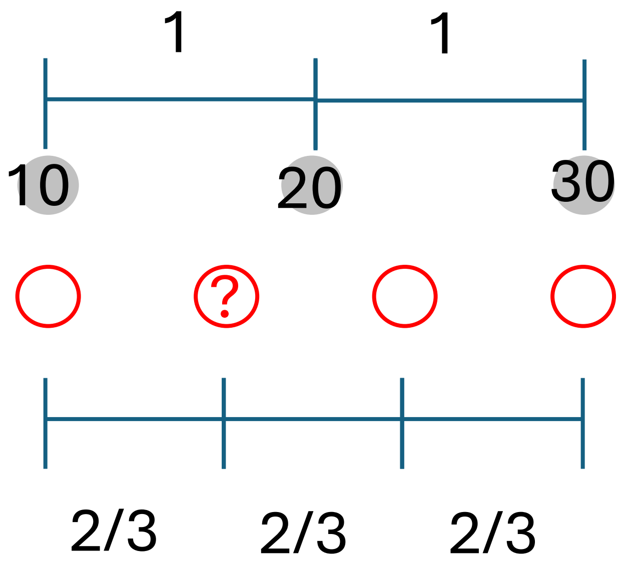
10			30
70			90

2-D 雙線性內插法 範例Example 2





計算縮放比例



長度2要平分3個間距

$$1個間距長度 = \frac{2}{3}$$

計算縮放比例

- 原始圖像尺寸:寬度$w_1 = 3$, 高度$h_1 = 3$
- 目標圖像尺寸:寬度$w_2 = 4$, 高度$h_2 = 4$

$$S_x = \frac{w_1 - 1}{w_2 - 1} = \frac{3 - 1}{4 - 1} = \frac{2}{3} \approx 0.667$$

$$S_y = \frac{h_1 - 1}{h_2 - 1} = \frac{2}{3} \approx 0.667$$

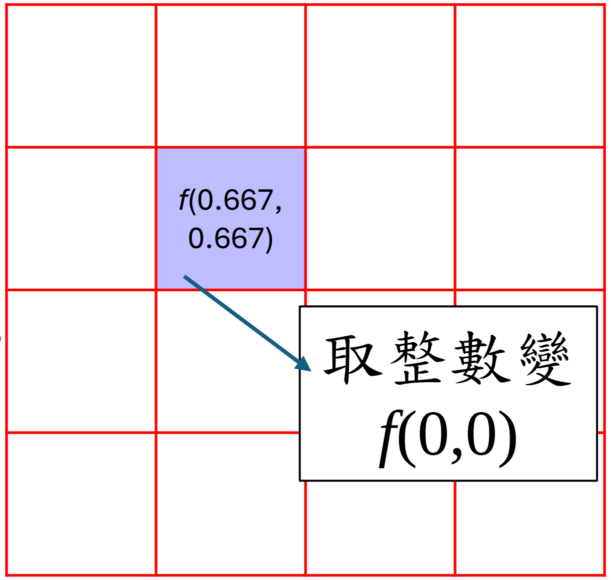
2-D 雙線性內插法 範例Example 2

1

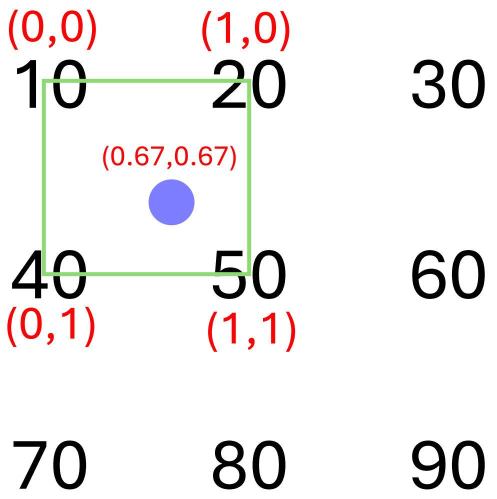
1

10	16.6	23.3	30
30	F(1,1)		50
50			70
70	76.6	83.3	90

×0.667



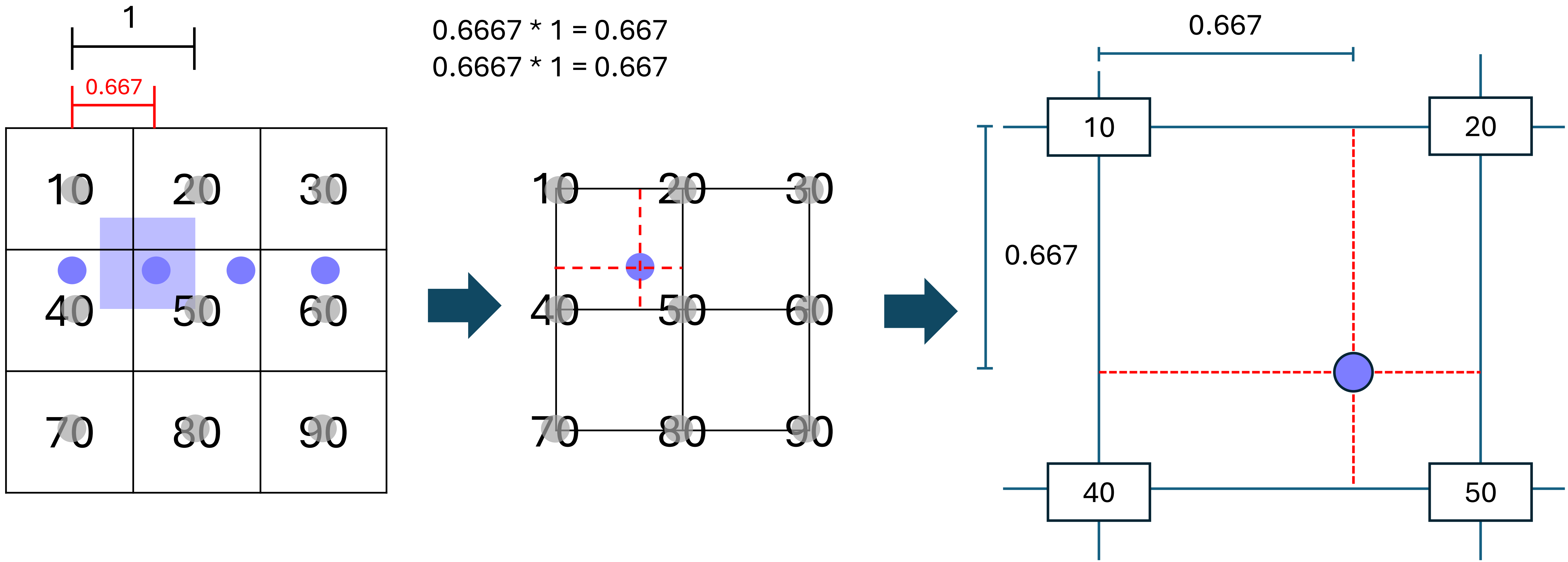
- $a_1 = f(x_1, y_1)$
- $a_2 = f(x_1 + 1, y_1)$
- $a_3 = f(x_1, y_1 + 1)$
- $a_4 = f(x_1 + 1, y_1 + 1)$



