MATERI 6

TUJUAN INSTRUKSIONAL KHUSUS

Setelah menyelesaikan pertemuan ini mahasiswa diharapkan :

- Dapat mengetahui definisi dan dapat menghitung perkalian vektor di ruang n-eucledian

Ruang-n Euclidean

(Euclidean n-space)

Review: Bab 3 membahas Ruang-2 dan Ruang-3

Ruang-n : himpunan yang beranggotakan vektor-vektor dengan n komponen

$$\{ \ldots, \mathbf{v} = (v_1, v_2, v_3, v_4, \ldots, v_n), \ldots.. \}$$

- Atribut: arah dan "panjang" / norma $||\mathbf{v}||$
- Aritmatika vektor-vektor di Ruang-n:
  1. Penambahan vektor
  2. Perkalian vektor dengan skalar
  3. Perkalian vektor dengan vektor

Norma sebuah vektor:

Norma Euclidean (*Euclidean norm*) di Ruang-n :

$$\mathbf{u} = (u_1, u_2, u_3, \ \ldots\ , u_n)$$

$$\boxed{\|\mathbf{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2 + \ \ldots + u_n^2}}$$

Penambahan vektor: di Ruang-n

$\mathbf{u} = (u_1, u_2, u_3, \ldots, u_n)$; $\mathbf{v} = (v_1, v_2, v_3, \ldots, v_n)$

$\mathbf{w} = (w_1, w_2, w_3, \ldots, w_n) = \mathbf{u} + \mathbf{v}$

$\mathbf{w} = (u_1, u_2, u_3, \ldots, u_n) + (v_1, v_2, v_3, \ldots, v_n)$

$$\boxed{\mathbf{w} = (u_1 + v_1, u_2 + v_2, u_3 + v_3, \ldots, u_n + v_n)}$$

$w_1 = u_1 + v_1$

$w_2 = u_2 + v_2$

....

$w_2 = u_n + v_n$

Negasi suatu vektor:

$\mathbf{u} = (u_1, u_2 , u_3, \ldots, u_n)$

$-\mathbf{u} = (-u_1, -u_2 , -u_3, \ldots, -u_n)$

Selisih dua vektor:

$$\mathbf{w} = \mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$$

$$= (u_1 - v_1, u_2 - v_2, u_3 - v_3, \ldots, u_n - v_n)$$

Vektor nol: $\mathbf{0} = (0_1, 0_2 , 0_3, \ldots, 0_n)$

Perkalian skalar dengan vektor:

$$\mathbf{w} = k\mathbf{v} = (kv_1, kv_2, kv_3, \ldots, kv_n)$$

$$(w_1, w_2, w_3, \ldots, w_n) = (kv_1, kv_2, kv_3, \ldots, kv_n)$$

$$w_1 = kv_1$$

$$w_2 = kv_2$$

……..

$$w_n = kv_n$$

Perkalian titik: (perkalian Euclidean)

$$\mathbf{u} \cdot \mathbf{v} = \text{skalar}$$

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 + \ldots + u_nv_n$$

$\mathbf{u} \cdot \mathbf{v} = 0$ jika **u** dan **v** ortogonal

Catatan: perkalian silang hanya di Ruang-3

Aritmatika vektor di Ruang-n:

Teorema 4.1.1.: **u, v, w vektor-vektor di Ruang-n**

***k*, *l* adalah skalar (bilangan *real*)**

- $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
- $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
- $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$
- $\mathbf{u} + (-\mathbf{u}) = (-\mathbf{u}) + \mathbf{u} = \mathbf{0}$
- $k(l\mathbf{u}) = (kl)\mathbf{u}$
- $k(\mathbf{u} + \mathbf{v}) = k\mathbf{u} + k\mathbf{v}$
- $(k + l)\,\mathbf{u} = k\mathbf{u} + l\mathbf{u}$
- $1\mathbf{u} = \mathbf{u}$

Teorema 4.1.2:

Vektor-vektor **u**, **v**, **w** di Ruang-n; k adalah skalar

- $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
- $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
- $k(\mathbf{u} \cdot \mathbf{v}) = (k\mathbf{u}) \cdot \mathbf{v} = \mathbf{u} \cdot (k\mathbf{v})$
- $\mathbf{v} \cdot \mathbf{v} > 0$ jika $\mathbf{v} \neq \mathbf{0}$

  $\mathbf{v} \cdot \mathbf{v} = 0$ jika dan hanya jika $\mathbf{v} = \mathbf{0}$

Teorema 4.1.3 - 4.1.5:

