MATERI 7

TUJUAN INSTRUKSIONAL KHUSUS

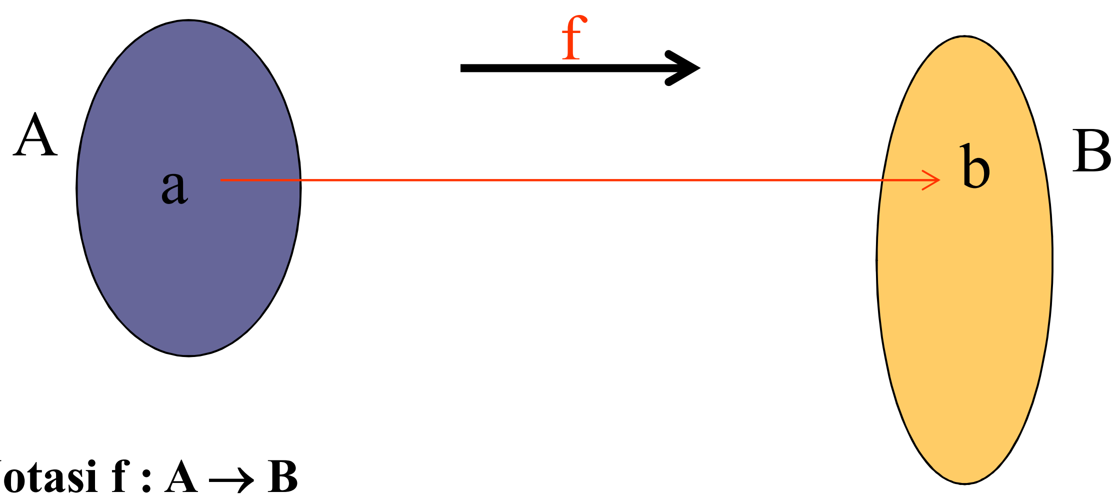
Setelah menyelesaikan pertemuan ini mahasiswa diharapkan :

- Dapat mengetahui matriks-matriks yang digunakan untuk transformasi linier
- Dapat mengetahui aplikasi transformasi linier

Transformasi Linier

Fungsi:

Pemetaan (*mapping*) dari himpunan A ke himpunan B



1. **Notasi f : A → B**
2. **Himpunan A disebut DOMAIN(f)**
3. **Himpunan B disebut CODOMAIN(f)**
4. **Tiap elemen A dipasangkan dengan (*associated with*) satu elemen B**
5. **Himpunan semua elemen b yang punya pasangan di A disebut RANGE(f)**
6. **Notasi f(a) = b, b disebut bayangan (*image*) dari a**

$f : R^n \rightarrow R^m$ disebut transformasi dan ditulis

$$T : R^n \rightarrow R^m$$

T adalah **transformasi linier** jika

1. $T(\mathbf{u} + \mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$
2. $T(c\mathbf{u}) = cT(\mathbf{u})$

Catatan: **u**, **v** vektor-vektor di Ruang-n

c adalah skalar

$T(\mathbf{u} + \mathbf{v})$, $T(\mathbf{u})$, $T(\mathbf{v})$, $T(c\mathbf{u})$, $cT(\mathbf{u})$ vektor-vektor di Ruang-m

$T : R^n \rightarrow R^m$

Transformasi T dapat "digantikan" oleh perkalian matrix

(matrix A berukuran m x n)

$$(x_1, x_2, x_3, \dots, x_n) \rightarrow (w_1, w_2, \dots, w_m)$$

jika $\mathbf{x} = (x_1, x_2, x_2, \dots, x_n)^T$ dan $\mathbf{w} = (w_1, w_2, \dots, w_m)^T$

maka transformasi dapat "digantikan" dengan

persamaan: $A\mathbf{x} = \mathbf{w}$

di mana A disebut matriks standar untuk transformasi linier T

Contoh:

Transformasi nol (*zero transformation*) dari R^3 ke R^2

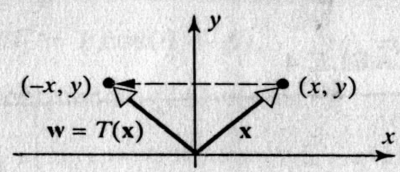
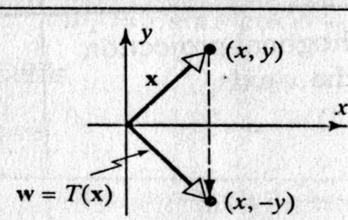
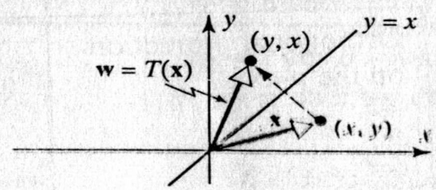
Transformasi nol (*zero transformation*) dari R^2 ke R^3

Refleksi (lihat Tabel 2 halaman 185)

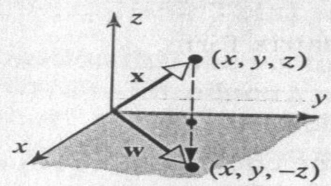
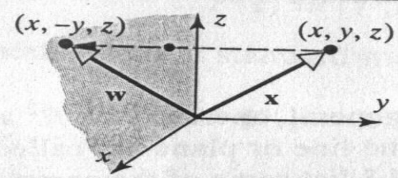
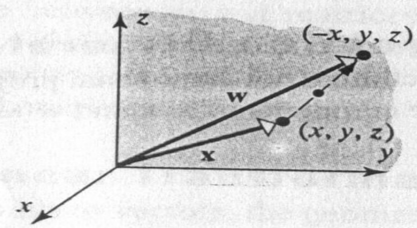
Proyeksi ortogonal (lihat Tabel 4 halaman 187)

Tabel Pencerminan

TABLE 2

Operator	Illustration	Equations	Standard Matrix
Reflection about the y-axis		$w_1 = -x$ $w_2 = y$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
Reflection about the x-axis		$w_1 = x$ $w_2 = -y$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
Reflection about the line $y = x$		$w_1 = y$ $w_2 = x$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

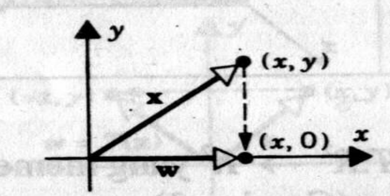
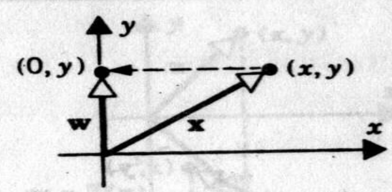
Tabel Pencerminan

Operator	Illustration	Equations	Standard Matrix
Reflection about the xy-plane		$w_1 = x$ $w_2 = y$ $w_3 = -z$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
Reflection about the xz-plane		$w_1 = x$ $w_2 = -y$ $w_3 = z$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Reflection about the yz-plane		$w_1 = -x$ $w_2 = y$ $w_3 = z$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

matrix for T is

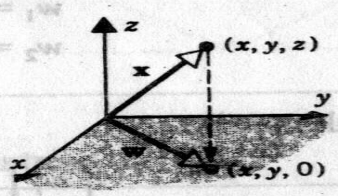
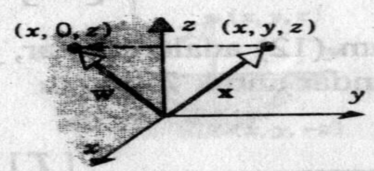
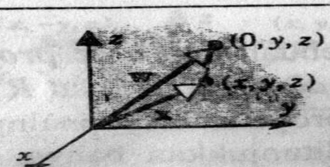
Tabel Proyeksi orthogonal

TABEL 4

Operator	Ilustrasi	Persamaan	Matriks Standar
Proyeksi ortogonal pada sumbu-x		$w_1 = x$ $w_2 = 0$	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Proyeksi ortogonal pada sumbu-y		$w_1 = 0$ $w_2 = y$	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