鄰近四個點

f(0,0) 10	f(1,0) 20	30
f(0,1) 40	f(1,1) 50	60
70	80	90

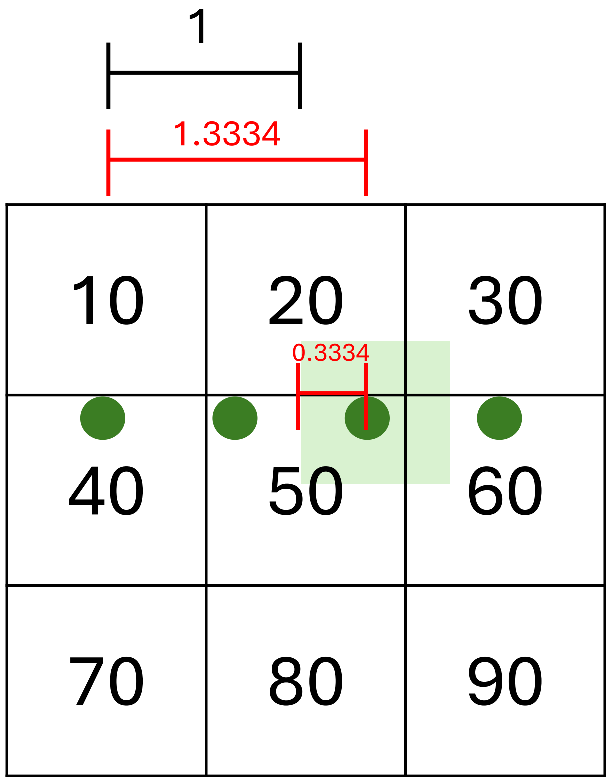
2-D 雙線性內插法 範例Example 2

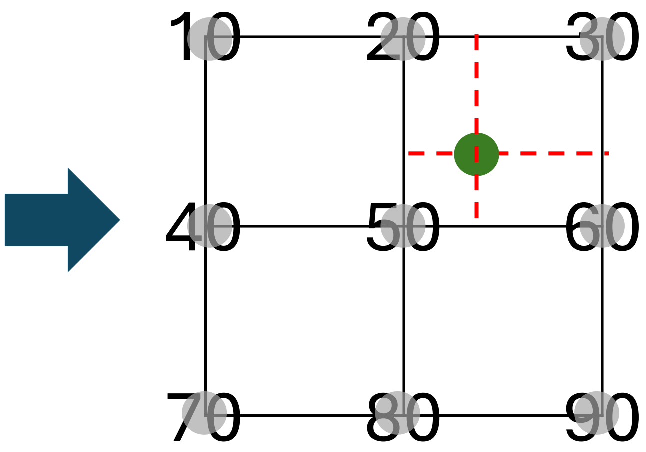


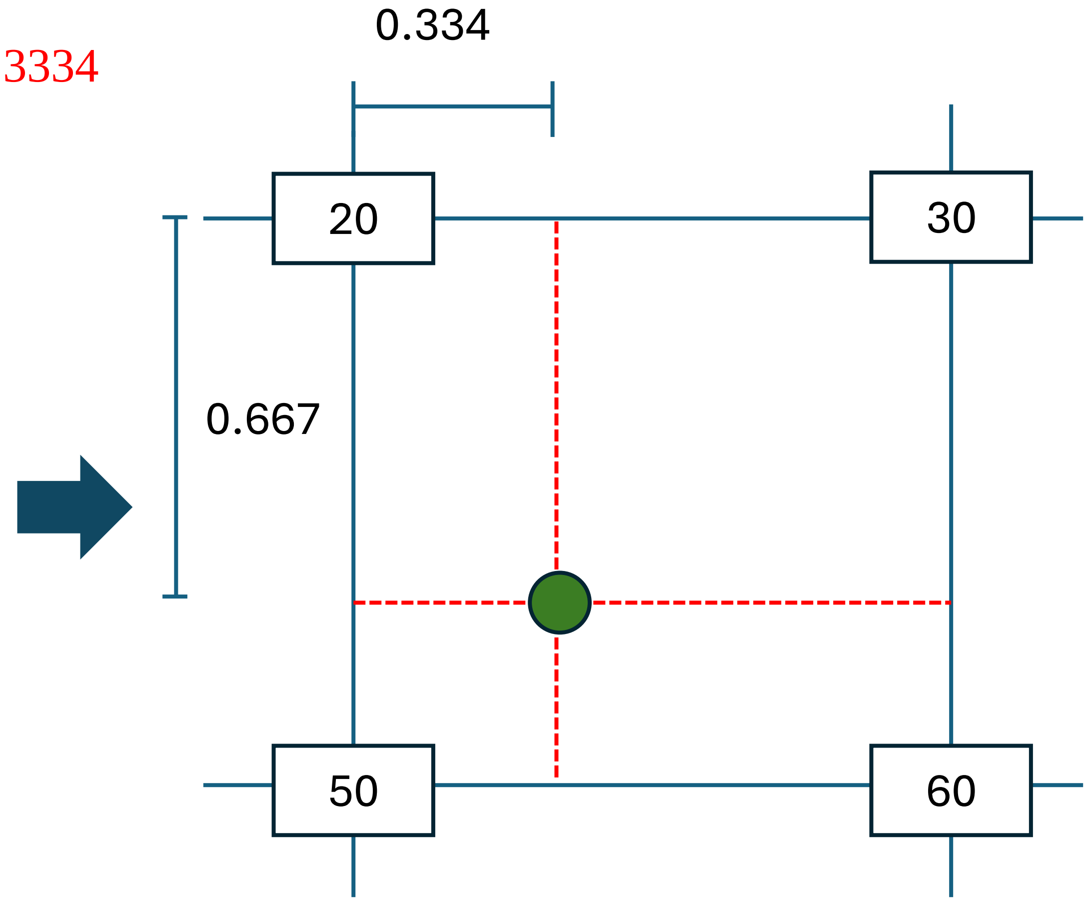
$$F(1,1) = (1-0.6667)\times[(1-0.6667)\times 10+0.6667\times 40]+ 0.6667\times[(1-0.6667)\times 20+0.6667\times 50] \quad =36.6672$$