$| \mathbf{u} \cdot \mathbf{v} | \leq \| \mathbf{u} \| \, \| \mathbf{v} \|$

$\| \mathbf{u} \| \geq 0$

$\| \mathbf{u} \| = 0$ jika dan hanya jika $\mathbf{u} = \mathbf{0}$

$\| k\mathbf{u} \| = | k | \, \| \mathbf{u} \|$

$\| \mathbf{u} + \mathbf{v} \| \leq \| \mathbf{u} \| + \| \mathbf{v} \|$

$d(\mathbf{u}, \mathbf{v}) \geq 0$

$d(\mathbf{u}, \mathbf{v}) = 0$ jika dan hanya jika $\mathbf{u} = \mathbf{v}$

$d(\mathbf{u}, \mathbf{v}) = d(\mathbf{v}, \mathbf{u})$

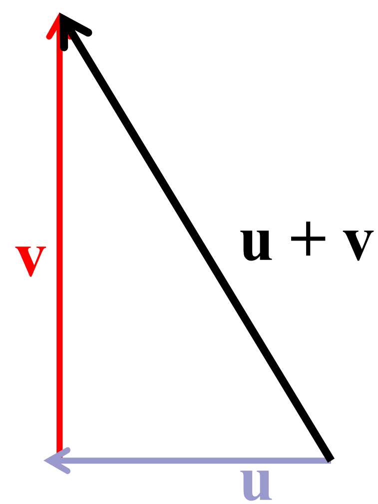
$d(\mathbf{u}, \mathbf{v}) \leq d(\mathbf{u}, \mathbf{w}) + d(\mathbf{w}, \mathbf{v})$

Teorema 4.1.6 – 4.1.7:

$$\mathbf{u} \cdot \mathbf{v} = \tfrac{1}{4} \| \mathbf{u} + \mathbf{v} \|^2 - \tfrac{1}{4} \| \mathbf{u} - \mathbf{v} \|^2$$

Teorema Pythagoras

$$\| \mathbf{u} + \mathbf{v} \|^2 = \| \mathbf{u} \|^2 + \| \mathbf{v} \|^2$$



Perkalian Titik (*dot product*) dikerjakan dengan perkalian matriks

$\mathbf{u} = (u_1, u_2, u_3, \ldots, u_n)$; $\mathbf{v} = (v_1, v_2, v_3, \ldots, v_n)$

$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 + \ldots + u_nv_n$

Kalau vektor u dan vektor v masing-masing ditulis dalam notasi matriks <u>baris</u>, maka

$$\mathbf{u} \cdot \mathbf{v} = (u_1 \;\; u_2 \;\; u_3 \;\; \ldots \;\; u_n) \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{pmatrix}$$

$$\boxed{\mathbf{u} \cdot \mathbf{v} = (\mathbf{u})\,(\mathbf{v})^T}$$

$\mathbf{u} = (u_1, u_2, u_3, \ldots, u_n)$; $\mathbf{v} = (v_1, v_2, v_3, \ldots, v_n)$

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 + \ldots + u_nv_n$$

Kalau vektor u dan vektor v masing-masing ditulis dalam notasi matriks <u>kolom</u>, maka

$$\mathbf{u} \cdot \mathbf{v} = (u_1 \quad u_2 \quad u_3 \quad \ldots \quad u_n) \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{pmatrix} = \mathbf{v} \cdot \mathbf{u}$$

$$\boxed{\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= (\mathbf{u})^T (\mathbf{v}) \\ \mathbf{v} \cdot \mathbf{u} &= (\mathbf{v})^T (\mathbf{u}) \\ \mathbf{u} \cdot \mathbf{v} &= (\mathbf{v})^T (\mathbf{u}) \end{aligned}}$$

Matriks A (n x n), **u** dan **v** masing-masing vektor kolom

$$A\mathbf{u} \cdot \mathbf{v} = (A\mathbf{u}) \cdot \mathbf{v} \stackrel{?}{=} \mathbf{u} \cdot (A^T\mathbf{v})$$

$$\mathbf{u} \cdot A\mathbf{v} \stackrel{?}{=} A^T\mathbf{u} \cdot \mathbf{v}$$

$$\begin{aligned}(A\mathbf{u}) \cdot \mathbf{v} &= \mathbf{v} \cdot (A\mathbf{u}) \\ &= \mathbf{v}^T(A\mathbf{u}) \\ &= (\mathbf{v}^T A)\mathbf{u} \\ &= (A^T\mathbf{v})^T \mathbf{u} \\ &= \mathbf{u} \cdot (A^T\mathbf{v})\end{aligned}$$

Contoh:

- Hitunglah eucledian norm dari vektor berikut :
  - (-2,5)
  - (1,-2,2)
  - (3,4,0,-12)
  - (-2,1,1,-3,4)
- Hitunglah eucledian inner product u.v
  - u = (1,-2) , v = (2,1)
  - u = (0,-2,1,1), v = (-3,2,4,4)
  - u = (2,-2,2), v = (0,4,-2)