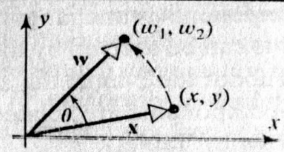
Tabel Proyeksi orthogonal

TABEL 5

Operator	Ilustrasi	Persamaan	Matriks Standar
Proyeksi ortogonal pada bidang-xy		$w_1 = x$ $w_2 = y$ $w_3 = 0$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
Proyeksi ortogonal pada bidang-xz		$w_1 = x$ $w_2 = 0$ $w_3 = z$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Proyeksi ortogonal pada bidang-yz.		$w_1 = 0$ $w_2 = y$ $w_3 = z$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Tabel Proyeksi orthogonal

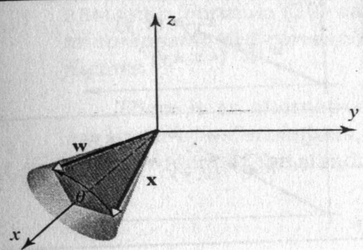
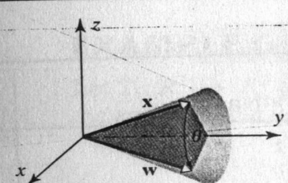
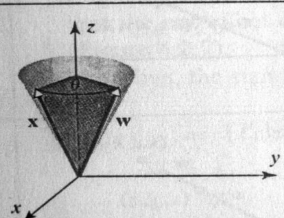
TABLE 6

Operator	Illustration	Equations	Standard Matrix
Rotation through an angle θ		$w_1 = x\cos\theta - y\sin\theta$ $w_2 = x\sin\theta + y\cos\theta$	$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Tabel Proyeksi orthogonal

4.2 *Linear Transformations from R^n to R^m* ••• 181

TABLE 7

Operator	**Illustration**	**Equations**	**Standard Matrix**
Counterclockwise rotation about the positive x-axis through an angle θ		$w_1 = x$ $w_2 = y\cos\theta - z\sin\theta$ $w_3 = y\sin\theta + z\cos\theta$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$
Counterclockwise rotation about the positive y-axis through an angle θ		$w_1 = x\cos\theta + z\sin\theta$ $w_2 = y$ $w_3 = -x\sin\theta + z\cos\theta$	$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$
Counterclockwise rotation about the positive z-axis through an angle θ		$w_1 = x\cos\theta - y\sin\theta$ $w_2 = x\sin\theta + y\cos\theta$ $w_3 = z$	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Komposisi dua transformasi:

$$
\begin{tikzcd}
\mathbf{u} \arrow[r, "T_1"] \arrow[rr, bend right, "T_2 \circ T_1"'] & \mathbf{v} \arrow[r, "T_2"] & \mathbf{w}
\end{tikzcd}
$$

$$\mathbf{v} = T_1(\mathbf{u})$$

$$\mathbf{w} = T_2(\mathbf{v}) = T_2(T_1(\mathbf{u})) = (T_2 \circ T_1)(\mathbf{u})$$

Komposisi dua transformasi:

$$
\begin{tikzcd}
\mathbf{u} \arrow[r, "T_1"] \arrow[rr, bend right, "T_2 \circ T_1"'] & \mathbf{v} \arrow[r, "T_2"] & \mathbf{w}
\end{tikzcd}
$$

Matriks standar untuk $T_1 = A_1$

Matriks standar untuk $T_2 = A_2$

Matriks standar untuk $T_2 \circ T_1 = (A_2)(A_1)$

Komposisi dua / lebih transformasi:

$$T_r \circ T_{r-1} \circ \ldots\ldots\ldots T_2 \circ T_1$$

Contoh: **u** = (–3, 4)

1. T_1 refleksi terhadap sumbu-y

   $A_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

2. T_2 proyeksi ortogonal pada sumbu-x

   $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

Hasilnya : (3, 0) ?

(cek dengan menghitung dan menggambar)

Komposisi dua / lebih transformasi:

Contoh: $\mathbf{u} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$

1. T_1 refleksi terhadap sumbu-y

$$A_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \qquad A_1\mathbf{u} = \mathbf{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

2. T_2 proyeksi ortogonal pada sumbu-x

$$A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad A_2\mathbf{v} = \mathbf{w} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$A_2 \circ A_1 = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \qquad (A_2 \circ A_1)\,\mathbf{u} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$