2-D 雙線性內插法 範例Example 2

0.6667 * 2 = 1.3334　扣除整數=0.3334
0.6667 * 1 = 0.667



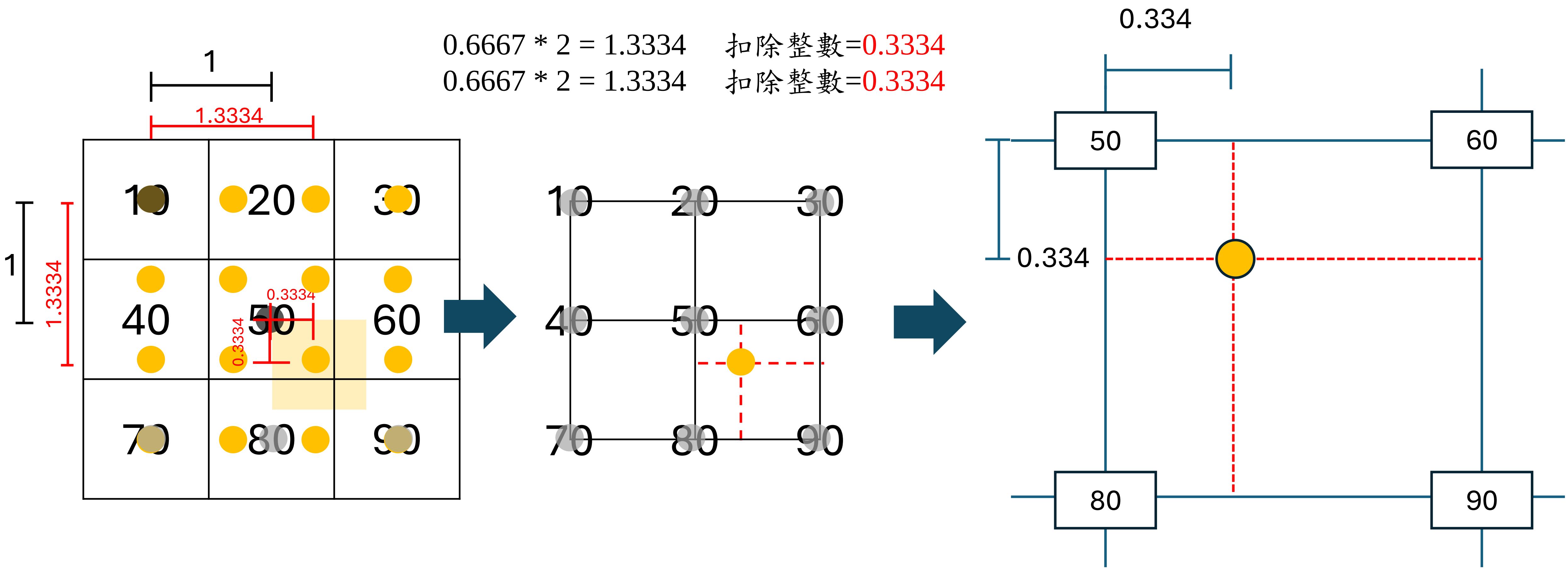




$$F(2,1) = (1-0.3334)\times[(1-0.6667)\times 20+0.6667\times 50]+ 0.3334\times[(1-0.6667)\times 30+0.6667\times 60] = 43.3352$$

2-D 雙線性內插法 範例Example 2

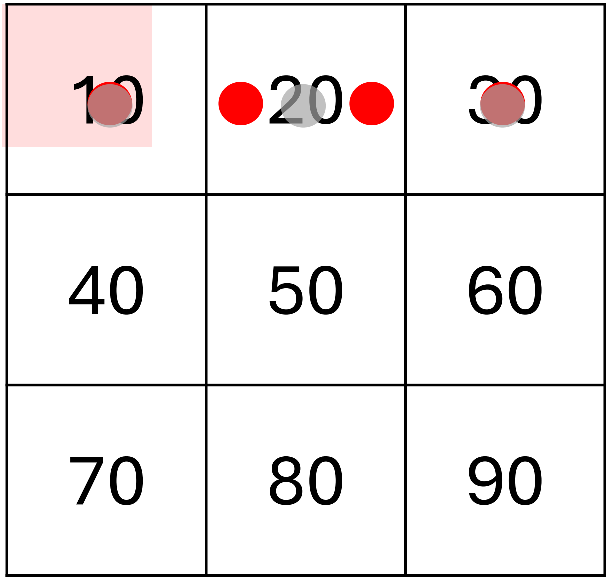
0.6667 * 2 = 1.3334　扣除整數=0.3334
0.6667 * 2 = 1.3334　扣除整數=0.3334

