Homework 5 Solution

May 20, 2007

1. Given the following information, try to prove that Garfield eats fish.

   (a) Cats like fish.

   (b) Cats eat everything they like.

   (c) Garfield is a cat.

   Ans. Define the following predicates:

   $isCat(x)$: x is a cat

   $like(x, y)$: x like y

   $eat(x, y)$: x eat y

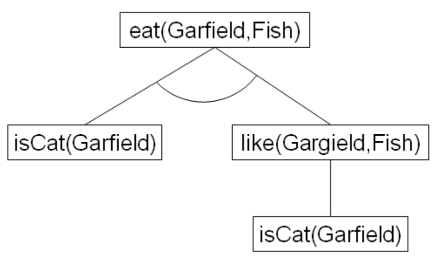
   Then, by the facts above, we can define the following sentences:

   $isCat(x) \Rightarrow like(x, Fish)$

   $isCat(x) \wedge like(x, y) \Rightarrow eat(x, y)$

   $isCat(Garfield)$

   We can prove that $eat(Garfield, Fish)$ is true by backward-chaining as following:



2. Consider the following axioms, try to establish the conclusion using the axioms by applying refutation resolution.

   (a) Anyone who rides any Harley is a rough character.

   (b) Every biker rides [something that is] either a Harley or a BMW.

   (c) Anyone who rides any BMW is a yuppie.

   (d) Every yuppie is a lawyer.

   (e) Any nice girl does not date anyone who is a rough character.

(f) Mary is a nice girl, and John is a biker.

(g) (Conclusion) If John is not a lawyer, then Mary does not date John.

Ans. Define predicates such as:

$rides(x, y)$: x rides y
$isBiker(x, y)$: x is a biker
$isRoughChar(x)$: x is a rough character
$isYuppie(x)$: x is a yuppie
$isLawyer(x)$: x is a lawyer
$isNiceGirl(x)$: x is a nice girl
$isBMW(x)$: x is BMW
$isHarley(x)$: x is Harley
$date(x, y)$: x dates y

Then these axioms can be transferred to FOL sentences:

(a) $\forall x \forall y \; rides(x, y) \wedge isHarley(y) \Rightarrow isRoughChar(x)$
(b) $\forall x \; isBiker(x) \Rightarrow \exists y \; rides(x, y) \wedge (isBMW(y) \vee isHarley(y))$
(c) $\forall x \forall y \; rides(x, y) \wedge isBMW(y) \Rightarrow isYuppie(x)$
(d) $\forall x \; isYuppie(x) \Rightarrow isLawyer(x)$
(e) $\forall x \forall y \; isNiceGirl(x) \Rightarrow \neg(date(x, y) \wedge isRoughChar(y))$
(f) $isNiceGirl(Mary) \wedge isBiker(John)$
(g) $\neg isLawyer(John) \Rightarrow \neg date(Mary, John)$

Where item (g) is what we want to prove.
Convert these sentences to CNF as:

$$\neg rides(x, y) \vee \neg isHarley(y) \vee isRoughChar(x) \tag{1}$$
$$\neg isBiker(x) \vee rides(x, F(x)) \tag{2}$$
$$\neg isBiker(x) \vee isHarley(F(x)) \vee isBMW(F(x)) \tag{3}$$
$$\neg rides(x, y) \vee \neg isBMW(y) \vee isYuppie(x) \tag{4}$$
$$\neg isYuppie(x) \vee isLawyer(x) \tag{5}$$
$$\neg isNiceGirl(x) \vee \neg date(x, y) \vee \neg isRoughChar(y) \tag{6}$$
$$isNiceGirl(Mary) \tag{7}$$
$$isBiker(John) \tag{8}$$
$$\neg isLawyer(John) \tag{9}$$
$$date(Mary, John) \tag{10}$$

The last two sentences come from the goal sentence being negated because we are going to proof by refutation using resolution.

Then, by resolution:

	Resolvent clause	Unification	
(2),(8)	$rides(John, F(John))$	$\{x/John\}$	(11)
(3),(8)	$isHarley(F(John)) \vee isBMW(F(John))$	$\{x/John\}$	(12)
(1),(11)	$\neg isHarley(F(John)) \vee isRoughChar(John)$	$\{x/John, y/F(John)\}$	(13)
(4),(11)	$\neg isBMW(F(John)) \vee isYuppie(John)$	$\{x/John, y/F(John)\}$	(14)
(6),(7)	$\neg date(Mary, y) \vee \neg isRoughChar(y)$	$\{x/Mary\}$	(15)
(10),(15)	$\neg isRoughChar(John)$	$\{y/John\}$	(16)
(5),(9)	$\neg isYuppie(John)$	$\{x/John\}$	(17)
(14),(17)	$\neg isBMW(F(John))$		(18)
(13),(16)	$\neg isHarley(F(John))$		(19)
(12),(18)	$isHarley(F(John))$		(20)
(19),(20)	$\{\}$		(21)

Thus if John is not a lawyer, Mary does not date John.