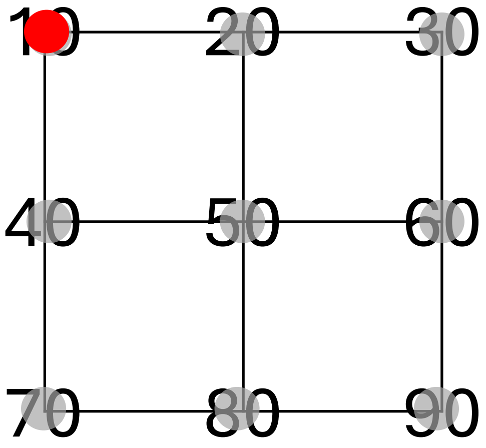


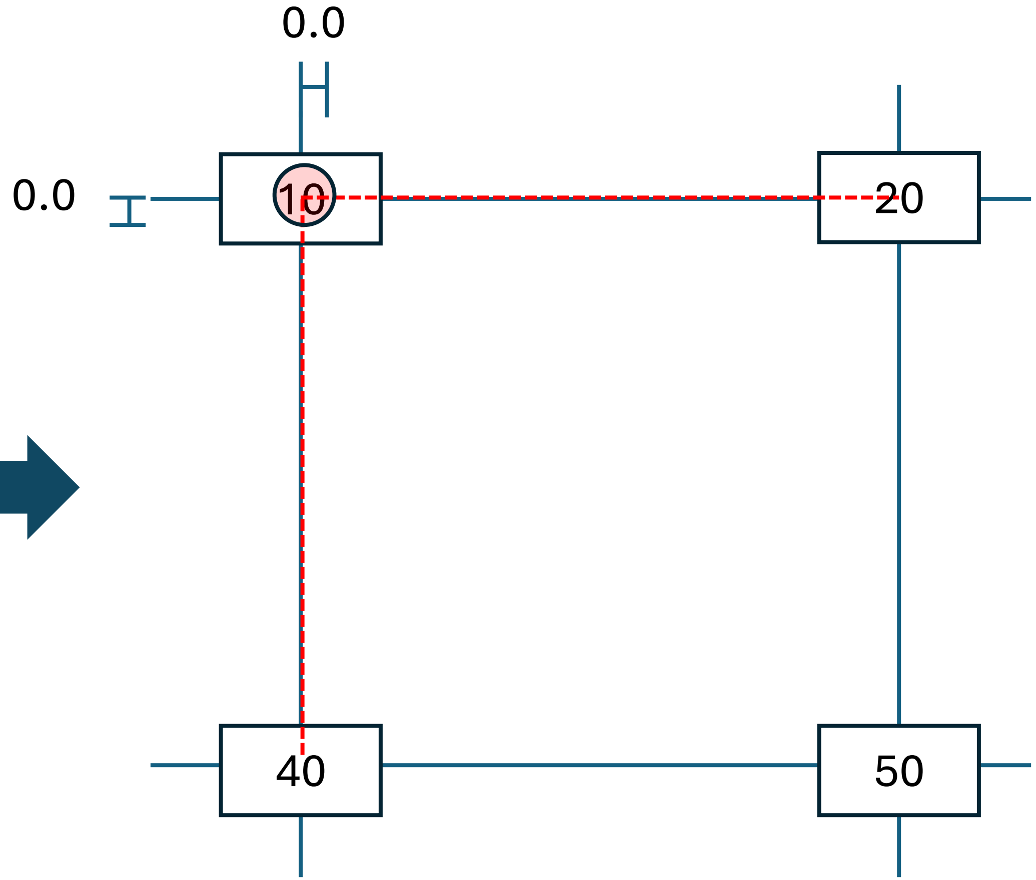
$$F(2,1) = (1-0.334)\times[(1-0.334)\times 50+0.334\times 80]+ 0.334\times[(1-0.334)\times 60+0.334\times 90] = 63.36$$

2-D 雙線性內插法 範例Example 2

$0.6667 * 0 = 0$
$0.6667 * 0 = 0$







$$F(0,0) = 1\times[0\times40+1\times10]+ 0\times[(0\times50+1\times20] = 10$$

2-D 雙線性內插法 範例Example 2

10	20	30
40	50	60
70	80	90

→

10	16.6	23.3	30
30	36.6	43.3	50
50	56.6	63.3	70
70	76.6	83.3	90

2-D 雙線性內插法 整理

Step1:縮放比例（Scaling Ratio）

- 原始圖像尺寸:寬度w_1，高度h_1
- 目標圖像尺寸:寬度w_2，高度h_2

$$S_x = \frac{w_1 - 1}{w_2 - 1} \qquad S_y = \frac{h_1 - 1}{h_2 - 1}$$

Step2:映射到原圖

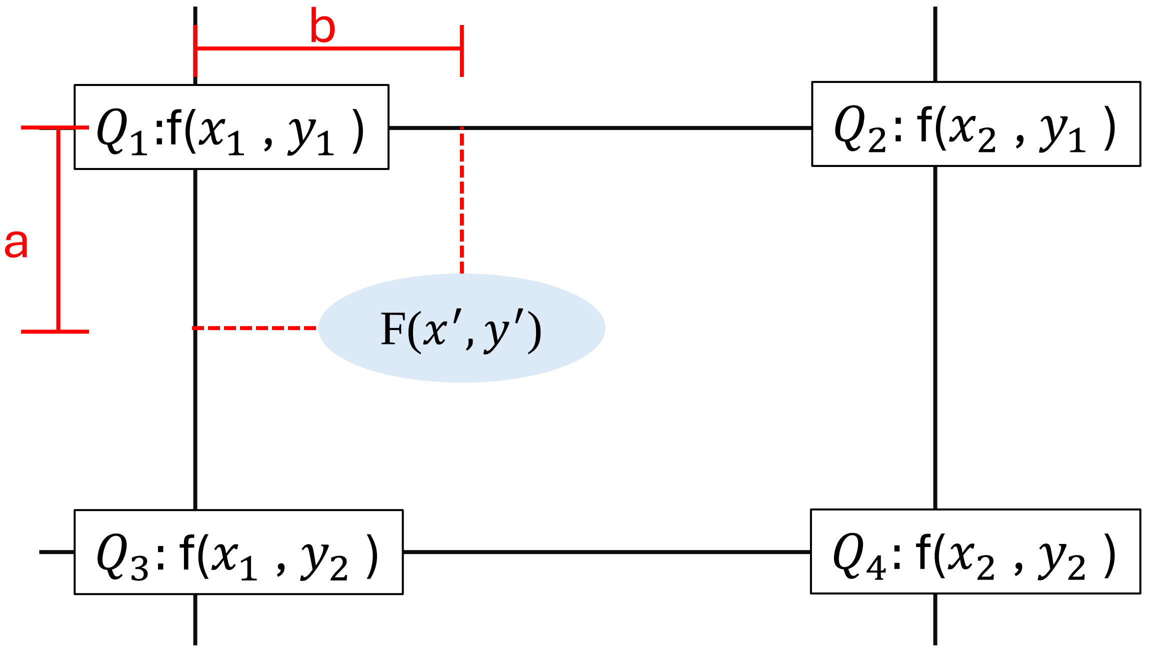
要算目標圖F(x',y')的值
先算映射到原圖的f(x,y)

$$x = x' \times S_x$$
$$y = y' \times S_y$$

Step3:計算目標圖值

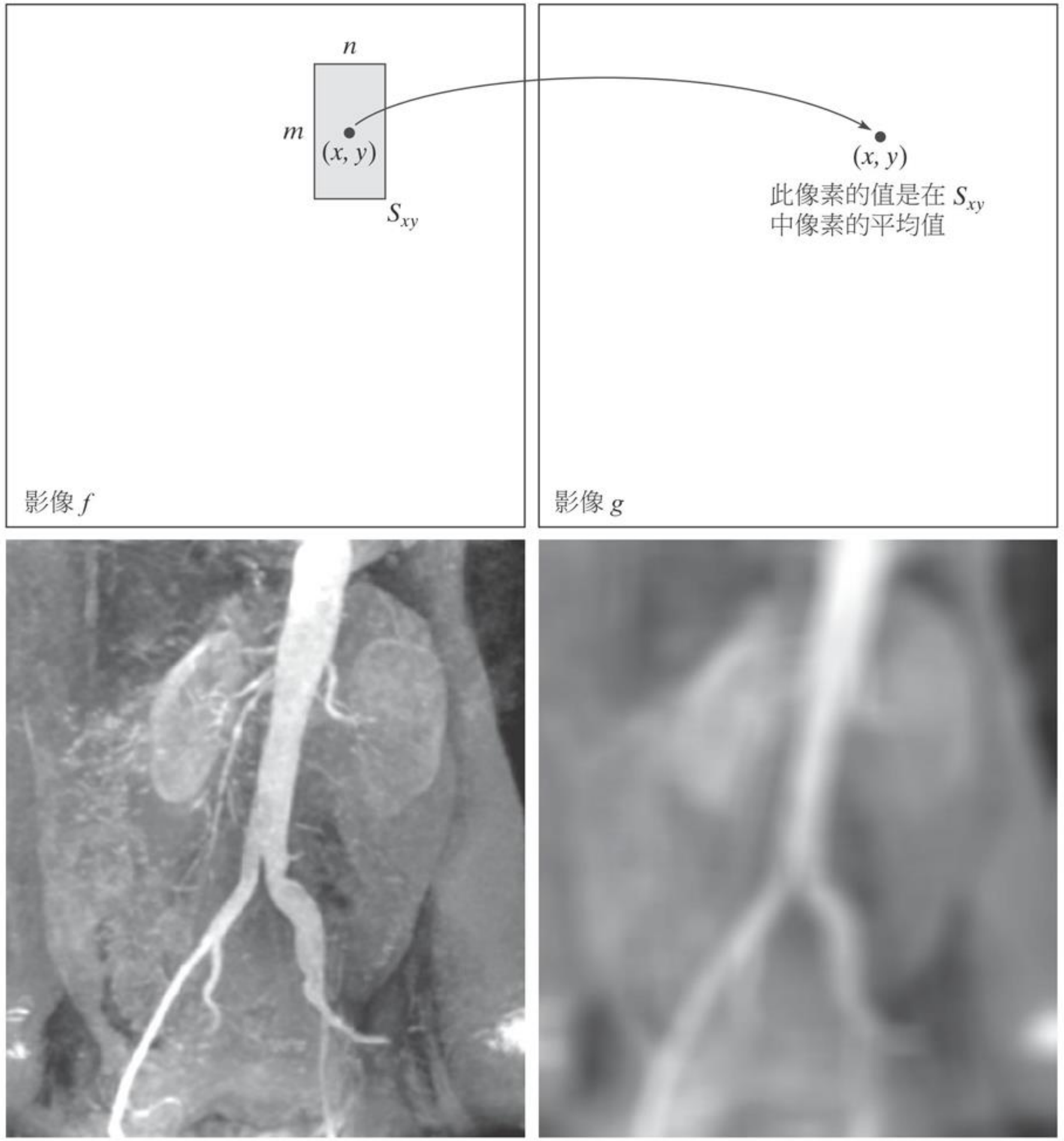
$x_1 = \lfloor x \rfloor$　$x_2 = \lfloor x \rfloor + 1$　b=$x - \lfloor x \rfloor$

$y_1 = \lfloor y \rfloor$　$y_2 = \lfloor y \rfloor + 1$　a=$y - \lfloor y \rfloor$



$$F(x',y') = (1-b)\times[(1-a)\times Q_1 + a\times Q_3] + b\times[(1-a)\times Q_2 + a\times Q_4]$$

2.5 空間運算: 鄰域運算

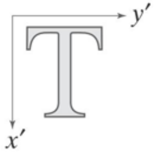
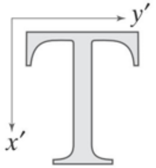
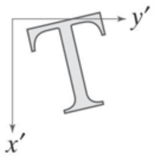
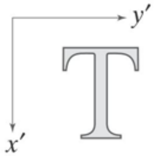
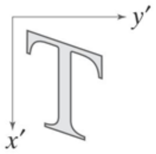
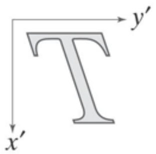


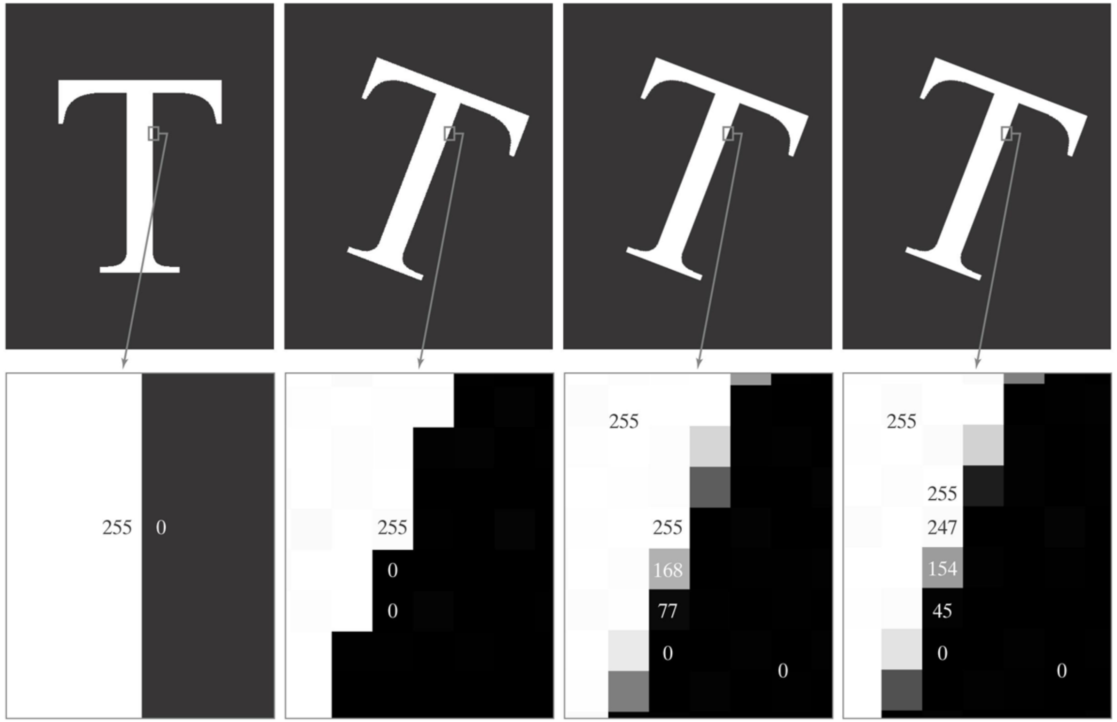
平均

$$g(x,y)=\frac{1}{mn}\sum_{(r,c)\in S_{xy}} f(r,c)$$

2.5 空間運算: 仿射轉換

表 2.1　依據 (2-23) 式的仿射轉換

轉換名稱	仿射矩陣 A	座標方程式	例　子
恆等式	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = y$	
尺度調整 / 翻轉 (對於翻轉，將一個尺度因子設成 −1 且其它設成 0)	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = c_x x$ $y' = c_y y$	
旋轉 (對原點)	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x\cos\theta - y\sin\theta$ $y' = x\sin\theta + y\cos\theta$	
平移	$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + t_x$ $y' = y + t_y$	
切變 (垂直)	$\begin{bmatrix} 1 & s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + s_v y$ $y' = y$	
切變 (水平)	$\begin{bmatrix} 1 & 0 & 0 \\ s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = s_h x + y$	



a b c d
e f g h

圖 2.29　(a) 字母 T 的 541×421 影像；(b) 用最近鄰插補法做強度指定給經旋轉 −21° 的影像；(c) 用雙線性插補法使影像旋轉 −21°；(d) 用雙立方插補法使影像旋轉 −21°。(e)~(h) 放大部分 (每個方形是一個像素，而所顯示的數字是強度值)。

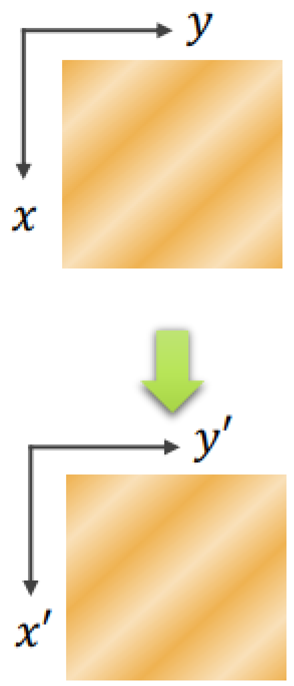
$$[x\ y\ 1] = [v\ w\ 1]T = [v\ w\ 1]\begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

- Identity transformation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Identity

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

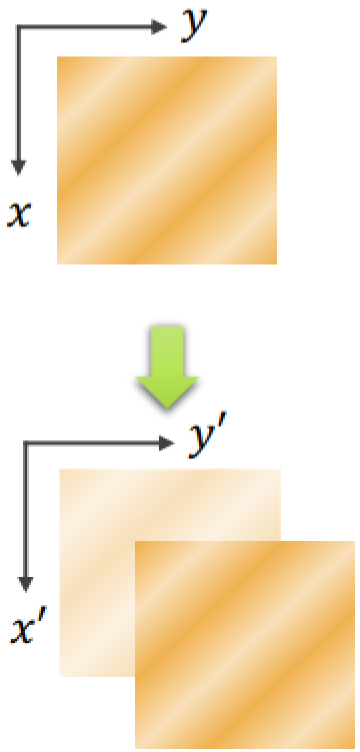


- Affine transformation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Translation

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

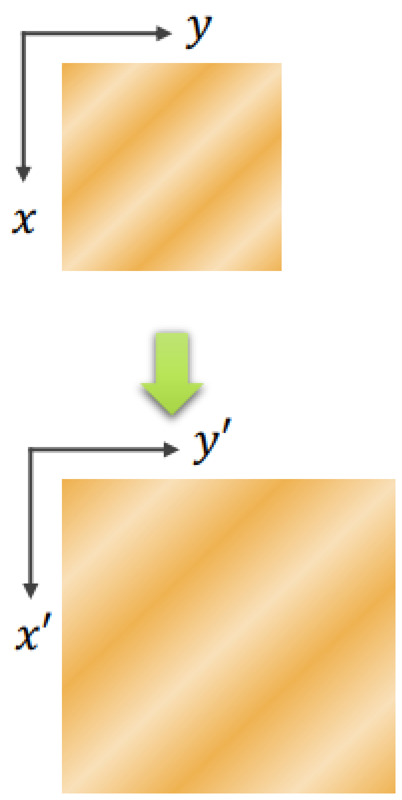


- Affine transformation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Scaling

$$\mathbf{A} = \begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

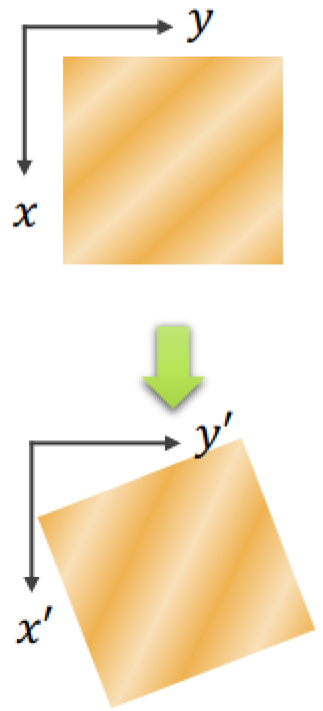


- Affine transformation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

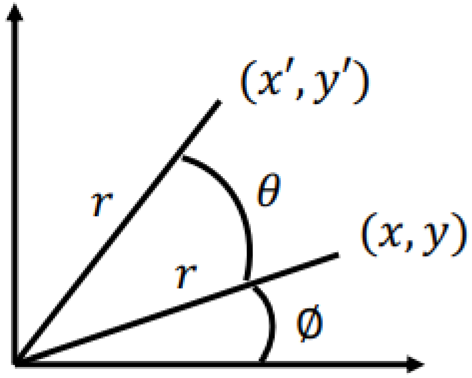
- Rotation

$$\mathbf{A} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



- Rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



$$x = r\cos\emptyset \qquad x' = r\cos(\emptyset + \theta)$$
$$y = r\sin\emptyset \qquad y' = r\sin(\emptyset + \theta)$$

$$x' = r\cos(\emptyset)\cos(\theta) - r\sin(\emptyset)\sin(\theta)$$
$$y' = r\sin(\emptyset)\cos(\theta) + r\cos(\emptyset)\sin(\theta)$$

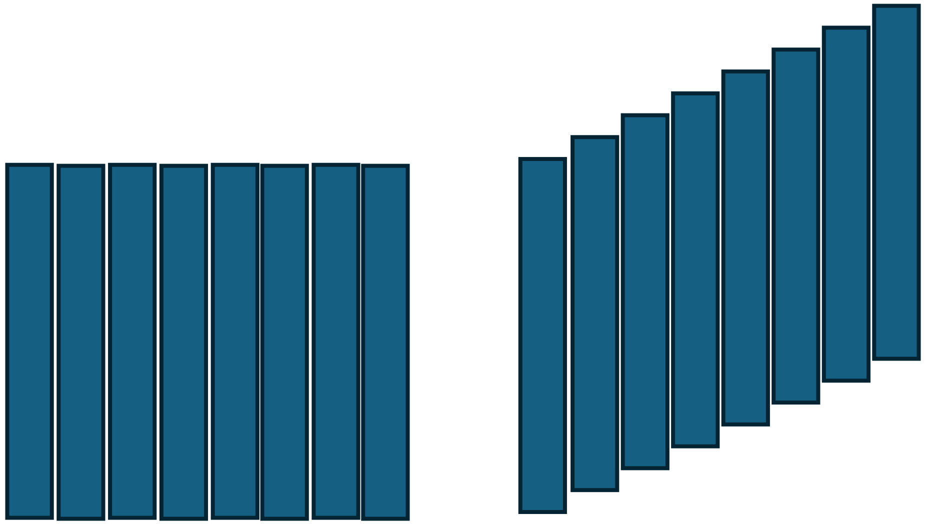
$$x' = x\cos(\theta) - y\sin(\theta)$$
$$y' = x\sin(\theta) + y\cos(\theta)$$

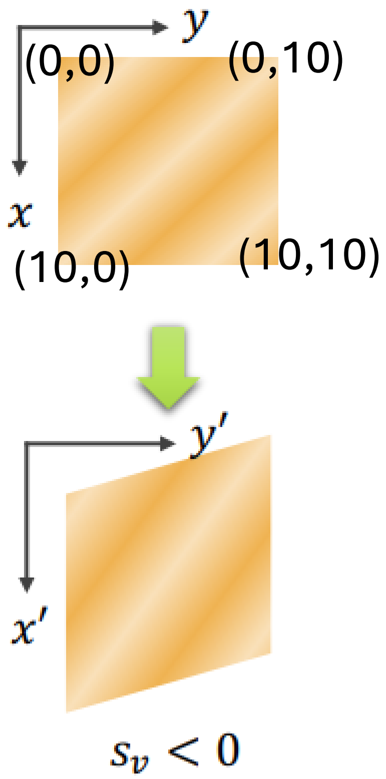
- Affine transformation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Shearing (vertical)

$$\mathbf{A} = \begin{bmatrix} 1 & s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

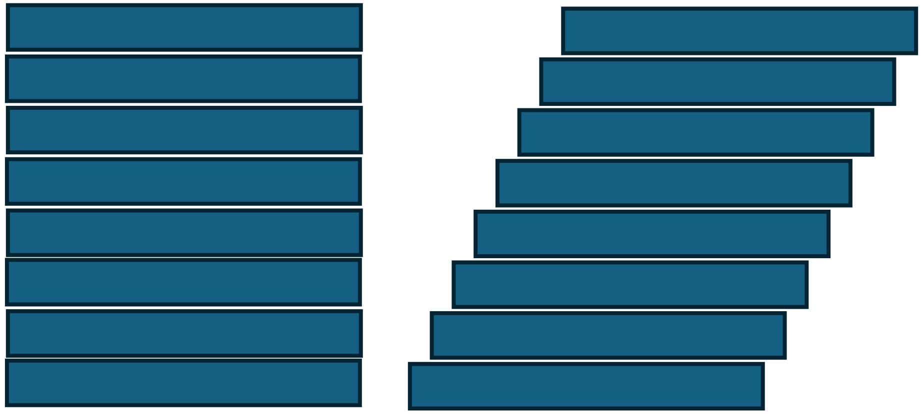




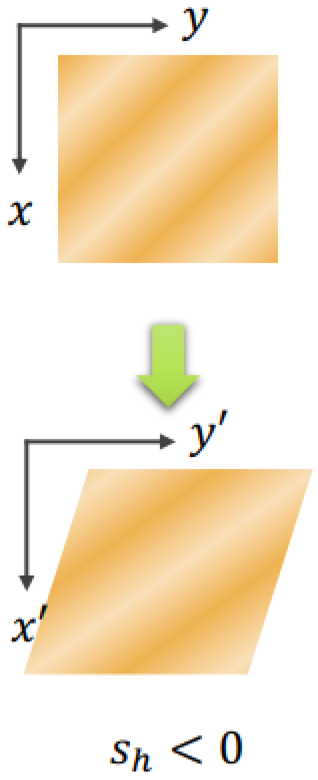
- Affine transformation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Shearing (horizontal)



$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



- Forward mapping

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{A} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Backward mapping

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Problem 2.37

(a) The forward scaling transformations is:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

and the corresponding inverse transformation is

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1/c_x & 0 & 0 \\ 0 & 1/c_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

(b) The forward translation transformation is

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

and the corresponding inverse transformation is

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

(c) The forward vertical shear transformation is

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

and the corresponding inverse transformation is

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & -s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Similarly for horizontal shear,

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

(d) The forward rotation transformation is

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

and the corresponding inverse rotation transformation is

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

(e) A composite translation/rotation transformation is

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

and the corresponding inverse transformation is

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & -t_x \cos - t_y \sin\theta \\ -\sin\theta & \cos\theta & t_x \sin\theta - t_y \sin\theta \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Note the order of the matrices in the forward *vs* the inverse composite transformations.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = A \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = A^{-1} \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

a	b
c	d

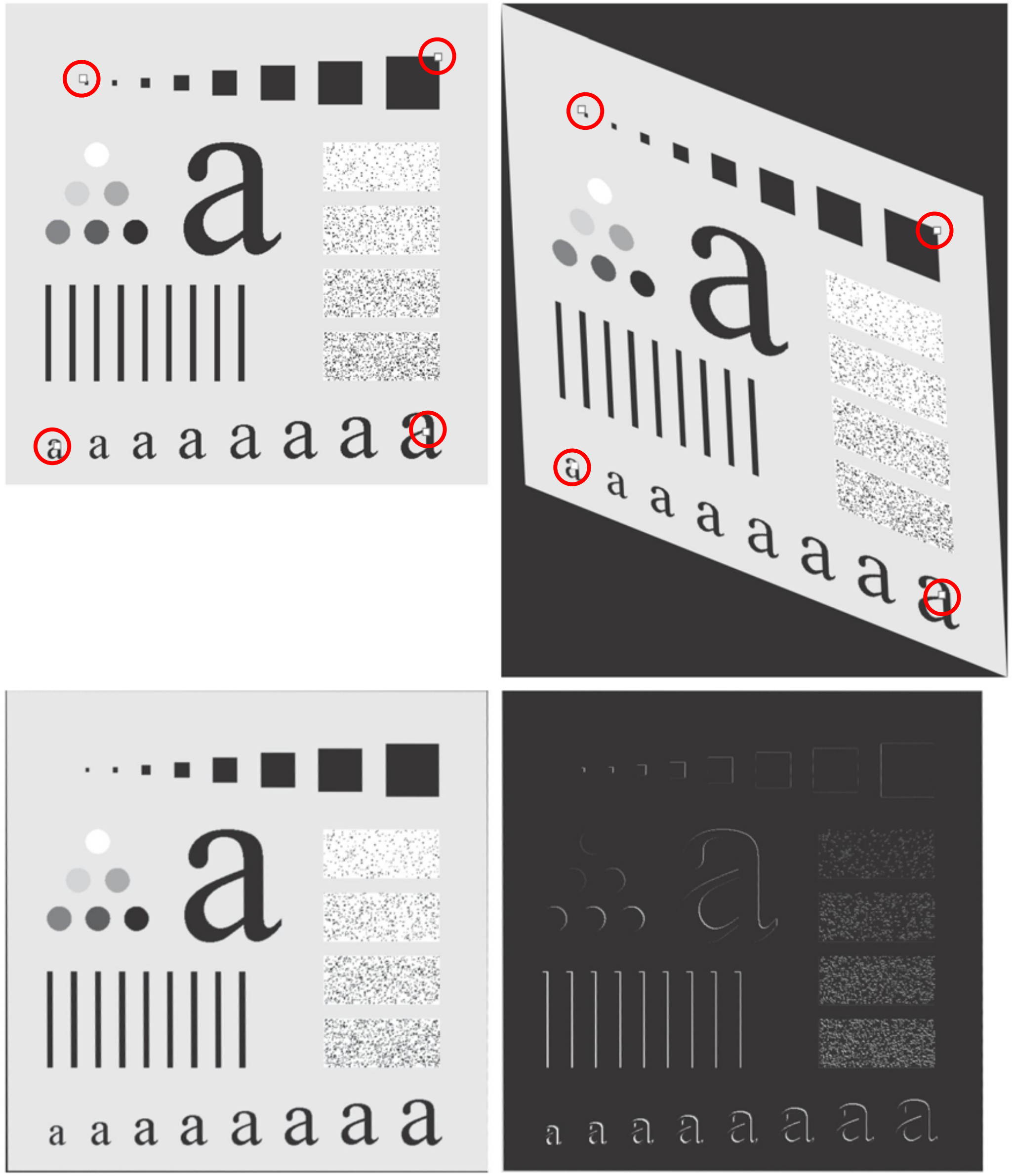


圖 2.31
影像對準。(a) 參考影像；(b) 輸入（幾何失真影像）。對應的連接點顯示成靠近角落的白色小方形；(c) 對準後的影像（注意到邊界上的誤差）；(d) (a) 和 (c) 之間的差異，顯示更多的對準誤差。

$$x = c_1 v + c_2 w + c_3 v w + c_4$$

$$y = c_5 v + c_6 w + c_7 v w + c_8$$

×4組

小考

2.6 (a) (b) 線性索引
2.14 (a)~(e) 仿射